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Abstract

Minimum size limits have become an increasingly popular management tool in recreational fisheries. This popularity stems from the potential of minimum size limits to accomplish the twin goals of limiting overfishing and improving fishing quality through increasing the average size of fish caught. The success of minimum size limits in achieving these objectives depends in a complicated way on both the behavior of anglers and the biological mechanisms that guide the growth of the fish population. This paper examines these relationships and also considers the welfare implications of size regulations.

Keywords

Fisheries Management, Recreational Angling, Minimum Size Limits

1 Introduction

Popular recreational fisheries have been beset by the same problems faced by open access commercial fisheries: high effort levels have put pressure on fish stocks. In the absence of regulations, stock levels are driven to low levels, leading to a higher probability of stock collapse, low catch rates, and low angler satisfaction levels. In order to prevent stock collapse and to bolster angler satisfaction, managers of recreational fisheries have focused their attention on limiting angler effort. Traditional means used to limit overexploitation include creel limits, enhancement, closed seasons, gear restrictions, area closures, catch and release regulations, and size limits. With healthier stocks, catch rates and angler satisfaction levels improve.

It is important to recognize, however, that catching fish is only a part of what anglers consider to be important. Research has shown that anglers value many features of the fishing experience, including being outdoors and being with others.[1] Also, there appear to be distinct types of anglers (e.g., specialists and casual anglers) who assign different levels of importance to the various dimensions of a fishing experience.[2] Depending on the type of angler, the emphasis placed on catching fish may be substantial or may be quite small.

The role of a management agency is primarily limited to managing fishing quality, and so we may justifiably focus exclusive attention on the contribution of fishing quality to angler satisfaction. Even so, it is essential to recognize that catch per unit effort may not be the sole criterion by which fishing quality is judged. The focus on catch rates as a measure of quality has been a common practice in the literature on valuation of recreational fisheries.[3] However, other dimensions of catch are important, and the importance of the different features will vary depending on the type of angler. While some anglers rate the quality of their experience by the number of fish caught and kept, others simply seek the experience of catching fish and are content to keep only enough to eat or to release all of their catch.[4] In addition, anglers may have preferences about fish size; they may prefer the average size of fish caught to be high, there may be a desire for fish in a particular size range for consumption, or they may want the chance to catch a trophy-sized fish.[5]

Fishery managers now also recognize the importance of providing a recreational experience for anglers that includes more than just an adequate number of fish. The choice of management tool is therefore likely to be governed by its ability to contribute to several different objectives. Size limits are one such option. Size limits may take the form of a minimum size limit, in which all fish smaller than a particular size must be released, or a slot limit, which combines a minimum size limit with a limit on the number of fish that can be kept in a particular "slot," or size range. They have the potential to enhance biomass by restricting the number of fish kept. In one instance in Minnesota, for example, a size limit has been quite successful in increasing biomass. In an account of this success, a fishery manager is quoted as having said: "It seems like there's magic to the slot limit, but if you reduce the harvest, there are just more fish out there for people to catch."(Eibler, 1997) A size limit may also enhance biomass by increasing the number of reproductive fish. The enhancement of biomass contributes to the objectives of maintaining a sustainable stock level and of providing anglers the opportunity to catch more fish. The minimum size limit is also attractive because more large fish survive, increasing the average size of fish caught and increasing anglers' chances of catching trophy-sized fish (see, e.g., Hoff, 1995).

The success of minimum size limits in achieving these objectives depends in a complicated way on both the behavior of anglers and the biological mechanisms that guide the growth of the fish population. To assess whether a minimum size limit will be successful, it is important to consider several key questions. Will minimum size limits reduce harvest in the short run? Will reductions in harvest lead to increases in the population, and how fast will increases occur? How will changes in the population affect harvest rates and the average size of fish caught in the long run? Finally, how might these changes affect angler welfare?

This paper presents a simple model designed to illuminate these questions. Our primary focus is a predictive model of angler behavior and population dynamics, but welfare considerations are also addressed. The model combines a behavioral model of angler participation with a simple biological model embedding a depiction of population size distributions. While purposely oversimplified, this model nevertheless addresses the above questions and points the way toward additional research issues.

In the next section, we outline a general model of angler behavior in which an angler derives utility from the number and size of fish kept. Then, we will explore the problem using the CES utility function. With these utility function, we will investigate the effects of imposing a binding minimum size limit both in the short run as the regulation is imposed and in the long run as the population responds.

2 General Model

We consider the choices of a representative angler in an open access fishery once he has chosen his level of participation in the fishery.[6] We assume that anglers derive utility from the number of fish they catch and keep, h, and the minimum size of fish they keep, s.[7] The catch function is specified as a standard Schaefer production function, so that catch is equal to qEN, where q is a catchability coefficient, E is the predetermined effort level, and N is the biomass level. Total catch, then, is qEN. The number of fish kept, h, is some fraction of total catch where the fraction is determined by the “keeper,” or minimum, size kept. This fraction is determined by the distribution of fish in the lake as follows. The probability density function that characterizes the distribution of fish in a lake is $f(s)$. The corresponding cumulative density function, $F(s)$, is the fraction of fish below a particular size. Then, $1-F(s)$ is the fraction of fish above a particular size, s; this is the fraction of total catch that will be kept as a function of the minimum size, s. Finally, an equation defining the production function between the two outputs, harvest (number kept) and minimum size, can be written:

$$h=qEN(1-F(s)).$$

The full specification of the angler's utility maximization problem is, then:

$$\max_{s} U(h,s)$$

$$\text{subject to } h = qEN(1-F(s))$$

Notice that the choice of s determines the number of fish kept, h, through the production function. Substituting the production function into the utility function and taking the derivative with respect to s yields:

$$-U_h\, qENf(s) + U_s = 0.$$

Rearranging this expression yields:

$$-U_h/U_s = -1/qENf(s).$$

This condition states that the marginal rate of substitution between h and s must be equal to the marginal rate of product transformation at the optimum. We can think of the price of h as $1/(qENf(s))$ so that a higher biomass level (N), a higher effort level (E), or a higher catchability coefficient (q) makes keeping fish relatively less costly than maintaining a higher minimum size.

