A GENERALISED McFADDEN COST FUNCTION FOR AUSTRALIAN AGRICULTURE

by

Denis Lawrence

Industries Assistance Commission
Canberra, Australia

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1 INTRODUCTION

Information on the ease with which rural producers can substitute between various inputs to production has long been of interest to agricultural economists. The scope for substituting between inputs will be an important determinant of the effects of price and policy changes. In spite of differing methodologies and data sets used, most previous econometric studies have shown that input demand in Australian agriculture exhibits relatively little price responsiveness. Selected own-price elasticities from previous Australian studies are presented in Table 1.

TABLE 1 : SELECTED ESTIMATES OF OWN-PRICE INPUT DEMAND ELASTICITIES FOR AUSTRALIAN AGRICULTURE

Study	Labour	Materials + Services	Capital	Livestock Input	Land
McKay, Lawrence and Vlastuin (1980)	-0.67	-0.98	-1.22	-0.19	-0.19
McKay, Lawrence and Vlastuin (1983)	-0.47	-0.10			
Lawrence and Zeitsch (1989)	-0.68	-0.53	-0.44	-0.20	-0.07

The aim of this paper is to estimate a Generalised McFadden cost function for Australian agriculture and derive from it a set of elasticities characterising input demand. Pooled time-series, cross-section data covering 6 States and 8 years in the period 1972/73 to 1986/87 are drawn principally from the Australian Bureau of Statistics' Agricultural Finance Survey. The work extends the earlier study of Lawrence and Zeitsch (1989), which used the same data set but estimated a profit function to examine the production relationships between outputs and inputs. The current study assumes that output levels are held fixed and examines input demand subject to that constraint.

Lawrence and Zeitsch (1989) found that own-price demand elasticities for the 5 variable inputs identified were all inelastic. Seven input categories are identified in the current study: hired labour, capital, land, services, materials, operator labour, and livestock. The demand elasticities reported here differ in two important respects to the corresponding profit function elasticities of Lawrence and Zeitsch. Firstly, no inputs are held fixed in the cost function framework, which would tend to allow more adjustment to take place than in the profit function case. It is for this reason that the cost function elasticities are often referred to as being long-run, whereas those of the profit function represent a medium-run response. Counteracting this, however, is the fact that output is held fixed in the cost function case, whereas it is able to respond to price movements in the profit function framework. This would tend to allow more scope for adjustment and hence a larger magnitude for the elasticities in the profit function case. Which of these countervailing influences will dominate is an empirical question.

2 COST FUNCTION METHODOLOGY

Denoting the N input quantities by the vector x, input prices by the vector $p >> 0$, output by y, and the firm's production function by $y = f(x)$, then the producer's cost function can be represented as the solution to the following constrained minimisation problem;

$$(1) \quad C\,(p;y) = \min_{x}.\ (p^T x \ :\ f(x) \geq y,\ x \geq 0_N)$$

With numerous small producers each having no control over the prices they pay for inputs, agricultural input demand is well modelled by the cost function framework where producers vary their input usage each period to minimise the cost of producing a given amount of output.

The cost function (1) will be linearly homogeneous, increasing and concave in input prices p. If the cost function is also differentiable with respect to p then the input demand functions can be derived by applying Shephard's (1953) Lemma:

$$(2) \quad x\,(p;y) = \nabla_p\, C\,(p;y).$$

In this study, a system of demand equations is estimated for 7 inputs (hired labour, capital, land, services, materials, operator labour, and livestock). Data covering 8 years in the period 1972-73 to 1986-87 are pooled across the 6 States to produce a total of 48 observations. To conserve degrees of freedom, constant returns to scale with respect to output are imposed. This facilitates estimation of a unit profit function where costs are minimised per unit of output.

