What Do Aggregate Agricultural Supply and Demand Curves Mean?

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A traditional view of empirical aggregation problems is summarized by Phlips:

> The attitude of most applied econometricians (including myself) is simply to ignore this aggregation problem and adopt a third approach by formulating aggregate relationships directly from the theory of the individual consumer.

That is, one pretends either that aggregation poses no problem or alternatively that no tractable solution to aggregation problems is available.

This also appears to be the view of most agricultural economists. A casual survey of this Journal and its predecessor since 1960 reveals over 100 empirical supply and demand studies, roughly 60 or 70 percent of which are based on data reported at something other than the firm or consumer level. Yet typically nothing is said or done about aggregation issues. Instead agricultural economists frequently focus very sophisticated and exotic econometric techniques on accurate measurement of aggregate relationships with little, if any, basis in economic theory. Behavioral relations derived for individuals are simply presumed to apply to highly aggregated data sets. Presumably, the general view is that "dealing" with aggregation issues is *a priori* cumbersome or inconsequential.

As an example of a practice endemic in empirical, agricultural economics[1], Alston and Sumner estimated an aggregate derived demand for U.S. tobacco as a function of a U.S. tobacco price, the price of imported tobacco, and an index of other input costs. This demand relationship was derived from a cost function. Alston and Sumner, therefore, presume that theoretical restrictions received from firm theory apply at an industry level. Is there a reason to expect this to be true? Our results suggest not. But the issue is more basic than whether aggregate demand relationships obey symmetry and homogeneity properties. The real question is whether it is *arithmetically* sensible to add, say, the supplies of two farms and then relate them to an average price. A logical, arithmetic analogue is--"Can you add apples and oranges?" Numerous studies of aggregate supply or demand response in other agricultural industries have assumed you can add apples and oranges in an unrestricted fashion (Rucker, Burt, and La France; Shumway, Saez and Gottret; Capalbo and Denny; Lowry, Glauber, Miranda and Helmberger; Brorsen, Chavas and Grant; Ball and Chambers; and Azzam and Yanagida, to name a few). All of these studies and many more reflect Phlips' view on aggregation. Unfortunately, however, it turns out that adding apples and oranges is not costless. It can end by predetermining the results of empirical analysis.

A more modern approach to dealing with aggregation problems empirically has emerged. Pioneered by Gorman, this approach has been more fully characterized and popularized by Deaton and Muellbauer. The theory of market demand is viewed as one of aggregating microeconomic behavior to allow consistent empirical work. Aggregation is not seen as a nuisance to be swept under the rug but rather as the essence of price theory. Its empirical manifestation is the practically ubiquitous AIDS system and its generalizations (see, e.g., Deaton and Muellbauer, Chapter 6). The modern consensus outside agricultural economics is: empirical attempts to study aggregation explicitly are proving worthwhile for those serious about empirical measurement (Nichols).

Though many different aggregation issues face agricultural researchers, our focus is on price aggregation when measuring producer derived demands and supplies. This issue is chosen for several reasons. Most importantly, price dispersion is the rule and not the exception in agricultural markets. Reflecting this reality is the fact that aggregate price indices are regularly produced by the US government and then used by agricultural researchers.

These reasons may not seem compelling to those unaccustomed to thinking uncritically about competitive firms. Indeed, one can always take the nihilistic position that if prices for a product are not collinear among suppliers, the researcher has no business doing aggregate work. Taken to its logical conclusion this position would condemn all empirical work carried out using anything but the most rudimentary cross-section data. In any case, this is clearly not standard practice in agricultural economics. Below is the correlation matrix for state annual average prices (Oklahoma, Texas, Washington, California, North Dakota) for wheat during the recent period 1980-85:

	OK	TX	WA	CA	ND
OK	1.00	.92	.91	.89	.67
TX		1.00	.92	.81	.70
WA			1.00	.68	.62
CA				1.00	.70
ND					1.00

The correlations are far from unity because of quality and weather differences. In spite of these differences, many empirical studies have used aggregate wheat quantities and price indices published by the USDA and other sources without attempting to determine the relevance of such a procedure (See e.g., Morzuch, Weaver, and Helmberger; Chambers and Just; Shumway; Sarris and Freebain; Lopez; Moschini; de Gorter and Meilke; and Gardner). In a similar vein, even though participants and nonparticipants in government programs typically face different prices, common practice is to ignore this reality in empirical work. And as numerous studies have shown, land prices and rents vary widely across individuals (Peterson). Perhaps the strongest testimony to the presence of price dispersion is the fact that many public resources are expended to calculate or measure U.S., State, County, and farm level prices of inputs and outputs.

Having argued that agricultural price distributions are not degenerate, the issue remains whether the consequences of price dispersion are substantive. In the end, a truly scientific answer can only be obtained inductively through empirical applications and analysis as in the case of AIDS. Yet some deductive analysis yields insight. The following simple argument shows that, *a priori*, one expects aggregation to be important.

If all firms face the same input price and aggregate input demand is defined as the sum of firm demands, the slope of the aggregate demand curve is the sum of the firm demand curve slopes. Using an *average* price in the aggregate demand relation, however, implies that all firms do not face the same price. The slope of the aggregate demand curve, therefore, is how aggregate demand changes in response to a change in *average* price. Generally this aggregate slope is not the sum of firm demand curve slopes. To see why, perform the simple thought experiment: there are two firms (A and B) in an industry, each has an identical Cobb-Douglas demand function but firm A faces a higher price than firm B. Lower A's price to the average while raising B's to the average. Average price is unchanged. The slope of aggregate demand curve fitted to the average price thus predicts no change. But simple computation reveals that industry demand does change. The slope of the aggregate demand curve cannot be used directly to produce estimates of the demand response.

This paper designs workable empirical procedures for constructing price aggregators which can capture accurately the information that empirical research is trying to uncover. Unlike Deaton and Muellbauer, however, complete demand systems are not considered. Only single equation aggregation problems are studied. Thus, the current approach is less ambitious than Deaton and Muellbauer. Two reasons lie behind this choice: It shifts the focus of the paper away from the study of optimizing firms toward the arithmetic requirements for adding up industry supplies and demands. Thus, our approach is robust to the differing optimization paradigms (profit maximization, expected-utility maximization, safety-first, etc.) underlying behavioral models of firms. (Restrictions across systems of such equations generally lead to pessimism about a single price index aggregating the entire system. For an analysis under profit maximization see Pope and Chambers). Second, most applied work in agricultural economics does not employ systems type restrictions, therefore, the method and results here should have wide applicability.

