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**A FOOD DEMAND SYSTEM BASED ON DEMAND FOR CHARACTERISTICS:  
IF THERE IS "CURVATURE" IN THE SLUTSKY MATRIX,  
WHAT DO THE CURVES LOOK LIKE AND WHY?**

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## INTRODUCTION

In his review of empirical estimates of food price and income elasticities which are disaggregated by income group, Alderman [1986] finds that elasticities, with a few exceptions, decline (in absolute value) with increasing income. This pattern occurs across a wide range of countries, for cross-section data sets collected at various frequencies and time intervals, using several estimation techniques. Timmer [1981], on whose article the subtitle of this paper is based, was the first to propose such a relationship, going so far as to speculate that the (utility constant) "pure substitution" term in the Slutsky equation declined (in absolute value) by approximately half as much as the income elasticity declined as incomes rose.

In the first section of this paper, a structure of consumer demand behavior for food is proposed which can explain why low-income groups are more price-responsive than higher income groups, as is apparent empirically from the Alderman review. While the practical importance of this empirical relationship in terms of policymaking is well understood, the underlying causes generating this demand behavior are not well understood, apart from the intuitive notion that persons in high income groups are not forced by necessity to be particularly budget conscious (price-responsive) in terms of food purchases and so eat, for the most part, what they prefer to eat.

A "demand for characteristics" approach is used to explain food acquisition behavior, in particular demand for bulk to alleviate hunger, demand for variety in the diet, and demand for tastes inherent in particular goods. The analysis will show that, in general, demand for variety increases price response, while demand for bulk and tastes decreases price response for food staples. The relative importance of shadow prices for these characteristics varies by income group, so that price response is highest for income groups which assign a relatively high weight in their utility functions to marginal increases in variety. The implication is that in most empirical studies thus far undertaken, then, it is the lowest income groups that place the highest relative weight on variety. However, in those fewer empirical studies where price elasticities were highest for middle income groups, it can be surmised that low income groups assign a relatively high weight to marginal increases in bulk.

The second section of this paper develops the bordered Hessian matrix, implied by the constrained maximization of the utility function specified in the first section of the paper, from which the demand matrix of price and income elasticities for various foods can be derived. Given the assumptions specified in the first section of the paper as to food consumption behavior, the mathematical derivations will show that only four parameters (shadow prices and/or price and income elasticities) need be specified to fill in the entire demand

elasticity matrix for a system of  $n$  foods and one non-food. All four standard restrictions derived from demand theory (adding up, homogeneity, symmetry, and negativity) hold for demand elasticities so derived.

In the third section of the paper, specific values suggested by the theory developed in first section of the paper and a data set from the Philippines, are substituted into the bordered Hessian for a demand system of four foods (one preferred and one non-preferred cereal, fruits/vegetables, and meat) and one non-food. Comparative static analysis is used to compare the relative magnitudes of price and income elasticities generated under various assumptions of relative preferences for bulk, variety, and tastes. The results show that a wide range of price and income elasticities can be generated by the model, depending on relative weights assigned to the three characteristics mentioned above.

A fourth section of the paper implements the characteristic demand framework to derive food demand matrices for urban and rural populations in the Philippines, using published data from the 1978 nationwide nutritional survey. When observed price and income changes between 1978 and 1982 are applied to the estimated food demand matrices, they are found to "predict" 1982 food consumption levels reasonably closely, as measured by a second nationwide nutritional survey. A fifth section of the paper draws the final conclusions.

## I. "CHARACTERISTIC" DEMAND SYSTEM FOR FOOD

### I.A. An Intuitive Approach

In a sense, the task of this paper is to work "backwards," from empirical results to a theory which can account for those results. It is perhaps fitting, then, to start with some demand estimates which do not conform to the usual monotonically-declining pattern and to develop some intuitive insight into why a different pattern might occur. This will hopefully add clarity to the theoretical discussion to follow.

Table 1 shows empirical estimates for rice and corn demand functions for three regions in the Philippines which were estimated using a cross-section, time series data set. The estimates (taken from Bouis [1982]) utilize data from sixteen household food expenditure surveys undertaken quarterly by the Ministry of Agriculture between 1974 and 1978.