To simplify the model, we assume that the distribution of fish is Uniform (0,1).[8] Therefore, $F(s)=s$ and $f(s)=1$. The production function becomes

$$h = qEN(1\text{-}s).$$

We can then characterize the optimal choices of h^* and s^* using an indifference curve diagram. See Figure 1. The production function is a linear function of *s*, and *s* takes on values between 0 and 1. As *q*, *E*, or *N* increase, the production function rotates up, pivoting around 1, the maximum value of *s*. Figure 1 provides one example of how h^* and s^* may change as biomass increases from N_0 (point a) to N_1 (point b) to N_2 (point c). In this illustration, h^* increases as biomass increases. The voluntarily chosen minimum size, s^*, falls as biomass grows from N_0 to N_1 then rises as biomass grows further to N_2.[9]

From the locus of optimal harvest levels with alternative biomass levels, we can determine $h^*(N)$. See Figure 2. Points a, b, and c correspond with those in Figure 1. With the introduction of a biomass growth function into this model, the equilibrium level of biomass can be characterized. The $h^*(N)$ function is upward sloping, and the biological yield function is concave, reaching a carrying capacity at *K*. Point b represents the equilibrium where the level of harvest ($h^*(N)$) equals the growth in biomass. The resulting biomass level is $\hat{N}^U$, where the hat denotes equilibrium and superscript denotes that the equilibrium is unregulated. Note that, on Figure 1, the location of the production function is not arbitrary. If it happens that biomass is at N_0, harvest ($h^*(N_0)$) will be less than the growth in biomass, the population will grow, and the production function will rotate up until the biomass reaches $N_1 = \hat{N}^U$. Similarly, if biomass is at N_2, harvest ($h^*(N_2)$) will be higher than biomass growth. The population size will fall and the production function will rotate down until the biomass reaches $N_1 = \hat{N}^U$.

Now consider the introduction of a binding minimum size limit, $\bar{s}$. The size limit must be larger than the voluntarily chosen minimum size in order to be effective. See Figure 3. When $\bar{s}$ is imposed, the angler is constrained to harvest according to the function $\bar{h}(N) = qEN(1\text{-}\bar{s})$. This function intersects the yield curve at a higher biomass level than does $h^*(N)$. In the short run, biomass will be at the unregulated equilibrium level $\hat{N}^U$, and harvest will fall to $\bar{h}(\hat{N}^U)$ at point d. Then, because harvest is lower than yield, biomass will grow until a new regulated equilibrium is reached at point e where the biomass level is $\hat{N}^R$ and harvest is $\bar{h}(\hat{N}^R)$.

We can also depict these changes on an indifference curve diagram. See Figure 4. Initially, the angler is at point b with the biomass at the original unregulated equilibrium level, $\hat{N}^U$. When the regulation $\bar{s}$ is imposed, the angler is constrained to be at point d, with a reduced harvest level. This constraint must lead to reduced utility; anglers could have chosen the regulated minimum size and reduced harvest level in the absence of regulation. Since they did not, the regulated combination of $\bar{h}(\hat{N}^U)$ and $\bar{s}$ must yield lower utility than h^* and s^*. However, since harvest is reduced, biomass will grow to the regulated equilibrium biomass level, $\hat{N}^R$, and the production function will rotate up. The angler will now be at point e. At this point, the angler achieves a higher utility level than before the regulation was imposed, even though harvest is lower and the angler is still constrained by the regulation.

There are many possible outcomes, depending on the initial equilibrium and the level at which the regulation is set. Consider, for example, an optimal harvest function that intersects the biological yield curve to the left of maximum sustainable yield. See Figure 5. The angler is at point a with biomass level $\hat{N}^U$. The introduction of a somewhat restrictive minimum size limit ($\bar{s}_1$) will reduce harvest initially, but will lead to increased biomass ($\hat{N}_1^R$) and increased harvest ($\bar{h}(\hat{N}_1^R)$) in the long run (point b). In the long run, both harvest and minimum size will be higher, leading to an unequivocal increase in utility. This outcome is represented by point b on Figure 6. With a more stringent regulation ($\bar{s}_2$) so that the restricted harvest function intersects to the right of maximum sustainable yield, biomass will grow even larger (to $\hat{N}_2^R$), minimum size will be higher, but harvest will fall relative to the initial harvest level (to $\bar{h}(\hat{N}_2^R)$). See point c on both figures. Still, utility is higher than both the initial utility and the utility level associated with $\bar{s}_1$ level since the higher minimum size contributes to utility and the reduced harvest is achieved at a lower relative cost due to the increase in biomass. If the regulation becomes very strict, it is possible that harvest falls so far that utility will also fall. See point d. In the limit, as the regulated minimum size approaches one, harvest approaches zero, utility approaches zero, and biomass approaches its carrying capacity. This is represented by point e on both figures.

