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Empirical Estimates of Cost Functions for Mixed Feed Mills in the Midwest

By Richard Phillips

Plants in agricultural marketing and processing industries, generally speaking, function in an imperfectly competitive environment and therefore do not necessarily face perfectly elastic supply functions for inputs, or perfectly elastic demand functions for outputs. Relevant coefficients in cost functions of such firms often are dependent on supply as well as production coefficients. Indeed, in contrast to farm production, here the more interesting relationships may arise from price functions rather than production functions. Determination of reliable cost-function estimates under such conditions presents a real challenge to researchers. The accompanying discussion of the estimation of empirical cost functions in such firms is published as journal paper number J–2860 of the Iowa Agricultural Experiment Station, Ames, Iowa, project number 1224.

THE PROBLEM of determining reliable cost functions may be approached either (1) by budgeting from relevant production and price data or (2) by observing cost and volume data from a representative sample of operating firms. Much could be said about the relative merits of each and the criteria for selecting the more efficient for a given situation, but discussion here is limited to the second approach. Suffice it to say that the budget approach has many advantages when the relevant coefficients can be accurately ascertained, especially with the application of linear programming or related techniques.

In estimating costs from observed cost and volume data the typical procedure is to estimate a long-run average cost function by observing, for a single period of time, plants that operate at varying volumes of output. Although the procedure is legitimate if it is based on accurate data from an efficient representative sample of plants in the industry under study, several questions arise as to models and methods of analysis used. A recent study of feed-mixing costs made in the Midwest by Iowa State College under contract with the Agricultural Marketing Service illustrates some of the questions.[1]

Purpose and Design of Study

The Iowa study was designed to obtain information regarding volumes and costs of feed mixing in plants selected to represent a large range in operating tonnage. The population of feed-mixing plants in Iowa and surrounding States was

[1] This article is based on the final unpublished report under the contract, PHILLIPS, RICHARD, HARRINGTON, DAVID N., AND SCOTT, J. T. RELATIONSHIPS BETWEEN VOLUME AND FEED MIXING COSTS IN SELECTED PLANTS IN THE MIDWEST.

For a report of the overall study by the Agricultural Marketing Service, see BRENSIKE, V. JOHN, AND ASKEW, WILLIAM R. COSTS OF OPERATING SELECTED FEED MILLS AS INFLUENCED BY VOLUME, SERVICES AND OTHER FACTORS. Marketing Research Report 79. Agricultural Marketing Service, U. S. Department of Agriculture, Washington, D. C. 1955.

stratified by volume and the several strata were sampled randomly at differential rates to insure an adequate volume distribution in the sample plants. Data were obtained from a total sample of 36 feed-mixing plants.

The study stressed the production and overhead costs of mixing feed. It did not consider such items as costs of ingredients and other raw materials, nor was it concerned with revenues and operating margins in the mixed feeds industry. It was directed toward the operation of feed-mixing *plants*, not toward the size, organization, or functions of the several types and varying sizes of feed *firms*. It was designed to establish the relationship between total mixing volume and cost efficiency in feed mixing, taking into account the degree of capacity utilized. In addition to the data on annual costs, volume, and capacity, monthly figures for production and major operating costs were obtained for many of the plants studied.

Annual Feed-Mixing Volume, Capacity and Cost

A summary of the annual data obtained by personal interview from the 36 sample feed-mixing plants is given in table 1. In this analysis the labor costs of obtaining ingredients, selling finished feeds, and trucking, were excluded also. Thus, the total cost figures include only wages and salaries in feed production and office, and overhead salaries chargeable to production, depreciation, heat, light and power, insurance, maintenance and repairs, interest, rent, and other items of expense. The volume of feed mixed includes the total tonnage of all types of feed mixed at each plant but excludes millfeeds, oil meals, and other ingredients bagged at or handled through the plant but not mixed there. The figures on unused capacity were computed by subtracting the actual production from the possible annual production for each plant if operated at capacity for 52 135-hour weeks.[2] The per ton costs shown in table 1 were obtained by dividing the total cost by the tonnage mixed without regard to unused capacity.