Table 2 shows that very little corn is eaten in the north in Luzon, while corn is a very important consumption item in the southern regions of the Visayas and Mindanao, especially among low income groups. Aggregate price elasticities for rice and corn are much higher in the southern regions because of the substitution possibili-



Table 1--Estimates of own-price elasticities for rice and corn, by region by income group

Cereal	Group	National <sup>a</sup>	Region		
			Luzon	Visayas	Mindanao
Rice	I	-0.7	-0.4	-0.9	-2.5
	II	-0.7	-0.2	-1.2	-2.0
	III	-0.7	-0.2	-1.4	-1.5
	IV	-0.4	-0.3	-0.8	-0.5
	All	-0.6	-0.3	-1.1	-1.4
Corn	I	-1.4		-1.3	-1.3
	II	-1.5		-1.5	-1.6
	III	-1.5		-2.1	-0.9
	IV	-1.3		-2.0	-0.6
	All	-1.4		-1.6	-1.2

Source: Bouis (1982).

<sup>a</sup> For corn, national estimates include observations for Cagayan in Luzon.

Table 2--Per capita consumption of rice and corn, by region by income group (kilograms per capita/week)

Region	Income Group				All
	1	2	3	4	
Rice					
Manila	1.84	1.83	1.95	1.82	1.86
Ilocos	2.61	2.61	2.63	2.33	2.55
Cagayan	1.97	2.20	2.37	2.37	2.21
Central Luzon	2.44	2.39	2.47	2.51	2.45
Southern Tagalog	2.25	2.29	2.27	2.15	2.24
Bicol	1.95	2.12	2.19	2.18	2.10
Western Visayas	1.76	1.98	2.05	2.10	1.96
Eastern Visayas	1.17	1.29	1.43	1.62	1.37
Northern Mindanao	1.05	1.35	1.73	1.97	1.49
Southern Mindanao	1.16	1.48	1.85	1.95	1.58
National Total	1.77	1.94	2.17	2.09	1.96
Corn					
Manila	0.02	0.02	0.01	0.03	0.02
Ilocos	0.05	0.03	0.04	0.04	0.04
Cagayan	0.66	0.39	0.27	0.22	0.40
Central Luzon	0.03	0.04	0.03	0.03	0.03
Southern Tagalog	0.03	0.03	0.04	0.04	0.03
Bicol	0.09	0.05	0.10	0.08	0.08
Western Visayas	0.65	0.54	0.41	0.28	0.48
Eastern Visayas	1.27	1.16	0.94	0.71	1.03
Northern Mindanao	1.39	1.17	0.83	0.54	1.02
Southern Mindanao	1.28	0.84	0.55	0.44	0.81
National Total	0.58	0.46	0.35	0.26	0.42

Source: Special Studies Division Surveys, Ministry of Agriculture, Philippines

ties between the two main staples, rice and corn. Note, however, that for the Visayas for both rice and corn, highest price response occurs for middle income groups. How can this result be explained?

Data to be presented below show corn to be a lower cost staple than rice. Intuitively, one can imagine a very low-income household having to spend a high proportion of its total income on food, and of its total food expenditures, having to spend almost the entire amount on corn to keep from going hungry. Given such a situation, suppose that the price of corn falls. How will the household react? The household could afford to substitute some rice for corn now, the preferred staple, without going hungry. A drawback, however, is that the diet would still consist almost entirely of bland cereals. The household may prefer instead to continue eating nearly the same amount of corn as before to satisfy hunger, and to supplement an essentially monotonous diet with some of inexpensive dried fish. If the latter situation is the case, if variety is more important to the household than the superior taste of rice, then the observed own-price elasticity for corn may be very low.

Now suppose that the low price of corn in the previous example prevails but that the income of our household has gone up on a permanent basis. The family can afford substantial variety in the diet represented by some meat at every meal (sometimes dried fish, less often more expensive fresh meat) and even can afford the "luxury" of some rice consumption. Again, suppose that price of corn rises. The family is wealthy enough now not to have to worry about the specter of hunger, despite the corn price increase. The household may be willing to substitute substantial amounts of rice for corn. Because it pays more for cereals now, bulk and variety may go down somewhat. However, although total utility goes down, the marginal utilities of bulk and variety have come down far enough that the least utility is lost by giving up some variety and some bulk, but recouping some utility from the superior taste of rice. A higher price-response is observed for the middle income household than for the low-income household.

