OPTIMIZATION OVER TIME FOR AGRICULTURE

AND RESOURCE MANAGEMENT

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ABSTRACT

This paper deals with determination of optimal decisions that reach over a time horizon. Four case studies taken from the literature are re-examined. One is concerned with mackerel fishing in European sea waters. It employs a dynamic nonlinear age cohort model. A further complicating aspect is that the changes in fishing effort from year to year are restricted to be within 20 per cent of previous year's level. In the literature it is mentioned that more than five and a half hours of computer time were needed to find the optimum. In contrast, our calculations take only about seven minutes to reach the same optimum. Clearly an effective procedure can save time and effort in such complicated situations. The second study examines the economics of cartelization of a commodity by producers. A model was reported which simulates cartels for petroleum oil, bauxite and copper. We discuss some of its complexities and obtain optimal decisions. In the case of oil our results are practically the same as reported in the literature. But, for copper we find an optimum which is nearly a third better, i.e. a considerably superior optimum can be achieved. The last illustration involves a model built to investigate the economics of soil conservation. We employ it to highlight the care needed in constructing such exercises. We point out that the objective function in this case is not sensitive to wide variations in the decision variable. Some comments are then included about control theory which also deals with decision making over time.

1. INTRODUCTION

Mathematical optimization is frequently employed for analyzing agriculture and resource management situations. Labys and Pollak (1984) give a survey of different models constructed to this end. In particular, the techniques of linear and quadratic programming have found numerous applications. Their advantage is that robust and reliable procedures exist for obtaining the optimal decisions. This is because well constructed linear and quadratic programming models have unique optimum. In contrast, nonlinear optimization is qualitatively quite different. Such models are characterized by the presence of multiple optima. Determining optima in nonlinear cases is far more difficult. However, many practical decision-making situations require nonlinear optimization. This is being reported increasingly in the recent literature.

The objective of this article is to highlight some of the complexities associated with nonlinear optimization. As well, it deals with the class of situations that involve decision-making over time. That is, decisions made during different time periods are interrelated. For illustration four case studies reported in the literature are re-examined. One relates to the harvesting of Western mackerel fish found in European sea shelf. To determine the optimal fishing pattern over a 40 year horizon, it is mentioned in the literature that more than 5½ hours of computer time were needed. In contrast, we present a slightly better pattern but which takes only about seven minutes for calculations.

This illustrates the wide variation in the effectiveness of different computation procedures. The next two examples deal with the economics of cartelization of commodities. In one case our results are close to those described in the literature. But in the other we obtain an optimum for which the gain is nearly a third higher. As well, our optimal decisions show a qualitative different pattern. This is also very much possible in nonlinear analysis. Our last illustration involves a model built to examine soil conservation economics. It helps to explain some other difficulties that may arise in nonlinear optimization.

These illustrations are presented in Sections 2 through 5. Since control theory also deals with decision-making over time, we briefly discuss its scope in Section 6. Some concluding comments are included in Section 7.

2. A MULTICOHORT FISHERY MODEL

We consider the fishery model reported in Horwood (1987). It analyses harvesting of mackerel found along the west coasts of France, Ireland and the U.K.

The fish stock is assumed to consist of 10 age groups. The first nine represent ages of years 1, 2,...., up to 9. The last group refers to fish 10 years in age or older.

The symbols used for different parameters and variables are as follows.

Parameters

n = 10, number of age groups

ws_i = weight at spawning of fish at age i

p_i = proportion of mature fish of age i

q_i = proportion selection in catch (age-specific selectivity or catchability) at age i

wm_i = mean weight in the catch of fish at age i

M = instantaneous natural mortality rate (0.15 per year)

α and γ = parameters entering the recruitment function

α = 0.00202 millions of recruits per tonne

γ = 1/(2,160,000)

h and c = parameters entering the objective function

h = 10^{-6}

c = 2.5

δ = discount rate

T = time horizon for planning (20 and 40 years here)

Endogenous Variables

State Variables

$x_i(t)$ = number of fish of age i in year t (millions)

Other Dependent Variables

$b(t)$ = spawning stock biomass in year t (tonnes)

$s_i(t)$ = survival of fish of age i in year t

$y(t)$ = yield from fishery in year t (tonnes)

<u>Control (Decision) Variable</u>

$u(t)$ = fishing effort in year t (Hence $q_i.u(t)$ = age - specific rate of instanteous fishing mortality).

<u>Independent Variable</u>

t = the discrete time variable (year)

= 0,1,2,....,T

<u>Model Equations</u>

The spawning biomass in year t is determined by

$$b(t) = \sum_{i=1}^{n} p_i.ws_i.x_i(t) \qquad (2.1)$$

That is, the fish numbers in each age group are multiplied by the weight per fish and a proportion of the biomass then counts towards spawning.

The survival of fish of age i in year t is given by the equation

$$s_i(t) = \exp\,[-M - q_i.u(t)] \qquad (2.2)$$

Here M is the instantaneous natural mortality and $q_i.u(t)$ the age-specific fishing mortality, $u(t)$ being the fishing effort across all age groups.

The yield (in tonnes) from fishing in year t is then obtained as

$$y(t) = \sum_{i=1}^{n} wm_i . q_i . u(t) . x_i(t) . \frac{1-s_i(t)}{M + q_i . u(t)} \qquad (2.3)$$

The age structure of the fish stock is determined by a number of simultaneous discrete-time dynamic equations. The recruitment equation is

$$x_{1(t+1)} = \frac{\alpha . b(t)}{1+\gamma\ b(t)} \qquad (2.4)$$

This is a nonlinear function of the current biomass b(t). The parameters α and γ have estimated values mentioned above.

Age groups 2 to (n-1) are governed by the survival rate $s_i(t)$. In year (t+1) the number of fish of age i are those surv'ving from age (i-1) the previous year. Thus,

$$x_i(t+1) = s_{i-1}(t) \ . \ x_{i-1}(t), \ i = 2,3, \ldots, (n-1) \qquad (2.5)$$

The numbers in the last age group is dependent upon its own survival rate as well as that of one year younger group, so that

$$x_n(t+1) = s_{n-1}(t) \ . \ x_{n-1}(t) + s_n(t) \ . \ x_n(t) \qquad (2.6)$$

The initial age distribution and values of the parameters $\{wm_i, ws_i, p_i, q_i\}$ are given in Horwood.

Equations (2.1) - (2.6) thus represent the biological recruitment process, the age structure and the physical yield obtainable from fishing. These are nonlinear dynamic equations. The single decision variable, viz. u(t), enters in a complicated fashion through the survival rate $s_i(t)$ and directly in the yield equation (2.3). However, this is the case of a scalar control variable since only one u(t) needs to be determined for each year.

Inequality Constraints on Fishing Effort

Horwood considers the practical situation in which fishing effort cannot change dramatically from year to year. He models this case by requiring that changes in the fishing effort be restricted to within 20 per cent of the previous year's level. Symbolically, the following inequality constraints on the control variable are stipulated.

$$0.8 \leq \frac{u(t+1)}{u(t)} \leq 1.2 \qquad (2.7)$$

In addition, bounds are specified within which the fishing effort remains, viz.

$$0 \leq u(t) \leq 10 \qquad (2.8)$$

And it is stipulated that fishing must leave a spawning biomass of at least one million tonnes at the end of the planning horizon; i.e

$$b(T+1) \geq 1{,}000{,}000 \qquad (2.9)$$

Optimization Criterion for Decision-Making

For optimal operation a criterion for selecting the appropriate control sequence is required. Horwood provides a rationale for choosing a linear objective function, viz.

$$\max \sum_{t=1}^{T} (1 + \delta)^{-t} [h.y(t) - c.u(t)] \qquad (2.10)$$

That is, the sum of discounted returns from fishing is to be maximized over a horizon of T years. The coefficient h converts yield to return in an arbitrarily selected unit. The coefficient c assigns a cost measure to the use of control.

The mathematical optimization problem then is to find the fishing pattern $\{u(1), u(2), \ldots, u(T)\}$ which maximizes the criterion (2.10) and conforms to the inequality constraints. The yield variable y(t) appearing in the objective function is, to recall, determined through the dynamic equations (2.1)-(2.6).

Calculation of Optimal Fishing Mortalities

Horwood reports alternative calculations of optimal fishing pattern for different time horizons (T) and discount rates (δ). He employed one of the available computer programs for solving these nonlinear optimization problems.