Dependable accuracy in the data for all plants is difficult to achieve in a study of this kind. Even though the information was taken by researchers directly from feed-plant accounts, seven of the plants shown in table 1 were excluded from the analysis because of questionable accuracy in the data. Four of the small plants (numbers 29, 31, 33, and 35) were excluded because the lack of separate accounts for the feed-mixing department meant that the plant managers had to do considerable estimating in order to allocate joint cost

[2] Actual peak performance in the past rather than rated capacity was used to establish the capacity weekly output.

TABLE 1.—*Annual volume and cost of feed mixing and unused capacity, by plants, Iowa and surrounding States*

Number of plant	Feed mixing			Plant capacity	
	Volume mixed	Cost: Total	Cost: Per ton	Unused	Percentage used
	Tons	*Dollars*	*Dollars*	*Tons*	*Percent*
1	1, 877	31, 002	16. 52	21, 523	8. 0
2	2, 191	21, 277	9. 71	7, 169	23. 4
3	6, 922	62, 097	8. 97	58, 910	10. 5
4	7, 856	76, 276	9. 70	27, 244	22. 4
5	8, 464	107, 442	12. 69	52, 948	13. 8
6	10, 818	102, 691	9. 49	82, 782	11. 6
7	13, 122	104, 437	7. 96	38, 046	25. 6
8 [1]	14, 326	177, 988	12. 42	NA	NA
9	18, 746	95, 046	5. 07	86, 554	17. 8
10	17, 103	175, 000	10. 23	26, 837	38. 9
11	18, 999	296, 826	15. 62	176, 001	9. 7
12	20, 000	189, 500	9. 47	54, 240	35. 6
13	23, 152	315, 118	13. 61	223, 744	9. 4
14	24, 848	238, 167	9. 58	115, 552	17. 7
15	26, 472	253, 169	9. 56	78, 828	25. 1
16	35, 271	228, 524	6. 48	1, 285	96. 5
17	40, 729	299, 996	7. 37	169, 871	19. 3
18 [1]	63, 813	841, 647	13. 19	12, 263	83. 9
19	59, 833	489, 495	8. 18	27, 454	68. 9
20	69, 219	441, 552	6. 38	141, 381	32. 9
21	66, 628	502, 412	7. 54	58, 692	52. 3
22	64, 783	517, 929	7. 99	119, 453	35. 2
23	[2]	[2]	8. 30	[2]	43. 0
24	[2]	[2]	7. 77	[2]	24. 2
25	466	7, 071	15. 17	20, 594	2. 2
26	555	4, 470	8. 07	7, 141	7. 2
27	358	4, 651	12. 99	6, 662	5. 1
28	678	9, 648	14. 23	13, 310	4. 8
29 [1]	915	5, 058	5. 53	13, 125	6. 5
30	1, 789	14, 828	8. 29	29, 827	5. 7
31 [1]	2, 500	12, 164	4. 87	11, 540	17. 8
32	554	6, 377	11. 51	6, 466	7. 9
33 [1]	1, 013	7, 346	7. 27	13, 027	7. 2
34	1, 500	16, 993	11. 33	5, 520	21. 4
35 [1]	1, 030	5, 355	5. 20	30, 586	3. 3
36 [1]	9, 885	39, 168	3. 96	18, 195	35. 2
Average	20, 762	173, 094	8. 34	78, 163	--------

[1] Plants excluded for reasons given in text.
[2] Large mills—data used but not shown in order to preserve plant identity.

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[2] Actual peak performance in the past rather than rated capacity was used to establish the capacity weekly output.

items. Plant number 8 was excluded because the lack of capacity performance in the past made it difficult to ascertain actual plant capacity. For plant 18, the accounting system did not permit an accurate separation of salaries in sales and other nonproduction activities from those in feed mixing. Plant 36 was excluded because some of the monthly production records had been lost so the manager had to estimate these figures in order to arrive at the annual production figure. The cost, volume, and capacity figures for the remaining 29 plants appeared to be accurate.