The approach developed enables researchers to delineate the class of supply or demand functions consistent with the actual aggregate price index employed. The emphasis here is on developing constructive rules to guide future studies and insure aggregation consistency in empirical practice. Though some of the analysis is quite general, particular attention is paid to the two most commonly published and used producer price indices, the simple and weighted averages.

In what follows, we first illustrate the kernel of our problem with two examples drawn from common empirical practice in agricultural supply studies. One of the most common empirical specifications, the Cobb-Douglas, is shown to produce systematic biases when used with either of the two most common price indices reported by USDA. The third section generalizes these examples and provides constructive rules for consistent aggregation in a tabular format. The fourth section examines the consequences of inappropriate aggregation procedures. The fifth section contains an empirical example of the methods and results in earlier sections. Significantly, we find no empirical support for the state-level technology implied by presently calculated U.S. season average prices. The sixth section concludes. We note explicitly that although for illustrative purposes our discussion is solely in terms of producer models the results apply directly to the consumer case. This follows from the symmetry between the producer cost function and the consumer expenditure functions.

Common Agricultural Price Aggregation and the Cobb-Douglas: An Example

The most common aggregate prices used by agricultural economists are simple averages or weighted averages. Prices reported in such familiar sources as *Agricultural Statistics*, *Agricultural Prices*, or the *Survey of Current Business* tend to be averages or weighted averages. For example, *Agricultural Prices* reports commodity prices received by farmers by states on a monthly basis. These prices usually are "...estimates of average prices received for all of the commodity sold during the entire month." Thus, the aggregation procedure is simple averaging. This same publication, as well as *Agricultural Statistics*, also reports annual U.S. season average prices for many commodities. These U.S. season average prices are computed by weighting ". . . State season average prices by the estimated quantity sold in each state." Here the aggregation procedure is weighted averaging. (We refer to this weighted average as the Laspeyres in what follows.) This section briefly considers the implications of using these two price aggregation schemes in Cobb-Douglas supply-response models.

Consider the price-aggregation problem: Agricultural economists wish to relate aggregate supply, Y (the sum of firm supplies), econometrically to the average industry price, $\bar{P} = \frac{1}{m} \sum_{j=1}^{m} p_j$ (p_j is the price of the j^{th} firm) and a vector of input prices, w, which are assumed to be constant across all firms.

To do so, the researcher first specifies an aggregate supply function which we denote by $G(\bar{P},w)$. Then using data on Y, $\bar{P}$, and w, the parameters of G are estimated. For many agricultural applications, G might be chosen as Cobb-Douglas, $G = A\bar{P}\, w^{-a}$. For notational simplicity, assume w is a scalar.

A basic questions is--Is this procedure consistent in the sense that G can be interpreted as the sum of firm supplies over all m firms? Differentiate both sides of the aggregate supply relationship $(Y = G(\bar{P},w))$ with respect to p_i and $p_j (j \neq i)$ using the definition of Y, G and $\bar{P}$ to find that a necessary condition for a positive answer is that

$$(1) \qquad \frac{\frac{\partial G}{\partial \bar{P}}\frac{\partial \bar{P}}{\partial p_i}}{\frac{\partial G}{\partial \bar{P}}\frac{\partial \bar{P}}{\partial p_j}} = \frac{\frac{\partial \bar{P}}{\partial p_i}}{\frac{\partial \bar{P}}{\partial p_j}} = \frac{\partial Y/\partial p_i}{\partial Y/\partial p_j} = \frac{\partial y_i/\partial p_i}{\partial y_j/\partial p_j} \qquad \text{for all } i,j=1,\ldots,m.$$

Because $\bar{P}$ is the simple average, expression (1) reduces to

$$\frac{\frac{\partial \bar{P}}{\partial p_i}}{\frac{\partial \bar{P}}{\partial p_j}} = 1 = \frac{\partial y_i/\partial p_i}{\partial y_j\ p_j}.$$

Own price supply slopes must be equal for all firms for aggregation to be consistent. Because each such slope can depend only upon p_i and w (firm supply only depends on the prices it faces), these slopes must be independent of the firm and hence depend only on w. Integrating with respect to p_i yields

$$y_i = \alpha_i(w) + \beta(w)p_i \qquad i = 1,\ldots,m.$$

That is, firm level supplies must be linear in own price. Summing, aggregate supply is also linear

$$Y = \sum_{i=1}^{m} y_i = \sum_{i=1}^{m} \alpha_i(w) + \beta(w) \sum_{i=1}^{m} p_i = \alpha(w) + \beta(w) \cdot m\bar{P}$$

where $\sum_{i=1}^{m} \alpha_i(w) = \alpha(w)$.

But G has been chosen as Cobb-Douglas so that

(2) $$Y = A\bar{P}^{\alpha} w^{-\alpha} = \alpha(w) + \beta(w)m\bar{P}.$$

This implies $A w^{-a} \bar{P}^{a} = \beta(w)m\bar{P}$ and $\alpha(w) = 0$. A regression with this commonly used form will tend toward a unit elasticity supply curve as the aggregation errors tend toward zero. *The aggregate supply elasticity must be unity!* This latter equality also requires that either $\alpha_i(w) = 0$ $(i = 1,\ldots,m)$ or that $\sum_{i=1}^{m} \alpha_i(w) = 0$ with some $\alpha_i \neq 0$. In the case that all $\alpha_i(w) = 0$, *micro and macro supply elasticities are unity*. Hence, there is no need to estimate the supply elasticity. In the case that $\sum_{i=1}^{m} \alpha_i(w) = 0$, technology must follow a very distinct dependence between firms; elasticities will rise [fall] with increasing p as $\alpha_i(w) < 0$ [> 0].