The present author obtained the numerical solutions using a gradient algorithm. For a 40 year planning horizon, the optimal fishing effort is plotted in Chart 1 and for a 20 year horizon in Chart 2. Charts 3 and 4 show the respective spawning biomass. These results correspond to a 10 per cent discount rate (δ = 0.1). They are similar to those reported by Horwood. In fact, our calculations yield somewhat better optimum value of the objective in each case. For comparison, these values are as follows:

Time Horizon T = 20 years, discount rate = 0.1

Horwood's optimum = -0.5065

Author's optimum = -0.474

Time Horizon T = 40 years, discount rate = 0.1

Horwood's optimum = -0.3750

Author's optimum = -0.3559

The big difference is in the computation time. Horwood (1987) states that his computations required 342 minutes for the 40 year horizon case (with inequality constraints on the control variable). In contrast, our calculations take only about 7 minutes of computer time to find the optimum. This case thus demonstrates that alternative computatuion schemes can vary greatly in their effectiveness. It also shows that the way available computer programs are put to use, can also vary significantly from user to user.

3. GAINS FROM CARTELIZATION OF PETROLEUM OIL

Pindyck (1978) analyses gains to producers from cartelization of exhaustible resources. He examines the cases of petroleum oil, bauxite and copper. For each commodity he constructed an econometric model. These models are used to calculate the optimal gains from cartel pricing policy. In this section we consider the model for petroleum oil. The base year for the analysis is 1974.

Endogenous Variables

TD(t) = total demand for oil in year t (billion barrels = bb)

D(t) = demand for cartel oil (bb/year)

S(t) = supply from competitive fringe producers not in cartel (bb/year)

CS(t) = cumulative supply from the competitive fringe (bb)

R(t) = oil reserves of the cartel (bb)

Decision (Control) Variable

P(t) = price of oil in constant 1975 dollars ($ per barrel)

Model Equations

The total demand for oil in year t is determined by the dynamic equation:

$$TD(t) = 1.0 - 0.13\,P(t) + 0.87\,TD(t-1) + 2.3\,(1.015)^{t} \qquad (3.1)$$

This is based on a total demand of 18 bb/year at a price of $6 per barrel in the base year. The short-run elasticity is 0.04 and the long-run 0.33 (with a Koyck adjustment pattern) at $6 per barrel. At a price of $12, these elasticities are 0.09 and 0.90, respectively. The last term in Eq. (3.1) incorporates an autonomous growth component at the rate of 1.5% per year, based on a long-run income elasticity of 0.5 and a 3% real rate of growth in income.

Some of the demand is met by supply from the competitive fringe producers. These producers are not members of the cartel. To account for the depletion of reserves of the competitive producers, a cumulative supply variable CS(t) is employed, viz.

$$CS(t) = CS(t-1) + S(t) \tag{3.2}$$

That is, the previous level plus the current supply make up the cumulative production so far.

The supply from the competitive fringe is modelled as:

$$S(t) = 0.75\ S(t-1) + [1.1 + 0.1\ P(t)]\ (1.02)^{-CS(t)/7} \tag{3.3}$$

This is a dynamic supply equation. In the base year the competitive supply is about 6.5 bb/year at $6 a barrel. Eq.(3.3) implies a short-run elasticity of 0.09 and a long-run elasticity of 0.35 at this price. At $12 a barrel, the corresponding figures are 0.16 and 0.52.

The effect of reserve depletion is to shift the supply function, Eq (3.3), to the left over time. Assuming a fixed price, competitive supply would fall to 55% of its original value after a cumulative production of 210 billion barrels (e.g., 7 bb/year for 30 years). It is assumed that no new technology or reserves add to the potential supply of competitive fringe producers.

The demand D(t) facing the cartel then is the total demand minus the supply from the competitive fringe, i.e

$$D(t) = TD(t) - S(t) \qquad (3.4)$$

It is assumed that the cartel can manipulate the price P(t) to satisfy this demand. Accordingly the cartel reserves get depleted; so that

$$R(t) = R(t-1) - D(t) \qquad (3.5)$$

<u>Cartel Decision-Making</u>

The cartel price P(t) is chosen so as to maximize the discounted sum of profits over a long time horizon. The mathematical objective function is:

$$\max W = \max \sum_{t=1}^{N} (1+\delta)^{-t} \left[P(t) - \frac{0.5 \times 500}{R(t)} \right] D(t) \qquad (3.6)$$

This stipulates that the cost of production increases as reserves fall.

In the base year the initial reserves are R(0) = 500 billion barrels and the initial average cost of production is $0.5 per barrel. The average cost of production rises hyperbolically as reserve R(t) depelete to zero. The difference between the cartel price and the cost of production is the profit margin per barrel. Eq. (3.6) then represents the maximization of the sum of discounted profits over the planning horizon.

The initial situation is specified as: TD(0) = 18.0, S(0) = 6.5 and CS(0) = 0. That is, the total demand in the base year is 18 bb/year, the competitive supply is 6.5 bb/year and the cumlative supply from the competitive fringe is initialized to zero.

This is a nonlinear mathematical optimization situation. The objective function (3.6) is nonlinear as is the set of dynamic equations (3.1) - (3.5). There is an additional complication. The variables S(t) and CS(t) appear or the right hand side as well as on the left hand side. In other words, Eq. (3.2) and (3.3) are in implicit form, not explicit. In econometric terminology, the set of dynamic equations (3.1) - (3.5) is in "structural form" and not in the "reduced form".

Calculation of Gains to Cartel Producers

Optimization can be performed using one of many computer programs that are available. Again, we employed a gradient procedure. In Table 1 we give the optimal cartel price over a 40 year time horizon (N=40) and for a 5 per cent discount rate (δ=0.05). For comparison we have also included the corresponding price trajectory obtained by Pindyck. Chart 5 plots the price trajectory of our calculations. It is seen that our results are quite close to those of Pindyck.

The maximum gain from cartelization over 40 years is found to be 2163 in contrast to 2092 in Pindyck's calculations. That is, our optimum is marginally (about 3%) better.

In the case of petroleum oil, we thus verify that our results are close to those of Pindyck, both quantitively and qualitatively. This implies that our calculations are satisfactory.

In the next section, we describe that for copper we find substantially better results than Pindyck.

4. GAINS FROM CARTELIZATION OF COPPER

This model is slightly larger than the previous one for petroleum. The base year is again 1974 however.

Endogenous Variables

TD(t) = total demand for copper in year t (million metric tons per year).

S(t) = total supply from the competitive fringe (mmt/year)

SP(t) = primary supply from competitive fringe (mmt/year)

CSP(t) = cumulative value of primary supply from the competitive fringe (mmt)

SS(t) = secondary supply derived from scrap by the competitive fringe (mmt/year).

K(t) = stock of copper in product form (mmt)

R(t) = reserves of the cartel (mmt)

Model Equations

The total demand for refined copper is determined by the dynamic equation.

$$TD(t) = 0.405 - 0.78\,P(t) + 0.9\,TD(t-1) + 0.91\,(1.03)^{t} \qquad (4.1)$$

In 1974 the total demand was estimated to be about 7.3 million metric tons at a price of $0.75 per pound. The short and long-run elasticities at this price are 0.16 and 0.80, respectively. An autonomous growth component appears in the demand equation (4.1) It reflects a 3.75% rate of growth, corresponding to a long-run income elasticity of 1.25 and a 3% real rate of growth in income.

Aluminium is a major substitute for copper. But Pindyck assumes a fixed price for it and therefore its price does not appear in the demand equation (4.1)

The total demand is satisfied by supply from the cartel as well as the fringe producers. The latter are not members and accept the price set by the cartel. In this sense they are considered competitive.

The total supply from the competitive fringe comprises two sources. One is the primary refined copper and the other from scrap. The cumulative value of the primary supply is given by the identity:

$$CSP(t) = CSP(t-1) + SP(t) \tag{4.2}$$

The supply equation for primary refined copper from the competitive fringe is as follows:

$$SP(t) = 0.88\ SP(t-1)+[-0.19+ 0.8613\ P(t)](1.015)^{-CSP(t)/4} \tag{4.3}$$

The relationship is based on a primary supply of 3.8 mmt/year at $0.75 per pound in the base year. The short-run and long-run price elasticities are 0.2 and 1.6, respectively. Even though the long-run elasticity is high, the adjustment time is quite considerable.