Alternative Models for Analysis of Annual Feed Plant Data

It is clear that the determination of the regression of the cost of mixing on some measure of the volume of mixing is appropriate, and likely to be more fruitful, than any tabular analysis of the data in table 1. But even in a problem as simple as this, questions arise as to both the selection of the regression model and the economic interpretation of the statistical results obtained.

Regression of costs on mixing volume.—Perhaps the most common and most simple procedure would be to fit a simple regression of total mixing cost per ton on total tonnage mixed. From the observed data for the 29 plants (table 1), it appears that a reasonably good fit could be obtained in this way. The average cost function would be one that decreases rapidly with increases in output for small outputs and flattens out substantially at the larger outputs. A model of the inverted type $\left[Y=a+b\left(\frac{1}{x}\right)\right]$ or a logarithmic model probably would be most suitable. The resulting simple correlation coefficient probably would be in the neighborhood of 0.6. If a simple regression of total (rather than per unit) mixing cost against mixing tonnage were fitted to the same data, the simple correlation coefficient probably would be about 0.9.

If correctly interpreted, a simple regression model such as this may be useful, as it shows the relationship between output and costs during the period studied. It is not an appropriate estimate of the long-run average cost function. Rather it shows relative per unit costs at various outputs, regardless of the size of plant that produces the output. As it does not consider the position of each plant on its short-run cost function for the period studied, this method provides an approximate estimate of the long-run average cost function only when observed plant size and plant output are perfectly correlated. It correctly estimates the long-run average cost function only when each plant studied is observed at a point on its short-run cost function that is tangent to the long-run average cost function.

It is possible to conceive of instances wherein the simple regression of cost on volume is more appropriate for the purpose at hand than an estimate of the long-run average cost function. But if generalized to periods other than the one studied, the former procedure has serious limitations as the short-run outputs observed are likely to change over time. Notice in table 1 that the plants with the smallest outputs are predominantly those with the smallest percentage use of plant capacity and those with the largest outputs are mostly those with the largest percentage utilization of plant capacity. If this situation were to be reversed in the year following the study, the regression line of cost on volume would be flatter than the one for the period studied. This would be true even though there were no change in the long-run average cost function between the 2 years.

Addition of the capacity variable.—One method of estimating more accurately the long-run average cost function from empirical data, such as those in table 1, is to adjust the observed cost and volume for each plant to full utilization of plant capacity. The simple regression of per unit costs on volume obtained from such adjusted data will provide an estimate of the long-run average cost function.[3] But this procedure requires not only an accurate separation of fixed and variable costs, but also a detailed separation, item by item, of cost elements that vary directly, but not proportionately, with output.[4]

[3] This estimate of the long-run average cost function will pass through the low points of the short-term average cost functions rather than the true tangency points. The seriousness of this divergence will depend on the shape of both the long-run and the short-run cost functions.

[4] Another disadvantage of this procedure arises when industry acceptance and utilization of the research is desired. This is the general questioning by trade people of adjustments in observed data.

By using a multiple regression model with some measure of capacity utilization as a second independent variable, the same end can be achieved without adjusting the observed data. Such a model considers the maintenance of idle plant capacity as an output that affects production costs apart from the cost of producing the output of the product. It thus has the effect of shifting each plant along its short-run average cost curve to its optimum short-run output in the determination of the long-run regression coefficient of cost on output.[5] The nature of the short-run average cost curve is specified by the multiple regression model itself.

Figure 1 illustrates such a model for an individual plant of OB capacity producing OA units of output. In this instance, the per unit cost incurred, OD, consists of two segments—cost per unit for OA units of maintaining AB units of idle capacity, CD, and cost per unit of output if no idle capacity were maintained, OC. If this plant fell on the regression line in both instances, a simple regression of per unit cost on output would pass through point S. The net regression of per unit cost on output in a multiple regression equation containing an unused capacity variable would pass through point T.