Now, consider the Laspeyres price index

$$P = \hat{P} = \frac{\sum_{i=1}^{m} p_i \, y_i}{\sum_{i=1}^{m} y_i} .$$

Again assume that $\sum_{i=1}^{m} y_i = Y$. $\hat{P}$ clearly is a function of Y and w. Because the denominator of $\hat{P}$ is Y, it seems clear that the numerator depends on Y and w. Pope and Chambers have shown that the numerator (revenue) of $\hat{P}$ is necessarily affine in Y. Thus, $\sum_{i=1}^{m} p_i \, y_i = \alpha(w) + \beta(w)Y$ with $p_i y_i = \alpha_i(w) + \beta(w)y_i$. Hence,

$$\hat{P} = \frac{\alpha(w) + \beta(w)Y}{Y}$$

and

$$Y = \frac{\alpha(w)}{\hat{P} - \beta(w)}.$$

Only forms satisfying this last equation are candidates for $G(\hat{P},w)$. For example, suppose as above that $G(\hat{P},w)$ is Cobb-Douglas $A\hat{P}^{a}w^{-a}$. Comparing this with the above then requires $\beta(w) = 0$, $A = 1$, $a = -1$. Hence, a Cobb-Douglas can only be used with the Laspeyres if aggregate supply has a supply elasticity of minus one! *That is, a regression with one of the most widely published and used price indices in one of the most widely utilized functional forms for supplies, will tend toward a downward sloping aggregate supply curve as the aggregation errors tend to zero.*

The consequences of choosing a Cobb-Douglas supply specification with either of the two most common price aggregates, the average and the quantity-weighted average, are clear: elasticity values are predetermined if supply is aggregable. Where then is the place for estimation if one is mainly interested in those elasticities? What is the meaning of elasticity estimates differing from these values? In choosing his or her own answer to these questions, the reader is urged to consider some advice King (p. 846) offered our profession just a decade ago:

> The consequences that follow when researchers choose among alternative models must be considered in terms of theoretical properties and uses to be made of these findings.

Single Equation Price Aggregation-General Results

The previous examples are important special cases of a more general price aggregation problem: Find functions G, P, and H satisfying

(3) $$G(P,w) = H(h_1(p_1,w),h_2(p_2,w),\ldots,h_m(p_m,w))$$
$$P = P(p_1,\ldots,p_m;w).$$

Here p_i remains the price faced by the i^{th} firm, and w is the vector of input and output prices which are common to all firms. The indexes $(i=1,\ldots,m)$ delimit the number of firms whose supplies or demands are aggregated. The functions h_i are the microeconomic response functions (e.g. supplies, demands, profits, or costs) to be aggregated. The function H represents the rule by which the aggregate quantity variable is constructed from these microeconomic response functions. For example, in both examples above, h_i is the supply of the i^{th} firm and H is the summation operator. The function G is the aggregate behavioral function upon which analysis is based. In the examples above, G is the Cobb-Douglas industry supply function. P is the representative price used in the aggregate relationship. In the first example, P is the average price while in the second it is the weighted average. Aggregation schemes which satisfy (3) are said *to aggregate consistently*.

Attention is restricted to strictly monotonic and differentiable H functions and G functions which are differentiable in both arguments and strictly monotonic in P. This represents only a slight loss in generality because virtually all commonly used functional forms satisfy these criteria. This assumption clarifies the intimate relationship between the aggregate price index and the microeconomic response functions.

Result 1 (The Fundamental Price Aggregation Result): The aggregate price index P, consistent with (3), can always be expressed as a differentiable and monotonic function of the microeconomic response functions of the following form:

(4) $$P = P^*(h_1(p_1,w),\ldots,h_m(p_m,w);w).$$

P* has partial derivatives

(5) $$\frac{\partial P^*}{\partial h_i} = \left(\frac{\partial G}{\partial P}\right)^{-1}\left(\frac{\partial H}{\partial h_i}\right)$$

$$\left.\frac{\partial P^*}{\partial w}\right|_{h_i} = \left(\frac{\partial G}{\partial P}\right)^{-1}\left(\frac{\partial G}{\partial w}\right)$$

where the notation $\left.\frac{\partial P^*}{\partial w}\right|_{h_i}$ denotes the partial derivative of P* with respect to w holding the h_i functions constant.

Expressions (4) and (5) are direct consequences of our assumption that both H and G are strictly monotonic and differentiable and the implicit function theorem (Courant, p. 117) applied to (3). The price-aggregation rule, P, inherits structural restrictions placed upon H. For example, suppose that the aggregate economic quantity is firmwise strongly separable in the microeconomic response functions. This includes the case where aggregate supplies, demands, shares, or profits are the sum of their micro counterparts. It also includes the case where the log of each micro entity was equal to the log of the macro entity or that the log of the sum of the micro entities was equal to the log of the macro entity and so on. Functionally,

$$(6) \qquad H(h_1,\ldots,h_m) = \bar{H}\left[\sum_{i=1}^{m} g_i(h_i(p_i,w))\right] = \bar{H}(g).$$

Applying (4) to (6) gives the price aggregation rule consistent with (6)

$$(7) \qquad P = \bar{P}*\left[\sum_{i=1}^{m} g_i(h_i(p_i,w));w\right] = \bar{P}*(g;w).$$

Expression (7) is firmwise strongly separable in the prices p_i. The reader can easily verify that expression (4) implies the following result (proved formally in Pope and Chambers):

Result 2: Let N be the set of integers $\{1,\ldots,m\}$ and let $\Omega = (\Omega_1,\ldots,\Omega_s,\ldots,\Omega_S)$ be a mutually exclusive and exhaustive partition of N. Let Ω_p and Ω_h be the set of prices, p, and functions, h, partitioned according to the indexes in Ω. Then, for H, P, and h, monotonic and twice differentiable, with $H(h_1(p_1,w),\ldots,h_n(p_n,w)) = G(P,w)$, P is strongly (weakly) separable in Ω if and only if H is strongly (weakly) separable in Ω.

Thus, one cannot divorce the structure of the price aggregate from the structure of the quantity aggregation rule. Once the researcher picks a price aggregate (quantity aggregation rule) the quantity aggregation rule (the price aggregate) cannot be chosen arbitrarily. Result 2 shows, for example, that choosing the price aggregate to be mean price (this rule is strongly separable) automatically rules out using quantity aggregation rules which are not strongly separable. Because most aggregation rules used in the construction of reported quantity aggregates are strongly separable aggregation rules, we consider further the practical implication of Result 2.