Depletion of reserves shifts the primary supply function of the competitive fringe to the left over time. Assuming a fixed price, the primary supply would fall to 55% of its original level after a cumulative production of 160 mmt (e.g. at the rate of 4 mmt/year for 40 years).

The secondary supply of copper from the competitive fringe is derived from scrap. This depends upon the stock of copper products available to be converted into scrap. The copper stock is governed by the equation:

$$K(t) = 0.98\ K(t-1) + TD(t) - SS(t) \tag{4.4}$$

This allows for losses at the rate of 2%. The secondary production SS(t) in the current year is subtracted to avoid double counting.

The equation for secondary supply from the fringe producers is as follows:

$$SS(t)/K(t) = 0.0094+0.005733\ P(t)-0.37[SS(t-1)/K(t-1)] \qquad (4.5)$$

In this relationship price of the primary copper, viz. P(t), appears because the price of secondary copper is highly correlated with it. The equation includes a stock-adjustment behaviour. It has a higher short-term price elasticity of 0.43 and a lower long-run elasticity of 0.31.

The total supply from the competitive fringe forms the identity:

$$S(t) = SP(t) + SS(t) \qquad (4.6)$$

The cartel supplies the rest of the demand, i.e.

$$D(t) = TD(t) - S(t) \qquad (4.7)$$

The cartel reserves deplete by this amount in year t, so that:

$$R(t) = R(t-1) - D(t) \qquad (4.8)$$

Cartel Decision Making

The objective of the exercise is again to maximize the sum of discounted profits for the cartel. The optimal trajectory for the cartel price P(t) needs to be determined to achieve that. The mathematical objective function is chosen to be:

$$\max W = \sum^{N} (1+\delta)^{-t} \; 2204 \left[P(t) \quad \frac{0.5 \times 135}{R(t)}\right] D(t) \qquad (4.9)$$

Here an initial reserve of 135 mmt and production cost of \$0.5 per pound are assumed. Again, the production cost rises hyperbolically as reserves R(t) fall. The coefficient 2204 appears in order to convert pounds to metric tons. And as before, δ is the discount rate.

The initial values for different variables are:

TD(0)=7.3, SP(0)=3.8, SS(0)=1.2, CSP(0)=0, K(0) = 120. They refer to 1974 as the base year.

Calculation of Gains to the Cartel

We again compare our results with Pindyck's for a 40 year time horizon (N=40) and a discount rate of five per cent (δ=0.05). In Table 2 the optimal price trajectories are given numerically. The cartel price obtained in our calculations is plotted in Chart 6. This pattern is qualitatively quite different from that of Pindyck. Our optimal trajectory shows a smooth pattern with a single spike in the middle of planning horizon. In contrast, Pindyck's trajectory follows an oscillatory, fluctuating time path, although the general envelope of the functuations is similar to our smooth trajectory. That is, it contains spikes throughout in its graph.

The optimal gain to producers is found to be 39,767 in our calculations. In contrast, Pindyck reports a figure of

28,988. We are thus able to find an optimum which is nearly a third higher. In other words, the procedure employed by Pindyck finds a local optimum which is substantially lower than our result.

5. A SOIL CONSERVATION MODEL

We now examine a model developed by Bhide, Arden Pope III and Heady (1982). It deals with the economics of soil conservation. According to these authors soil use is analogous to the use of an exhaustible resource. If soil loss from farming exceeds the natural soil formation, then exhaustion will occur. They give numerical models for three different soil types. We consider one of those models here.

Endogenous (state) variable

SD(t) = soil depth in acre-inches in year t

Control (decision) variable

SL(t) = soil loss in tons/acre in year t

Model relationships

There is only one dynamic equation in this model, viz

$$SD(t+1) = SL(t) - 0.0069\,[SL(t) - 5.0] \qquad (5.1)$$

Here the constant 5.0 is the rate of natural soil formation and 0.0069 is the inverse of the bulk density of soil (tons/acre inch). Eq. (5.1) represents the dynamics of soil depth which is affected by the natural soil formation and loss from farming.

Initially, in year 0, the soil depth level is 7 acre-inches, i.e., $SD(0) = 7$.

The single decision variable, viz. soil loss, is restricted to be in the range 0 to 16 tons per acre; i.e.

$$0 \leq SL(t) \leq 16 \qquad (5.2)$$

Criterion for Optimization

The objective of the exercise is to maximize the sum of discounted net returns from farming. The mathematical objective function in this case is as follows.

$$\max W = \max \sum_{t=0}^{N} (1+\delta)^{-t} \left[225.297(1+0.25t)\left(1-0.5269^{SL(t)}\right)\right.$$

$$\left. - \left\{4.8584 + (t+35)(0.6803 - 0.122SD(t) + 0.0043\ SD(t)^2)\right\}\right] \qquad (5.3)$$

This is a fairly complicated function of the single state variable SD(t) and the single control variable SL(t). The discrete time variable t is included to reflect the effect of technological changes.

Calculation of Optimal Returns

Compared to the models of Horwood and Pindyck considered in previous sections, this is a much simpler model. There is only one dynamic equation. Also, there is only a single endogenous variable and a single decision variable for each time period. Only the mathematical objective function is nonlinear. And the decision variable is restricted within the inteval [0,16]. However, this model can also be solved using a gradient procedure.

Bhide *et al.* report the optimal solution for a 5 percent discount rate (δ = 0.05) and a 35 year time horizon (N=35). Note that there are 36 decision variables, SL(0), SL(1), ..., SL(35), since the optimization criterion extends from t=0 to t=35. The optimal soil loss obtained in our calculations is plotted in Figure 7 and it is close to that reported by Bhide *et al.* Also, our optimal gain at 4986.9 compares well with their value of 4986.05.

This model is however seriously flawed and that has not been pointed out by Bhide *et al.* From Figure 7 it can be seen that the optimal soil loss remains in the vicinity of 10 from year 0 to year 25. In fact, it remains within the interval [9½, 10½] in this period. After t=25 the discount factor becomes very effective and large variations in the decision variable make little contribution. In other words, the optimal soil loss pattern of Figure 7 gives a clue to the situation that the objective function (5.3) is not very sensitive to large changes in the decision variable. This can be verified numerically. To do this we keep the decision variable at a fixed value over the whole planning horizon (e.g. SL(t) = 9, t = 0,1,2,...,35) and calculate the corresponding value of the objective function (which is not optimized). The following table lists some alternative values of the soil loss and corresponding value of the discounted net returns:

Soil Loss Constant at	Discounted Net Returns	Departure from Optimum (4986.9)
8	4975.2	-0.24%
9	4984.2	-0.06%
10	4986.2	-0.02%
11	4984.6	-0.04%
12	4980.9	-0.12%

In Figure 8 we plot similar pairs for soil loss kept constant at 1,2,3,...,16. The above table and Figure 8 bring out that the particular objective function in the present case is not sensitive to wide variations in the decision variable. In fact, beyond a soil loss of 7, the objective function is quite insensitive to wide variations. In other words, this objective function is not helpful in identifying the optimal pattern of the decision variable. That is, this objective function is not meaningful for determining optimal control actions.

6. CONTROL THEORY AND MATHEMATICAL OPTIMIZATION

These case studies highlight some of the complexities associated with nonlinear mathematical optimizat. They demonstrate that those who construct models in an application area, are not always able to obtain solutions most effectively. Someone else with expertise in nonlinear optimization could produce better performance or identify difficulties. Here availability of computer programs does not appear to be the overriding factor. The resource cartelization models examined here were developed at the Massachusetts Institute of Technology where powerful computer software is readily available. Similarly, Horwood employed a standard mathematical software package. In the case of nonlinear mathematical optimization, considerable skill and ingenuity are required even in using available computer software. It is quite possible that two persons will produce different results using the same software.

(This quizz heard on the radio is of some relevance here. Question : Does the availability of a word-processing package make one an accomplished author? The answer is obvious. Similarly, mere possession of a computer package would not make one accomplished in nonlinear mathematical optimization.)

Mathematical optimization is a broad and demanding discipline (see, e.g., Shapiro 1979, Simmons 1975). Nonlinear optimization in particular is quite an involved process. Only in rare cases would experts in agriculture and resource economics have great expertise in nonlinear optimization and numerical methods as well. In normal, pedestrian cases a cooperative approach amongst experts in different disciplines seems advisable.