Average versus total cost models.—As in the case of the simple regression of cost on volume, either an average or a total cost equation can be used for the multiple regression model including idle plant capacity. But one must be sure that they are so stated as to be comparable models if they are to result in comparable parameter estimates from the same data. For example, a total-cost model with total cost taken as a function of tons mixed and tons of idle capacity is comparable to an average-cost model with cost per ton taken as a function of tons mixed and the ratio of idle capacity to feed output. It is not comparable to an average-cost model with cost per ton taken as a function of tons mixed and tons of idle capacity.

Regression coefficients obtained by comparable average cost and total cost models fitted to the same data are not directly comparable in any case. If the true average cost curve approaches a straight horizontal line, it is possible to obtain

Figure I.

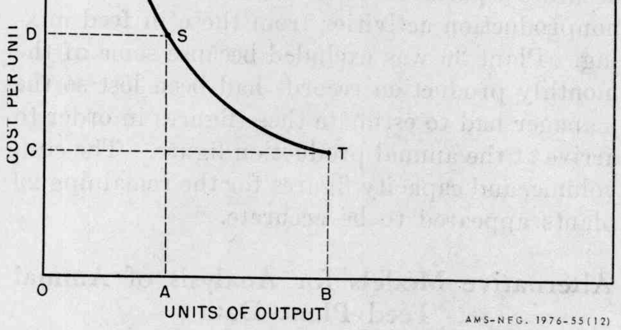

ILLUSTRATION OF THE BREAKDOWN OF OBSERVED TOTAL COST PER UNIT FOR A PLANT OPERATING AT LESS THAN FULL CAPACITY

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an R^2 of near 1.0 with a total-cost model and an R^2 of near 0 with a comparable average-cost model fitted to the same observed data. The reverse would be the case if the true total-cost curve approached a straight horizontal line. As the former is more reasonable over any substantial range, and more nearly the usual empirical situation, one can normally expect to obtain higher R^2's with total cost models than with average cost models. Statistical tests of significance need to be interpreted with this in mind.[6]

Selection of the total-cost function—the volume variable.—When plotted against feed output on arithmetic paper, the total mixing costs shown in the third column of table 1 show a slight curvature that is characteristic of other cost studies of the agricultural marketing industries at Iowa State College and elsewhere. Total costs of mixing appear to increase at a slightly decreasing rate as output increases.[7] The curvature is so slight that a linear regression of total cost on the output variable will provide a reasonably good fit.

If the solution with this linear model results in a positive Y-intercept of reasonable magnitude, the corresponding average cost function will be nonlinear, decreasing at a diminishing rate as output increases (the apparent relationship in table

[5] The divergence pointed out in footnote 3 applies here as well, but this causes little or no difficulty with the family of cost curves shown in figure 2.

[6] David Harrington plans to develop, more fully, comparisons of average- and total-cost models as applied to costs of grain storage in a thesis problem for the Ph.D. degree at Iowa State College. Mr. Harrington is on the staff at the University of Missouri.

[7] In other words, mixing costs per ton appear to decrease at a diminishing rate as output increases.

