ECONOMICS OF WILD OATS CONTROL:

AN APPLICATION OF A STOCHASTIC DYNAMIC PROGRAMMING MODEL

Sushil PANDEY

The University of Western Australia

Nedlands WA 6009

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1 INTRODUCTION

Weeds impose a considerable burden on the Australian farmer. Total losses due to weeds for 1985/86 have been estimated to be approximately $1500 million (Combellack 1987). In 1984, farmers in Australia spent over $220 million on herbicides (Blacklow, Pearse and Humphries 1984), a large proportion of which was for weed control in wheat. Despite such massive amounts of expenditure involved, current recommendations are based on a simplified concept of threshold cast on a deterministic framework.

The effects of uncertainty within the crop-weed-herbicide systems cannot be satisfactorily evaluated within such a simplified framework. Uncertainty arises mainly from the variability in the performance of control measures, the variability of weed density, the variability of the effects of weed competition on crop yields, and the variability of crop prices.

Benefits from weed control have a multiperiod dimension due to (a) the effects of current level of control on future buildup of infestation, (b) development of resistance, and (c) pesticide carryover. Even though current profits may not be adequate to recoup the current costs, some treatment may be justifiable if the possible prevention of future losses is also taken into account. Crop rotation decisions in many cases are governed by such long term considerations. Similarly, if resistance to herbicide is likely to develop, recommendations based on current period effects only will be suboptimal. Despite the importance of such long term effects, advice given to farmers is mostly ad hoc and is based on 'gut-feelings' rather than on a systematic analysis of the problem in a stochastic multiperiod framework. The level of control currently applied, hence, is unlikely to be optimal.

In this paper, optimal policy is obtained by taking into account the effect of current treatment on future buildup of infestation. The dynamically optimal solution is obtained by employing a dynamic programming model in conjunction with a bioeconomic simulation model. Uncertainties in the performance of herbicide and crop yields are modelled and optimal decision rules are derived for risk-neutral farmers. A continuous cropping system with wheat infested by wild oats (Avena fatua) is analysed.

2 AN ECONOMIC MODEL OF WEED MANAGEMENT

Consider a farm infested with a single species of annual weed spread uniformly over the farm. Assume the farm to represent a closed system in the sense that movement of weed seeds in and out of the farm is negligible. The farm may be large or small but is assumed to be managed by a single farmer. The economic output of the farm is the grain yield of an annual crop. The farmer wishes to maximise profits over a planning horizon of T periods. The profit function is assumed to be stationary in the sense that its parameters are constant over time. The decision problem is to derive an optimal strategy for chemical control of the annual weed. The solution is the degree of control applied in each time period such that the present value over the planning horizon is maximised.

The solution to the problem can be found by using the tools of the optimal control theory. Let $B(SD_t, X_t)$ define implicit profit in time period 't' as a function of the number of weed seeds (SD) and herbicide quantity (X). In the parlance of optimal control theory, SD and X are the state and decision variables, respectively. The change in seed number from one time period to the next depends on the initial seed number which determines the potential for seed production during the current time period and the quantity of herbicide used which determines the actual number of seeds produced. Let $G(SD_t, X_t)$ be a function measuring the change in seed number. The function G represents the equation of motion. Also, let $S(SD_T)$ represent the terminal value of the stock of weed seeds at the end of the planning horizon. The objective function is to maximise present value (PV)

[1] $$\text{Max PV} = \sum_{t=1}^{T-1} BV(SD_t, X_t)\delta^t + \delta^T S(SD_T) \, ,$$

$$\text{subject to } SD_{t+1} - SD_t = G(SD_t, X_t),$$

where δ^t is the discount factor for time period 't'. Applying Pontryagins' Principle of Maximum, one of the first order conditions requires that

[2] $$\delta^t \partial B/\partial X_t + \lambda_{t+1} \partial G/\partial X_t = 0; \quad t=0,1,\ldots,T-1.$$

The first term in equation [2] is the marginal current profit. If future effects are to be ignored, the static solution is obtained by solving the condition $\partial B/\partial X_t = 0$. The second term measures the marginal benefit resulting from the effects of current level of control on future infestation. The costate variable λ_{t+1} represents the marginal change in present value caused by a marginal change in the number of seeds at the beginning of time period 't'. An increase in number of seeds (and hence weeds) reduces profits; hence, $\lambda_{t+1} < 0$. Also, $\partial G/\partial X_t \leq 0$ because weed population is reduced by herbicides. The second term in equation [2] is hence non-negative. Due to the beneficial effect of the current level of control on future profits, the marginal productivity of control inputs is increased. This results in a higher level of control than when current profits are maximised.