In dynamic optimization situations the additional dimension of time is involved. Some convey the impression that such models can be handled routinely by control theory. But control theory itself is a broad discipline and not just a handful of techniques or procedures (see, e.g , Dorf 1980, Franklin and

Powell 1980). It deals in general with the design and construction of systems so as to make them more reliable or perform better. In physical systems its domain varies from simple (such as air-conditioning) to quite complicated (such as automatical pilots for aircrafts, and even guidance and control of space vehicles).

It is however true that in mathematical form some of the situations considered in control theory are similar to the case studies included here. A typical discrete-time control model involving nonlinear dynamics and nonlinear performance criterion can be written as follows:

$$x(t+1) = f[x(t), u(t+1)]$$

$$\max \sum_{t=1}^{N} g[x(t), u(t), t]$$

where x(t) is the state vector, u(t) the control (decision) vector, and g[....] is the contribution to the objective function in period t. But this control model itself requires the theory of non-linear optimization and suitable computer software for solution (see e.g. Bryson and Ho 1969).

One particular instance of control theory models seems to have become well-known in other disciplines. It is the so-called linear quadratic regulator (LQR) case. In discrete time domain the (constant parameter) LQR can be represented as follows:

$$x(t+1) = A\ x(t) + B\ u(t+1)$$

$$\min \sum_{t=1}^{N} x'(t)\ P\ x(t) + u'(t)\ Q\ u(t)$$

where x(t) is the nx1 state vector, u(t) the mx1 control vector, and P and Q are (positive definite) weighting matrices. The great simplifying feature of the LQR formulation is that the optimal control is a linear function of the state, viz.

$$u^*(t+1) = -K\ x(t)$$

where K is the feedback matrix. This means that the optimal control actions can be determined readily once the matrix K is known. In the literature several alternative ways for calculating K have appeared (see, e.g., Franklin and Powell 1980). These are based on analytical derivations. It is also possible to find K directly by numerical methods once the system matrix A, the control matrix B and the weighting matrices P and Q are specified. This LQR formulation has found several applications in economics, particularly in economic stablization situations (see, e.g., Chow 1975, Vishwakarma 1974 for case studies of macroeconomic stabilization).

This LQR model is deterministic. It also assumes that the state vector x(t) is measured directly. In a more general formulation this is not the case. That is, the state is not supposed to be measured directly. Instead, only a linear transformation of the state is assumed available as the observation (output, measurment) vector y(t). Furthermore, it is assumed that a measurement noise $\eta(t)$ corrupts the observation. Similarly, the dynamic relationship describing the transition of the state from period to period is thought to contain a random noise component $\xi(t)$. Thus, the following discrete-time linear stochastic quadratic regulator case arises:

$$x(t+1) = A\ x(t) + B\ u(t+1) + C\ \xi(t)$$

$$y(t+1) = D\ x(t+1) + \eta(t+1)$$

and

$$\min E\ [\ \sum_{t=1}^{N} x'(t)\ P\ x(t) + u'(t)\ Q\ u(t)\ ]$$

Here C and D are additional coefficient matrices. Since the state $x(t)$ is a random process in this case, the objective function involves minimization of the expected value of the quadratic. And, it is usual to assume that the random disturbance processes $\xi(t)$ and $\eta(t)$ are normally distributed (Gaussian) white noise processes. In spite of these generalizations, the optimal control is again a linear function of the state with the qualification that the conditional expectation of the state is now involved, so that

$$u^*(t+1) = -K\ \ x^*(t).$$

Since the state is a random process and not directly measured, its expectation needs to be estimated from the observation process $y(t)$. The celebrated Kalman filter enables that. Briefly, the principle of certainty equivalence applies and the Kalman filter is needed to obtain the expected value of the state which in turn facilitates calculation of the optimal stabilizing control. See i.e. Chow (1975) and Vishwakarma (1974) for application of this formulation in macroeconomic stabilization policy.

In simple terms therefore, control theory encompasses the Kalman filter. Applications of this filter in economic analysis are numerous. For forecasting by means of the Kalman filter see, for example, Vishwakarma (1970, 1974). In practical situations the coefficient matrices A,B,C,D and the covariances of noise processes $\xi(t)$ and $\eta(t)$ are of course unknown. They need to be estimated from available time series data. An illustration of such estimation for simultaneous forecasting of interest rate, money supply and bank loans is given in Vishwakarma (1987).

To be sure, there are many other models and formulations in control theory as well as mathematical optimization theory which have yet to find applications in economic analysis.

In this context reference needs to be made to the well-known method of dynamic programming (e.g. see Nemhauser 1966). This is a particular optimization procedure which utilizes the so called principle of optimality. To quote (Bellman 1961, p. 56), this principle states: "An optimal policy has the property that whatever the initial state and the initial decision are, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision" This is a broad stateme which applies to other techniques as well as to dynamic programming. Unfortunately, the term "dynamic programming" is a misnomer. This method is better described as a recursive optimization approach which leads to the optimum through a sequence of steps. As such dynamic programming does not deal with optimization over time involving dynamic systems, although it may be applicable in these situations. The other major difficulty with it is what is termed the "curse of dimensionality". When several decision variables are simultaneously involved, use of dynamic programming becomes impractical. Horwood's model presents such a situation and dynamic programming would not be an effective procedure for it.

Theoretical literature on mathematical optimization and control theory is voluminous. Computer software for performing calculations in this regard is also plentiful. For example, Shittkowski (1980) presents a comparative evaluation of some 20 different optimization programs. Well-known software libraries, such as IMSL and NAG, now include a number of routines. And some particular packages, MINOS (Murtagh and Saunders 1987) for example, are becoming widely known. But, as Schittkowski elaborates, each program has its strengths and weaknesses. In fact, some of the programs are simply of poor quality. And, there is as yet no single program which is superior to others in all respects. This is not surprising. The variety of nonlinear optimization models that can arise in practice is very great indeed. Programs that are geared to a given type of situations will naturally perform better than those which cater for other scenarios. In fact, choosing an appropriate program for the situation in hand is a useful skill. It is unlikely that the same program will be suitable in all situations.

An associated aspect is the care needed in making use of available computer software. Nonlinear optimization programs typically require the user to provide a subroutine or a function to evaluate the objective functions and constraints. As a rule these are given in the FORTRAN language. An ability to write FORTRAN programs in itself is a nontrivial skill. In this context the dictum "garbage in, garbage out" is very much relevant. Both in writing of the subprograms and in the use of the software, the possibility of making nonsensical calculations exits. The model of Bhide, Arden Pope III and Heady provides an example. To recall, its objective function is not a good one since it is insensitive to wide variations in the decision variable. No optimization software can check whether such flaws exist in a model. In other words, just because available software is run on a computer to arrive at the results, there is no guarantee that those results are sound.

Not only for optimization but use of mathematics in general is not so trivial a process. Even inserting numbers into established formulae is not a trivial endeavour. The study conducted by Dewald, Thursby and Anderson (1986) is a case in point. They examined the issue of replicating the results published in articles on practical economic analysis. They found that in many cases the authors did not correctly use even regression analysis software. Calculation of optima in nonlinear dynamic systems is a more involved task and requires considerable expertise on the part of the analyst.

This discussion should not be taken to mean that Horwood's or Pindyck's analyses are incorrect. Quite the contrary, they ought to be commended for their articles. This author is aware that both Pindyck and Horwood have great expertise in optimization methodology as well as control theory. That has enabled them to devise and operate the models cited here. The description in their articles is complete. That makes it possible for others to replicate their calculations. Such is not always the case with articles appearing in the literature. Again, the Dewald, Thursby and Anderson (1986) study throws light on this issue. It mentions that very many published analyses are not amenable to replication. Such results could only be acceptable as a matter of faith. Our scrutiny of Horwood's and Pindyck's models indicates that the analysis of even experts could benefit from cooperation with others. Those who have lesser training would certainly be better off seeking help. As a matter of fact, there are professional consultants who render such services.

7. CONCLUSION

Division of labour and synergy are well-known concepts. They signify that a complex activity is better performed by dividing it into areas of expertise. Applications of mathematical optimization and co rol theory are in this category. The case studies examined here illustrate that agriculture and resource management involve complicated decision-making. Cooperation of experts in control and optimization theory with agriculture and resource experts should result in more efficient analysis.