As this analysis suggests, many long-run results are possible relative to the initial position: both harvest and utility may be either higher or lower in the long run equilibrium. Furthermore, even though increases in harvest lead to unequivocal increases in long run utility levels, decreases in harvest do not necessarily lead to reductions in utility in the long run. Apart from questions about welfare, it is interesting to look at how changes in biomass may change the voluntarily chosen minimum size, making the minimum size constraint either more or less binding. The implied constraint on harvest (that is, harvesting at $\bar{h}(\hat{N}^R)$ rather than at $h^*(\hat{N}^R)$ may also become more or less binding. At this level of generality, however, it is difficult to determine the outcomes that would emerge from any particular size regulation. In order to look at these questions in more detail, we investigate the problem using a CES utility function.

3 The Constant Elasticity of Substitution Utility Function

The CES allows the elasticity of substitution between the number of fish kept, *h*, and the minimum size of fish kept, *s*, to range between zero and infinity. As it turns out, the nature of the solution, using the CES, hinges whether the elasticity of substitution between *h* and *s* is greater or less than one.

The CES utility function is specified as:

$$U(h,s)=(\alpha h^{\rho} + \beta s^{\rho})^{1/\rho}$$

The solutions for h^* and s^* are:

$$s^* = \frac{1}{1+\left(\frac{\beta}{\alpha}\right)^{\frac{1}{\rho-1}}(qEN)^{-\frac{\rho}{\rho-1}}}$$

$$h^* = \frac{qEN}{1+\left(\frac{\alpha}{\beta}\right)^{\frac{1}{\rho-1}}(qEN)^{\frac{\rho}{\rho-1}}}.$$

Harvest is an increasing function of biomass, and minimum size may be either an increasing or a decreasing function of biomass. The elasticity of substitution (for the CES, $\sigma=1/(1-\rho)$) between h and s determines whether the $h^*(N)$ function is convex or concave in N and whether $s^*(N)$ is downward or upward sloping.

By taking the second derivative of the $h^*(N)$ function, we find that h^* is convex in N if ρ is greater than zero. If ρ is greater than zero, the elasticity of substitution between h and s is greater than 1. If ρ is less than zero, so that the elasticity of substitution is less than one, then h^* is concave in N. Taking the first derivative of $s^*(N)$ shows that if ρ is greater than zero, s^* is decreasing in N while if ρ is less than zero, s^* is increasing in N. If ρ is zero (so that the elasticity of substitution between h and s is one), the CES utility function simplifies to a Cobb-Douglas. In the Cobb-Douglas, $h^*(N)$ is linear and s^* is invariant to changes in biomass.[10]

The intuition behind these results relies on a consideration of the income and substitution effects of a price change. As N increases, the relative price of h falls. The substitution effect encourages a move from minimum size (*s*) to the number of fish kept (*h*), where the income effect causes both *h* and *s* to rise. If the two goods are highly substitutable, the income effect is not large enough to counteract the strong substitution effect so that the minimum size falls. The income effect just reinforces the substitution effect for the number of fish kept, so the increase in *h* is substantial. On the other hand, if the two goods are not highly substitutable, the substitution effect is weak. The income effect is strong enough to counteract the negative substitution effect for *s* so that *s* increases. The number of fish kept, *h*, also increases but the increase is not as dramatic as with a strong substitution effect. Therefore, the $h^*(N)$ function is concave.

3.1 Minimum Size Limits

Now we consider the effects of the imposition of a minimum size limit. To be effective, the regulated minimum size $\bar{s}$ must be larger than the voluntarily chosen minimum size, s^*. In the short run, biomass will remain at the unregulated level and harvest will fall to $\bar{h}(\hat{N}^U)$=qE $\hat{N}^U(1-\bar{s})$. Biomass will eventually rise, to $\hat{N}^R$=(a-qE(1-$\bar{s}$))/b, and so harvest will rise in the long run to $\bar{h}(\hat{N}^R)$=qE$\hat{N}^R(1-\bar{s})$.

In the Cobb-Douglas case, the difference between the voluntarily chosen minimum size and the regulated minimum size will not change as biomass grows; anglers remain as constrained (in terms of minimum size) as before the regulation is imposed, regardless of changes in the biomass, since the optimally chosen minimum size depends only on the parameters of the utility function. Since both $h^*(N)$ and $\bar{h}$(N) are increasing linear functions of biomass, the difference between the two grows as biomass grows. In a sense, anglers will feel more constrained (in terms of numbers of fish kept, *h*) in the long run than immediately after the regulation is imposed even though the long run (constrained) harvest level $\bar{h}(\hat{N}^R)$ is higher than the initial constrained harvest level ($\bar{h}(\hat{N}^U)$). This is because anglers would choose a much higher *h* with the long run biomass level than the constrained harvest level. (See Figure 7: the distance between A and A^1 is smaller than the distance between B and B^1.)

Looking at the CES case, we see that the choice of minimum size is no longer invariant to changes in biomass. Therefore, the minimum size choice is likely to differ in the new equilibrium. A relevant question is whether the minimum size constraint remains binding: does the choice of minimum size increase to the constrained size? First, consider the harvest function when the elasticity of substitution between *h* and *s* is greater than one. In this case, the regulation becomes even more binding. If the elasticity of substitution is greater than one, the voluntarily chosen minimum size becomes smaller with increased biomass. So, as the regulation is effective in increasing biomass, the gap between the regulated minimum size and anglers' voluntarily chosen minimum size becomes wider. In addition, since the $h^*(N)$ function is convex, the gap between the voluntarily chosen harvest level and the constrained harvest level also becomes wider. Whether the utility level rises or falls in the long run is an open question, but the angler certainly will feel more constrained as biomass grows.

Recall that if the elasticity of substitution between *h* and *s* is less than one, the $s^*(N)$ function is increasing in *N* and the $h^*(N)$ function is concave. A binding regulation will raise the biomass level and consequently the voluntarily chosen minimum size. The gap between the regulated minimum size and the voluntarily chosen minimum size is reduced; anglers will feel less constrained as the regulation becomes effective in increasing the size of the population.