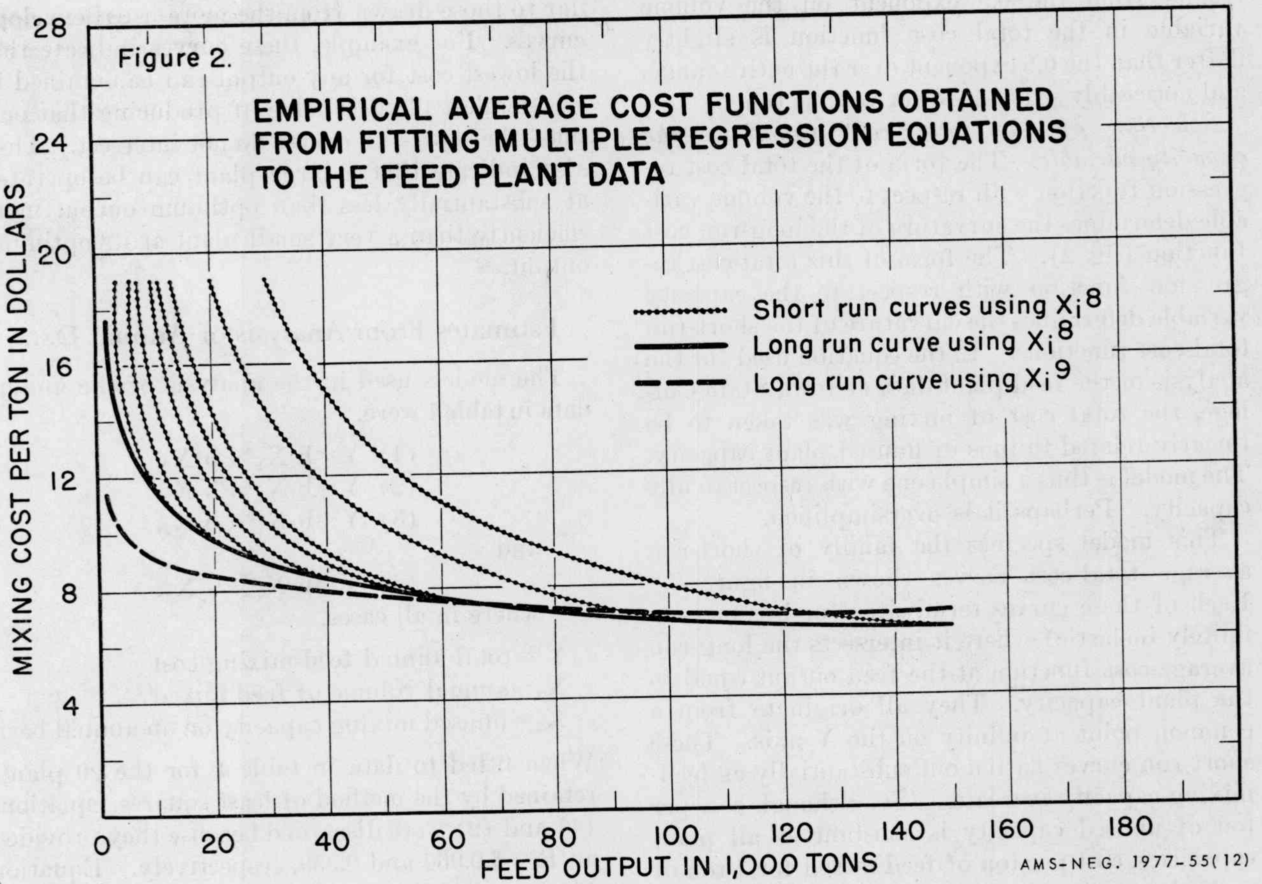


Figure 2.

EMPIRICAL AVERAGE COST FUNCTIONS OBTAINED FROM FITTING MULTIPLE REGRESSION EQUATIONS TO THE FEED PLANT DATA

1). If the solution results in a negative Y-intercept, however, the corresponding average-cost function increases at the smaller outputs as output increases, possibly well into the relevant output range. Therefore the dependability of the results obtained with such a model depends heavily on the accuracy of the observations at the extreme lower end of the output range.

A total-cost regression equation nonlinear in the volume variable which passes through the origin avoids this difficulty. Such a model is logical because total cost should be zero when both output and unused capacity are zero. But some difficulty is encountered in selecting the final equation most suitable. The apparent curvilinearity is such that an exponent on the volume variable of slightly less than one is required. The precise value of this exponent establishes the curvature not only of the total cost function but of the average cost function as well. Its value is not determined by the usual least-squares solution in fitting the regression equation but must be taken as given, for example, $Y = b_1X_1^{.8} + b_2X_2^{.8}$

In the analysis of data in table 1 by members of the staff of Iowa State College, 4 different exponents on the volume variable were used, 0.5, 0.7, 0.8, and 0.9. The 0.5 exponent resulted in the R^2 of 0.960, the 0.7 exponent in an R^2 of 0.959, the 0.8 exponent in an R^2 of 0.979, and the 0.9 exponent in an R^2 of 0.986. The precise exponent in this instance appears to be somewhere between 0.8 and 0.9.[9] The difference in the curvature of the corresponding long-run average-cost functions when the 0.8 and the 0.9 exponents are used can be seen in figure 2. The average cost function that

[8] Dr. Herman O. Hartley of the Department of Statistics at Iowa State College is currently developing a workable method of solving for the value of the exponent in a comparable model applied to the costs of grain storage.

