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Estimation of Marginal Risks with Seemingly Unrelated Regression and Panel Data*

G. H. Wan, W.E. Griffiths and J. R. Anderson[†]

In this paper, seemingly unrelated regressions (SUR) which consist of production functions with composed errors for section and time and heteroscedastic disturbances are proposed. These functions are distinguished from others in that they allow the risks (indicated by variances) of outputs change in any direction in response to input changes. The SUR are then applied in the analysis of cross-section time-series data for rice, wheat and maize production in China.

1 Introduction

It is reasonable to propose that changes in some inputs, e.g., investment in improving environmental conditions, are inversely or negatively related to changes in risks of crop outputs. However, a positive relationship may exist between other inputs, e.g., areas sown with modern cultivars (Anderson, Findlay and Wan 1989), and output variabilities of agricultural crops. Just and Pope (1978) showed that these relationships cannot be correctly taken into account by the commonly-used functions, no matter whether the function is of additive error or multiplicative error and no matter whether the function is linear or nonlinear. For example, the widely-used Cobb-Douglas, transcendental and CES functions, restrict the marginal product and marginal variance to be of the same sign, normally positive. Other restrictions of these functions are detailed by Just and Pope (1978).

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[†]Faculty of Economics Studies, University of New England, Armidale, NSW 2351.

To relax these restrictions, models with heteroscedastic disturbances such as (1) to (3) are proposed:

$$Y = f(X) + h(X)\epsilon, \quad (1)$$

$$Y = f(X) + h(X, \epsilon), \quad (2)$$

$$Y = f(X, \epsilon), \quad (3)$$

where Y and X are dependent and independent variables, respectively. ϵ is usually a vector of random disturbance, and h , f represent functional forms. Since equations (2) and (3) are rather too general to discuss insightfully their estimation, Just and Pope (1978) focus on equation (1) and suggest a four-step procedure for estimating (1), where both f and h are assumed to be log-linear in parameters. Using the Cobb-Douglas function as f and h , Griffiths and Anderson (1982) considered an error component version of equation (1) and developed corresponding estimation techniques.

This paper presents an extension of the model considered by Griffiths and Anderson (1982) into seemingly unrelated regressions (SUR). The SUR specification is made in section 2, followed by discussion on an econometric estimation procedure in section 3. Some empirical results based on Chinese data are provided in section 4 to illustrate the possible superiority of the extended model over more conventional ones. The paper is concluded with a summary in section 5.

2 The model

If there are N cross-sectional entities over T time periods, a set of M nonlinear stochastic equations of the form

$$Y_m = \gamma_m \prod_{k=1}^K X_{mk}^{\beta_{mk}} + \epsilon_m \circ \prod_{k=1}^K X_{mk}^{\alpha_{mk}} \quad (4)$$

can be established, where $m = 1, 2, \dots, M$, Y_m is the $NT \times 1$ vector of observations on the dependent variable, ϵ_m is a $NT \times 1$ disturbance vector, X_{mk} is

the $NT \times 1$ vector of observations on the k -th explanatory variable of the m -th equation and α, β s are parameters to be estimated. The symbols, \circ and \prod , denote component multiplication of matrices.

Assume $i = 1, 2, \dots, N$ and $t = 1, 2, \dots, T$, let

$$h_{mit} = \prod_{k=1}^K X_{mkit}^{\alpha_{mk}}, \quad (5)$$

$$H_m = \text{diag}(h_{m1}, h_{m2}, \dots, h_{mNT}), \quad (6)$$

$$H = \text{diag}(H_1, H_2, \dots, H_M), \quad (7)$$

$$\epsilon_m = Z_\mu \mu_m + Z_\lambda \lambda_m + \nu_m, \quad (8)$$

where

$$Z_\mu = I_N \otimes e_T, \quad (9)$$

$$Z_\lambda = e_N \otimes I_T, \quad (10)$$

and \otimes denotes the Kronecker operation; I_N, I_T denote $N \times N$ and $T \times T$ unit matrices, e_N, e_T are $N \times 1$ and $T \times 1$ vectors of ones. The model can then be written as

$$Y_m = \gamma_m \prod_{k=1}^K X_{mk}^{\beta_{mk}} + u_m, \quad (11)$$

$$\nu_m = H_m (Z_\mu \mu_m + Z_\lambda \lambda_m + \nu_m), \quad (12)$$

where the i -th element of the vector $\mu_m = [\mu_{m1}, \mu_{m2}, \dots, \mu_{mN}]'$ and the t -th element of the vector $\lambda_m = [\lambda_{m1}, \lambda_{m2}, \dots, \lambda_{mT}]'$ represent the error components specific to the i -th entity and t -th period in the m -th equation, respectively; the $NT \times 1$ vector $\nu_m = [\nu_{m1}, \nu_{m2}, \dots, \nu_{mNT}]'$ contains the error component which is random over time and space for the m -th equation. Further, define

$$Y = \begin{pmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_M \end{pmatrix}, \quad u = \begin{pmatrix} u_1 \\ u_2 \\ \vdots \\ u_M \end{pmatrix}, \quad \gamma = \begin{pmatrix} \gamma_1 \\ \gamma_2 \\ \vdots \\ \gamma_M \end{pmatrix}$$

and

$$X_c = \text{diag} \left(\prod_{k=1}^K X_{1k}^{\beta_{1k}}, \prod_{k=1}^K X_{2k}^{\beta_{2k}}, \dots, \prod_{k=1}^K X_{Mk}^{\beta_{Mk}} \right),$$

the SUR models can be written as

$$Y = X_c \gamma + u \quad (13)$$

Following Avery (1977) and Baltagi (1980), the three components of u (i.e., μ , λ and ν) are, as seems reasonable, assumed to be stochastically independent from each other and

$$E(\mu_{mi}) = E(\lambda_{mt}) = E(\nu_{mit}) = 0.$$

Under these assumptions, it can be shown that

$$\begin{aligned} E(\mu_{mi} \mu_{lj}) &= \sigma_{\mu ml} \quad i = j, \\ &= 0 \quad i \neq j; \end{aligned} \quad (14)$$

$$\begin{aligned} E(\lambda_{mt} \lambda_{ls}) &= \sigma_{\lambda ml} \quad t = s, \\ &= 0 \quad t \neq s; \end{aligned} \quad (15)$$

$$\begin{aligned} E(\nu_{mit} \nu_{ljs}) &= \sigma_{\nu ml} \quad i = j \text{ \& } t = s, \\ &= 0 \quad i \neq j \text{ or } t \neq s, \end{aligned} \quad (16)$$

or in matrix notation,

$$E \begin{pmatrix} \mu_m \\ \lambda_m \\ \nu_m \end{pmatrix} (\mu'_l \lambda'_l \nu'_l) = \begin{bmatrix} \sigma_{\mu ml} I_N & 0 & 0 \\ 0 & \sigma_{\lambda ml} I_T & 0 \\ 0 & 0 & \sigma_{\nu ml} I_{NT} \end{bmatrix}, \quad (17)$$

where i, j are entity subscripts, m, l equation subscripts and t, s time subscripts. for m and $l = 1, 2, \dots, M$.