4 Welfare Implications of Size Limits

With a set of parameter estimates, it is a straightforward exercise to calculate the utility levels that would prevail at alternative regulated equilibria. It is only necessary to substitute the regulated harvest level at the steady state, $\bar{h}\,(N^R(\bar{s}))$, and the minimum size limit, $\bar{s}$, into the utility function to find the long-run utility level. In the Cobb-Douglas case, it is even possible to optimize the resulting expression with respect to minimum size to find the minimum size that would yield the highest long-run utility level. To find the size limit that would maximize long-run utility with more complex utility functions (such as the CES), numerical solution methods are required.

Of course, the long-run utility level is not achieved instantaneously. Figure 8 shows how harvest rates and the minimum size changes with time. Anglers first experience a sudden drop in utility levels as harvest falls in response to the implementation of the regulation. Since minimum size contributes to utility, the increase in minimum size (to $\bar{s}$) partially compensates for the loss in utility from a decrease in harvest, but not completely. The immediate impact on utility is certainly negative.

As the biomass grows, the restricted harvest level also grows, implying an increase in utility along the path to a new equilibrium. Anglers continue to fish and to derive utility as biomass grows. The welfare effect of a particular regulation, therefore, would be summarized in the discounted sum of utility levels that would emerge as biomass adjusts to the new equilibrium. Different regulations will lead to different adjustment paths and to different long run equilibria. It is therefore correct to judge alternative size limits based on the sum of discounted utility levels that would emerge from alternatives.

The expression to evaluate would be:

$$\int_0^{\infty} e^{-rt} U(\bar{h}(N(t,\bar{s})),\bar{s})dt.$$

Note that, along the path to the restricted equilibrium, anglers keep fish according to $\bar{h}$ which is a function of the growing biomass and of minimum size. To find the path of biomass, the differential equation describing the growth of biomass must be solved. With a logistic growth function, this solution is:

$$N(t) = \frac{\dfrac{a - qE(1-\bar{s})}{b}}{1 + \left(\dfrac{\dfrac{a - qE(1-\bar{s})}{b} - N_0}{N_0} \right) e^{(a+qE(1-\bar{s}))t}}.$$

5 Summary and Conclusions

This paper has used a comprehensive model to address questions surrounding the use of a minimum size regulation to improve fishing quality. Using simplified behavioral and biological models, both the short and long run implications of minimum size restrictions were investigated. In the short run, such a regulation diminishes harvest levels and angler utility. However, as the biomass responds to the reduced harvest, the harvest level recovers. If the fishery starts at a point to the left of MSY, this move unequivocally increases angler welfare. If the starting point is to the right of MSY, harvest falls in the long run. Still, the increase in minimum size due to the regulation may compensate for this decreased harvest and anglers may still be better off in the long run.

This paper also investigated the degree to which the regulation would be binding in the long run, depending upon the form of the utility function. If the elasticity of substitution between the number of fish kept and the minimum size is greater than one, the gap between the voluntarily chosen minimum size and harvest level and the regulated minimum size and harvest level widens as the regulation becomes successful in increasing biomass. If the elasticity of substitution is equal to one (the Cobb-Douglas case), the voluntarily chosen minimum size remains constant and so the gap between the voluntarily chosen and regulated minimum size remains the same. However, the gap between the voluntarily chosen harvest level and the regulated harvest level widens. Finally, if the elasticity of substitution is less then one, the minimum size constraint becomes less binding as biomass grows.

The simplified nature of the model has made the links between angler behavior and population dynamics transparent. While we have assumed that the size distribution of fish is unaffected by regulations, it is likely that the distribution will, in fact, shift as a result of size limits. Further research will employ a size-structured model of population dynamics to investigate the implications of this possibility.

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Endnotes

1. A substantial literature supports this conclusion. See Fedler and Ditton (1994) and the references cited therein.

2. Chipman and Helfrich (1988) first used principal components to group anglers into types in a study of Virginia river anglers. Fedler and Ditton (1994) examined seventeen of these studies: the studies found distinct subgroups that expressed different preferences for aspects of fishing trips.

3. See, for example, Samples and Bishop (1985), Johnson and Adams (1989), and Huppert (1989).

4. Anderson (1993) does include both landings (fish kept) and catch per day in a benefit function.

5. For example, Petering, Isbell, and Miller, (1995) in a survey of anglers, found that fish length and fish numbers both affected fishing satisfaction.

6. We choose this approach to highlight the angler's decision to keep or release fish. A more general model would explain the participation decision, but would obscure the keep/release decision.

7. It may be more reasonable to think of angler utility as a function of the average size of fish kept. This average size can easily be translated into the minimum size, so the two specifications are equivalent. We use the minimum size for analytical convenience.

8. To translate into actual sizes, multiply *s* by the difference between the size of the largest and smallest fish and add to the size of the smallest fish.

9. It may be helpful to think of this model as an analog to the standard labor/leisure model in which the budget constraint pivots around the maximum amount of leisure on the horizontal axis according to the wage rate. In this case, minimum size takes the place of leisure where the maximum minimum size is 1 and the production function pivots according to the values of *q*, *E*, and *N*. As the wage increases in the labor/leisure model, the consumption of goods always increases but the consumption of leisure may rise or fall depending on the relative strengths of the income and substitution effects. This model has analogous results.