3 DYNAMIC PROGRAMMING MODEL

Dynamic programming is a computationally efficient method for obtaining optimal control policies in a multiperiod context. The application of this method in models of agricultural resource management has been reviewed by Kennedy (1986). This method has been used for deriving optimal weed control strategies by Fisher and Lee (1981), Shoemaker (1982) and Taylor and Burt (1984). The advantage of dynamic programming is that risk can be incorporated relatively easily compared to other programming methods and globally optimal solutions can be found even if the objective function may be non-concave and discontinuous. The solution procedure consists of dividing the total lanning horizon into different periods (or 'stages' in the dynamic programming paralance) and deriving the optimal solution for each stage. The interdependence of decision between stages is captured by using the concept of the state. The state variables completely summarise the state of the system at the beginning of each stage. Thus the effects of decisions in one stage on the following stage is transmitted through the state variable. State variables need to be defined so that all information relevant to the current decision problem are captured by the state variables. This requirement of dynamic programming is called the condition of Markovian independence (Nemhauser 1966).

When a post-emergent herbicide is the control agent, weed density at the spraying time is one of the state variables. If seeds exhibit dormancy, as in the case of wild oats, the number of viable seeds in the soil is another state variable. However, weed density can be dropped out if recruitment is assumed to be a constant proportion of the size of the seed bank. Thus, in the deterministic model in which all stochastic variables are replaced by their mean values, the size of the seed bank is the only state variable. The stochastic model is properly specified as a two-state variables model. However, to save computation time, Taylor and Burt's (1984) decomposition method was used for solving the stochastic model. The decomposition procedure is explained in a later section.

The uncertain variables included in the model are herbicide performance and weed free yields. All other variables such as crop price, weed density, spray efficiency are assumed to be known with certainty. It has been the experience of farmers and researchers that the variability in herbicide performance is one of the dominant sources of risk in weed control. Similarly, variability in weed free yield as determined by climatic variability can make weed control decisions profitable or unprofitable.

The solution procedure involved in the dynamic programming model is described by the recursive equation:

$$[3] \qquad V_t\left(SD_t,W_t\right) = \underset{S_t}{\mathrm{Max}}\left[E\Pi\left(SD_t,W_t,S_t\right) + \beta EV_{t-1}\left(SD_{t-1},W_{t-1}\right)\right] \quad , \; t=1,2,\ldots,T,$$

where $V_t(SD_t,W_t)$ is the optimal value function at stage 't' given seed number (SD) and weed density (W); Π is the current profit if decision S is implemented; β is the discount factor, and E is the expectations operator. In accordance with the dynamic programming method, time subscripts are specified in reverse order. Thus, the last year of the planning horizon is labelled as stage 1, the second last year as stage 2, and so on.

The length of the planning horizon, T, may be finite or infinite. An approximate solution of a finite horizon problem can be derived by first solving the model for an infinite horizon such that $V_t = V_{t-1}$ and using the optimal decision rule corresponding to V_t for deriving solutions for a finite horizon problem.

In the deterministic model, weed density was dropped out as a state variable because weed density is assumed to be a constant proportion of seed number. The stochastic model was solved in two steps. First, the optimal value function for an infinite horizon problem was derived by dropping weed density as in the case of a deterministic model. In the second step, the optimal value function derived in the first step was substituted for V_{t-1} and an additional iteration solved. The recursive equation for the second step is written as:

$$[4] \qquad V_t\left(SD_t,W_t\right) = \underset{S_t}{\mathrm{Max}}\left[E\Pi\left(SD_t,W_t,S_t\right) + \beta EV_{t-1}\left(SD_{t-1}\right)\right] \; .$$

The two-state variable problem is solved in the second stage by specifying current profit as a function of weed density and seed number. Although V_{t-1} is specified as a function of SD_{t-1} only, the decision rule derived is not myopic because the dynamic effects of current decisions are reflected in the size of the seed bank which appears in V_{t-1}. It is also assumed that weed density in the current period does not have any significant effect on weed density next period. This is a reasonable assumption because any such effect is likely to be dominated by the size of the seed bank.