By defining

$$\epsilon = \begin{pmatrix} \epsilon_1 \\ \epsilon_2 \\ \vdots \\ \epsilon_M \end{pmatrix}, \quad (18)$$

the covariance matrix for (13) can be expressed as

$$\begin{aligned}\Phi &= E(uu') = HE(\epsilon\epsilon')H \\ &= H\Omega H,\end{aligned}\quad (19)$$

where

$$\Omega = \begin{bmatrix} \Omega_{11} & \Omega_{12} & \cdots & \Omega_{1M} \\ \vdots & \vdots & \ddots & \vdots \\ \Omega_{M1} & \Omega_{M2} & \cdots & \Omega_{MM} \end{bmatrix}.\quad (20)$$

The typical element of Ω denoted by Ω_{ml} has the form

$$\begin{aligned}\Omega_{ml} &= E(\epsilon_m \epsilon_l') \\ &= \sigma_{\mu ml} A + \sigma_{\lambda ml} B + \sigma_{\nu ml} I_{NT},\end{aligned}\quad (21)$$

where $A = I_N \otimes e_T e_T'$ and $B = e_N e_N' \otimes I_T$. Let

$$Q = I_{NT} - \frac{A}{T} - \frac{B}{N} + \frac{J_{NT}}{NT},\quad (22)$$

$$J_{NT} = e_{NT} e_{NT}',\quad (23)$$

then equation (21) can be alternatively put as (Baltagi 1980)

$$\begin{aligned}\Omega_{ml} &= \sigma_{3ml} \frac{J_{NT}}{NT} + \sigma_{1ml} \left(\frac{A}{T} - \frac{J_{NT}}{NT} \right) \\ &+ \sigma_{2ml} \left(\frac{B}{N} - \frac{J_{NT}}{NT} \right) + \sigma_{\nu ml} Q,\end{aligned}\quad (24)$$

where

$$\sigma_{1ml} = \sigma_{\nu ml} + T\sigma_{\mu ml},\quad (25)$$

$$\sigma_{2ml} = \sigma_{\nu ml} + N\sigma_{\lambda ml},\quad (26)$$

$$\sigma_{3ml} = \sigma_{\nu ml} + N\sigma_{\lambda ml} + T\sigma_{\mu ml},\quad (27)$$

and $\sigma_{\nu m}$ are the distinct characteristic roots of Ω_{ml} of multiplicity 1, $N-1$, $T-1$ and $(N-1)(T-1)$, respectively. These eigenvalues of Ω_{ml} can be computed according to Verlove (1971), if necessary.

After obtaining the values of σ_{1ml} , σ_{2ml} , σ_{3ml} and $\sigma_{\nu ml}$ by equations (25) to (27) for $m, l = 1, 2, \dots, M$, Baltagi (1980) shows that

$$\begin{aligned}\Omega &= \Omega_3 \otimes \frac{J_{NT}}{NT} + \Omega_1 \otimes \left(\frac{A}{T} - \frac{J_{NT}}{NT} \right) \\ &+ \Omega_2 \otimes \left(\frac{B}{N} - \frac{J_{NT}}{NT} \right) + \Omega_\nu \otimes Q,\end{aligned}\quad (28)$$

where

$$\Omega_3 = [\sigma_{3ml}], \quad (29)$$

$$\Omega_2 = [\sigma_{2ml}], \quad (30)$$

$$\Omega_1 = [\sigma_{1ml}], \quad (31)$$

$$\Omega_\nu = [\sigma_{\nu ml}], \quad (32)$$

all of dimension $M \times M$. As shown later, this expression will be useful for computing Ω^{-1} .

Under the above model specification, β_s represent production elasticities and α_s "risk elasticities" or risk effects of inputs, where risk is defined as the variance of Y . Since α_s can be of any sign, the proposed SUR are distinguished from more conventional ones in that they allow risks of output to change in any direction in response to input changes. Also, the three error components in the model are all heteroscedastic in the sense that variances of $H_m Z_\mu \mu_m$, $H_m Z_\lambda \lambda_m$ and $H_m \nu_m$ depend on the input levels. This implies that the magnitude of both entity and time effects will be influenced by the measured input levels, which may be more realistic than otherwise.

3 The multi-stage estimation procedure

Given the covariance matrix of (13) in (19), it can be seen that to estimate $\gamma = (\gamma_1, \gamma_2, \dots, \gamma_M)'$ and $\beta = (\beta_1, \beta_2, \dots, \beta_M)'$, where $\beta_m = (\beta_{m1}, \beta_{m2}, \dots, \beta_{mK})'$, the objective function to be minimise is

$$\Psi = u' \Phi^{-1} u$$

$$\begin{aligned}
&= u'H^{-1}\Omega^{-1}H^{-1}u \\
&= \hat{u}'\Omega^{-1}\hat{u},
\end{aligned} \tag{33}$$

where $\hat{u}' = u'H^{-1}$.

However, H cannot be computed without the estimates of $\alpha = (\alpha_1, \alpha_2, \dots, \alpha_M)'$, which, in turn, requires the estimation of u . To proceed in this direction, the first step is to minimise

$$u'u = \sum_{m=1}^M \sum_{i=1}^N \sum_{t=1}^T \left(Y_{mit} - \gamma_m \prod_k X_{mkit}^{\beta_{mk}} \right)^2 \tag{34}$$

and obtain $\hat{\gamma}$ and $\hat{\beta}$. Since cross-equation error is not considered here, the estimation can be undertaken for each m separately. Due to the existence of heteroscedasticity and cross-equation error, the estimates will be asymptotically inefficient. But they are generally consistent. Therefore, the estimated residual $\hat{u}_{mit} = Y_{mit} - \hat{\gamma}_m \prod_k X_{mkit}^{\hat{\beta}_{mk}}$ will converge in distribution to u_{mit} under appropriate assumptions.

The second step is to estimate α . To do so, rewrite equation (12) in a slightly different form as

$$u_{mit} = h_{mit} (\mu_{mi} + \lambda_{mi} + \nu_{mit}). \tag{35}$$

Squaring the above equation and taking logarithms yields

$$\ln u_{mit}^2 = \ln(\mu_{mi} + \lambda_{mi} + \nu_{mit})^2 + 2 \sum_{k=1}^K \alpha_{mk} \ln X_{mkit}. \tag{36}$$

Let

$$\alpha_{m0} = E[\ln(\mu_{mi} + \lambda_{mi} + \nu_{mit})^2], \tag{37}$$

$$\xi_{mit} = \ln u_{mit}^2 - E[\ln u_{mit}^2], \tag{38}$$

then

$$E(\ln u_{mit}^2) = \alpha_{m0} + 2 \sum_{k=1}^K \alpha_{mk} \ln X_{mkit}, \tag{39}$$

$$\xi_{mit} = \ln(\mu_{mi} + \lambda_{mi} + \nu_{mit})^2 - \alpha_{m0}. \tag{40}$$

Thus

$$\ln(\mu_{mt} + \lambda_{mt} + \nu_{mt})^2 = \alpha_{m0} + \xi_{mt} \quad (41)$$

and equation (36) reduces to

$$\ln u_{mt}^2 = \alpha_{m0} + 2 \sum_{k=1}^K \alpha_{mk} \ln X_{mkt} + \xi_{mt}. \quad (42)$$

Combining the set of M equations,

$$\dot{Y} = \dot{X}\alpha + \xi \quad (43)$$

is obtained, where $\alpha = (\alpha_1, \alpha_2, \dots, \alpha_M)'$, $\alpha_m = (\alpha_{m0}, 2\alpha_{m1}, \dots, 2\alpha_{mK})'$, $\xi = (\xi_1, \xi_2, \dots, \xi_{MNT})'$, $\dot{X} = \text{diag}(\dot{X}_1, \dot{X}_2, \dots, \dot{X}_M)$, \dot{X}_m is a $NT \times (K+1)$ matrix with 1.0s in the first column and $\ln X_{mk}$ in the other columns. \dot{Y} is defined similarly to \dot{X} with $\dot{Y}_m = (\ln u_{m1}^2, \ln u_{m2}^2, \dots, \ln u_{mNT}^2)'$.