What the previous example has done is to introduce some of the concepts to be developed more formally below and to raise the possibility that low income groups may be constrained in responding to price changes by the need to consume large amounts of a low-priced staple.

### I.B. Model Specification

Utility is a function of bulk, variety, and tastes, characteristics of quantities of food consumed, and of non-food purchases. Utility from the characteristics of foods consumed and from non-foods is separable. Thus:

$$(1) \quad U = w_b U_b(B) + w_v U_v(V) + w_t \sum_{i=1}^n w_i U_{ti}(q_i) + w_{nf} U_{nf}(q_k)$$

where:

$U$  = total utility from all food and non-food goods,  
 $q$  = quantity of a good,  
 $i$  = 1, ..., n are the n foods consumed,  
 $B$  = a measure of bulk in the diet,  
 $V$  = a measure of variety in the diet,  
 $U_b$  = utility derived from bulk,  
 $U_v$  = utility derived from variety,  
 $U_{ti}(q_i)$  = utility derived from the taste of  $q$  units of good  $i$ ,  
 $U_{nf}(q_{nf})$  = utility derived from  $q$  units of the non-food good,  
 $w_b$  = weight placed on utility from bulk,  
 $w_v$  = weight placed on utility from variety,  
 $w_{ti}$  = weight placed on taste from individual food  $i$ ,  
 $w_{nf}$  = weight placed on utility from the non-food good.

Next, explicit functions are developed for measures of bulk and variety in the diet, and first and second derivatives taken for use later in the paper.

#### I.B.1. Utility From Bulk

$$(2) \quad B = \sum_{i=1}^J z_i q_i$$

where  $z_i$  = factor converting quantity of the  $i$ th food into bulk (assume that all food quantities,  $q_i$ , are expressed in kilograms).

Intuitively, the  $z_i$ 's are relatively high for "dense" foods such as cereals and fats and relatively low say for most fruits. Calories will be used as a proxy for the true  $z_i$ 's in the empirical work presented later. In this paper,  $B$  is total calories consumed per adult equivalent.<sup>1</sup>

$$(3) \quad U_b(B) = b_1 + b_2 B + b_3 B^2$$

where  $b_2 > 0$  and  $b_3 < 0$

At low levels of total bulk, each additional unit of bulk increases utility, but at a decreasing rate. The functional form chosen, however, allows for marginal decreases in utility from additional units of bulk at sufficiently high intakes of bulk.

$$(4) \quad B_i = w_b(b_2z_i + 2b_3Bz_i) > 0 \text{ for low income groups}$$

$$\text{where } B_i = \frac{\partial U}{\partial U_b(B)} \frac{\partial U_b(B)}{\partial q_i}$$

$$(5) \quad B_{ij} = 2w_b b_3 z_i z_j < 0$$

$$\text{where } B_{ij} = \frac{\partial^2 B_i}{\partial q_i \partial q_j}$$

Analagous notation will be used below for  $V_i$ ,  $V_{ij}$ ,  $T_i$ , and  $T_{ij}$ .

### I.B.2. Utility From Taste

$$(6) \quad U_{ti}(q_i) = \log(q_i)$$

$$(7) \quad T_i = w_{ti}(1/q_i) > 0$$

$$(8) \quad T_{ii} = -w_{ti}(1/q_i)^2 < 0$$

$$(9) \quad T_{ij} = 0$$

Each additional unit of taste of good  $i$ , no matter what the quantity, adds additional utility, but at a decreasing rate. The first derivative is positive and the second derivative negative, then, which is similar to bulk for low income groups. However, for taste the "across food" second derivative is zero.