The functional form used for the cost function is the Generalised McFadden (GM) developed by Diewert and Wales (1987). The GM function is superior to earlier flexible forms such as the translog in that curvature conditions can be imposed on the model without loss of flexibility. The 7 input GM unit cost function is given by:

$$(3) \quad C\,(p,y)/y = \tfrac{1}{2} \Sigma_{i=1}^{6}\ \Sigma_{j=1}^{6}\ b_{ij} p_i p_j / p_7 + \Sigma_{i=1}^{7}\ b_i p_i$$

$$+ \Sigma_{i=1}^{7}\ b_{it} p_i t + b_{tt}\ (\Sigma_{i=1}^{7}\ \gamma_i p_i) t^2 \quad ,$$

where the b_{ij} parameters are estimated subject to the following symmetry restrictions;

$$(4) \quad b_{ij} = b_{ji} \text{ for all } i,j=1,\ldots,6;$$

t is an index of technology and the γ_i are exogenous constants set equal to the respective mean unit input quantities to conserve degrees of freedom.

By applying Shephard's Lemma the following set of unit input demand equations is obtained;

$$(5) \quad x_i/y = b_i + \Sigma_{j=1}^{6} b_{ij}p_j/p_7 + b_{it}t + b_{tt}\gamma_i t^2; \; i=1,\ldots,6;$$

$$(6) \quad x_7/y = b_7 - \tfrac{1}{2} \Sigma_{i=1}^{6} \Sigma_{j=1}^{6} b_{ij}p_ip_j/p_7^2 + b_{7t}t + b_{tt}\gamma_7 t^2.$$

The estimating system would normally consist of equations (5) and (6) with vectors of error terms attached and assumed to be independently distributed with a multivariate normal distribution with zero means and covariance matrix Ω. The cost function (3) is excluded from estimation as it adds no additional information.

In this application, time-series and cross-section data are pooled, and this needs to be allowed for in the estimation process as State differences in input mixes and production efficiency will make the sample nonhomogeneous. The theory of this situation is set out in detail in Fuss (1977). One option is to assume that the parameters of the unit input demand equations are State specific. Degrees of freedom considerations would limit the implementation of this to the intercept terms. The alternative is to assume that State effects are stochastic and that error terms consist of two components: a State-specific component and an overall remainder. There are two techniques for handling such a specification - covariance and error components estimators. Covariance estimation is computationally equivalent to the use of State-specific intercepts. While error components estimators have some theoretically more desirable properties than covariance estimators, Swamy and Arora (1972) show that when the sample is small and the number of States is less than 10 the covariance estimator is to be preferred. Consequently, in this study an analogous approach to Fuss (1977) is adopted. The estimating system then becomes:

$$(7) \quad x_i/y = \Sigma_{k=1}^{6} d_{ki}D_k + \Sigma_{j=1}^{6} b_{ij}p_j/p_7 + b_{it}t + b_{tt}\gamma_i t^2 + \nu_i; \; i=1,\ldots,6;$$

$$(8) \quad x_7/y = \Sigma_{k=1}^{6} d_{k7}D_k - \tfrac{1}{2} \Sigma_{i=1}^{6} \Sigma_{j=1}^{6} b_{ij}p_ip_j/p_7^2 + b_{7t}t + b_{tt}\gamma_7 t^2 + \upsilon_7$$

subject to the symmetry restrictions (4). The D_k are State-specific dummy variables taking the value one for an observation in State k and zero otherwise. The error vectors are now independently multivariate normally distributed with zero means and covariance matrix Ω. The system (7) - (8) can be estimated using Zellner's (1962) iterative seemingly unrelated regressions estimator. This can be carried out using the SYSTEM command in SHAZAM (White 1978).

The technology index, t, was represented by an instrumental variable formed from the State-specific productivity indexes. This specification was chosen as use of a simple time trend fails to capture the impact of seasonal conditions which can significantly influence output, particularly across States, and use of a seasonal conditions index fails to capture the importance of advances in productivity and technology over time. An instrumental variable is required to avoid simultaneity problems. It was formed by regressing the productivity indices on an index of pasture growth, transformations of a time trend and various dummy variables.

A limitation of applied duality theory models in the past has been the failure of many models to satisfy the necessary curvature conditions. Jorgenson and Fraumeni (1981) attempted to overcome this problem by imposing semi-definiteness conditions on the matrix of second-order coefficients from translog functions. However, this procedure can introduce large biases in the estimated elasticities and hence destroys the constrained translog's flexibility (Diewert and Wales 1987). In the GM case, if the matrix of estimated quadratic terms, $B = [b_{ij}]$, is negative semi-definite then the cost function is globally concave in prices. If B is not negative semi-definite then it can be reparameterised using the Wiley, Schmidt and Bramble (1973) technique of replacing B by minus the product of a lower triangular matrix and its transpose:

(9) $$B = -AA^T \text{ where } A = [a_{ij}];\ i,j=1,..,6;\ \text{and } a_{ij} = 0 \text{ for } i<j.$$

The GM function will then be globally concave in prices without having lost its flexibility properties (Diewert and Wales 1987). The cost of this procedure is that computer-intensive non-linear regression techniques have to be used.