When u_{mt} is replaced by its consistent estimator \hat{u}_{mt} , equation (42) can be used for estimation of α_m . However, properties of ξ_{mt} have to be investigated in order to discover the properties of the estimates and to employ an appropriate estimation technique.

If μ_{mt} , λ_{mt} and ν_{mt} are assumed to be normally distributed, the random variables defined as

$$q_{mt} = (\mu_{mt} + \lambda_{mt} + \nu_{mt}) / \sigma_m, \quad (44)$$

$$q_{it} = (\mu_{it} + \lambda_{it} + \nu_{it}) / \sigma_i, \quad (45)$$

where

$$\begin{aligned} \sigma_m &= \sqrt{\sigma_{\mu mm} + \sigma_{\lambda mm} + \sigma_{\nu mm}} \\ &= \sqrt{\sigma_{mm}} \end{aligned}$$

become standard normal variables with zero means and unit variance. Moreover, q_{mt}^2 , $m = 1, 2, \dots, M$ are each χ^2 random variables with one degree of freedom.

Taking the logarithm of the square of equation (44) produces

$$\begin{aligned} \ln q_{mt}^2 &= \ln(\mu_{mt} + \lambda_{mt} + \nu_{mt})^2 - \ln \sigma_m^2 \\ &= \alpha_{m0} + \xi_{mt} - \ln \sigma_m^2. \end{aligned} \quad (46)$$

where the second equality is obtained by use of equation (41). This variable is thus distributed as the logarithm of a χ^2 distribution with one degree of freedom. Since both α_{m0} and $\ln \sigma_m^2$ are constant and ξ_{mit} is defined by equation (38), it can be shown (Harvey 1976) that

$$\text{Var}(\ln q_{mit}^2) = \text{Var}(\xi_{mit}) = 4.9348, \quad (47)$$

$$\begin{aligned} E(\ln q_{mit}^2) &= -1.2704 \\ &= \alpha_{m0} - \ln \sigma_m^2. \end{aligned} \quad (48)$$

According to equations (38) and (47), ξ_{mit} has zero mean and a constant variance. Therefore, α_m can be estimated by applying OLS to equation (42) for $m = 1, 2, \dots, M$ separately and this produces no bias or inconsistency. But, it does result in inefficiency since the M sets of equations are related and each of them has a composite error structure similar to that of (17) as shown below.

When $i = j$ and/or $t = s$, q_{mit} and q_{ijs} will be correlated. This implies that $\ln q_{mit}^2$ and $\ln q_{ijs}^2$ will be also correlated when $i = j$ and/or $t = s$. It can be shown that

$$E[\ln q_{mit}^2 \ln q_{ijs}^2] = E(\xi_{mit} \xi_{ijs}) + 1.2704^2, \quad (49)$$

i.e.,

$$E(\xi_{mit} \xi_{ijs}) = E[\ln q_{mit}^2 \ln q_{ijs}^2] - 1.2704^2. \quad (50)$$

Since

$$\begin{aligned} E(q_{mit} q_{ijs}) &= \frac{\sigma_m \sigma_i}{\sigma_m \sigma_i}, \quad i = j \\ &= 0, \quad i \neq j, \end{aligned} \quad (51)$$

$$\begin{aligned} E(q_{mit} q_{ijs}) &= \frac{\sigma_m \sigma_i}{\sigma_m \sigma_i}, \quad t = s \\ &= 0, \quad t \neq s, \end{aligned} \quad (52)$$

$$\begin{aligned} E(q_{mit} q_{ijs}) &= \frac{\sigma_m \sigma_i}{\sigma_m \sigma_i}, \quad t = s \text{ \& } i = j \\ &= 0, \quad t \neq s \text{ or } i \neq j, \end{aligned} \quad (53)$$

where $\sigma_{mit} = \sigma_{\mu mit} + \sigma_{\lambda mit} + \sigma_{\nu mit}$, the following can be derived (Griffiths and Anderson 1982, Johanson and Kots 1972):

$$\begin{aligned}\delta_{\mu mit} &= E[\xi_{mit}\xi_{lit}] \\ &= \sum_{h=1}^{\infty} \left(\frac{\sigma_{\mu mit}}{\sigma_m \sigma_l} \right)^{2h} \frac{h! \Gamma(\frac{1}{2})}{h^2 \Gamma(h + \frac{1}{2})}\end{aligned}\quad (54)$$

$$\begin{aligned}\delta_{\lambda mit} &= E[\xi_{mit}\xi_{lit}] \\ &= \sum_{h=1}^{\infty} \left(\frac{\sigma_{\lambda mit}}{\sigma_m \sigma_l} \right)^{2h} \frac{h! \Gamma(\frac{1}{2})}{h^2 \Gamma(h + \frac{1}{2})}\end{aligned}\quad (55)$$

$$\begin{aligned}\delta_{\nu mit} &= E[\xi_{mit}\xi_{lit}] \\ &= \sum_{h=1}^{\infty} \left(\frac{\sigma_{\nu mit}}{\sigma_m \sigma_l} \right)^{2h} \frac{h! \Gamma(\frac{1}{2})}{h^2 \Gamma(h + \frac{1}{2})}\end{aligned}\quad (56)$$

$$\delta_{\nu it} = \delta_{\mu mit} - \delta_{\lambda mit} \quad (57)$$

$$0 \quad \text{if } i \neq s \text{ or } i \neq j.$$

Thus, ξ_{mit} can be viewed as having an error components structure similar to that of ϵ_{mit} and (42) can be estimated by the technique known as least squares with dummy variables (LSDV) (Maddala 1971). If μ_m^* , λ_m^* and ν_m^* are used to denote vectors containing these components, then

$$E \begin{pmatrix} \mu_m^* \\ \lambda_m^* \\ \nu_m^* \end{pmatrix} (\mu_i^{*'} \lambda_i^{*'} \nu_i^{*'}) = \begin{bmatrix} \delta_{\mu mit} I_N & 0 & 0 \\ 0 & \delta_{\lambda mit} I_T & 0 \\ 0 & 0 & \delta_{\nu mit} I_{NT} \end{bmatrix}. \quad (58)$$