For deriving numerical solutions, the state and decision variables were represented by 63 and 17 discrete values, respectively. The decision alternatives considered are different doses (including non-use) of Hoegrass. For each starting value of the state variable, profits and ending value of the state variable were calculated for all discrete decision alternatives. For the ending value of the state variable falling between the two grid points, the optimal value function was approximated by linear interpolation between the adjacent grid points.

Risk is introduced in the model through random variables. Two random variables are required to incorporate risks in herbicide performance and weed free yields. In the estimated weed kill function, herbicide performance was found to depend on soil moisture condition which can take any of the three ranked values. A discrete probability distribution for the soil moisture condition was derived by analysing the climatic data. Values from this distribution were selected by Monte-Carlo sampling. In the case of weed free yield, the simulated values were used directly as a sample from its distribution.

When more than one random variable is incorporated correlation between the variables also needs to be considered. In the present study, a positive correlation is expected because both the weed free yield and the herbicide performance vary directly with soil moisture content. The correlation is built into the model by adjusting the sampling procedure. The simulated data on weed free yields were first classified into three groups corresponding to good, average and poor moisture conditions. Within each category, the probability distribution of yield was represented by a discrete cumulative distribution function. When the sampled moisture condition was good, yields were sampled from the probability distribution of yield corresponding to good soil moisture condition. The same procedure was applied to soil moisture conditions in other categories.

4 BIOECONOMIC SIMULATION MODEL

A bioeconomic simulation model was developed to trace the effect of weed control decisions on both the current and the future profits. The overall model is comprised of submodels for weed population dynamics, yield response to weed infestation, weed kill function, weed free yield of crop, and climatic and economic factors.

A life cycle model of wild oats is used for predicting seedling recruitment, plant survival, seed production and seed survival. The recruitment of wild oats is not synchronised and occurs in waves during its life cycle (Quail and Carter 1968, Amor 1985). If a non-residual post-emergent herbicide is the control agent, it would be useful to divide seedlings into three cohorts. Plants which emerge before sowing belong to cohort 1. The cohort 2 has weeds emerging after sowing but before the post-emergent herbicide is applied. The third cohort has weeds emerging after the application of post-emergent herbicide.

The seed bank is assumed to be homogeneous. The recruitment in each cohort is specified to be a constant proportion of the seed bank. Seedlings in cohort 1 are assumed to be killed by presowing cultivations. Due to the competitive effects exerted by seedlings upon each other, only a proportion of seedlings survive to maturity. Empirical evidence indicates that most of the cohort 2 seedlings which die before maturity are dead before the biologically appropriate time for post-emergent herbicide application (Medd 1988, personal communication). Thus, it is assumed that the full effect of density-dependent mortality is realised before the application of post-emergent herbicides.

The number of seeds produced by mature plants is also density dependent and is described by a hyperbolic function. Some proportion of new seeds produced is assumed to be removed by the combine and and killed by straw burning. Also, a proportion of the existing seed bank is lost due to natural mortality. Thus, the seed dynamics can be described by the following identity:

[5] $$SD_{t+1} = SD_t - G_t - M_t + N_t$$

where SD_{t+1} is the size of the seed bank at the start of the period t+1, SD_t is the starting stock of seed bank, G is the loss due to recruitment, M is the loss due to mortality, and N is new seed added to the seed bank. Most of the parameters for the model are obtained from experimental work at Orange, NSW (Medd and Ridings 1988). Values of the parameters unavailable from this source were obtained from experimental work in the United Kingdom (Cousens et al 1986).

The yield (Y) of a weedy crop is specified as

[6] $$Y = Y^* g(W)$$

where Y^* is a parameter representing the maximum attainable yield in a weed free situation given the level of environmental and management inputs, W is the weed density, and $g(\cdot)$ is a function often called the 'relative yield response' (Lanzer and Paris 1981). By definition, $g(0) = 1$ and $g(\infty) = c^*$, where $c^* \geq 0$. Thus the function $g(\cdot)$ provides a scaling factor. It is usual to represent $g(\cdot)$ in the case of pests by a linear or sigmoidal function of pest density (Feder 1979, Zimdahl 1980). However, in the case of weeds, $g(\cdot)$ is more accurately represented as a hyperbola (Cousens 1985). The specific form used is:

[7] $$g(W) = 1 - W/(a^{-1} + Wb^{-1})$$

where W is the weed density and 'a' and 'b' are the parameters. Since crops and weeds exert competitive effects on each other, yield loss due to weeds also depends on crop density. Based on Australian data, Martin, Cullis and McNamara (1987) found 'a' to be proportional to crop density. Their parameter estimates have been used in the present study.