### I.B.3. Utility From Variety

#### I.B.3.a. Defining Variety

While the utility of variety in the diet is perhaps a universally appealing concept intuitively, it is far from intuitive how to mathematically define measures of variety in the diet, and even more difficult to sort out what functional properties utility derived from variety should take. So as not to burden the presentation of the model with this complex subject, a relatively simple mathematical formulation for variety will be used in this paper. Although the results of using more complex functions are not presented, they do not qualitatively affect the properties of the overall model. Some of the issues related to choosing a particular mathematical formulation for variety are discussed in Appendix A.

### I.B.3.b. Mathematical Formulation of Utility from Variety

$$(10) \quad U_V(V) = M/T$$

where:

M = non-staple kilograms of food consumed per adult equivalent,

T = total kilograms of food consumed per adult equivalent.

The more non-staples one consumes, the greater the variety in the diet. The more staples one consumes, the less variety.

$$(11) \quad V_i = -w_V M/T^2 < 0 \quad \text{for } i \leq s$$

$$V_i = w_V (I/T) (1 - [M/T]) > 0 \quad \text{for } s < i < n$$

where  $i=1, \dots, s$  are staple foods.

$$(12) \quad V_{ij} = 2M/T^3 > 0 \quad \text{for } i, j \leq s$$

$$V_{ij} = (w_V/T^3) [2M - T] \quad \text{for } i \leq s \text{ and } s < j \leq n$$

$$V_{ij} = (2w_V/T^3) [M - T] < 0 \quad \text{for } s < i, j \leq n$$

For all three sets  $i \neq j$  and  $j$ ,  $V_{ij} = V_{ji}$ .

### I.B.3.c. An Empirical Look at the Relationship Between Variety and Income

The particular index of variety just presented, is based on a dichotomy between staple and non-staple foods. Since alternative concepts of variety may involve more than two food groupings, at this point it will be helpful to examine a data set to see what variation in the data is being lost by such an aggregation. The data are taken from four surveys taken at four-month intervals of 148 households in 1984 and 1985 in Bukidnon province of Mindanao in the Philippines [see Bouis and Haddad forthcoming]. Nearly all of the households were engaged in corn production to some extent.

Tables 3, 4, and 5 show a breakdown of food expenditures, calorie intake, and pesos spent per calorie by expenditure quintile and by seven broad food groups. The food groups were chosen to examine in some detail how demand for certain types of food might vary by income. The striking result from Table 3 is that most of the increase in variety with increasing income comes fairly consistently from the meat food group. That is, there is no apparent trend (say) to increase variety through vegetable consumption at low incomes, and then to increase meat consumption at middle incomes, and then to add fruits to the diet at high incomes. The dichotomy between staples and non-

Table 3--Weekly per capita food expenditures and budget shares, by broad food group by per capita total expenditure quintile, all rounds (constant Round 1 pesos)

Food Group	Total Expenditure Quintile					All
	1	2	3	4	5	
Rice	2.01 (7.6)	3.43 (10.2)	4.14 (11.2)	4.13 (9.2)	9.22 (15.8)	4.57 (11.4)
Corn	9.48 (35.8)	9.70 (28.8)	8.95 (24.2)	9.14 (20.3)	4.36 (7.5)	8.35 (20.9)
Other staples	2.12 (8.0)	3.65 (10.9)	3.49 (9.4)	3.96 (8.8)	5.10 (9.7)	3.66 (9.2)
Meat/eggs/fish pulses	7.37 (27.8)	9.93 (29.5)	12.38 (33.5)	18.03 (40.1)	28.99 (67.1)	15.16 (37.9)
Vegetables and legumes	2.62 (8.9)	3.07 (9.1)	3.62 (9.8)	3.59 (8.7)	3.59 (6.2)	3.36 (8.4)
Fruits, snacks desserts, beverages	0.86 (3.3)	1.72 (5.1)	1.81 (4.9)	2.48 (5.5)	3.75 (6.4)	2.12 (5.3)
Cooking ingredients	2.00 (7.6)	2.13 (6.3)	2.62 (7.1)	3.31 (7.4)	3.86 (6.6)	2.78 (7.0)
Total food	26.46 (100.0)	33.63 (100.0)	37.01 (100.0)	44.95 (100.0)	58.33 (100.0)	39.99 (100.0)