Because the system of equations represented by (43) is of composite error structure, α can be more efficiently estimated by modifying the procedure and formulae in Baltagi (1980). This eventually produces generalised least squares (GLS) estimates of α , namely $\hat{\alpha}$, where

$$\begin{aligned}\hat{\alpha} &= \left[X' \left(\Lambda_1^{-1} \otimes \left(\frac{A}{T} - \frac{J_{NT}}{NT} \right) \right) X + X' \left(\Lambda_2^{-1} \otimes \left(\frac{B}{N} - \frac{J_{NT}}{NT} \right) \right) X \right. \\ &\quad \left. + X' \left(\Lambda_3^{-1} \otimes \frac{J_{NT}}{NT} \right) X + X' (\Lambda_4^{-1} \otimes Q) X \right]^{-1} \\ &\quad \times \left[X' \left(\Lambda_1^{-1} \otimes \left(\frac{A}{T} - \frac{J_{NT}}{NT} \right) \right) Y + X' \left(\Lambda_2^{-1} \otimes \left(\frac{B}{N} - \frac{J_{NT}}{NT} \right) \right) Y \right.\end{aligned}$$

$$+ \dot{X}' \left(\Lambda_s^{-1} \otimes \frac{J_{NT}}{NT} \right) \dot{Y} + \dot{X}' (\Lambda_v^{-1} \otimes Q) \dot{Y} \quad (59)$$

The Λ_s in the above expression are similar to Ω_s defined by (25) to (32) with σ_s replaced by \dot{s}_s . The Λ_s can be estimated after calculating \dot{s}_s according to (54) to (57). However, such a calculation requires the estimation of the σ_s , as is discussed in (68) to (72). Alternatively, one can obtain the best unbiased estimates of Λ_s directly (Baltagi 1980) by

$$\hat{\Lambda}_v = \frac{1}{(N-1)(T-1)} \zeta' Q \zeta, \quad (60)$$

$$\hat{\Lambda}_1 = \frac{1}{(N-1)} \zeta' \left[\frac{A}{T} - \frac{J_{NT}}{NT} \right] \zeta, \quad (61)$$

$$\hat{\Lambda}_2 = \frac{1}{(T-1)} \zeta' \left[\frac{B}{N} - \frac{J_{NT}}{NT} \right] \zeta, \quad (62)$$

$$\hat{\Lambda}_3 = \hat{\Lambda}_1 + \hat{\Lambda}_2 - \hat{\Lambda}_v, \quad (63)$$

where $\zeta = (\zeta_1, \zeta_2, \dots, \zeta_M)$ is the $NT \times M$ matrix of residuals, which can be obtained in two ways: (a) applying OLS to equation (42) for each m separately and calculating the corresponding residuals; or (b) performing LSDV on (42) for each m separately and computing the corresponding residuals (Amemiya 1971). It is noted that both sets of residuals can be used to replace ζ for estimating the Λ_s and the resulting α has the same asymptotic efficiency in each case. However, Λ_s estimated from the LSDV residuals are asymptotically more efficient than those from OLS residuals (Prucha 1984). Thus, LSDV is used in this study to obtain ζ .

Referring to both Baltagi (1980) and Prucha (1984), it can be shown that

$$\begin{aligned} \hat{\Lambda}_\mu &= \left[\hat{\delta}_{\mu mt} \right] = \frac{1}{T} \left(\hat{\Lambda}_1 - \hat{\Lambda}_v \right), \\ \hat{\Lambda}_\lambda &= \left[\hat{\delta}_{\lambda mt} \right] = \frac{1}{N} \left(\hat{\Lambda}_2 - \hat{\Lambda}_v \right), \\ \hat{\Lambda}_\nu &= \left[\hat{\delta}_{\nu mt} \right]. \end{aligned}$$

Once $\hat{\alpha}$ is obtained, ξ_{mst} can be estimated as

$$\hat{\xi}_{mst} = \ln \hat{u}_{mst}^2 - \hat{\alpha}_{m0} - 2 \sum_{k=1}^K \hat{\alpha}_{mk} \ln X_{mkt}. \quad (64)$$

It is now possible to find efficient estimates of β , which correct for heteroscedasticity, error components and correlation across equations. This is the task of the third step.

According to equations (46) and (48),

$$\begin{aligned}\ln \hat{q}_{mit}^2 &= \hat{\alpha}_{m0} + \hat{\xi}_{mit} - \ln \hat{\sigma}_m^2 \\ &= \hat{\xi}_{mit} - 1.2704,\end{aligned}$$

i.e.,

$$\hat{q}_{mit} = \sqrt{\exp(\hat{\xi}_{mit} - 1.2704)}. \quad (65)$$

It can be shown that

$$\begin{aligned}E(q_{mit} q_{lit}) &= \frac{\sigma_{\mu ml} + \sigma_{\lambda ml} + \sigma_{\nu ml}}{\sigma_m \sigma_l} \\ &= \frac{\sigma_{ml}}{\sigma_m \sigma_l} \\ &= \rho_{ml},\end{aligned} \quad (66)$$

where $\sigma_{ml} = \sigma_{\mu ml} + \sigma_{\lambda ml} + \sigma_{\nu ml}$ and ρ_{ml} is the correlation coefficient between ϵ_m and ϵ_l , which can be estimated by

$$\hat{\rho}_{ml} = \frac{1}{NT} \sum_{i=1}^N \sum_{t=1}^T \hat{q}_{mit} \hat{q}_{lit}. \quad (67)$$

From equation (48),

$$\hat{\sigma}_m^2 = \exp(\hat{\alpha}_{m0} + 1.2704). \quad (68)$$

These give

$$\hat{\sigma}_{ml} = \hat{\rho}_{ml} \hat{\sigma}_m \hat{\sigma}_l. \quad (69)$$

Now, $\sigma_{\mu ml}$, $\sigma_{\lambda ml}$ and $\sigma_{\nu ml}$ can be estimated by

$$\hat{\sigma}_{\mu ml} = \frac{2}{NT(T-1)} \sum_{i=1}^N \sum_{s=1}^{T-1} \sum_{t=s+1}^T \frac{\hat{u}_{mit} \hat{u}_{lis}}{\hat{h}_{mit} \hat{h}_{lis}}, \quad (70)$$

$$\hat{\sigma}_{\lambda ml} = \frac{2}{NT(N-1)} \sum_{t=1}^T \sum_{i=1}^{N-1} \sum_{j=i+1}^N \frac{\hat{u}_{mit} \hat{u}_{ljt}}{\hat{h}_{mit} \hat{h}_{ljt}}, \quad (71)$$

$$\hat{\sigma}_{\nu ml} = \hat{\sigma}_{ml} - \hat{\sigma}_{\mu ml} - \hat{\sigma}_{\lambda ml}, \quad (72)$$

where

$$\tilde{h}_{mit} = \prod_{k=1}^K \lambda_{mkit}^{\hat{\alpha}_{mk}}. \quad (73)$$

If these computations are being made with a view towards using (54) to (57), then the estimate for $\hat{\alpha}_{mk}$ would be from OLS or LSDV, rather than GLS, because (54) to (57) are required before GLS estimation is employed. Substituting $\hat{\sigma}_{vml}$, $\hat{\sigma}_{\mu ml}$, and $\hat{\sigma}_{\lambda ml}$ into equations (25) to (27) enables the computation of $\hat{\Omega}$ via (28) to (32). According to equation (19),

$$\hat{\Phi} = \hat{H} \hat{\Omega} \hat{H}', \quad (74)$$

where \hat{H} can be obtained through equations (5) to (7) with h_{mit} replaced by \tilde{h}_{mit} . Thus, to obtain efficient estimates of β , represented by $\hat{\beta}$, it is a matter of minimising

$$\begin{aligned} \hat{\Psi} &= u' \hat{\Phi}^{-1} u \\ &= u' \hat{H}^{-1} \hat{\Omega}^{-1} \hat{H}^{-1} u \\ &= \hat{u}' \hat{\Omega}^{-1} \hat{u}, \end{aligned} \quad (75)$$

where $\hat{u} = \hat{H}^{-1} u$.