TABLE 1. COMPARISON OF OPTIMAL CARTEL PRICE TRAJECTORY FOR PETROLEUM OIL

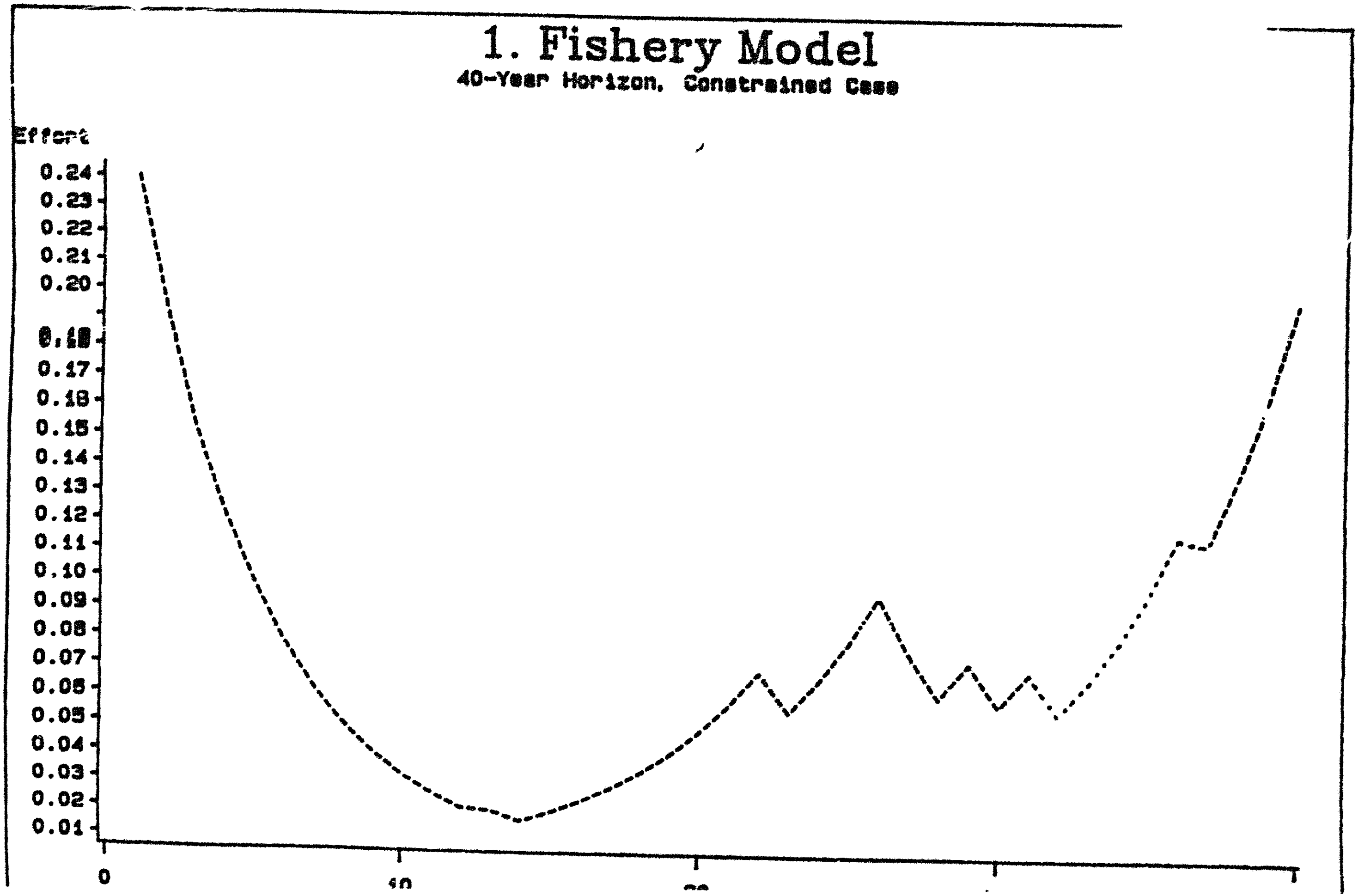
(Discount Rate = 5 per cent)

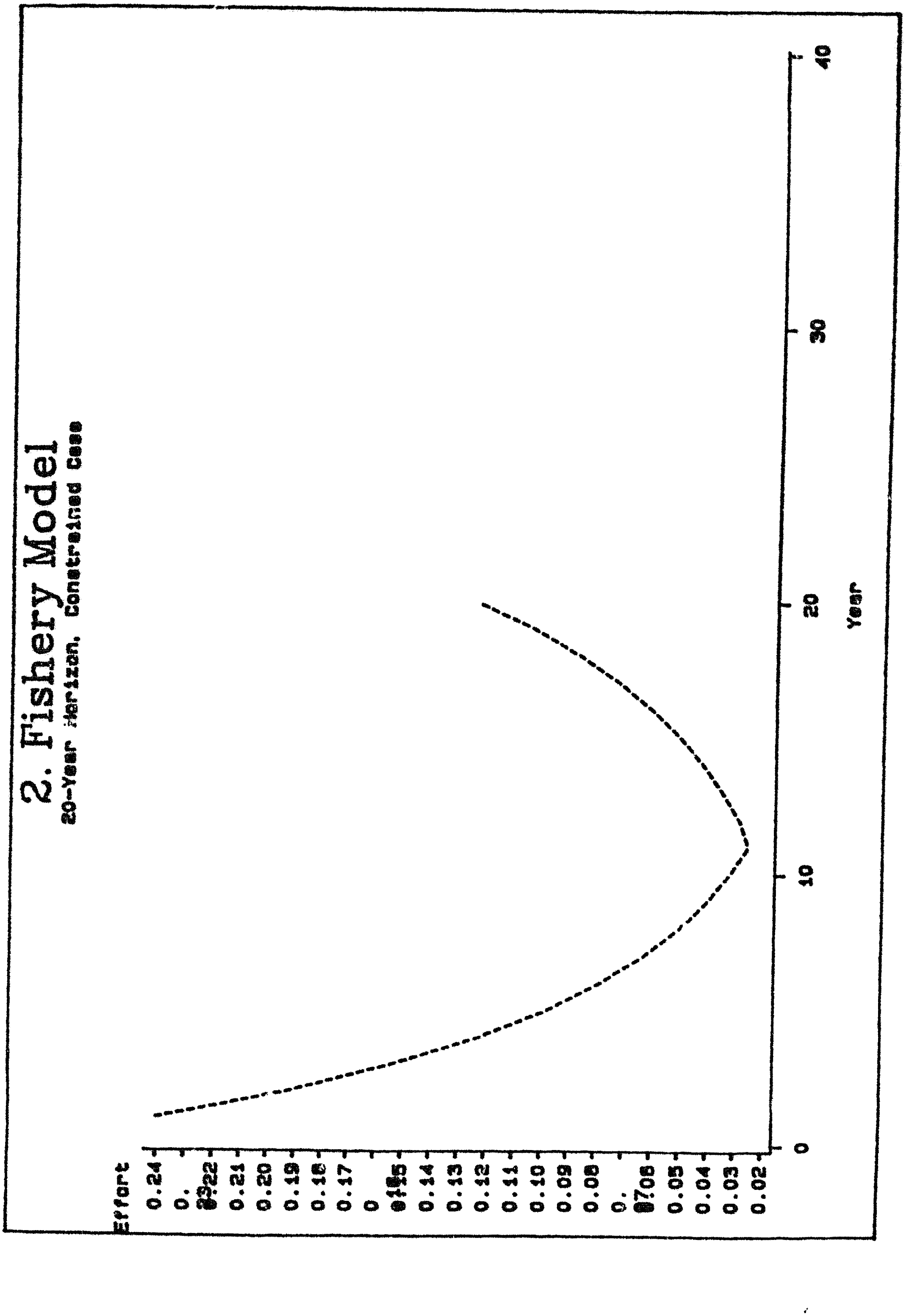
Year	Author's Results	Pindyck's Results
1975	13.26	13.24
1976	11.31	11.19
1977	10.47	10.26
1978	10.17	9.90
1979	10.13	9.82
1980	10.33	9.88
1985	11.06	10.84
1990	12.13	11.98
1995	13.84	13.18
2000	15.09	14.46
2005	15.79	15.92
2010	20.36	20.29
OPTIMAL GAIN	2,163	2,092