[9] From the calculations made, one cannot be certain that the exact exponent isn't slightly greater than 0.9 but the range in the R^2's for the four exponents used suggests that it is slightly less.

results from the 0.9 exponent on the volume variable in the total cost function is slightly flatter than the 0.8 exponent over the entire range, and noticeably so at the lower volume range.

Selection of the total cost functions—the capacity variable.—The form of the total cost regression function with respect to the volume variable determines the curvature of the long-run cost function (fig. 2). The form of this total-cost regression function with respect to the capacity variable determines the curvature of the short-run total-cost functions. In the equation used for the analysis of the feed-plant data at Iowa State College, the total cost of mixing was taken to be linearly related to tons of unused plant capacity. The model is thus a simple one with respect to idle capacity. Perhaps it is oversimplified.

This model specifies the family of short-run average total-cost curves shown in figure 2.[10] Each of these curves terminates (or becomes infinitely inelastic) where it intersects the long-run average-cost function at the feed output equal to the plant capacity. They all originate from a common point at infinity on the Y axis. These short-run curves flatten out substantially as feed-mixing capacity increases. So, although cost per ton of unused capacity is constant at all plant capacities, cost per ton of feed mixed for 1 ton of unused capacity decreases as plant capacity (or output) increases.

Logical questions might be raised concerning the nature of the short-run average-cost curves in figure 2 and the use of the model that gives rise to it. Certainly the appearance of these curves differs substantially from that of the usual "envelope" curve. Short-run curves of the same slope for all capacities would require a different multiple regression model, for example, introduction of an interaction term between output and unused capacity. However, except for the fact that the curves in figure 2 do not show a range of costs that increase at an increasing rate beyond the optimum point,[11] they lead to conclusions similar to those drawn from the more usual envelope curves. For example, these curves indicate that the lowest cost for any output can be obtained in the smallest plant capable of producing that output—the short-run curves do not intersect. They also indicate that a large plant can be operated at substantially less than optimum output more efficiently than a very small plant at its optimum output.

Estimates From Analysis of Annual Data

The models used in the analysis of the annual data in table 1 were,

(1) $Y=b_1X_1^{.5}+b_2X_2$,

(2) $Y=b_1X_1^{.7}+b_2X_2$,

(3) $Y=b_1X_1^{.8}+b_2X_2$,

and

(4) $Y=b_1X_1^{.9}+b_2X_2$,

where in all cases,

Y = total annual feed-mixing cost

X_1 = annual volume of feed mixed

X_2 = unused mixing capacity on an annual basis

When fitted to data in table 1 for the 29 plants retained by the method of least squares, equations (1) and (2) were discarded because they provided an R^2 of 0.960 and 0.959, respectively. Equation (3) provided the regression equation,

(5) $Y=70.04\,X_1^{.8}+0.301\,X_2$, and an R^2 of 0.979, while equation (4) provided the regression equation,

(6) $Y=22.702\,X_1^{.9}+0.30\,X_2$, and an R^2 of 0.986.

The two estimated long-run average feed-mixing cost curves in figure 2 were computed by setting $X_2=0$ in equations (5) and (6), respectively, solving for a series of total costs associated with a given series of values for mixing volume and dividing the result in each case by the mixing volume. The estimated short-run cost functions for the several capacities plotted in figure 2 were computed from equation (5) by calculating the decrease in estimated total cost resulting from a given decrease in X_1 and the corresponding increase in X_2 and dividing the result by the remaining value of X_1 in each case.