For the purpose of programming, it is necessary to find a transformation of the error term, say $p\hat{u}$, such that $\hat{u}' p' p \hat{u} = \hat{u}' \hat{\Omega}^{-1} \hat{u}$. When $\hat{\Omega}$ is of small dimension, one of the methods is to find c and Λ such that $p = \Lambda^{-\frac{1}{2}} c'$, where c is an orthogonal matrix consisting of the characteristic vectors of $\hat{\Omega}$ and Λ is a diagonal matrix consisting of eigenvalues of $\hat{\Omega}$. However, $\hat{\Omega}$ is of order of $(MNT \times MNT)$, which could well be exceeding dimension 200. In this case, solving $\hat{\Omega}$ for c and Λ requires solving a polynomial equation of degree of over 200. This is a difficult task and unreliable results may incur. To tackle this problem, a two step procedure is developed: (a) decomposing $\hat{\Omega}^{-1}$ according to the suggestion of Baltagi (1980), which gives

$$\hat{\Omega}^{-1} = \hat{\Omega}_3^{-1} \otimes \frac{J_{NT}}{NT} + \hat{\Omega}_1^{-1} \otimes \left(\frac{A}{T} - \frac{J_{NT}}{NT} \right)$$

$$+ \hat{\Omega}_2^{-1} \otimes \left(\frac{B}{N} - \frac{J_{NT}}{NT} \right) + \hat{\Omega}_v^{-1} \otimes Q, \quad (76)$$

where $\hat{\Omega}_1$, $\hat{\Omega}_2$, $\hat{\Omega}_3$ and $\hat{\Omega}_v$ can be calculated according to equations (25) to (27) and (29) to (32) with σ s replaced by their estimated counterparts. It is noted that these matrices only have dimensions of $M \times M$. (b) let $\Omega_v^{-1} = P_4 P_4'$ and defining

$$\hat{\Omega}_i^{-1} = P_i P_i' \quad (77)$$

for $i = 1, 2, 3, 4$. Further defining

$$D_1 = \frac{A}{T} - \frac{J_{NT}}{NT}, \quad (78)$$

$$D_2 = \frac{B}{N} - \frac{J_{NT}}{NT}, \quad (79)$$

$$D_3 = \frac{J_{NT}}{NT}, \quad (80)$$

$$D_4 = Q. \quad (81)$$

Then, since the D_i s are all idempotent and $D_i D_j = 0$ for $i \neq j$, equation (76) can be written as

$$\begin{aligned} \hat{\Omega}^{-1} &= \sum_{i=1}^4 (P_i P_i' \otimes D_i D_i) \\ &= \left(\sum_{i=1}^4 P_i \otimes D_i \right) \left(\sum_{i=1}^4 P_i' \otimes D_i \right). \end{aligned} \quad (82)$$

Therefore, an equivalent operation of minimising $\hat{\Psi}$ is to minimise $\tilde{u}'\tilde{u}$, where

$$\tilde{u} = \left(\sum_{i=1}^4 P_i' \otimes D_i \right) u. \quad (83)$$

To summarise, the estimation of seemingly unrelated regression models, which carry risk implications and incorporate composite errors, normally takes the following steps:

(1) Find $\tilde{\beta}$ and $\tilde{\gamma}$ by using nonlinear least squares either to minimise $u_m' u_m$ for $m = 1, 2, \dots, M$, respectively, or to minimise $u'u$; denote the corresponding residuals by \tilde{u} .

(2) Obtain $\hat{\alpha}$ by applying the GLS technique on the SUR models with error components, where $\ln \hat{u}_{mit}^2$ is regressed linearly on the $\ln X_{mit}$ s; denote the corresponding residuals by ξ .

(3) Use $\hat{\xi}$ to estimate q via (65) and then ρ_{ml}, σ_{mm} via (67), (68). This enables the estimation of σ_{ml} via (69).

(4) Use $\hat{\sigma}_{ml}$ and $\hat{\alpha}$ to find \hat{h}_{mit} from (73) and subsequently $\hat{\sigma}_{\mu ml}, \hat{\sigma}_{\lambda ml}$ and $\hat{\sigma}_{\nu ml}$ from (70) to (72). Meanwhile, H can be estimated via (6) and (7).

(5) Construct $\hat{\Omega}_1, \hat{\Omega}_2, \hat{\Omega}_3$ and $\hat{\Omega}_\nu$ by replacing σ s in (25) to (27) and (32) by their estimated counterparts computed in step (4).

(6) Find P_i of $\hat{\Omega}_i$ for $i = 1, 2, 3, 4$ and then obtain $\hat{\Omega}^{-1}$ from (76).

(7) Use \hat{H} from step (4) and $\hat{\Omega}^{-1}$ from step (6) to find $\hat{\gamma}$ and $\hat{\beta}$ by employing nonlinear least squares to minimise $u' \hat{H}^{-1} \hat{\Omega}^{-1} \hat{H}^{-1} u$.

4 Empirical application

Chinese survey data for 28 regions (i.e., entities) for a 4-year period from 1980 to 1983 are utilised to estimate the disturbance-related production functions, as proposed in preceding sections. The data, covering three crops (rice, wheat and maize), comprise output (*jin*), sown-area (*mu*), organic fertiliser (*yuan*), chemical fertiliser (*yuan*), machinery cost (*yuan*), irrigation cost (*yuan*), labour input (persondays) and other costs (*yuan*). Those variables in value terms are deflated by a weighted index of agricultural prices in state and free markets.

The Marquardt-Levenberg-Nash approach is adopted here to find the non-linear least squares estimates of β s (Marquardt 1963, Nash and Walker-Smith 1987). The estimates for the mean output function and the output variance function are, respectively, presented in Tables 1 and 2. These results are obtained from several different sets of starting values.

In Table 1, estimated coefficients of the SUR heteroscedastic models are reported in the third column. For comparison only, results from assuming $u_{it} =$

ϵ_{ij} are also presented.

From Table 1(a), it is seen that, among the eight variables included in the model, four of them have coefficients with negative signs. That is, rice production elasticities with respect to labour, chemical fertiliser, animal cost and machinery cost are less than zero. Since rice is mainly planted in Southern China, where substantial underemployment or over-supply of labour exists in the rural areas, it may be possible that negative returns with respect to labour starts occurring, particularly after the resumption of double cropping (after triple cropping) since the late 1970s. The negative elasticity with respect to chemical fertiliser is consistent with the findings of Wiens (1982). Large increases in the application of nitrogen without corresponding increase in potassium and phosphorus might be one of the most important reasons for the negative elasticity (Stone 1986). The negative elasticity associated with machinery cost is plausible as replacement of labour by machines "destroys" the traditional labour-intensive farming technique. This is particularly true with rice production since rice requires fine soil preparation and flat land but machine operation cannot meet these requirements as well as labour does. As for the animal cost, the negative sign is implausible. However, except for labour, all the negative coefficients have 95 per cent confidence intervals which include a positive range.