TABLE 2. COMPARISON OF OPTIMAL CARTEL PRICE TRAJECTORY FOR COPPER

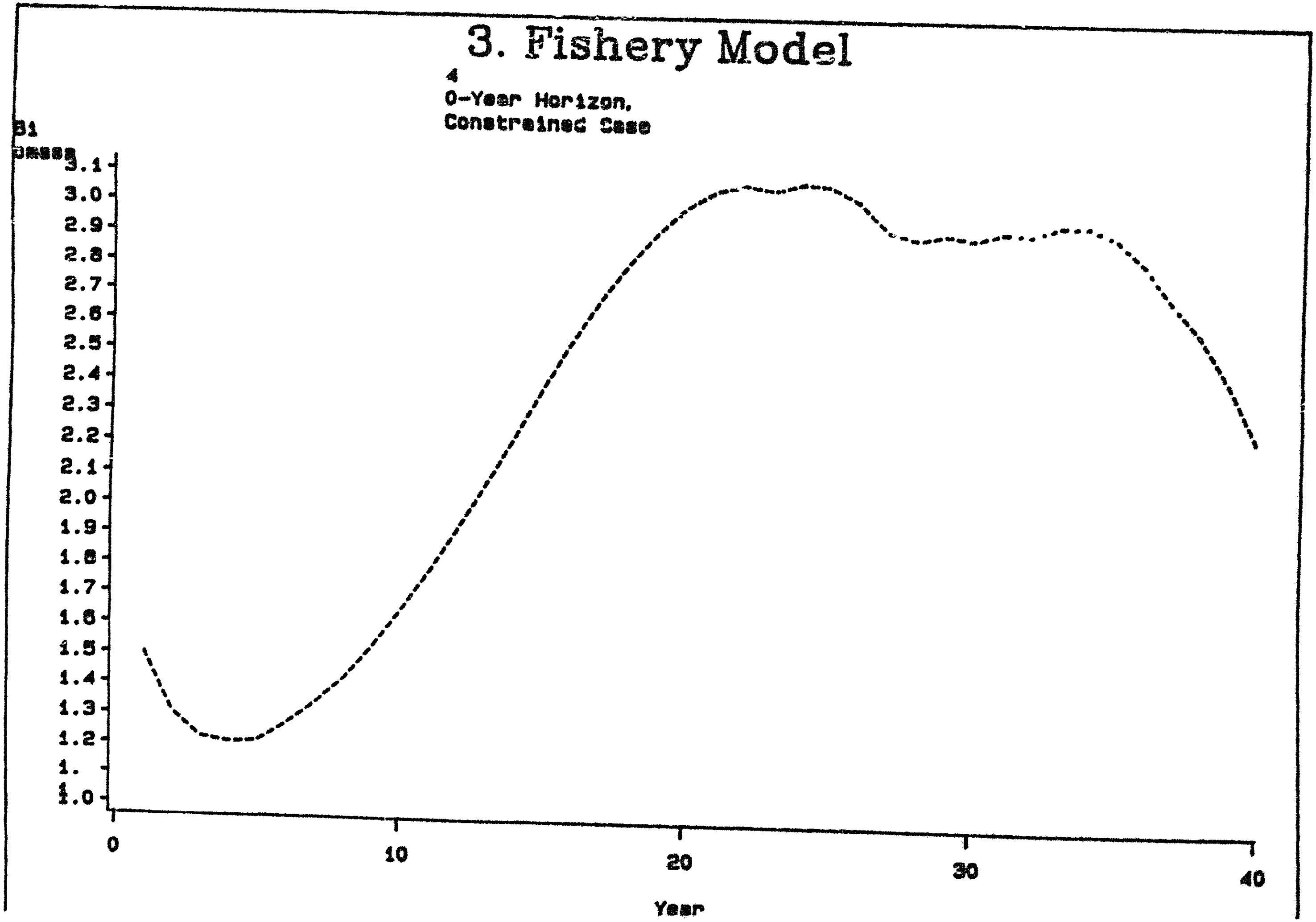
(Discount Rate = 5 per cent)

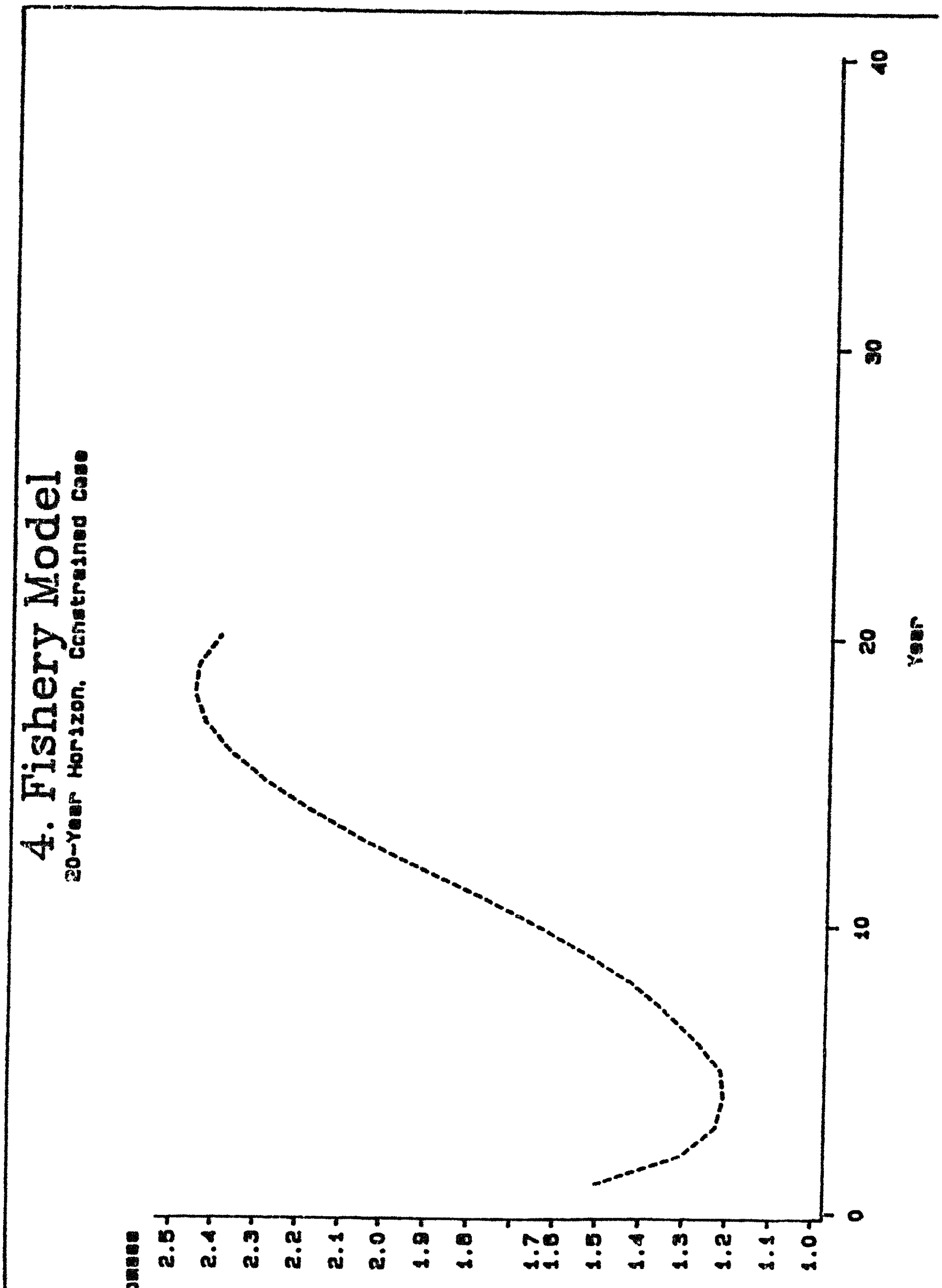
Year	Author's Results	Pindyck's Results
1975	1.01	1.23
1976	0.98	0.78
1977	0.94	1.02
1978	0.92	0.73
1979	0.92	0.98
1980	0.93	0.75
1985	1.00	1.01
1990	1.11	0.97
1995	1.45	1.15
2000	1.40	1.20
2005	1.59	1.36
2010	1.88	1.49
OPTIMAL GAIN	39,767	28,988


1. Fishery Model
40-Year Horizon, Constrained Case
Effort
0.24
0.23
0.22
0.21
0.20
0.17
0.15
0.14
0.13
0.12
0.11
0.10
0.09
0.08
0.07
0.05
0.04
0.03
0.02
0.01
0