Constructing Firmwise Strongly Separable Aggregates

To examine the implications of Result 2, imagine that one chooses a price-aggregation rule

$$(8) \qquad P = \bar{P}\left[\sum_{i=1}^{m} k_i(p_i,w);w\right] = \bar{P}(k;w)$$

which is firmwise strongly separable in the prices p. (The average price is a special case of (8).) Result 2 now implies H must be also firmwise strongly separable in the h functions, i.e., it can be expressed as (6), to satisfy consistent aggregation. (Alternatively, if one chooses a quantity aggregation rule satisfying (6), the price aggregation rule must satisfy (8).) Use this fact and differentiate both sides of (3) to get in this case that

$$\frac{\partial G}{\partial p_i} = \frac{\partial G}{\partial P}\frac{\partial \bar{P}}{\partial k}\frac{\partial k_i}{\partial p_i} = \frac{\partial \bar{H}}{\partial g}\frac{\partial g_i}{\partial p_i} \qquad i = 1,\ldots,m.$$

Taking ratios gives after a slight manipulation that

$$(9) \qquad \frac{\partial k_i/\partial p_i}{\partial g_i/\partial p_i} = \frac{\partial k_j/\partial p_j}{\partial g_j/\partial p_j} \qquad \text{for all } i \text{ and } j.$$

Expression (9) implies that these ratios must be the same for all i and j. By construction, each ratio can only depend upon the vector w and the price specific to the i^{th} firm. Because they must be the same regardless of the firm considered (and thus the price), these ratios can only depend upon w--call this common ratio $\gamma(w)$. Integrating the differential equations implied by (9) gives

$$(10) \qquad g_i(h_i(p_i,w)) = \gamma(w)k_i(p_i,w) + v_i(w) \qquad i = 1,\ldots,m$$

and

$$h_i(p_i,w) = g_i^{-1}\left[\gamma(w)k_i(p_i,w) + v_i(w)\right] \qquad i = 1,\ldots,m.$$

To aggregate consistently, the i^{th} response function must be a monotonic transformation of an affine transformation of $k_i(p_i,w)$. The implication of (10) is: *choice of a price or quantity (micro response) aggregation rule dictates the general class of micro response (price) functions that can be consistently aggregated.*

Limitations on Strongly Separable Aggregates

Table 1 presents a guide to constructing firmwise strongly separable price indexes and quantity indexes which are mutually consistent. The families of price indexes in the second column exhaust the possible alternatives consistent with the quantity indexes in the corresponding row of the first column. If the researcher chooses a price aggregate belonging to the second column of that Table, the structure of the associated micro-response functions is predetermined. Notice, in particular, that this relationship does not follow from any optimization hypotheses but from the arithmetic necessity of having both sides of aggregate equations consistent with one another. Violating these principles in empirical work is, to use an old saw, "adding apples and oranges." Just as two apples and two oranges do not make four apples or four oranges, using inconsistent aggregation procedures violates the simple arithmetic laws upon which aggregation is purportedly based.

The limiting nature of these results can be seen clearly by considering the apparently sensible procedure of using a geometric average of firm-level prices as a price index (row 7, column 2) to explain industry supply (the sum of firm supplies). From Table 1, the firm-level supply functions consistent with aggregation belong to the family of functions

$$\alpha_i(w) + \hat{\beta}(w)\ \theta_i(w)\ell np_i \qquad i = 1,\ldots,m.$$

which is homogeneous of degree zero in all prices, as required by the theory of the profit maximizing firm, only if it is independent of p_i. So, if the researcher is aggregating over the supply price this means that at the firm level, either that profit maximization has been eschewed entirely or that industry level supply must be perfectly price inelastic. (Most aggregate studies purport to be based on the profit maximization hypothesis--clearly many really are not because they repeatedly violate these general principles). Using a geometric average for a price index and the sum of firm supplies as the quantity aggregate jointly is inadmissible. In fact, the whole family of quantity aggregates consistent with the geometric average is inconsistent with homogeneity.

Why Quantity (Price) Aggregates Inherit the Structure of Price (Quantity) Aggregates

When *either* the price or the quantity index is specified to be firmwise strongly separable (excepting the use of aggregate shares considered in a separate paper this is the most common empirical practice), its counterpart in the aggregation scheme inherits both this structural property as well as the general functional structure imposed. Why is this result obtained? The key lies in recognizing that the three level sets associated with the aggregate response function (G), the aggregate price index (P), and the aggregate quantity index (H), respectively:

$$\{p:G(P(p_1,\ldots,p_m);w) = G\}$$

$$\{p:P(p_1,\ldots,p_m;w) = P\}$$

$$\{p:H(h_1(p_1,w),\ldots,h_m(p_m,w)) = H\}$$

Table 1

Aggregate Function (H)	Class of Price Aggregators P
	General Structure
$\sum_{i=1}^{m} h_i(p_i,w)$	$P^*\left[\sum_{i=1}^{m} (\alpha_i(w) + \beta(w)h_i(p_i;w));w\right]$
$H\left[\sum_{i=1}^{m} h_i(p_i,w)\right]$	$P^*\left[\sum_{i=1}^{m} (\alpha_i(w) + \beta(w)h_i(p_i;w));w\right]$
$H\left[\sum_{i=1}^{m} \hat{\alpha}_i(w) + \hat{\beta}(w)k_i(p_i,w)\right]$	$\sum_{i=1}^{m} k_i(p_i;w)$
$H\left[\sum_{i=1}^{m} \hat{\alpha}_i(w) + \hat{\beta}(w)k_i(p_i,w)\right]$	$P^*\left[\sum_{i=1}^{m} k_i(p_i;w);w\right]$
	Specific Examples
$\sum_{i=1}^{m} A_i\, p_i^{\alpha}\, w^{1-\alpha}$	$P^*\left[\sum_{i=1}^{m} \alpha_i(w) + \beta(w)A_i\, p_i^{\alpha}\, w^{1-\alpha};w\right]$
$H\left[\sum_{i=1}^{m} \hat{\alpha}_i(w) + \frac{\hat{\beta}(w)}{m}\, p_i\right]$	$\frac{1}{m} \sum_{i=1}^{m} p_i$
$H\left[\sum_{i=1}^{m} \hat{\alpha}_i(w) + \hat{\beta}(w)\theta_i(w)\ln p_i\right]^*$	$\prod_{i=1}^{m} p_i^{\theta_i(w)} = \exp\left[\sum_{i=1}^{m} \theta_i(w)\ln p_i\right]$
$\sum_{i=1}^{m} (a_i\, p_i^{\alpha_i} + b_i\, w^{\alpha_i})^{1/\alpha_i}$	$P^*\left[\sum_{i=1}^{m} \alpha_i(w) + \beta(w)(a_i\, p_i^{\alpha_i} + b_i\, w^{\alpha_i})^{1/\alpha_i};w\right]$
$H\left[\sum_{i=1}^{m} \hat{\alpha}_i(w) + \hat{\beta}(w)c_i\, p_i^{\alpha(w)}\right]$	$\left[\sum_{i=1}^{m} c_i\, p_i^{\alpha(w)}\right]^{1/\alpha(w)}$