For simplicity of presentation, only the conventional input demand elasticities are discussed in this paper. For the cost function these elasticities represent the change in the demand for input i with respect to a change in the price of input j subject to the quantity of output produced remaining constant. They are given by:

(10) $$E_{ij} = d \ln x_i / d \ln p_j = DP_{ij} p_j / x_i;\ i,j = 1,\ldots,7;$$

where DP_{ij} is the second-order price derivative of the cost function and x_i is the estimated unit input quantity obtained from the system of equations (7) and (8).

3 RESULTS

Initial estimation of the system of input demand equations ((7) and (8) subject to (4)) produced estimates which failed to satisfy the concavity in prices property with one of the eigenvalues of the matrix $B = [b_{ij}]$ being positive. One of the estimated own-price elasticities from this system was also positive. Subsequent estimation of the system was undertaken imposing negative semi-definiteness on the B matrix using equation (9). The

TABLE 2 : ESTIMATED UNIT INPUT DEMAND EQUATIONS[1]

	Coefficient													
	State dummy variables[2]						Second-order price terms (non-linear)						Technology Terms	
Equation i	d_{i1}	d_{i2}	d_{i3}	d_{i4}	d_{i5}	d_{i6}	a_{i1}	a_{i2}	a_{i3}	a_{i4}	a_{i5}	a_{i6}	b_{it}	b_{tt}
Hired	0.299	0.275	0.320	0.239	0.301	0.310	0.111	0.007	-0.006	-0.136	-0.006	-0.041	-0.258	0.704
Labour	(12.62)	(11.82)	(13.29)	(12.08)	(12.28)	(13.32)	(3.21)	(0.17)	(-0.20)	(-2.09)	(-0.12)	(-1.80)	(-6.87)	(4.48)
Capital	0.115	0.114	0.120	0.125	0.126	0.111		-0.076	0.029	0.086	-0.057	-0.017	-0.098	"
	(10.87)	(10.77)	(11.54)	(11.38)	(11.69)	(10.48)		(-1.22)	(0.89)	(1.21)	(-0.90)	(-0.58)	(-5.89)	
Land	0.397	0.430	0.387	0.411	0.405	0.393			-0.100	0.049	0.114	-0.077	-0.347	"
	(12.07)	(13.05)	(11.92)	(12.51)	(12.42)	(12.06)			(-4.56)	(1.04)	(2.78)	(-2.74)	(-6.56)	
Services	0.534	0.508	0.511	0.512	0.527	0.513				-0.219	0.212	0.001	-0.436	"
	(12.03)	(11.54)	(11.31)	(11.43)	(11.59)	(11.81)				(-10.89)	(12.46)	(0.06)	(-6.17)	
Materials	0.359	0.354	0.356	0.358	0.379	0.365					-0.000	0.000	-0.293	"
	(12.11)	(12.00)	(11.90)	(11.99)	(12.58)	(12.49)					(-0.00)	(0.00)	(-6.13)	
Operator	0.528	0.522	0.592	0.616	0.532	0.561		Symmetric				0.000	-0.474	"
Labour	(10.69)	(10.61)	(12.07)	(12.39)	(10.73)	(11.50)						(0.00)	(-6.06)	
Livestock	0.370	0.371	0.350	0.335	0.344	0.353							-0.306	"
Input	(12.00)	(12.17)	(11.39)	(10.68)	(10.64)	(11.71)							(-6.76)	
System log likelihood			1099.31											

1 t-statistics in parentheses.

2 States 1,...,6 are NSW, Victoria, Queensland, South Australia, Western Australia and Tasmania, respectively.

non-linear regression algorithm of the SHAZAM package (White 1978) was used with starting values set equal to the mean of the dependent variable for the State dummy coefficients and zero for all other coefficients. The constrained system estimates are presented in Table 2. The elasticities obtained from the constrained estimates were only marginally different from those obtained from the unconstrained system, with the exception of the livestock own-price elasticity which becomes negative with the imposition of the curvature conditions.