Dose response function relates the quantity of herbicide applied to the proportion of weeds killed. The dose response relationship is typically sigmoidal when plotted on an appropriate scale and has the properties of a probability distribution function (Finney 1971, Lichtenberg and Zilberman 1986). Since the response to herbicides is a binomial variable (with the plants being considered as dead or alive), probit and logit regressions are the appropriate methods for efficient estimation of dose response relationships (Finney 1971, Hewlett and Plackett 1979). The logit specification is used in the present study. The dose response relationship for Hoegrass was specified as:

$$[8] \qquad \log\left[P_i/(1 - P_i)\right] = \alpha_0 + \alpha_1 \log X_i + \alpha_2 SM_i + \alpha_3 A_i + u_i$$

where P is the proportion of weeds killed, X is the quantity of herbicide applied, SM and A are measures of soil moisture and additives which also affect the performance of herbicides, and u is the random disturbance term. All parameters except α_0 are expected to be positive.

Experimental trials on the control of wild oats by Hoegrass conducted by Hoechst in Western Australia, Victoria and New South Wales were used in this study. The data were adjusted for the effects of natural mortality using Abbott's formula (Finney 1971). Based on the description of the season in the trial report as good, average and dry, the soil moisture conditions was rated as 3, 2 and 1 respectively. The variable 'A' was specified as a dummy (A=1 if wetting agent added, A=0 otherwise).

Weed free yield of wheat was predicted by using a wheat growth simulation model developed at the West Australian Department of Agriculture. The model uses daily climatic data as inputs. Wheat yields for 74 years from 1912 to 1985 were predicted for Merredin by using the simulation model. Since all management-specific inputs are assumed non-limiting in the simulation, predicted yields were adjusted to reflect the level of input usage in the region.

5 RESULTS AND DISCUSSION

For deriving a deterministic solution, the weed free yield and the proportion of weeds killed were set at the respective average values. The optimal decision rule derived by solving the infinite horizon problem is presented in Figure 1. Although the number of seeds in the soil is the

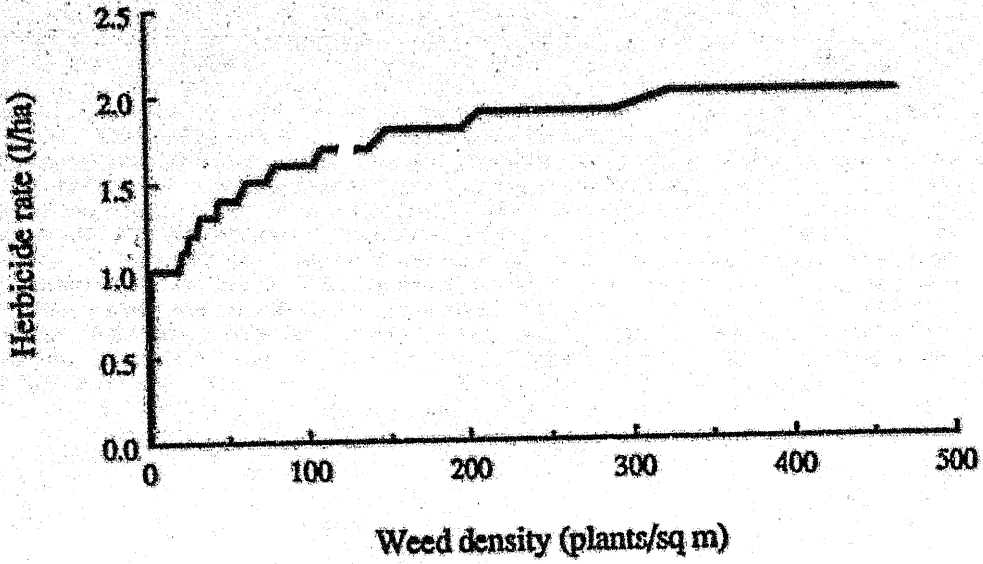

Figure 1: Optimal)ecision Rule (Deter istic)