10. The Cobb-Douglas utility function can be written:

$$U(h,s)=h^{\alpha}s^{\beta}$$

Carrying out the optimization, we get solutions for harvest and minimum size:

$$s^{*}=\frac{\beta}{\alpha+\beta},$$

and

$$h^* = \frac{\alpha}{\alpha+\beta} qEN.$$

The choice of *s* is a function only of the parameters α and β, and is invariant to levels of biomass and effort. Harvest is an increasing linear function of biomass, effort, and catchability. With a logistic biological growth function,

$$\dot{N} = g(N) = aN - bN^2,$$

we can solve for the equilibrium biomass and harvest levels, $\hat{N}^U$ and $h^*(\hat{N}^U)$:

$$\hat{N}^U = \frac{1}{b}\left[a - qE\left(\frac{\alpha}{\alpha+\beta}\right)\right],$$

$$h^*(\hat{N}^U) = qE\left(\frac{\alpha}{\alpha+\beta}\right)\frac{1}{b}\left[a - qE\left(\frac{\alpha}{\alpha+\beta}\right)\right].$$

These closed-form solutions are unavailable with the CES.

Figure 1

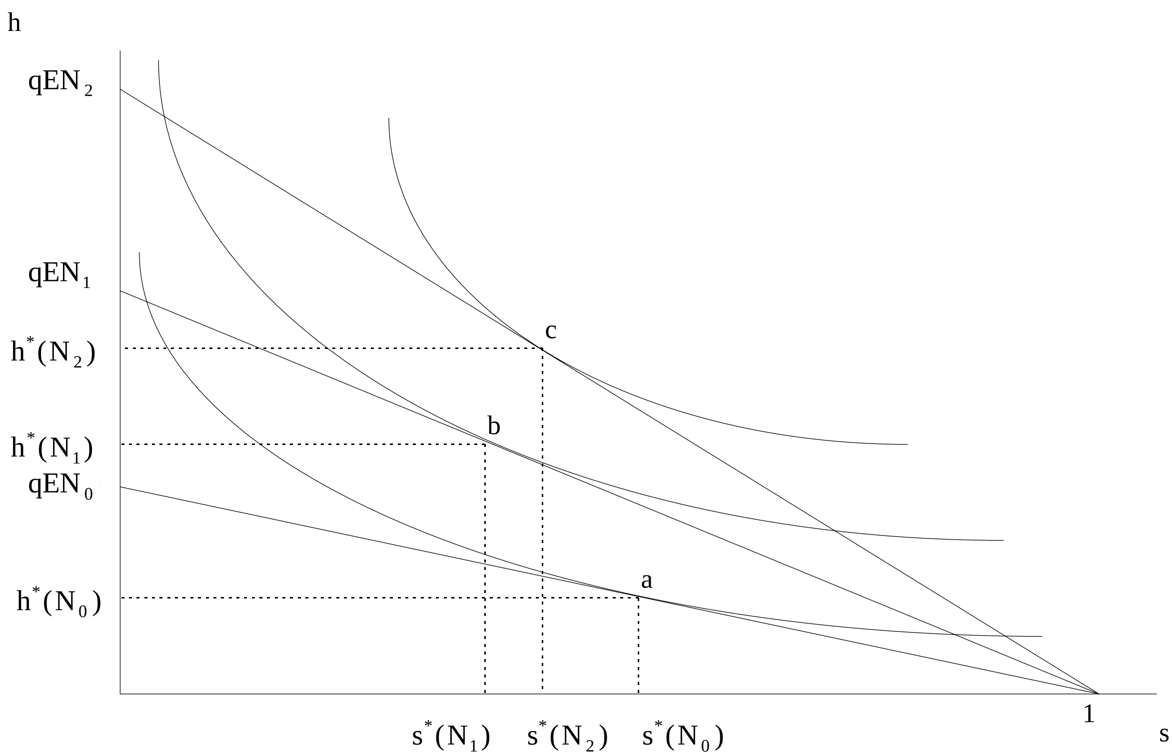


Figure 2