Input demand elasticities for Australia calculated at the means of the exogenous variables are presented in Table 3. The elasticities for Australia were obtained by weighting together the individual State elasticities according to shares in the mean input quantity.

TABLE 3 : INPUT DEMAND ELASTICITIES FOR AUSTRALIA, EVALUATED AT MEANS OF EXOGENOUS VARIABLES

		With respect to price of:						
		Hired Labour	Capital	Land	Services	Materials	Oper. Labour	Livestock Input
Change in Quantity of:	Hired Labour	-0.210	-0.014	0.013	0.256	0.013	0.019	-0.078
	Capital	-0.034	-0.266	0.115	0.331	-0.214	-0.011	0.080
	Land	0.008	0.028	-0.157	0.021	0.179	-0.022	-0.057
	Services	0.124	0.063	0.016	-0.617	0.406	-0.000	0.008
	Materials	0.010	-0.062	0.210	0.616	-0.940	0.025	0.142
	Operator Labour	0.044	-0.010	-0.084	-0.000	0.078	-0.019	-0.008
	Livestock Input	-0.088	0.037	-0.106	0.029	0.224	-0.002	-0.095

In line with the findings of earlier studies, the notable feature of Table 3 is the lack of input price responsiveness in Australian agriculture. Only materials and services inputs show any real degree of own-price responsiveness, with elasticities of -0.9 and -0.6, respectively. As is to be expected, operator labour shows the least price responsiveness with an own-price elasticity close to zero. This would appear to support the choice of operator labour as the fixed input in the Lawrence and Zeitsch profit function study. Capital shows some responsiveness with an own-price elasticity of -0.3, while hired labour and land have elasticities close to -0.2. Consistent with earlier findings, livestock input has an own-price elasticity of around -0.1.

The own-price elasticities for materials, services, livestock and land estimated here are broadly consistent with those estimated in the earlier cost function study of McKay, Lawrence and Vlastuin (1980). Labour and capital are both found to be considerably less responsive in the current study. It should be noted, however, that there are significant differences in data sources and methods of calculating the labour and capital variables between the two studies.

The estimated own-price elasticities obtained here for hired labour, capital, and livestock are all below the corresponding profit function estimates of Lawrence and Zeitsch. This reflects the fact that the elasticities of the present study are compensated estimates whereas those obtained from the profit function also allow some adjustment of outputs. The estimates for land responsiveness are slightly higher in the present study reflecting the scope for greater adjustment when all inputs are free to adjust in the cost function framework. Although the individual materials and services elasticities are higher than the aggregate materials and services elasticity from the profit function study, more responsiveness was found in the earlier study when the aggregate was broken down into 4 components.

Turning to the cross elasticities in Table 3, hired labour is found to be slightly complementary with both capital and livestock although the cross elasticities are close to zero. It is slightly substitutable with the other 4 inputs, with the relationship being strongest for services reflecting the scope to employ labour to perform tasks on-farm or to have them performed by contractors. Capital is slightly complementary with operator labour and more so with materials, reflecting the need for more fuel and electricity as more capital is used. Capital is substitutable with both land and services, with the relationship again being strongest with services. Land has generally weak interactions with the other inputs, with the cross elasticities all being close to zero with the exception of that relating to materials (reflecting the greater use of fertiliser and seed as more land is used).

The largest cross relationship is the substitutability between materials and services. This is consistent with the earlier profit function result and reflects the scope to provide some inputs directly or to have services carried out by contractors. Operator labour has very weak relationships with the other inputs reflecting its status as the most fixed of all inputs. Livestock has generally little interaction with the other inputs although some substitutability exists between it and materials reflecting the scope to use drenches and chemicals in varying proportions with livestock.

4 CONCLUSIONS

In line with the findings of earlier studies, Australian agriculture is found to exhibit little price responsiveness in input demand. Operator labour and livestock input demands are particularly unresponsive to price changes with own-price demand elasticities smaller than -0.1. Hired labour, capital, and land all have own-price elasticities of around -0.2. The input components which exhibit most price responsiveness are materials and services with own-price elasticities of -0.9 and -0.6, respectively. The major opportunities for substituting between inputs are found to exist between materials and services, and between services and capital.