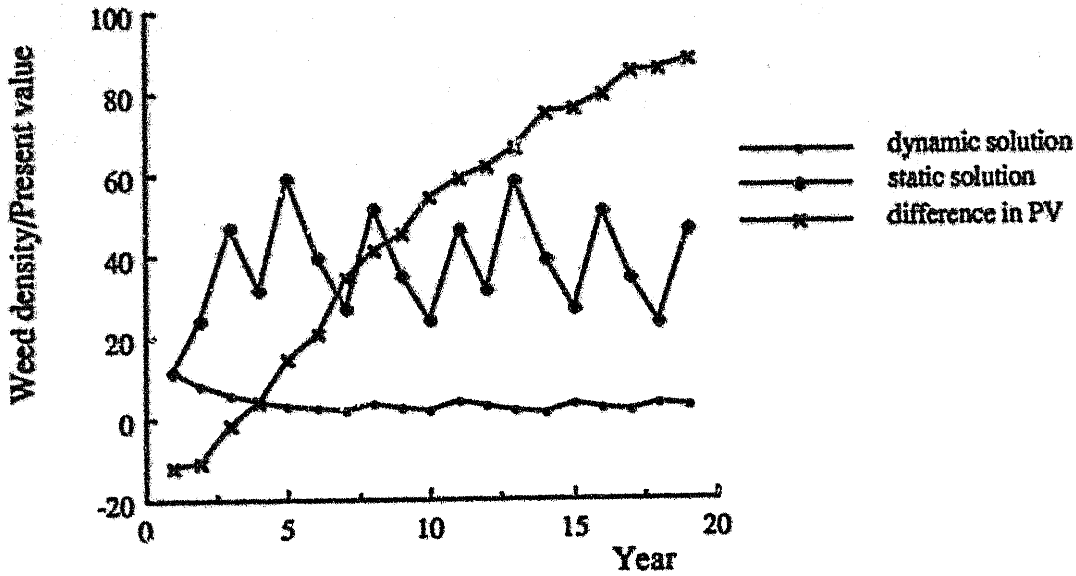

Figure 2: Optimal Trajectory of Weed Density
(initial seed number 100)

state variable, results are presented in terms of weed density. The decision rule illus'rated is quite simple and can be a useful decision tool. The optimal :ose of herbicide depends on weed density and is updated as weed density changes over time.

The time traces of weed number when the optimal decision rule is applied repeatedly are shown in Figures 2 and 3. These traces were derived

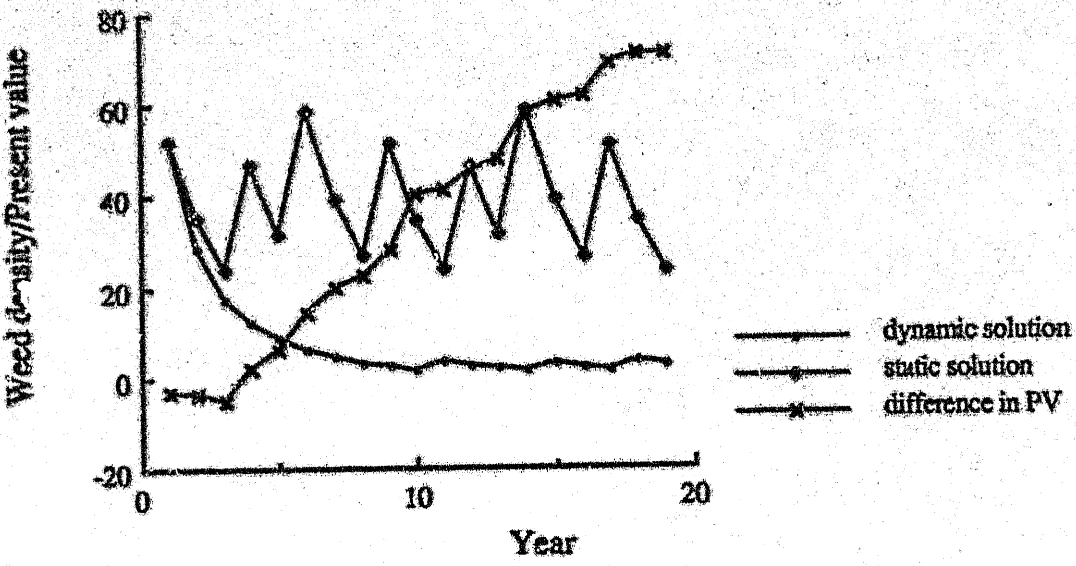

Figure 3: Optimal Trajectory of Weed Density (initial seed number 5000)