Table 4--Daily calorie availability per adult equivalent and shares by broad food group and total expenditure, all rounds

Food Group	Total Expenditure Quintile					All
	1	2	3	4	5	
Rice	233 (11.3)	382 (17.0)	477 (20.7)	484 (19.6)	1,148 (44.5)	542 (23.2)
Corn	1,502 (73.0)	1,458 (64.8)	1,339 (58.2)	1,385 (56.0)	660 (25.6)	1,272 (54.5)
Other staples	109 (5.3)	115 (5.1)	165 (7.2)	161 (6.5)	200 (7.7)	150 (6.4)
Meat/eggs/fish pulses	84 (4.1)	111 (4.9)	127 (5.5)	181 (7.3)	260 (10.1)	152 (6.5)
Vegetables and legumes	28 (1.4)	33 (1.5)	35 (1.5)	41 (1.7)	38 (1.5)	35 (1.5)
Fruits, snacks desserts, beverages	41 (2.0)	71 (3.2)	65 (2.8)	74 (3.0)	84 (3.3)	67 (2.9)
Cooking ingredients	61 (3.0)	79 (3.5)	93 (4.0)	149 (6.0)	191 (7.4)	114 (4.9)
Total food	2,058 (100.0)	2,249 (100.0)	2,302 (100.0)	2,474 (100.0)	2,591 (100.0)	2,332 (100.0)



Table 5--Pesos spent per 1,000 calories by broad food group by per capita total expenditure quintile, all rounds

Food Group	Total Expenditure Quintile					All
	1	2	3	4	5	
Rice	1.72	1.76	1.73	1.69	1.72	1.73
Corn	1.31	1.36	1.34	1.34	1.34	1.34
Other staples	8.64	10.54	9.01	6.86	7.68	8.55
Meat/eggs/fish/ pulses	18.50	19.29	19.29	19.80	21.54	19.67
Vegetables and legumes	21.46	19.11	21.65	19.30	20.88	20.47
Fruits, snacks, desserts, beverages	5.77	6.77	6.39	7.04	10.17	7.22
Cooking ingredients	8.12	6.26	5.60	5.16	4.11	5.86
Total food	1.89	2.14	2.28	2.56	3.28	2.43

staples, therefore, is largely a dichotomy between the staple and meat food groups.

Table 4 shows that the absolute number of calories consumed from staples remains remarkably constant across income groups, although the mix of particular staple goods changes. Table 5 shows that high income groups pay about twice as much per calorie for all foods as low income groups. At the margin as income increases, households buy foods which are calorie expensive. Interestingly, however, within the broad food groups selected, calorie prices do not vary much by income.

## I.C. Some Implications of the Characteristic Demand System

### I.C.1. Substitutability and Separability

$$(13) \quad \frac{w_b \beta_i(q_i) + w_v \vartheta_i(q_i) + w_{tj} \tau_i(q_i)}{w_b \beta_j(q_j) + w_v \vartheta_j(q_j) + w_{tj} \tau_j(q_j)} = \frac{P_i}{P_j}$$

(11) states the well-known relationship that the ratio of the marginal utilities should equal the ratio of the prices for goods  $i$  and  $j$  (let  $\beta_i = B_i/w_b$ ,  $\vartheta_i = V_i/w_v$ ,  $\tau_i = T_i/w_{tj}$ ). One way of thinking about the substitutability of two goods is to reason that when two goods are perfectly substitutable, a peso (using Philippine currency) spent on good  $j$  instead of good  $i$  leaves the consumer at exactly the same level of utility. Two goods are rarely ever perfect substitutes, so one must judge utility losses on a relative scale.

Consider the case of two staples where good  $i$  is corn and good  $j$  is rice for the data set of Tables 3 through 5. At low levels of income, where bulk is presumably an important consideration in the consumption decision (where  $w_b B$  is high), because rice is a more expensive source of bulk, substituting one peso's worth of rice for one peso's worth of corn would represent a relatively large loss in utility. The large loss in bulk utility would be compensated for somewhat by gains in variety utility (in our particular mathematical formulation, variety goes up when staple consumption goes down), and by gains in taste utility if say three-fourths of a kilo of rice delivers more "taste" than a one kilo of corn. But overall, utility goes down by enough that corn and rice are not very substitutable, so that price response is low.