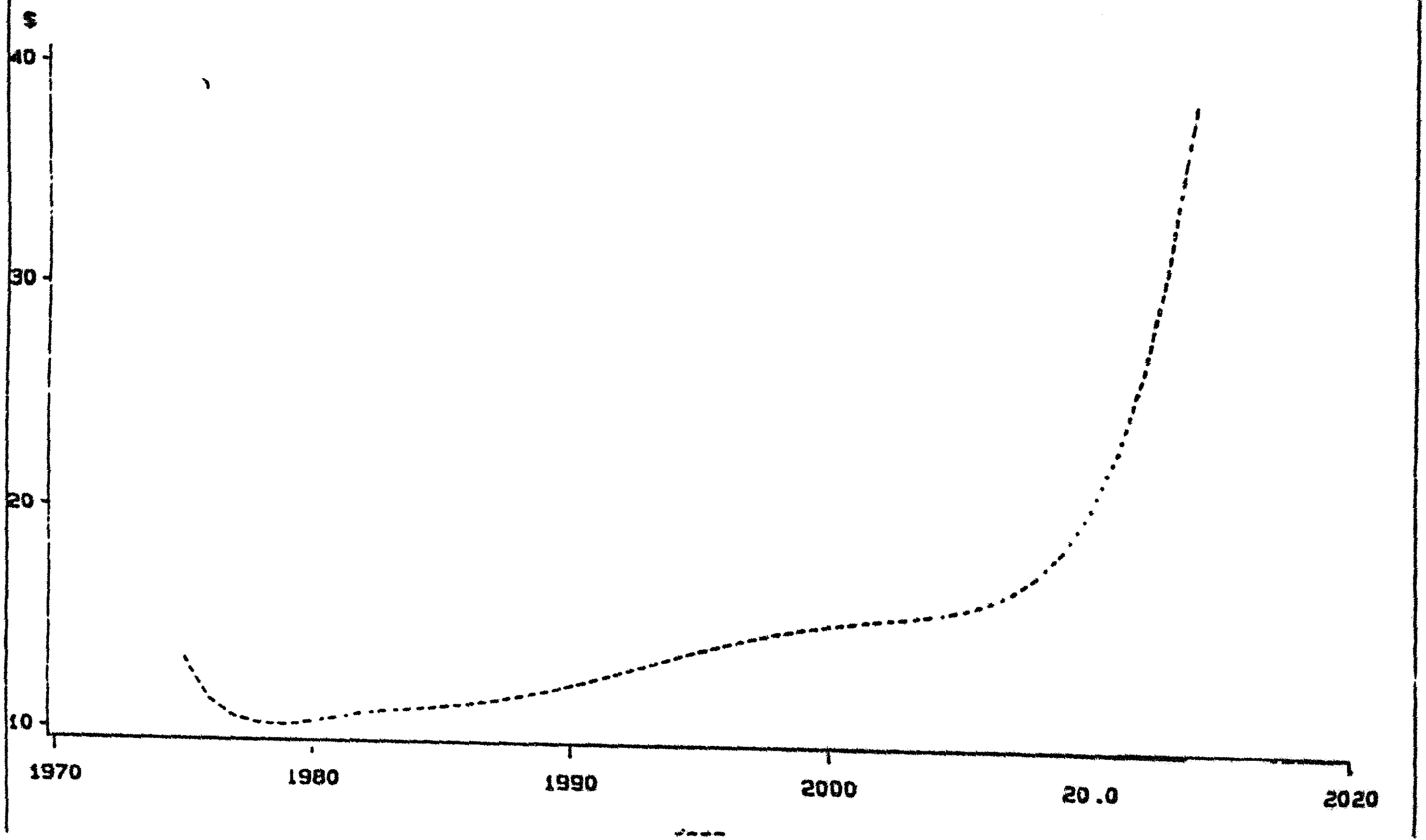
2. Fishery Model
20-Year Horizon, Constrained Case
Effort
0.24
0.22
0.21
0.20
0.19
0.18
0.17
0.15
0.14
0.13
0.12
0.11
0.10
0.09
0.08
0.06
0.05
0.04
0.03
0.02
0
10
20
30
40
Year

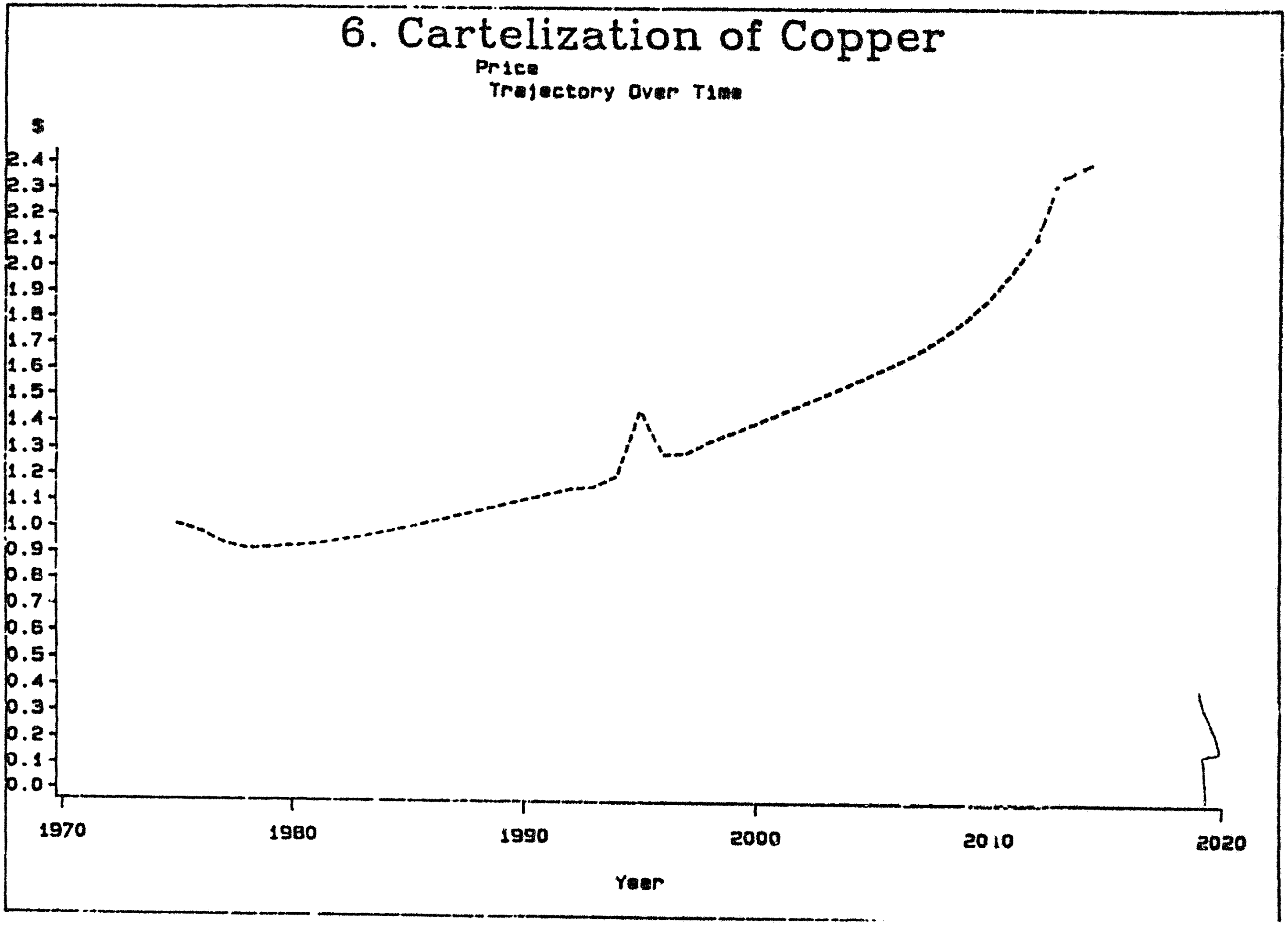

3. Fishery Model
4
0-Year Horizon,
Constrained Case
Bi
omass
3.1
3.0
2.9
2.8
2.7
2.6
2.5
2.4
2.3
2.2
2.1
2.0
1.9
1.8
1.7
1.6
1.5
1.4
1.3
1.2
1.
1.0
0
10
20
30
40
Year