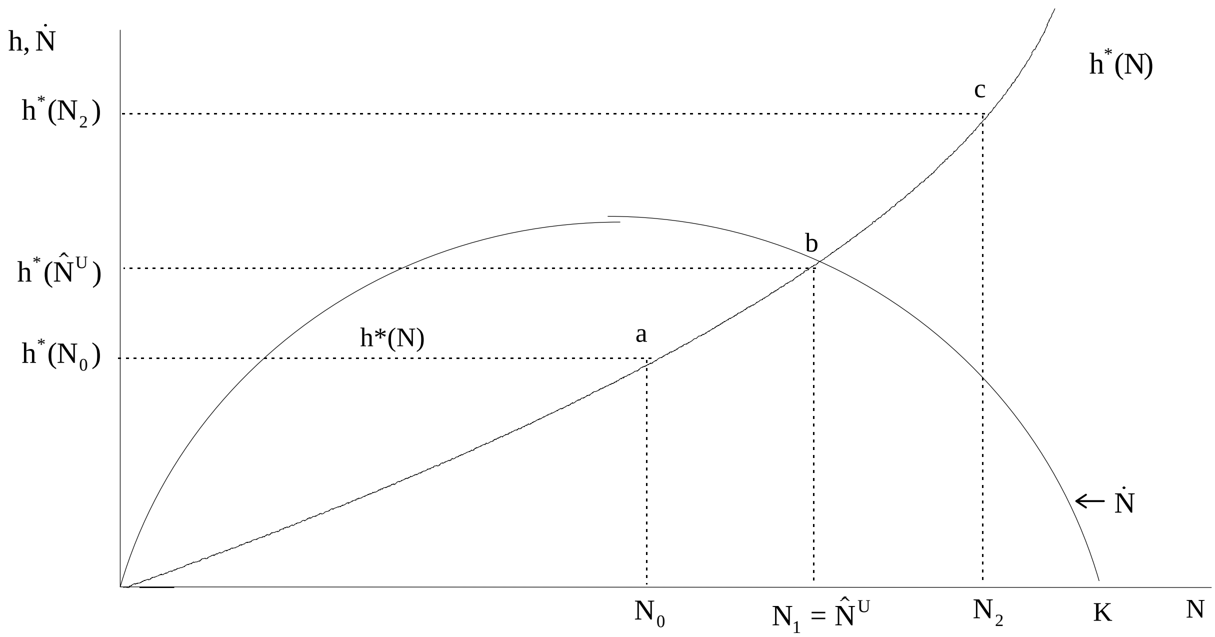


Figure 3

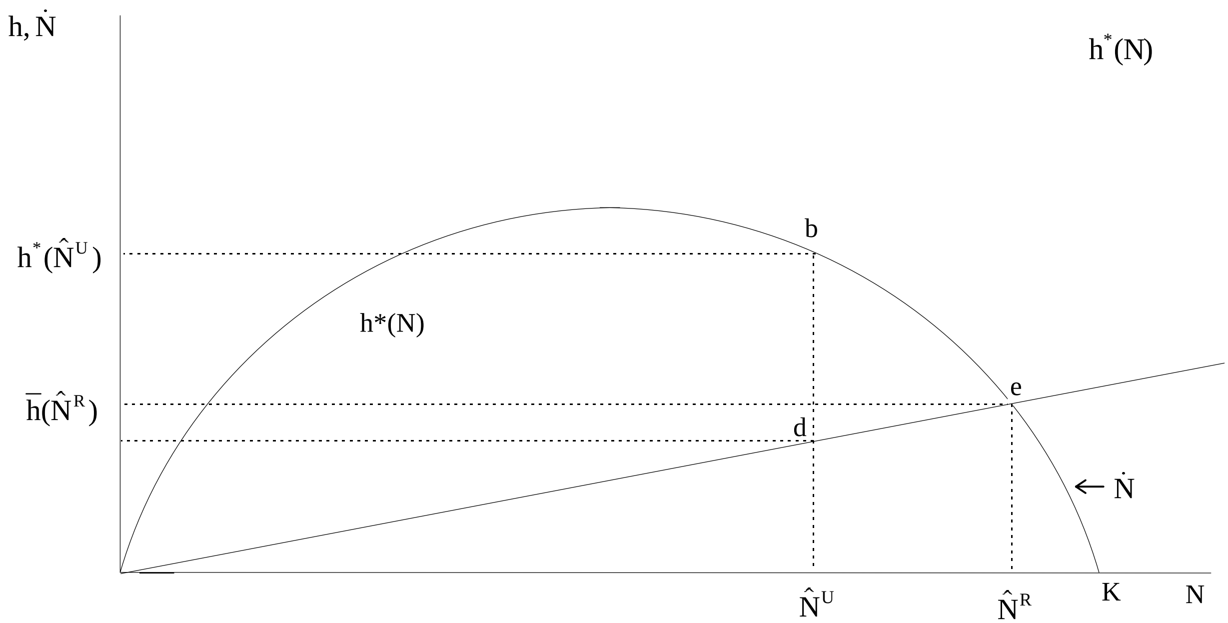


Figure 4

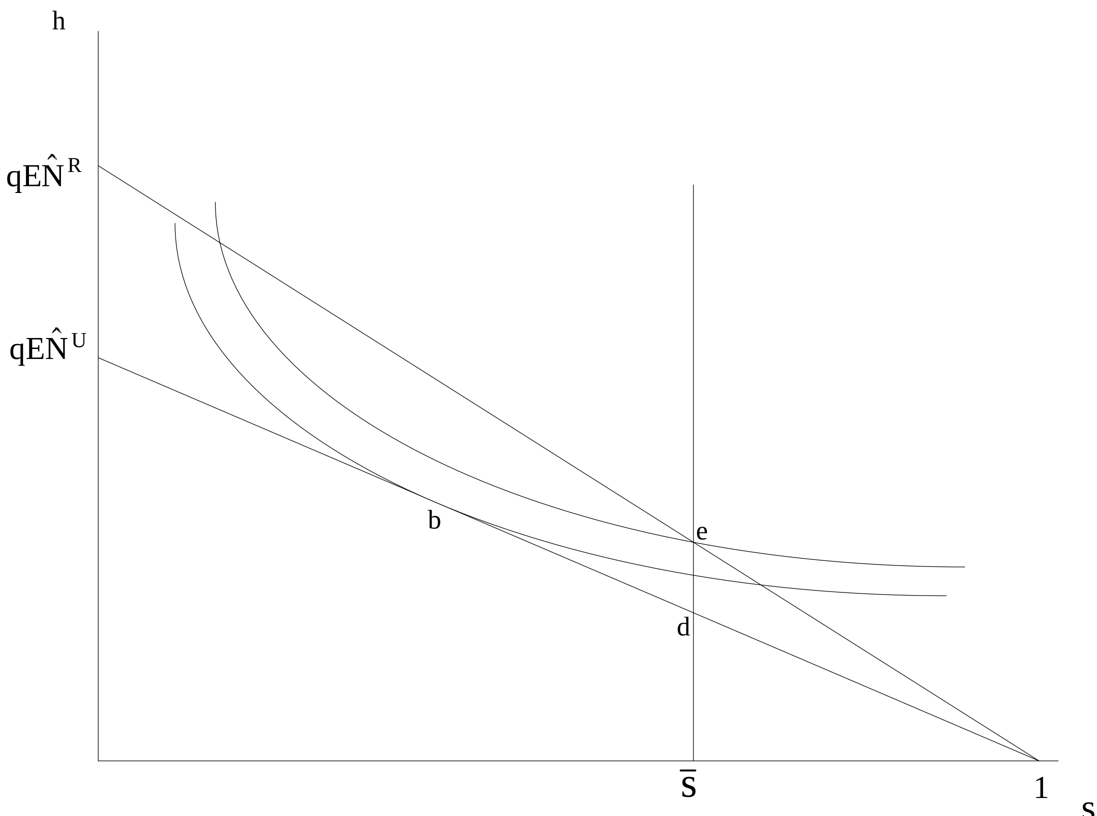
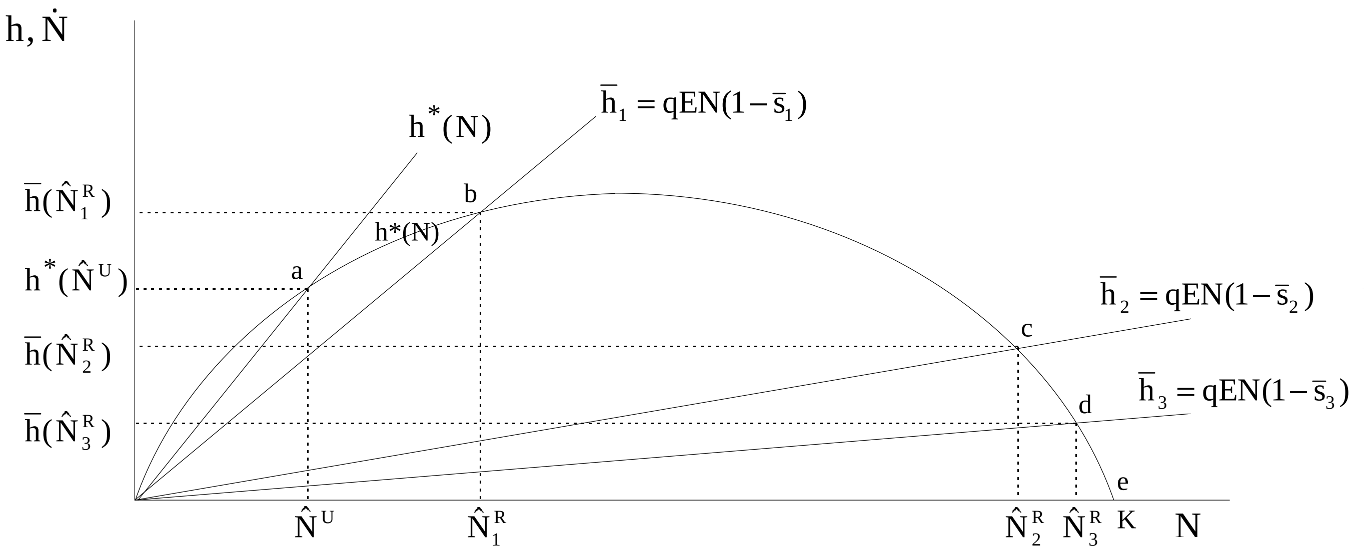


Figure 5



$h, \dot{N}$
$h^*(N)$
$\bar{h}_1 = qEN(1-\bar{s}_1)$
$\bar{h}(\hat{N}_1^R)$
$h^*(\hat{N}^U)$
$\bar{h}(\hat{N}_2^R)$
$\bar{h}(\hat{N}_3^R)$
b
h*(N)
a
$\bar{h}_2 = qEN(1-\bar{s}_2)$
c
$\bar{h}_3 = qEN(1-\bar{s}_3)$
d
e
$\hat{N}^U$
$\hat{N}_1^R$
$\hat{N}_2^R$
$\hat{N}_3^R$
K
N