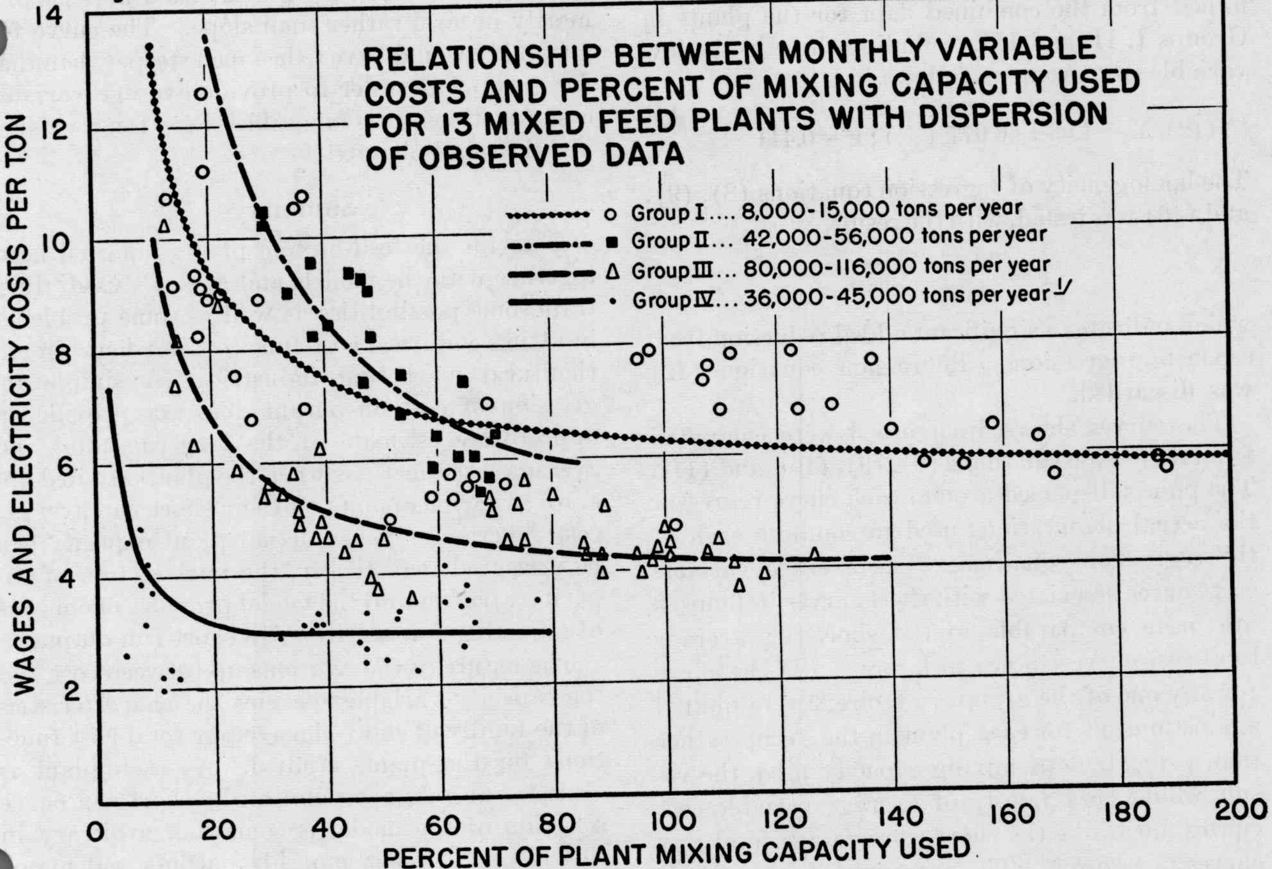
Monthly Volumes and Variable Costs

Reliable monthly figures for tonnage of feed mixed and costs of two major variable cost items—production labor and electricity—were obtained

[10] The short-run curves shown are those obtained with the regression equation, $Y=b_1X_1^{.8}+b_2X_2$. Those obtained with the equation using $X_1^{.9}$ are very similar. The b_2's were 0.301 and 0.300 respectively.

[11] This situation might be expected, in view of the physical production in mixed-feed manufacture. It is borne out by the average variable cost functions, discussed in a later section of this paper.

Figure 3.



1/ All three of the plants in this group exhibited a high ratio of fixed to variable costs compared to the other plants studied.