*Whole family will be inconsistent with homogeneity when $\hat{\beta}\,\theta_i \neq 0$.

have the same graph in p-space for fixed w. Thus, once one specifies the curvature properties of, say, the price aggregation rule in p-space one also specifies the curvature of the level set of the quantity aggregation rule and the aggregate response function. Put another way, each of these functions when considered as functions of the prices over which one is aggregating are monotonic transformations of the others (see Result 1). Hence, once one of these functions is specified (say H) the whole family of possible functions available for the two other functions (G and P) is known. Because these latter families correspond to monotonic transformations of the original aggregation rule specified (there are an infinity of such possible transformations), there are an infinity of possible rules which are consistent with aggregation over prices. This is reflected in Table 1, for example, by the fact that the average price equation will aggregate firm level supplies of the general form for any possible H function that is monotonic in its argument.

However, once a specific H function is chosen, say the sum, then choice of a specific price aggregator places direct restrictions on the form of the functions to be aggregated. There are no longer an infinity of P functions available because attention has been restricted to a particular form. Only special classes of h() functions can then be consistent with both H, P, and the requirement that the latter be expressible as a monotonic transformation of the former. This is why once exact forms of P and H are specified, we can infer exactly the type of supply or demand functions (if any exist) that are consistent with these aggregators. For example, the Laspeyres price index is not generally firmwise strongly separable. But $H = \sum_{i=1}^{m} h_i$ is firmwise strongly separable. Thus, if this H is to be used with a Laspeyres index, only firm-level supply functions which make the Laspeyres index firmwise strongly separable in the p_i's are consistently aggregable. Pope and Chambers demonstrate that these functions all belong to the family

$$h_i(p_i,w) = \frac{\alpha_i(w)}{p_i - \beta(w)}.$$

Researchers adopting US season average prices as published in *Agricultural Statistics* or *Agricultural Prices*, therefore, have prejudged that firm-level responses must be of this form. Moreover, as we have already shown, the aggregate relation must also belong to the same family. The reader is left to answer the question of how many of these empirical studies have actually obeyed these requirements of not adding apples and oranges. Those which have not are internally inconsistent and invalid empirically.

These ideas are portrayed geometrically in Figure 1. The level curves for G, P, and H are represented by the curve (set) a-b. The marginal rate of substitution of the curve is $(\partial P/\partial p_j)/(\partial P/\partial p_i)$ or equivalently $(\partial h_j/\partial p_j)/(\partial h_i/\partial p_i)$ which is nonnegative. In general the marginal rate of substitution may rise or fall with increasing p_j. For strongly separable aggregation rules, the marginal rate of substitution is unaffected by variations in $p_k (k \neq i,j)$. When P equals average price, $\bar{P}$, the marginal rate of substitution is constant at unity. In the Laspeyres case, $\hat{P}$, the slope is nonconstant and equal to $\alpha_j(w)(p_j-\beta(w)^{-2}/\alpha_i(w)(p_i-\beta(w))^{-2}$. The marginal rate of substitution is diminishing and hence aggregate price increases with a mean preserving spread in the distribution of prices.

Finally, exact knowledge of P, H, and the h functions lets one infer G(P,w) because differentiating (3) gives

$$\text{(11)} \qquad \frac{\partial G}{\partial P}\frac{\partial P}{\partial p_i} = \frac{\partial H}{\partial h_i}\frac{\partial h_i}{\partial p_i}.$$

If $\frac{\partial P}{\partial p_i}$ and $\frac{\partial H}{\partial h_i}$ $\frac{\partial h_i}{\partial p_i}$ are known, (11) can be solved for $\partial G/\partial P$ which can, in principle, be integrated to obtain G.

A simple example illustrates: Suppose that $H = \sum_{i=1}^{m} h_i$ and P is an average price. By Table 1, the h functions are of the general form

$$h_i(p_i,w) = \alpha_i(w) + \frac{\hat{\beta}(w)}{m} p_i \qquad i = 1,\ldots,m.$$

Differentiating the resulting (11) with respect to p_j for $(j \neq i)$ implies

$$\frac{\partial^2 G}{\partial P^2} = 0$$

or that the aggregate response function must be linear in the aggregate price. As long as H is linear G always inherits the same form as the h_i functions. As another example, we have already shown above for the Laspeyres price index and H linear that

$$G(P,w) = \frac{\alpha(w)}{P - \beta(w)}$$

which is to be compared with the associated h_i's shown above.

Consequences of Using Inappropriate Aggregators: An Example

Earlier results show that if a Laspeyres price index is used with a quantity aggregate that is the sum of the firm-level quantities the correct form for the aggregate relation is

$$G(P,w) = \frac{\alpha(w)}{P - \beta(w)}.$$

This section considers the error involved in approximating this correct relation with two widely-used forms--the linear in P (affine) and the Cobb-Douglas. Before proceeding, some regularity conditions must be imposed. In doing so treat G(P,w) as if it represented an aggregate supply relationship If G(P,w) is to be upward sloping then

$$G_p(P,w) = \frac{\alpha(w)}{(P - \beta(w))^2} \geq 0$$

meaning $\alpha(w) \geq 0$. To insure a nonnegative supply then requires that $P - \beta(w) > 0$ for all P.