Among the remaining variables, all but irrigation are significant contributors to rice output. Examination of the magnitudes of the estimates indicates that sown area change asserted the greatest positive impact on rice output, followed by organic fertiliser. The insignificance of irrigation may result from the fact that almost all the rice area sown is irrigated and thus irrigation is not a particularly limiting factor in rice production.

The estimates of the mean maize output function are tabulated in Table 1 (b). Judging by the asymptotic *t*-ratios, all the positive estimates are statistically significant at 0.05 level. On the other hand, all the three negative coefficients have 95 per cent confidence intervals which include a positive range.

Furthermore, the three negative values for labour, chemical fertiliser and animal cost are implausible. Unlike in the case of rice, machinery cost is positively related to maize production. A possible explanation is that, for maize production, machine operation is mainly involved with cultivation and planting. Thus, there is less post-harvest loss than harvesting by machines. More importantly, timing of planting is more crucial for maize production than for rice and the requirement for seedbed preparation is not as great as for rice. Maize is mainly grown in the central and north of China, where farming techniques are relatively poorer than in the south. In other words, replacement of labour by machines is likely to create a positive impact on maize output. Moreover, in the far north the excess labour problem is less severe if it exists at all. This may also help explain the positive sign of $\hat{\alpha}_g$.

Area sown is the dominant source of change of maize output. The production elasticity with respect to sown area is 0.68, followed by 0.16 with respect to organic fertiliser and 0.15 with respect to other costs. The elasticity is only 0.02 for machinery cost and 0.016 for irrigation.

The wheat production function seems to be estimated most successfully (Table 1(c)). The only negative estimate is the elasticity of irrigation. Wheat is largely planted in the far north of China, where water supply relies heavily on rainfall. It is noted that the negative value has a small *t*-statistic. Thus, the true elasticity of wheat output with respect to irrigation might be very small and its estimate could well turn out to be nonpositive.

Contrary to both rice and maize, the coefficients of labour, chemical fertiliser and animal cost are all positive, although the estimate associated with animal cost is not significant at the 5 per cent level. Chemical fertiliser has the smallest positive elasticity and organic fertiliser has the largest elasticity. The elasticity with respect to labour is not only positive, but substantial relative to that for other inputs. This comes as no surprise since wheat is predominantly planted in the far north of China where labour is relative scarce. The above-mentioned

reason could also explain the relatively large elasticity of machinery cost in wheat production.

Overall, Table 1 indicates that, where labour is relatively scarce, machinery generates a positive and significant impact on crop yield. For example, when labour input has negative returns in rice production, machinery creates a negative effect on production and the effect is significant at a 10 per cent level. In the case of maize production, labour had no significant impact and machinery generated a limited, though a significant effect on yield (the coefficient is only 0.02), whereas when the labour effect is significantly positive in wheat production, the machinery effect becomes positive, significant and substantial (the elasticity is 0.08).

The parameters determining the signs of marginal risks are presented in Table 2. Although attention will be focused on the estimates given by GLS, parameters estimated by other techniques are also shown in Table 2. The goodness of fit for the *SUR* system is calculated according to

$$R_{SUR}^2 = 1 - \frac{\hat{e}'(\Sigma^{-1} \otimes I_{NT})\hat{e}}{Y'(\Sigma^{-1} \otimes D_{NT})Y} \quad (84)$$

where Σ^{-1} is the variance covariance matrix of the *SUR* models, \hat{e} is a $MNT \times 1$ vector containing the GLS residuals, and $D_{NT} = I_{NT} - J_{NT}/NT$. The F_{SUR} statistic is obtained based on R_{SUR}^2 (Judge et al. 1985, p. 478). The R_{SUR}^2 and F_{SUR} are not reported in the tables. The result shows that R_{SUR}^2 equals 0.55 and F_{SUR} equals 16.01. Noting that data used for estimation are basically cross-sectional (the time span is relatively short), 0.55 indicates a reasonable goodness of fit. The F_{SUR} is statistically significant at any conventional level, which suggests the existence of heteroscedasticity. This may imply the inadequacy of conventional functions or the superiority of the heteroscedastic *SUR* models.

Machinery, organic fertiliser and other costs seem to have stabilising effects on rice output (Table 2(a)). The coefficient for machinery is insignificant. It is reasonable to have a negative $\hat{\alpha}_8$ since the major component of other costs is expenditure on management. The significance of both positive $\hat{\beta}_7$ and negative

$\hat{\alpha}_7$ imply the importance of organic fertiliser in achieving a high and stable yield in China's rice production. The variance of production is positively related to chemical fertiliser application, though there is a lack of statistical significance. This is in line with the expectation of Hazell (1984), who suspected that, as seed-fertiliser technology advances with the adoption of high yielding varieties, increased use of chemical fertiliser may bring about higher production variability. As far as animal cost is concerned, the significantly positive sign is implausible and thus needs further investigation. While irrigation is expected to help stabilise production, the empirical result here does not seem supportable. Noting that $\hat{\alpha}_3$ is significant and of considerable magnitude, better management of the irrigation system in China is implied to be urgently needed. This is because the positive $\hat{\alpha}_3$ and negative $\hat{\beta}_3$ could well be the result of malfunctioning of the irrigation system due to a collapsed management of water resource and irrigation facilities after the introduction of the agricultural production responsibility system in late 1978. The impact of area sown on production risks basically depends on the correlation coefficients among rice outputs of different seasons and on the management skills. In general, a positive relationship is expected. Finally, labour does not produce a significant impact on production risk. This is primarily because the labour input in China was near "saturation" long before 1980. Thus its changes may not generate any effect on either mean output or output risk.

Contrary to the case of rice production, animal cost and irrigation were estimated to be stabilising factors in maize production in China (Table 2(b)). This may be due to the relative insensitivity of maize to water supply in timing, quantity and frequency. In other words, irrigation can help to stabilise maize production and, while there are problems of irrigation in China, these may generate only very limited impact on maize yield variability. The variables other than animal cost and irrigation are all positively related to maize production variance. This is plausible for labour, area sown and chemical fertiliser for the

reasons discussed earlier. The positive signs of machinery, organic fertiliser and other costs are implausible. However, all the positive estimates have 95 per cent confidence intervals which include negative values.

The relationship estimated between wheat output variance and inputs can be found in Table 2(c). All the slope parameters are insignificant at the 5 per cent level. The negative value for area sown is unexpected as is that for organic fertiliser. The estimates associated with labour and other costs are not only positive, but also quite large in magnitude. It should be stressed that all the slope coefficients could well be zeros in accordance with the asymptotic *t*-ratios.

It is difficult to generate findings from the estimates of the three equations because (a) most of the estimates are not encouraging in terms of statistical significance; and (b) the magnitudes of and especially the signs of parameters are so inconsistent across equations. However, as far as the relationship between the 'green revolution' and production risks is concerned, the empirical results indicate that there is a positive link between seed-fertiliser technology and output variability. This may be due to the introduction of modern cultivars which have a narrower genetic base than their predecessors (Hazell 1984). The nature of irrigation in the context of output variability crucially depends on the reliability of the water supply. Taking into account the fact that the irrigation systems in many parts of China are severely damaged, a nonnegative effect of irrigation on output risk may be understandable. The machinery input possibly brought about higher risks, which could arise from the poor quality of both tools and operations.