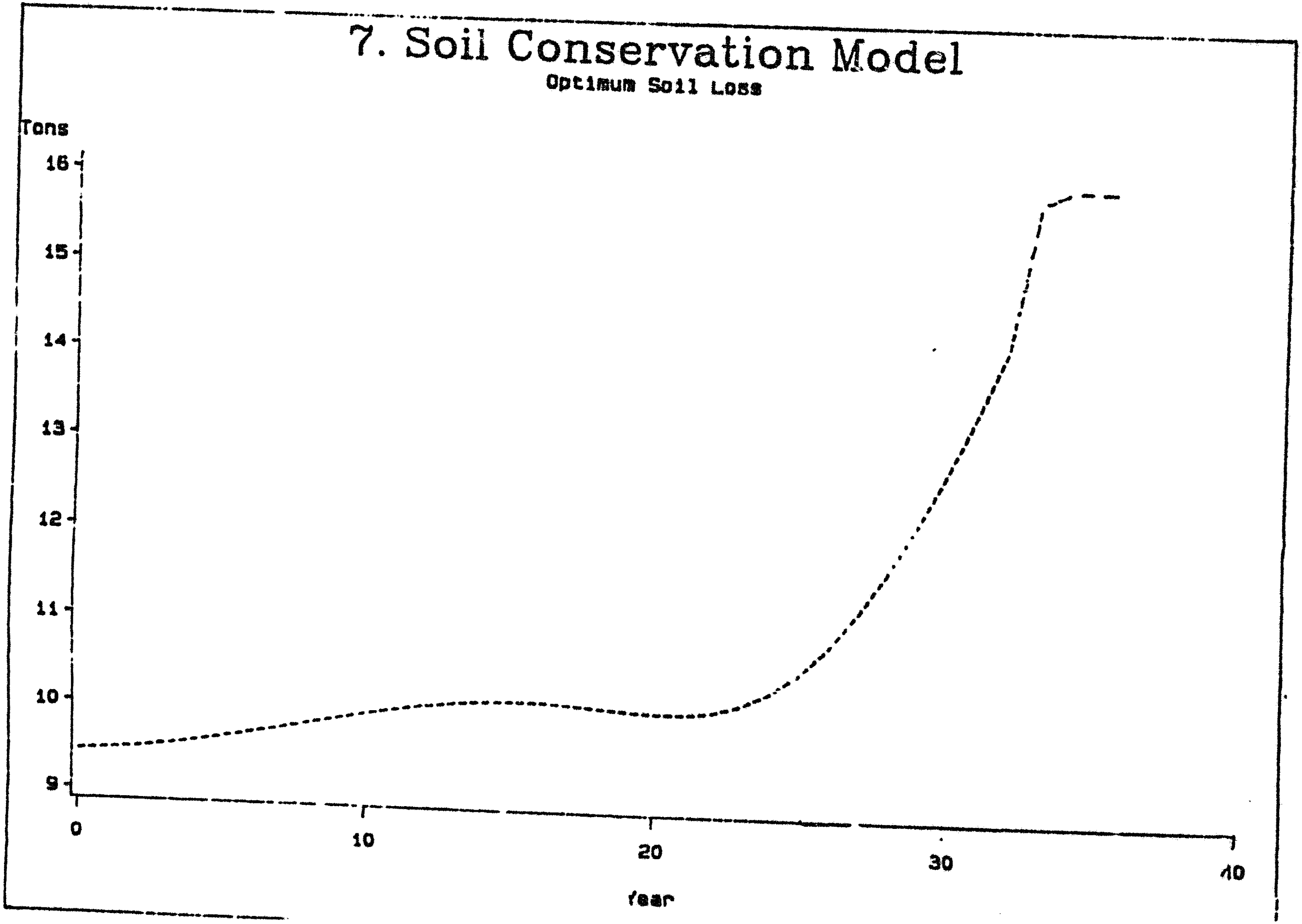

4. Fishery Model
20-Year Horizon, Constrained Case
Biomass
2.5
2.4
2.3
2.2
2.1
2.0
1.9
1.8
1.7
1.6
1.5
1.4
1.3
1.2
1.1
1.0
0
10
20
30
40
Year

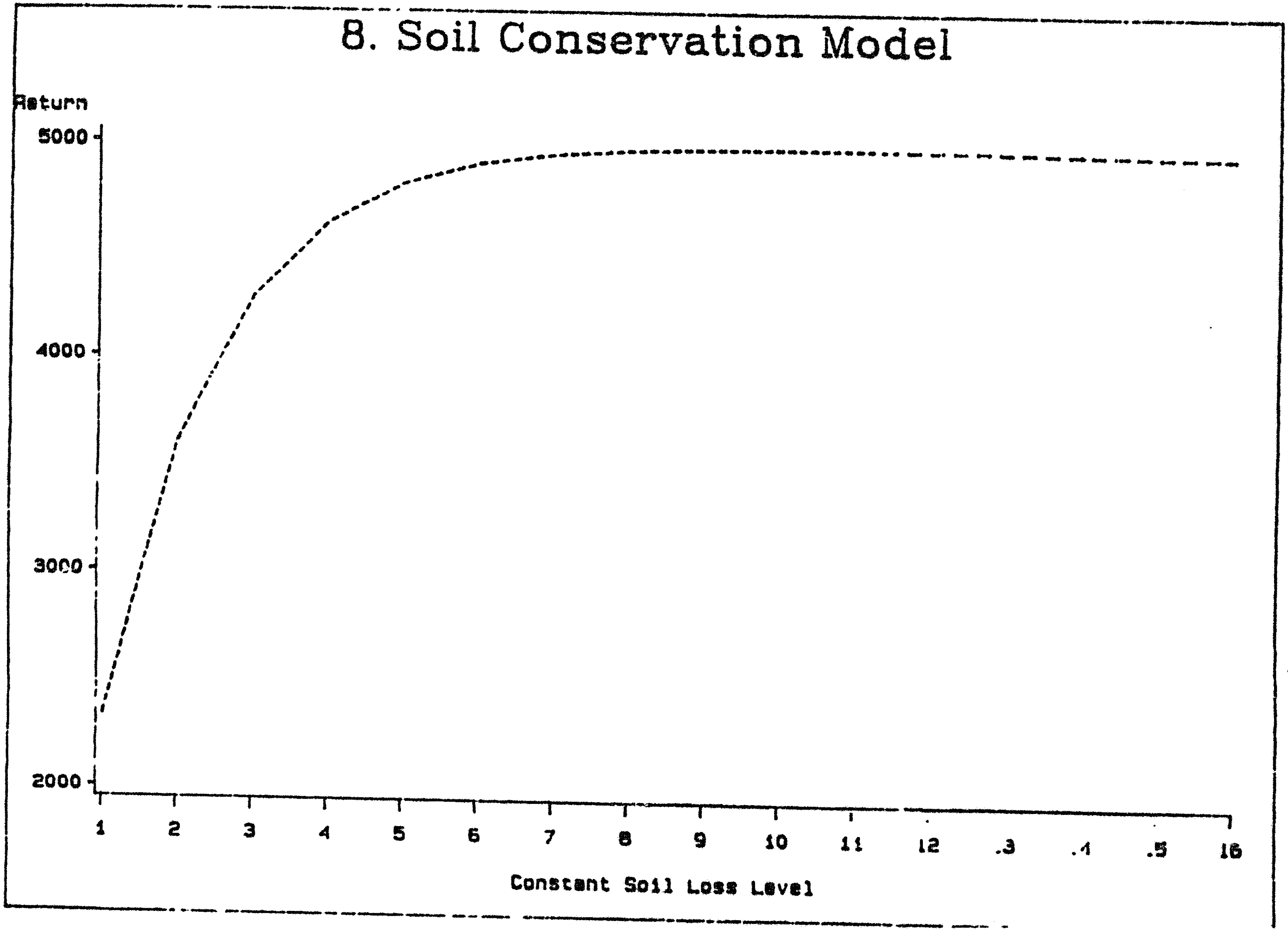
5. Cartelization of Petroleum

Price Trajectory Over Time
Discount Rate = 5%




6. Cartelization of Copper
Price
Trajectory Over Time
$
2.4
2.3
2.2
2.1
2.0
1.9
1.8
1.7
1.6
1.5
1.4
1.3
1.2
1.1
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
1970
1980
1990
2000
2010
2020
Year


7. Soil Conservation Model
Optimum Soil Loss
Tons
16
15
14
13
12
11
10
9
0
10
20
30
40
Year


8. Soil Conservation Model
Return
5000
4000
3000
2000
1
2
3
4
5
6
7
8
9
10
11
12
.3
.4
.5
16
Constant Soil Loss Level

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