To serve as a point of reference we shall require that both the affine and Cobb-Douglas forms be interpretable as a first-order differential approximation in P. The function H(P) is a first-order differential approximation to G(P,w) at P^o if

(12)
$$H(P^o) = G(P^o,w)$$
$$H_p(P^o) = G_p(P^o,w).$$

Notice in particular that equation (12) also implies H(P) is a first-order differential approximation only if

$$\frac{H_p(P^o)P^o}{H(P^o)} = \frac{G_p(P^o,w)P^o}{G(P^o,w)}.$$

The affine function $H(P) = a + bP$ is a first-order differential approximation to G(P,w) at P^o only if

$$a = \frac{\alpha(w)}{P^o - \beta(w)} \left[1 - \frac{P^o}{P^o - \beta(w)}\right]$$

$$b = \frac{\alpha(w)}{(P^o - \beta(w))^2}.$$

When the affine function is used to approximate the effect on aggregate supply of a small change in the aggregate price at any point away from P^o, say P^*, the measurement error can be computed as

$$G_p(P^*,w) - b = \frac{\alpha(w)[(P^o - \beta(w))^2 - (P^* - \beta(w))^2]}{(P^* - \beta(w))^2(P^o - \beta(w))^2}.$$

If $P^* > P^o$ the linear function underestimates the supply response while the reverse happens if $P^o < P^*$. Thus using a linear relationship leads to a systematic bias in estimating the responsiveness of aggregate supply to changes in the aggregate price.

The Cobb-Douglas function $H^{CD}(P) = AP^F$ provides a first-order differential approximation to G(P,w) in this case at P^o only if

$$F = \frac{P^o}{P^o - \beta(w)}$$

$$A = \left(\frac{\alpha(w)}{P^o - \beta(w)}\right) P^{o\,\frac{P^o}{[\beta(w) - P^o]}}.$$

Performing a similar calculation reveals that

$$\frac{G_P(P*,w)\ P*}{G(P*,w)} - F = \frac{\beta(w)(P* - P^o)}{(P* - \beta(w))(P^o - \beta(w))}.$$

Hence, if $P* > P^o$ the Cobb-Douglas underapproximates the true supply price elasticity while if $P* < P^o$ the Cobb-Douglas overapproximates it.

Figure 2 illustrates the divergence between exact and "approximate" aggregation. The curve labeled LP is the aggregate supply function corresponding to exact aggregation with the Laspeyres form. The other curves express the first-order approximations of LP at $P^o = 20$. Both the linear and Cobb-Douglas underestimate the true response which is convex in P. It is apparent that both the linear and Cobb-Douglas approximation do very poorly when P* diverges from the approximation point. For example, at approximately P* = 40, the approximations are 50% of the true response. Thus, in any sample of time series (e.g. post-war) data, the approximation error could be substantial as price moves throughout its sample domain.

Now suppose alternatively that statistical methods were used to fit either a linear or Cobb-Douglas function to the scatter of points generated by the true function consistent with the Laspeyres. Visual inspection of Figure 2 indicates that this would result in an estimated model which consistently underestimates the true value for low prices, overestimates it for intermediate prices, and then underestimates it again for high prices.

Empirical Application

To illustrate the empirical implications of the preceding developments recall that U.S. season average prices for crops as reported in *Agricultural Prices* and *Agricultural Statistics* "...are computed by weighing State season average prices by the estimated quantity sold in each state." Thus, the U.S. season average price is an index of the Laspeyres form:

$$\hat{P} = \frac{\sum_{j=1}^{m} p_j y_j}{\sum_{j=1}^{m} y_j}$$

If $\hat{P}$ is to be used in an aggregate supply-response equation where aggregate supply is taken to be the sum of firm supplies then Result 2 implies that aggregation is possible if and only if there exists a family (or families) of firm-level supplies which make $\hat{P}$ strong separable in the firm-level prices. Pope and Chambers have shown that the only family of supply functions which will make $\hat{P}$ strongly separable is

(13) $$y_i(p_i, w) = \alpha_i(w) / (\beta(w) - p_i) \qquad i = 1, \ldots\ldots, n.$$

Several comments need to be made about this result: first, *using a Laspeyres index presumes that the supply-response technology is given by this last equation*; no other technology is consistent with the Laspeyres price index. Second, differences in supply response across firms are restricted to the term $\alpha_i(w)$; each firm must have the same common expression in the denominator of its supply-response equation. If, for example firms have supply-response equations of the general form:

(14) $$y_i(p_i, w) = \alpha_i(w) / (\beta_i(w) - p_i) \qquad i = 1, \ldots\ldots, n.$$

technical differences across firms prevent consistent aggregation and the use of the published U.S. season average prices in U.S. supply response equations.

Thus, an empirical procedure for determining whether use of $\hat{P}$ in U.S. supply-response equations is clear: first, test the general validity of (14) against a more general supply-response specification; and then test the validity of (13) against (14). The first test represents a test of the validity of the general family of technologies while the second represents a test for a sufficient degree of commonality across firm technologies to permit aggregation.

We illustrate this procedure using annual USDA data (available upon request) on wheat production and wheat prices for two wheat producing states, Oklahoma and South Dakota, for the period 1959-1987. Specifically, the goal is to determine empirically whether it is legitimate to sum Oklahoma and South Dakota wheat supplies and then use this aggregate in a behavioral relation with a weighted average of the price in South Dakota and the price in Oklahoma. (We leave to the side the nontrivial issue of whether the state level prices and quantities have been appropriately aggregated. However, please see the next section.) Because our purposes here are mainly illustrative, our general supply-response specification is a simple generalization of (14)

(15) $$y_i(p_i,w) = \sigma_i(w) + \alpha_i(w) / (\beta_i(w) - p_i) \qquad i = 1, \alpha$$

The terms, $\sigma_i(w)$, $\alpha_i(w)$, and $\beta_i(w)$ were treated parametrically and equations (15) were estimated after appending intertemporally independent, mean-zero error terms by systems, nonlinear least squares (in implicit form) with the following results:

Oklahoma

$$y = \underset{(7.196935)}{143.096941} + \underset{(5.596082)}{27.181352} / (\underset{(.50611)}{2.300478} - p)$$

South Dakota

$$y = 39.[illegible]64 \;+\; 2.331802 \;/\; (2.481225 - p)$$
$$(2.800659) \qquad (2.766963) \qquad (.157022)$$

Terms in parentheses are asymptotic standard errors.