The estimated matrices $\{\hat{\delta}_{\mu ml}\}$, $\{\hat{\delta}_{\nu ml}\}$, $\{\hat{\delta}_{\lambda ml}\}$ and $\{\hat{\delta}_{\epsilon, \cdot}\}$ are given in Table 3. It is clear that the time effect may be negligible by comparing the corresponding values of $\{\hat{\delta}_{\lambda ml}\}$ and $\{\hat{\delta}_{\mu ml}\}$. The existence of cross-equation covariance is seen by the possibly significant off-diagonal values of $\{\hat{\delta}_{ml}\}$. If the assumption of a normal distribution for each of the three (time, region and random) errors holds, the diagonal elements of $\{\hat{\delta}_{ml}\}$ should be close to 4.9348. Statistical tests

(F statistics) reveal found that all three values are not significantly different from 4.9348 at a 5 per cent level. In passing, it is noted that the negative values on the diagonals of these matrices are possible and they can be set to zero in practice if necessary (Fuller and Battese 1974, p. 72).

The variance-covariance matrices of the mean output functions are given in Table 4. The lack of time effects is again seen by the small ratios of $\hat{\sigma}_{\lambda ml}/\hat{\sigma}_{\mu ml}$. The contemporary covariances across equations are all positive and substantial.

5 Summary

In this paper, SUR models which incorporate time-specific and region-specific error components and permit the marginal variances of outputs to be of either positive or nonpositive sign are presented. An estimation procedure is suggested. This attempt is of empirical significance, particularly in agro-economic research, since outputs of various agricultural activities tend to be influenced by some common factors, notably weather and policy changes. Also, increases of different inputs can either enhance or reduce output risks. Conventional SUR models restrict the marginal risks to be positive.

Using combined time-series (4 years) and cross-section (28 regions) data on Chinese rice, maize and wheat production, heteroscedastic SUR production functions were estimated. The results indicate that, as chemical fertiliser, sown area and irrigation cost increase, output variances generally rise. On the other hand, organic fertiliser, machinery cost and the other costs may help stabilise Chinese cereal production. Labor input does not create significant impacts on either mean outputs or output variances. These results suggest the possible superiority of the heteroscedastic SUR over more conventional ones.

It must be noted that most of the inputs considered in the models are not necessarily significantly related to production risks. This is not to suggest that these and other inputs are, in fact, unimportant or unnecessary, in production

and its riskiness. It may, however, imply the importance of weather and government intervention in agriculture in determining the variability of Chinese cereal production.

References

- Amemiya, T., 1971. The estimation of the variances in a variance-components model, *International Economic Review* 12, 1 - 13.
- Anderson, J.R., C.J. Findlay and G.H. Wan, 1989, Are modern cultivars more risky? a question of stochastic efficiency, in: Anderson, J.R. and P.B.R. Hazell, eds., *Variability in grain yields: implications for agricultural research and policy in developing countries* (Johns Hopkins University Press, Baltimore) pp. 470 - 482 of draft manuscript.
- Avery, R.B., 1977. Error components and seemingly unrelated regressions, *Econometrica* 45, 199 - 209.
- Baltagi, B.H., 1980, On seemingly unrelated regressions with error components, *Econometrica* 48, 1547 - 1551.
- Fuller, W.A. and G.E. Battese, 1974, Estimation of linear models with crossed-error structure, *Journal of Econometrics* 2, 67 - 78.
- Griffiths, W. J. and J. R. Anderson, 1982 Using time-series and cross-section data to estimate a production function with positive and negative marginal risks, *Journal of the American Statistical Association* 77, 529 - 536.
- Harvey, A. C . 1976, Estimating regression models with multiplicative heteroscedasticity, *Econometrica* 44, 461 - 465.
- Hazell, P.B.R., 1984. Sources of increased instability in Indian and US cereal production, *American Journal of Agricultural Economics* 66, 302 - 311.
- Johnson, N.L. and S. Kotz, 1972, *Distribution in statistics - continuous multivariate distributions* (Wiley New York) p. 226.

- Judge, G.G., W.E. Griffiths, R.C. Hill, H. Lütkepohl and T.C. Lee, 1985, *The theory and practice of econometrics*, 2nd edn.(Wiley, New York).
- Just, R.E. and R.D. Pope, 1978, Stochastic specification of production functions and economic implications, *Journal of Econometrics* 7, 67 - 86.
- Maddala, G.S., 1971, The use of variance components models in pooling cross-section and time-series data, *Econometrica* 39, 341 - 358.
- Marquardt, D.W., 1963, An algorithm for least squares estimation of nonlinear parameters, *Journal of the Society for Industrial and Applied Mathematics* 11, 431 - 441.
- Nash, J.C. and M. Walker-Smith. 1987, *Nonlinear parameter estimation*, (Marcel Dekker, New York).
- Nerlove, M., 1971, A note on error components models, *Econometrica* 39, 383 - 396.
- Prucha, I.R., 1984, On the asymptotic efficiency of feasible Aitken estimates for seemingly unrelated regression models with error components, *Econometrica* 50, 203 - 207.
- Stone, B., 1986, Chinese fertiliser application in the 1980s and 1990s: issues of growth, balance, allocation, efficiency and response, in: *China's economy looks towards the year 2000*, vol 1. selected papers submitted to the joint economic committee, Congress of the United States (U.S. Government Printing Office, Washington) pp. 453 - 496.

Table 1: Parameter Estimates for the Mean Output Function

	(a) Rice	
	Specifications of Error Structure	
	$u_{it} = \nu it$	$u_{it} = (\mu_i + \lambda_i + \nu it)h_{it}$
$\hat{\gamma}$	6.836 (3.29)	269.663 (3.37)
$\hat{\beta}_1$ (Area)	-0.181 (-2.47)	0.728 (8.78)
$\hat{\beta}_2$ (Labour)	0.347 (10.52)	-0.125 (-2.38)
$\hat{\beta}_3$ (Chemical Fertiliser)	-0.007 (-0.16)	-0.005 (-0.43)
$\hat{\beta}_4$ (Animal Cost)	-0.031 (-1.67)	-0.035 (-1.43)
$\hat{\beta}_5$ (Irrigation)	-0.010 (-0.75)	0.017 (0.73)
$\hat{\beta}_6$ (Machinery Cost)	-0.031 (-2.59)	-0.073 (-1.84)
$\hat{\beta}_7$ (Organic Fertiliser)	0.005 (11.98)	0.379 (5.48)
$\hat{\beta}_8$ (Other Costs)	0.120 (7.30)	0.098 (3.00)

Note. Figures in brackets are asymptotic t-ratios

Table 1: Parameter Estimates for the Mean Output Function

	(b) Maize	
	Specifications of Error Structure	
	$u_{it} = \nu it$	$u_{it} = (\mu_i + \lambda_t + \nu it)h_{it}$
$\hat{\gamma}$	325.277 (4.02)	415.365 (6.42)
$\hat{\beta}_1$ (Area)	0.584 (15.10)	0.676 (13.24)
$\hat{\beta}_2$ (Labour)	0.012 (0.32)	-0.027 (-0.70)
$\hat{\beta}_3$ (Chemical Fertiliser)	0.0002 (0.01)	-0.001 (-0.13)
$\hat{\beta}_4$ (Animal Cost)	-0.062 (-5.72)	-0.017 (-0.81)
$\hat{\beta}_5$ (Irrigation)	0.013 (1.79)	0.016 (1.94)
$\hat{\beta}_6$ (Machinery Cost)	0.009 (0.88)	0.022 (2.13)
$\hat{\beta}_7$ (Organic Fertiliser)	0.106 (2.71)	0.161 (4.55)
$\hat{\beta}_8$ (Other Costs)	0.570 (3.61)	0.147 (5.36)

Note: Figures in brackets are asymptotic t-ratios.