for initial seed numbers 100 and 5000, representing respectively low and high infestations. Shown in the diagrams are also the resulting time trace when policies which maximise profits in each time period are applied (ie, time trace corresponding to the static solution). The weed density corresponding to the approximate steady-state is lower in the case of dynamic optimisation. Hence, at the steady-state, profits are higher under dynamic optimisation. The present value of profits is also higher under dynamic optimisation and the gain over the static solution increases over time. The results show that, in the case of wild oats, significant gains can be realised in future periods by reducing the weed burden early in the planning horizon even if current gains from such actions might be negative. It is also observed that weeds are not eradicated at the optimal steady state.

In the stochastic version, the solution procedure is conceptually similar except that transitions are probabilistic. The stochastic solutions were derived using the two-step method described earlier. Farmers were assumed to maximisе expected profits. The results are presented in Figure 4.

The optimal decision depends on weed density as well as on the size of the seed bank. For a given weed density, the optimal herbicide dose decreases with an increase in the size of the seed bank. This is expected because the size of the seed bank can be manipulated by varying

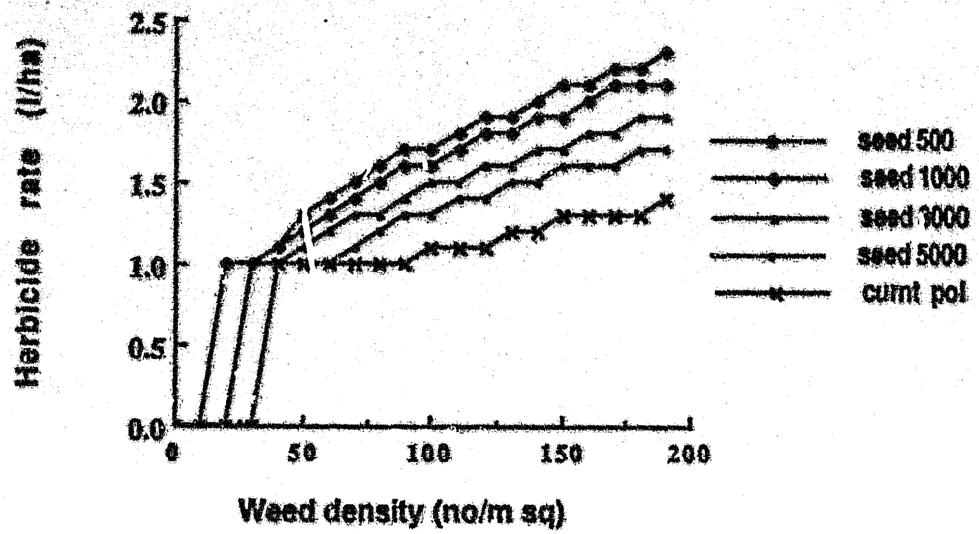

Figure 4: Optimal Herbicide Rate According to Seed Bank and Weed Density

the herbicide quantity only when there are a few seeds in the soil to start with. On the other hand, when the initial seed bank is large, varying the addition to the seed bank by killing more weeds is unlikely to affect the seed bank substantially. If the seed bank is very large, the dynamic solution will approach the static solution.

The economic threshold at which herbicide is applied decreases with a decrease in the seed bank. When the seed bank consists of about 500 seeds, the dynamic economic threshold is approximately 10 weeds/m^2. The static economic threshold is approximately 30 weeds/m^2.

If farmers are assumed to be risk averse, then both the mean and variability in profits enter the objective function. If herbicide performance is assumed to be stochastic but the weed free yield is deterministic, an increase in herbicide will reduce the variance. Thus a risk averse farmer would tend to apply more herbicide compared to a risk neutral farmer. If herbicide performance is assumed to be deterministic but the weed free yield is stochastic, the opposite result will hold because the variability of profits increases with an increase in herbicide rate. The expected direction of change is ambiguous if both the weed free yield and herbicide performance are assumed to be stochastic simultaneously.

6 SUMMARY AND CONCLUSIONS

In this paper dynamically optimal dose of a post-emergent herbicide was derived using the method of stochastic dynamic programming. A bioeconomic simulation model was used to generate return matrix and transition probabilities. The results indicate that the dynamically optimal solution is to maintain a lower steady-state weed population in comparison to the static solution. The dynamic economic threshold is also lower than the static econoaic threshold.

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