The next step in our procedure is to test if representation (14) represents an acceptable null hypothesis against the maintained model (15). To test this hypothesis statistically we reestimated the supply-response equations for Oklahoma and South Dakota using representation (14) again by systems, nonlinear least squares with the following results:

$$y = 15.891579 \;/\; (2.571662 - p)$$
$$(14.35400) \qquad (.200817)$$

South Dakota

$$y = .270932 \;/\; (2.515280 - p)$$
$$(4.326164) \qquad (.203139)$$

Because (14) is a special case of (15) (the constant in the latter has been set to zero), an asymptotic, likelihood ratio test can be used. The test is to compare the computed value of -2 times the log likelihood ratio derived from the two estimated versions of the model with a tabulated χ^2 with two degrees of freedom. The computed log-likelihood ratio statistic is 67.6726 while the tabulated χ^2 with two degrees of freedom at the .005 level is 10.6. Not surprisingly, given the relatively restrictive nature of (14), the null hypothesis that state-level supply functions belong to the general class which can be restricted in a fashion consistent with using the commonly computed Laspeyres price index reported in *Agricultural Prices* is rejected.

Although our evidence now indicates that (14) cannot be maintained in favor of (15) and by implication (13) does not pertain to these two states, we continue to illustrate our procedure by conducting a test for aggregation across states *given that (14) is the presumed state-level technology*. (This should not be interpreted as a sequential test.) Our aggregation test is then a test of whether the β term in the denominator of each of these equations is common across states. Reestimating specification (14) again by systems, nonlinear least squares subject to this restriction gives:

Oklahoma

$$y = \underset{(10.660259)}{10.394166} \;/\; \underset{(.191352)}{(2.529964 - p)}$$

South Dakota

$$y = \underset{(3.8[illegible]2994)}{.627282} \;/\; \underset{(.191352)}{(2.529964 - p)}$$

In this instance, the null hypothesis of aggregability involves one parametric restriction. The computed log-likelihood ratio statistic is .1118 while the tabulated χ^2 with one degree of freedom at the .1 level is 2.71. We cannot reject the null hypothesis that given that (14) Oklahoma and South Dakota wheat supply response equations are aggregable.

An Inherent Inconsistency in Current Practice?

Monthly commodity prices updated by *Agricultural Prices* are ". . . estimates of average prices." To be used with a state-level quantity aggregate measuring the sum of firm supplies, this choice of a price index implies by Table 1 that both the firm and the state-level supply must be affine in price, i.e.,

$$\begin{aligned} y_{ij}(p_i, w) &= \alpha_{ij}(w) + \beta_j(w)p_i \\ Y_j &= \alpha_j(w) + \beta(w)m\bar{P}_j \end{aligned} \qquad (16)$$

where the i subscript now stands for individuals and the j subscript stands for the state. US season average prices, however, are calculated as a weighted average of the state season average prices. Thus, the national and state supply relations must assume the forms, respectively,

$$Y = \frac{\delta(w)}{\psi(w) - \hat{P}}$$

$$(17) \qquad Y = \frac{\gamma_j(w)}{\psi(w) - P_j}$$

Thus, if $\bar{P}_j$ is used in the calculation of the state season average prices the implied state-level supply relations in (16) and (17) are mutually inconsistent.

Conclusion

Aggregation of prices is commonplace in empirical work on agricultural markets. The most common aggregations are averages and weighted averages of the Laspeyres type. Since researchers either do not have access to the microdata or rely exclusively on aggregate data, the practical implications of using these aggregates is an important area of inquiry.

We have shown that each of these rules imposes a definite structure on supplies, demands, or profits, if microquantities are to add up or more generally conform to a strongly separable aggregation rule. These forms are limiting but are viable for research. Any other form of the supply or demand equations will not add up. Forms routinely used in applied agricultural research are internally inconsistent. If standard practice is pursued, such as estimating a Cobb-Douglas equation, one cannot conclude whether an over- or underestimate of the elasticity occurs.

The question of the importance of our approach for price aggregation can only be determined empirically and rather inductively. We have shown by way of an empirical example, which rejects the validity of the most common U.S. season average price formula for wheat, that it is a viable empirical approach which should be considered by agricultural economists in their research.

We conclude by noting that this line of research my well suggest that the two most reported indices, the mean and the weighted mean price, be abandoned. That is, there may be an inherent contradiction between observed behavior and these indices. The search for appropriate aggregation rules should logically start with an identification of appropriate versions of the micro technologies.

ENDNOTE

[1]All studies mentioned in this paper were drawn from a group of over 100 papers using similar aggregation procedures solely for the sake of having concrete illustrations to which readers could refer. We want to emphasize that this is a practice endemic in applied studies and not specific to those cited. Thus, our comments should be construed as a criticism of this general practice (to which the authors themselves plead guilty) and not as a specific criticism of the cited papers.