Table 1: Parameter Estimates for the Mean Output Function

	(c) Wheat	
	Specifications of Error Structure	
	$u_{it} = \nu_{it}$	$u_{it} = (\mu_i + \lambda_i + \nu_{it})h_{it}$
$\hat{\gamma}$	103.304 (3.79)	136.950 (3.75)
$\hat{\beta}_1$ (Area)	0.382 (4.41)	0.198 (1.96)
$\hat{\beta}_2$ (Labour)	0.212 (4.95)	0.140 (2.14)
$\hat{\beta}_3$ (Chemical Fertiliser)	-0.014 (-0.82)	0.049 (1.98)
$\hat{\beta}_4$ (Animal Cost)	-0.044 (-2.37)	0.064 (1.79)
$\hat{\beta}_5$ (Irrigation)	0.011 (0.80)	-0.024 (-1.21)
$\hat{\beta}_6$ (Machinery Cost)	0.074 (2.90)	0.084 (3.29)
$\hat{\beta}_7$ (Organic Fertiliser)	0.230 (7.60)	0.261 (4.09)
$\hat{\beta}_8$ (Other Costs)	0.148 (2.78)	0.187 (3.13)

Note: Figures in brackets are asymptotic t-ratios.

Table 2: Parameter Estimates for the Output-Variance Function

(a) Rice			
	Estimation Technique		
	OLS	LSDV	GLS
$\hat{\alpha}_0$	22.432 (14.90)	- -	18.813 (7.66)
$2\hat{\alpha}_1$ (Area)	3.443 (8.02)	1.339 (0.67)	2.072 (2.56)
$2\hat{\alpha}_2$ (Labour)	-0.910 (-3.03)	0.651 (0.44)	0.174 (0.32)
$2\hat{\alpha}_3$ (Chemical Fertiliser)	0.303 (2.82)	1.017 (1.16)	0.515 (1.92)
$2\hat{\alpha}_4$ (Animal Cost)	0.470 (2.63)	1.228 (1.36)	1.005 (2.84)
$2\hat{\alpha}_5$ (Irrigation)	0.273 (1.69)	1.049 (1.27)	0.796 (2.40)
$2\hat{\alpha}_6$ (Machinery Cost)	0.101 (1.08)	-0.179 (-0.33)	-0.005 (-0.03)
$2\hat{\alpha}_7$ (Organic Fertiliser)	-2.003 (-5.59)	-1.987 (-1.22)	-1.350 (-2.87)
$2\hat{\alpha}_8$ (Other Costs)	-0.450 (-1.89)	1.894 (-1.89)	-1.433 (-3.17)
R^2	0.637		
F-ratio	22.609		

Note: Figures in brackets are asymptotic t-ratios.

Table 2: Parameter Estimates for the Output-Variance Function

(b) Maize			
	Estimation Technique		
	OLS	LSDV	GLS
α_0	9.138 (6.77)	- -	9.107 (3.91)
$2\hat{\alpha}_1$ (Area)	9.377 (0.83)	1.675 (0.45)	1.056 (1.34)
$2\hat{\alpha}_2$ (Labour)	0.368 (1.03)	0.013 (0.004)	0.058 (0.11)
$2\hat{\alpha}_3$ (Chemical Fertiliser)	0.040 (0.35)	0.054 (0.08)	0.099 (0.56)
$2\hat{\alpha}_4$ (Animal Cost)	-0.055 (-0.39)	-0.785 (-0.70)	-0.440 (-1.77)
$2\hat{\alpha}_5$ (Irrigation)	-0.160 (-2.22)	-0.436 (-0.79)	-0.182 (-1.28)
$2\hat{\alpha}_6$ (Machinery Cost)	0.282 (2.87)	0.390 (0.51)	0.349 (1.86)
$2\hat{\alpha}_7$ (Organic Fertiliser)	0.340 (1.08)	0.338 (0.16)	0.437 (0.76)
$2\hat{\alpha}_8$ (Other Costs)	0.250 (0.82)	0.484 (0.29)	0.261 (0.49)
R^2	0.533		
F-ratio	14.684		

Note: Figures in brackets are asymptotic t-ratios.

Table 2: Parameter Estimates for the Output-Variance Function

(c) Wheat			
	Estimation Technique		
	OLS	LSDV	GLS
α_0	8.310 (6.99)	-	7.226 (2.54)
$2\hat{\alpha}_1$ (Area)	0.016 (0.04)	-0.709 (-0.37)	-0.407 (-0.41)
$2\hat{\alpha}_2$ (Labour)	0.886 (3.40)	-0.207 (-0.17)	0.700 (1.26)
$2\hat{\alpha}_3$ (Chemical Fertiliser)	-0.331 (-3.01)	0.290 (0.96)	0.018 (0.09)
$2\hat{\alpha}_4$ (Animal Cost)	-0.005 (-0.03)	0.003 (0.01)	-0.073 (-0.24)
$2\hat{\alpha}_5$ (Irrigation)	-0.002 (-0.02)	0.184 (0.42)	0.056 (0.24)
$2\hat{\alpha}_6$ (Machinery Cost)	0.444 (4.14)	-0.011 (-0.03)	0.247 (1.13)
$2\hat{\alpha}_7$ (Organic Fertiliser)	-0.141 (-0.47)	0.682 (0.67)	0.113 (0.19)
$2\hat{\alpha}_8$ (Other Costs)	0.482 (1.64)	1.270 (1.58)	0.878 (1.65)
R^2	0.596		
F-ratio	19.010		

Note: Figures in brackets are asymptotic t-ratios.

Table 3: Covariance Matrices of Output-Variance Functions

	-0.010	-0.043	-0.168
$[\hat{\sigma}_{\lambda ml}]$	-0.043	0.058	-0.046
	-0.168	-0.046	0.148
	1.660	1.336	-0.697
$[\hat{\sigma}_{\mu ml}]$	1.336	3.373	-0.402
	-0.697	-0.402	3.321
	5.293	-0.227	-0.088
$[\hat{\sigma}_{\nu ml}]$	-0.227	2.803	0.122
	-0.088	0.122	2.009
	6.093	0.833	-0.576
$[\hat{\sigma}_{ml}]$	0.833	5.035	-0.118
	-0.576	-0.118	3.981

Table 4: Covariance Matrices of Mean Output Functions

	15928.522	5809.082	840678.539
$[\hat{\sigma}_{\mu ml}]$	5809.082	4593.849	271162.822
	840678.539	271162.822	186857666.081
	1570.784	619.503	-131845.170
$[\hat{\sigma}_{\lambda ml}]$	619.503	246.119	-277397.578
	-131845.170	-277397.578	76840813.036
	14630.159	3448.611	1982192.815
$[\hat{\sigma}_{\nu ml}]$	3448.611	56.530	985374.936
	1982192.815	985374.936	263829703.787