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from 13 of the sample mixed-feed plants visited.[12] For purposes of regression analysis, monthly tonnage figures were stated in terms of percentages of the monthly 54-hour week capacity of each plant. The plants were grouped according to annual capacities. Group I included 4 plants ranging in capacity from 8,000 to 15,000 tons. Group II had 2 plants of 42,000 and 56,000 tons capacity. Group III included 4 plants with capacities ranging from 80,000 to 116,000 tons. In Group IV there were 3 plants ranging in capacity from 36,000 to 45,000 tons. All three of the plants in group IV showed a high ratio of fixed to variable costs as compared with the other plants studied.

Regression equations were fitted to the figures for percentage of monthly capacity utilized and the sum of monthly production labor and electricity costs for each of these four groups. The model used was (7) $Y_a=b_o+b_1\left(\frac{1}{x}\right)$, where,

Y_a=monthly cost per ton of labor and electricity
X =percentage of monthly capacity used

The resulting equations were, for the 8,000 to 15,000 ton capacities (Group I),

(8) $Y_a=5.50+78.36\left(\frac{1}{x}\right)$; $r^2=0.647$ for 42,000- to 56,000-ton capacities (Group II)

(9) $Y_a=3.329+213.845\left(\frac{1}{x}\right)$; $r^2=0.775$ for 80,000- to 116,000-ton capacities (Group III)

(10) $Y_a=3.545+65.455\left(\frac{1}{x}\right)$; $r^2=0.850$ for low variable cost plants (Group IV)

(11) $Y_a=2.756+8.732\left(\frac{1}{x}\right)$; $r^2=0.445$

[12] Because of their bulkiness, the tables reporting these monthly figures for 13 plants are not reproduced here. The observed monthly data are charted in figure 3.

A pooled simple regression equation was determined from the combined data for the plants in Groups I, II, and III, excluding those in the low variable cost group as follows,

$$(12)\ Y_a = 4.96 + 66.977\ \left(\frac{1}{x}\right);\ r^2 = 0.441$$

The homogeneity of regression equations (8), (9), and (10) was tested, with the result,

$$F = \frac{32.67}{2.07} = 15.78,$$

which indicates a significant added reduction from separate regressions. Regression equation (12) was discarded.

The curves shown in figure 3 were computed from regression equations (8), (9), (10), and (11). The points dispersed around each curve represent the actual observations used to compute each of the regression equations. The average variable cost curves associated with the 4 curves in figure 3 are quite comparable to the short-run average total-cost curves shown in figure 2. If the curve for any one of the groups in figure 3 were plotted against output for each plant in the group rather than percentage of mixing capacity used, the result would be a family of average variable cost curves much like the short-run average total cost curves in figure 2. The average variable cost regression equations give support to the selection of the annual model used. Although this results partly from the selection of the independent variable in the average variable-cost model used, apparently it is supported by the monthly data. A comparison of the regression curves in figure 3 for groups I and III (as well as the observed points dispersed around them) indicates a difference primarily in level rather than slope. The curve for group III would have to be much steeper than that for group I in order to provide average variable cost functions of comparable slope for the large and the small plants.[13]

Summary

Data for the feed-mixing plants reported here, together with the models and analysis used, illustrate some possibilities as well as some problems, in fitting empirical cost functions for firms in agricultural marketing industries. A simple regression of cost on output does not provide an appropriate estimate of the long-run total and average cost functions when the plants studied operate at various points on their short-run average cost functions. When actual plant capacity can be measured realistically, the introduction of capacity variable into the model provides one means of adjusting for variations in short-run output.

The nature of the relationship between cost and the capacity variable specifies the characteristics of the family of short-run average total-cost functions for the plants studied. As each plant is observed at only one point on its short-run curve selection of the model is somewhat arbitrary in this respect. When monthly outputs and major variable costs can be observed for the plants studied, regression analysis can be applied to these data as a check on the model used for the long-run analysis.

[13] A given variation in percentage of mixing capacity used, say from 80 to 100, represents a much greater change in output in the case of the curve for group III than in that for group I.