APPENDIX 1: DATA SOURCES

The principal data source used in this study is the Australian Bureau of Statistics' Agricultural Finance Survey (ABS Cat. No. 7507.0). The first survey year used is 1972-73. The survey then had a sample of approximately 10 000 farms. It was carried out on an annual basis until 1977-78 and then again in 1980-81 and 1986-87. The survey presents data on the value of farm outputs and inputs. Stock values for the 3 durable inputs are presented from 1974-75 onwards. The survey value data are combined with ABARE (1988) prices received and prices paid indices to produce price and implicit quantity indices for output and 5 input categories (hired labour, capital, materials, services, and livestock). A further 2 input categories (land and operator labour) are created from survey value data and ABS and ABARE quantity series. The data used here are at the aggregate level for each of the 6 States and 8 years producing a total of 48 observations.

For the components of output (crops and livestock) and the 3 non-durable inputs (hired labour, materials, and services), the ABARE State price indices were used along with the implicit quantities obtained by dividing the Survey values by the price indices as outlined in Appendix Table A.

For the 3 durable inputs (capital, land, and livestock inputs), a user cost value has been derived from the stock value by assuming that farmers aim to make a given rate of real return on their assets. A real opportunity cost rate of 4 per cent has thus been used along with a depreciation rate for each asset class. This approach differs from earlier studies such as Lawrence and McKay (1980), where a nominal opportunity cost was used. While the nominal opportunity cost rate varied from year to year it neglected the increasingly important role that capital gains have played. As no information was available on capital gains, in this study the alternative of assuming a constant real rate of opportunity cost was opted for. This is equivalent to a constant difference between the nominal rate of opportunity cost and the rate of capital gains. A depreciation rate of 5 per cent was assumed for capital and 1 per cent for land and improvements. The average unit of livestock was assumed not to depreciate but this required inputs in the form of livestock purchases. Consequently, the livestock input user cost consists of the real opportunity cost and the value of purchases. As no reliable land price series was available a land price index was formed by dividing the land value by the area of agricultural land in each State (ABS Cat. No. 7321.0).

No capital stock values were collected for 1972-73 or 1973-74, although capital purchase data was. The capital stocks for these years were estimated by deflating the stock for the following year after allowing for purchases and depreciation. Similarly no total livestock value was collected for 1972-73 or 1973-74. Livestock values for these years were estimated from the 1974-75 values by allowing for sales, purchases and an average rate of net natural increase observed for the rest of the sample period.

TABLE A: VALUE, PRICE AND QUANTITY SOURCES

Group	AFS Category	ABARE price index	ABS AND ABARE quantities
Output	Sales from crops	crops	
	Sales from livestock	livestock	
	Sales from livestock products	) livestock	
	Other miscellaneous revenue	) products	
Hired labour	Wages, salaries and supplements	wages	
	Payments to contractors	contracts	
Capital	0.09 x Value of machinery and equivalent	repairs and maintenance	
Land[a]	0.05 x Value of land and improvements		ABS area of agricultural land
Services	Marketing expenses	marketing	
	Water and drainage charges	) repairs and	
	Repairs and maintenance	) maintenance	
	Other selected expenses	)	
	Rates and taxes	rates and taxes	
	Insurance payments	insurance	
	Other expenses	other expenses	
Materials	Payments for fertiliser	fertiliser	
	Chemical and veterinary supplies	chemicals	
	Payments for fuel	fuel	
	Payments for electricity	electricity	
	Payments for seed and fodder	seed and fodder	
Livestock inputs	0.04 x Value of livestock	) livestock	
	Purchases of livestock	)	
Operator and family labour			ABARE number of operators and unpaid family helpers allocated to States by ABS proportions.

The most reliable estimates of the number of farm operators and unpaid family helpers were considered to be those of ABARE (1987, p. 4). However, these are only presented for Australia as a whole. Estimates for the States were obtained by allocating this total according to the proportions observed from ABS employment data for March of each year. The return to this input is taken to be the residual between total receipts and total costs associated with the other 6 input categories.

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