Figure 6

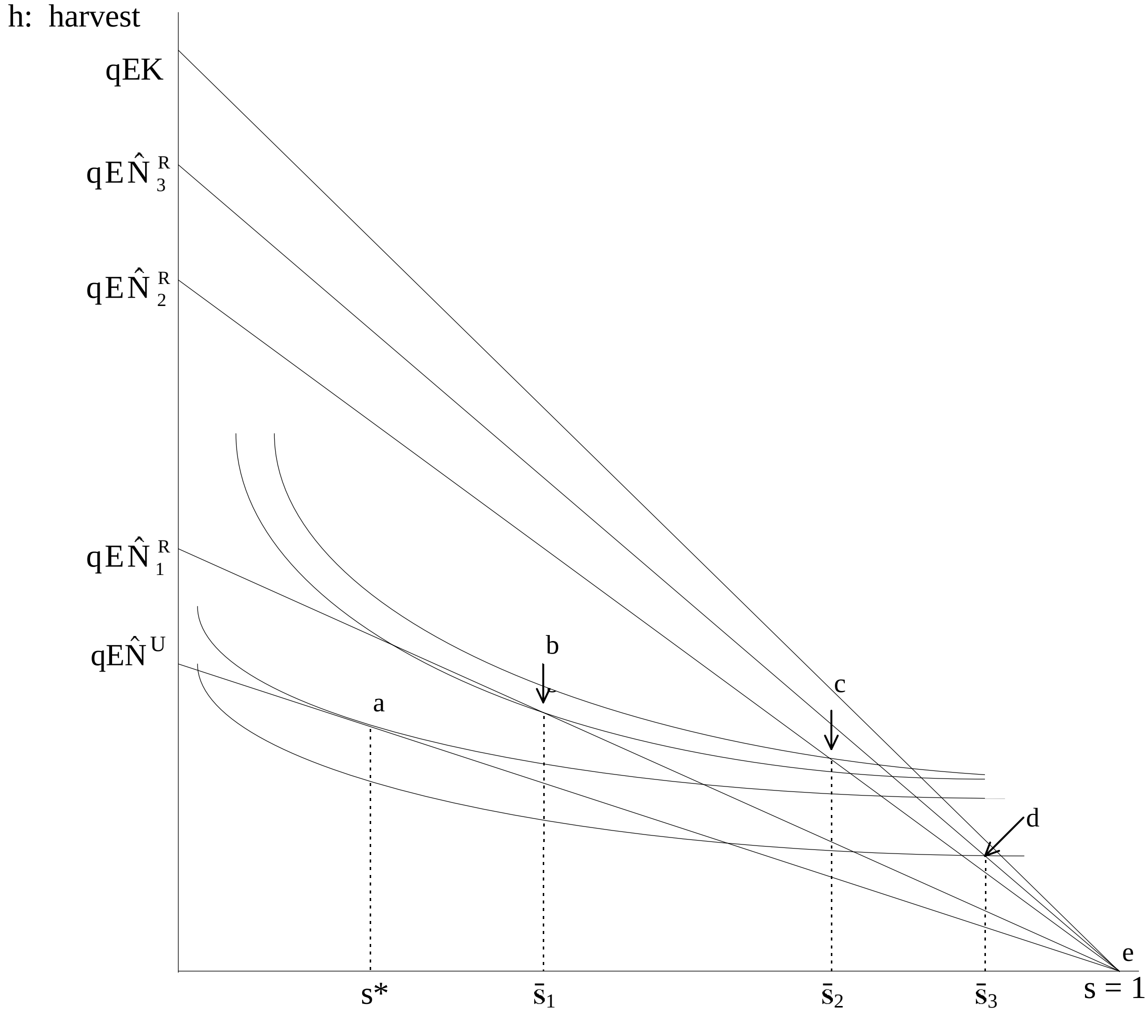
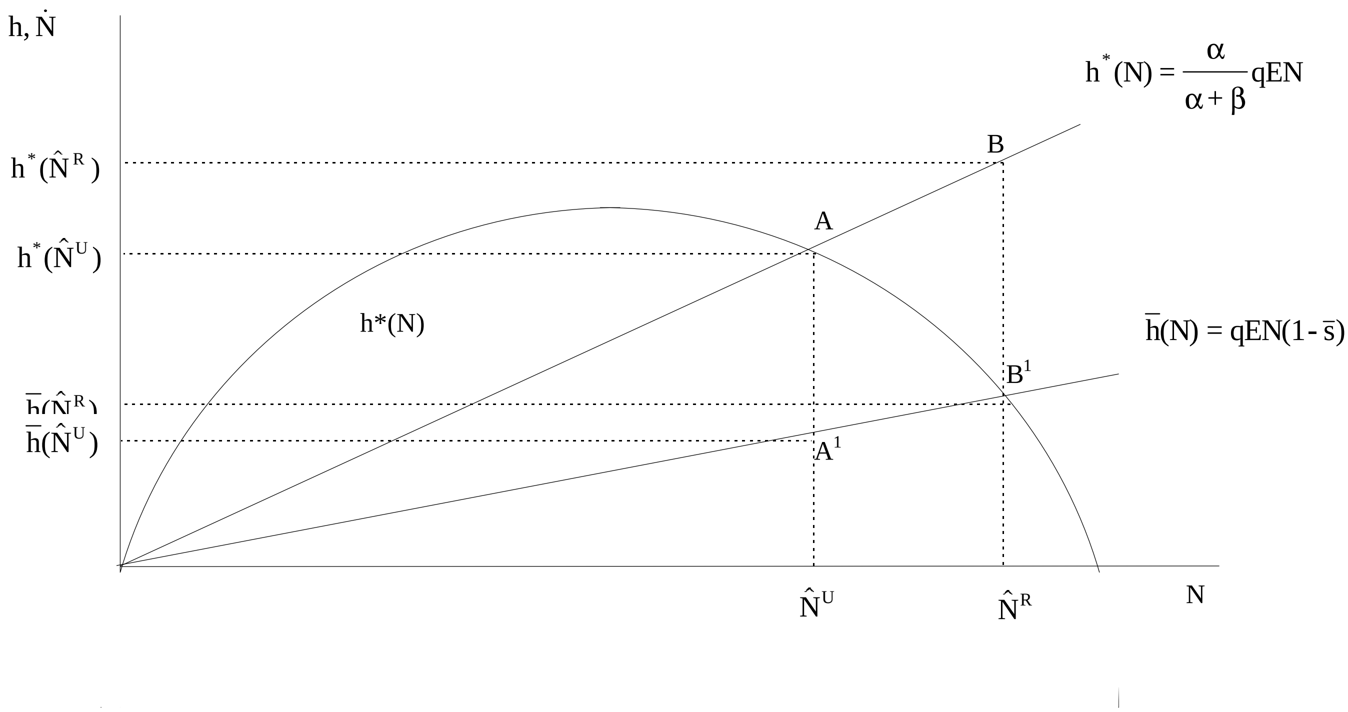


Figure 7



h, $\dot{N}$
$h^*(N) = \frac{\alpha}{\alpha+\beta} qEN$
$h^*(\hat{N}^R)$
B
A
$h^*(\hat{N}^U)$
h*(N)
$\bar{h}(N) = qEN(1-\bar{s})$
B^1
$\bar{h}(\hat{N}^R)$
$\bar{h}(\hat{N}^U)$
A^1
$\hat{N}^U$
$\hat{N}^R$
N