Now suppose that  $w_b = 0$  and the one peso expenditure is re-allocated as above (setting  $w_b = 0$  involves an initial re-allocation of food expenditures before the one peso is re-spent). Rice and corn become highly substitutable. One gains from variety, loses on the taste from corn, but gains on the taste from rice (declining overall marginal utility insures that there is a net loss of utility). The need for

bulk, then, reduces the substitutability of goods with different costs per unit of bulk, and so presumably reduces price response.

Taste has the same effect of reducing price-response, since by the definition of taste, two goods cannot substitute for one another. Setting  $w_b=0$  and  $w_v=0$  and re-allocating one peso of expenditure always results in a larger loss of utility with a high  $w_{ti}$  as compared with a low  $w_{ti}$ .

Variety has the opposite effect, that of increasing price response. Consider the case of substituting one peso of meat for one peso of corn at low incomes. There is an even larger loss in utility from bulk than from substitution of rice above. However, while the net overall loss is negative, the consumer could be compensated to a considerable degree by increased variety, depending on the magnitude of  $w_v$ .

It should be evident at this point that the structure of demand presented here is, in a sense, the very anti thesis of the idea of separability (even weak separability). The marginal rate of substitution between two staples, for instance, is very much dependent on the level of meat consumption (and vice-versa). To the extent that the analysis presented so far is a true reflection of the food expenditure decision-making process, it brings into question food demand parameter estimates derived from various formulations of linear or quadratic expenditure systems, all of which depend to some degree on the assumption of strong or weak separability.

## I.C.2. Characteristic Demand and Hedonic Prices

Ladd and Suvannant [1976] developed the following relationship between the market price paid for good  $i$  and the shadow prices of the characteristics provided by that good:

$$(14) \quad p_i = \sum_{r=1}^h \hat{p}_r \frac{dH_r}{dq_i}$$

where:

$U$  = utility =  $U(H_1, \dots, H_h)$

good  $i$  possesses one or all of  $h$  characteristics

$\hat{p}_r$  = the shadow price of the  $r$ th characteristic which is the ratio of the marginal utility of  $H_r$  and the marginal utility of income

$H_r$  is defined as the total amount of characteristic  $r$  which is present in the bundle of goods purchased. Typically there is a very well-defined relationship between  $H_{r,i}$ , the amount of  $H_r$  provided by  $q_i$ , the total units of good  $i$  which are purchased, which can be calculated from a constant, a technical coefficient, which converts each unit of  $q_i$  into its  $H_r$ th equivalent.

For example, with respect to food purchases, if calories are a desired characteristic which motivates food purchases, then the total amount of calories available from a certain bundle of goods can be calculated from a vector of  $z_i$ 's (to use the same notation as in (2) above), which converts each unit of  $q_i$  into its calorie equivalent.  $dH_r/dq_i$  in (14) is this technical coefficient. If one has data for several different foods along with the technical coefficients for converting these foods into calories, proteins, and so on, then one can run a regression with prices for these foods as the dependent variable and the technical conversion coefficients for the various nutrients as the "data" on the right-hand side to estimate shadow prices for the various nutrients.

Since the shadow price is an estimated constant, this implies that for successively higher increments of characteristic  $r$  that can be added to a unit of good  $i$ , the consumer would always be willing to pay a successively higher price for  $i$  equal to the shadow price for  $r$  times the amount of the increment. One can think of several situations, however, where diminishing marginal utility for that characteristic would lower the amount of each price increase that one would be willing to pay.