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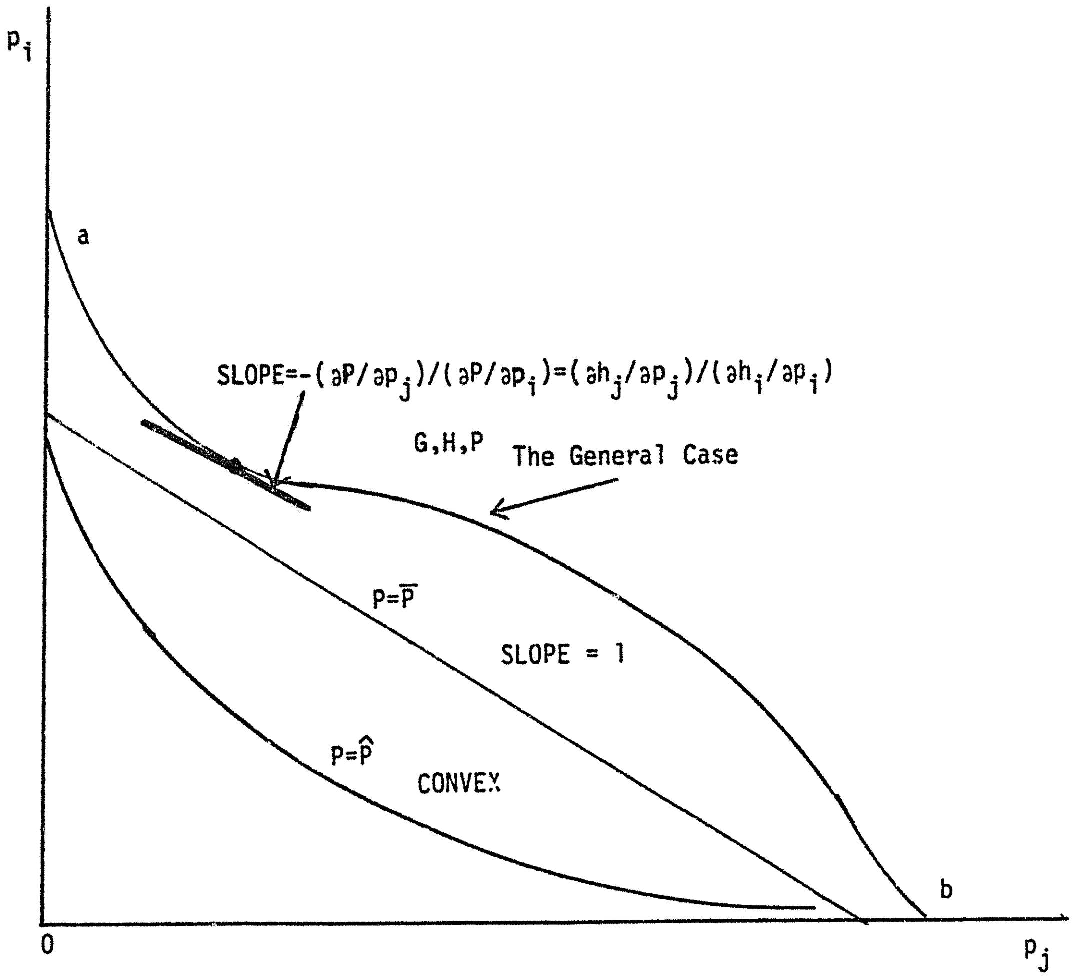
FIGURE 1

LEVEL CURVES AND AGGREGATION



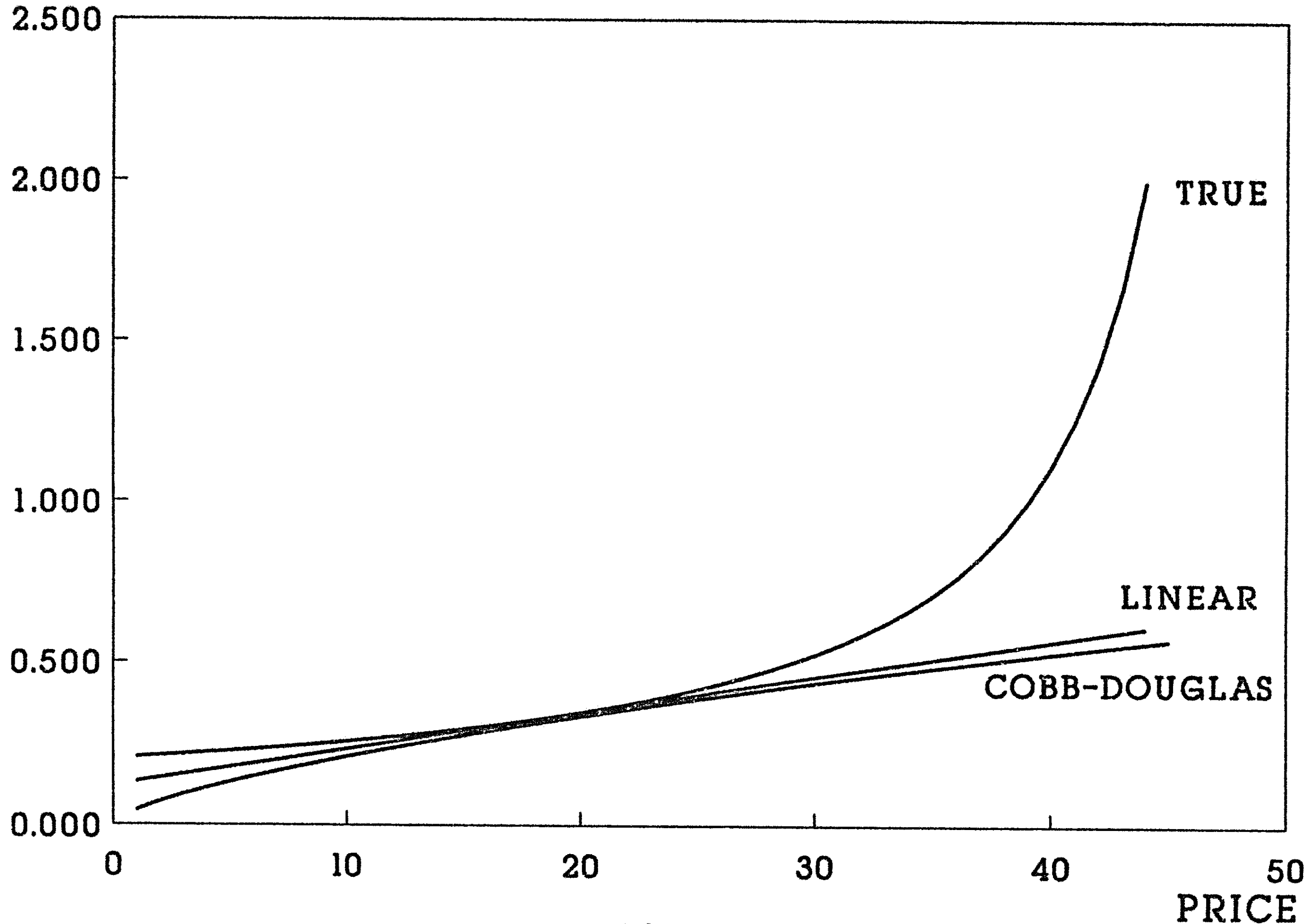

2.500
2.000
1.500
1.000
0.500
0.000
0
10
20
30
40
50
PRICE
TRUE
LINEAR
COBB-DOUGLAS

FIGURE 2.