In the demand structure presented in (1),  $B$  and  $T$ , the total amounts of bulk and taste if you will, are each "filtered" through a function first (in the case of bulk a quadratic function and in the case of taste a log function) so that doubling the amount of a characteristic in a unit of any good does not double the price that one is willing to pay for that good (an argument was developed for not filtering  $V$  through such a function; see Appendix A). The implication for estimation of the hedonic price function in (14) for the demand structure presented in this paper, is that instead of technical coefficients being the "data" on the right-hand side, our right-hand side variables are functions to be calculated before substitution into the regression. These functions are the first derivatives,  $B_i$ ,  $V_i$ , and  $T_i$ :

$$(15) \quad p_i = \hat{p}_b B_i + \hat{p}_v V_i + \hat{p}_t T_i \quad i = 1, \dots, n$$

where:

$$\hat{p}_b = \frac{\partial U / \partial U_b}{\lambda} = \frac{w_b}{\lambda}$$

$$\hat{p}_v = \frac{\partial U / \partial U_v}{\lambda} = \frac{w_v}{\lambda}$$

$$\hat{p}_{ti} = \frac{\partial U / \partial U_{ti}}{\lambda} = \frac{w_{ti}}{\lambda}$$

$\lambda$  = the marginal utility of income.

(15) may be rewritten as (15'), say for the first good in the food demand system:

$$\begin{aligned} (15') \quad p_1 &= \hat{p}_b(b_2z_1 + 2b_3Bz_1) + \hat{p}_v(-M/T^2) + \hat{p}_{t1}(-1/q_1) \\ &= \hat{a}_1 + \hat{a}_2(Bz_1) + \hat{p}_v(-M/T^2) + \hat{p}_{t1}(-1/q_1) \end{aligned}$$

Even though  $\hat{p}_b$ ,  $\hat{p}_v$ , and  $\hat{p}_{t1}$  may remain constant across income groups (apart from changes in  $\lambda$ ), the first derivatives inside the parentheses in (15') will vary by income group, altering the component shares of each characteristic (the shadow prices), which sum to the market price of the first good. Note that in estimating (15'), no direct estimate of  $\hat{p}_b$  is obtained. It turns out that estimates of  $\hat{p}_b(2b_3)$  (which is  $\hat{a}_2$ ) and of  $\hat{a}_1/\hat{a}_2$  are sufficient for deriving the matrix of demand elasticity estimates described in the next section of the paper.

## II. SOLVING THE UTILITY FUNCTION TO DERIVE A COMPLETE A DEMAND SYSTEM OF PRICE AND INCOME ELASTICITIES

The objective of this and the following section of the paper is to show that a hypothetical demand system based on the structure outlined in first section of the paper can be constructed which leads to increasing and then decreasing (in absolute value) compensated price elasticities or a monotonically decreasing pattern across income groups depending on assumptions as to the weights,  $w_b$ ,  $w_v$ , and  $w_{ti}$ , in the utility function. However, where appropriate, values will be used from the data set used for constructing Tables 3 to 5 to add relevancy to the comparative static analysis presented below.

### II.1. The Bordered Hessian Matrix for a Four Food and One Non-Food Good System

The demand system to be experimented with consists of four foods (an inexpensive but less preferred staple (corn) and a preferred

staple (rice), a relatively inexpensive source of variety (fruits/vegetables) and a more preferred source of variety (meat) and one non-food. To solve for the complete matrix of demand elasticities, what needs to be done is to fill in values for the bordered Hessian matrix of second derivatives given below and then to compute the appropriate determinants from this matrix (see Henderson and Quandt [1971], p. 31-32). While all of the analysis undertaken in this section of the paper is for a four food and one non-food good system, the results hold more generally for a system of  $n$  foods and one non-food good.

$$\begin{bmatrix} B_{11}+V_{11}+T_{11} & B_{12}+V_{12} & B_{13}+V_{13} & B_{14}+V_{14} & 0 & -P_1 \\ B_{21}+V_{21} & B_{22}+V_{22}+T_{22} & B_{23}+V_{23} & B_{24}+V_{24} & 0 & -P_2 \\ B_{31}+V_{31} & B_{32}+V_{32} & B_{33}+V_{33}+T_{33} & B_{34}+V_{34} & 0 & -P_3 \\ B_{41}+V_{41} & B_{42}+V_{42} & B_{43}+V_{43} & B_{44}+V_{44}+T_{44} & 0 & -P_4 \\ 0 & 0 & 0 & 0 & NF_{55} & -P_5 \\ -P_1 & -P_2 & -P_3 & -P_4 & -P_5 & 0 \end{bmatrix}$$

The zeros in the fifth row and fifth column of the above matrix are the result of an assumption of additivity between consumption of foods and non-foods.

Assumptions are made in Table 6 about the prices, quantities, and calorie conversion coefficients of the four foods and one non-food which are necessary for solving above demand system. The household is purchasing about 9075 total calories per capita per day for a family size of 7.0, or 1920 calories per adult equivalent per day for a household of 5.0 adult equivalents. [All figures used are more or less consistent with the data for the lowest expenditure group in Tables 3 through 5.]

Returning to the bordered Hessian matrix above, note that the four by four matrix formed by the first four rows and columns can be written as the sum of three separate matrices each involving only the bulk, variety, or taste second derivative terms. Each of these three four by four matrices,  $[B]$ ,  $[V]$ , and  $[T]$ , is symmetrical about the diagonal ( $[T]$  has non-zero values only along the diagonal).  $[B]$  and  $[V]$  can be rewritten by factoring out  $-B_{11}$  and  $V_{11}$  such that the resulting matrices are "normalized" about the first element in the first row as shown below. Given the assumptions about prices, quantities, and calorie conversion rates given above, the values in the matrices can now be filled in (only the diagonal and above are shown below to make it easier to discuss the data which are filled in; since the matrices are symmetrical it would be redundant to fill in the matrices below the diagonal).

Table 5--Assumptions as to prices,\* quantities, calorie conversion rates, and adult equivalents for a hypothetical demand system of four foods and one non-food

Commodity	Price/ Kilogram	Quantity (Kilograms)	Calories/ Kilogram	Budget Share	Calories/ Unit of Currency
Corn	1.00	2.08	3,600		3,600
Rice	1.30	0.37	3,600		2,770
Veg/Fruits	1.50	0.50	500		333
Meat	7.25	0.25	2,100		290
Non-Food				0.30	

Ratio of number of household members/adult equivalents = 1.4.

\* Prices normalized on corn.

$$(16) \quad [B] = K_b[B] = w_b(-2b_3z_1z_1) \begin{bmatrix} -1.00 & -1.00 & -0.139 & -0.583 \\ & -1.00 & -0.139 & -0.583 \\ & & -0.019 & -0.081 \\ & & & -0.340 \end{bmatrix}$$

$$(17) \quad [V] = K_v[V] = w_v(2M/T^3) \begin{bmatrix} 1.00 & 1.00 & -1.133 & -1.133 \\ & 1.00 & -1.133 & -1.133 \\ & & 3.266 & 3.266 \\ & & & 3.266 \end{bmatrix}$$

Note that  $K_b$  and  $K_v$  are both positive ( $b_3$  is hypothesized to be negative while all other "factored-out" parameters are positive). From the derivations of (15) and (15'), we may substitute  $\hat{p}_b\lambda$  for  $w_b$ ,  $\hat{p}_v\lambda$  for  $w_v$ , and  $\hat{a}_2$  for  $\hat{p}_b(2b_3)$  to obtain (16') and (17'):

$$(16') \quad [B] = \hat{a}_2\lambda(-z_1z_1)[B]$$

$$(17') \quad [V] = \hat{p}_v\lambda(2M/T^3)[V]$$

Assuming that (15') has been estimated, the only remaining unknown in (16') and (17') is  $\lambda$  which needs to be identified before the elements for  $[B]$  and  $[V]$  can be filled in. It turns out that the choice of a value for  $\lambda$  is arbitrary and does not affect the elasticity estimates. This result is shown later in section II.3 of the paper.

Turning now to the derivation of  $[T]$ , let  $K_t = w_t w_1 = \hat{p}_{t1}\lambda$ .  $[T]$  may be written as follows:

$$(18) \quad [T] = K_t[T] = w_{t1} \begin{bmatrix} -1/(q_1)^2 & 0 & 0 & 0 \\ 0 & -(w_{t2}/w_{t1})[1/(q_2)^2] & 0 & 0 \\ 0 & 0 & -(w_{t3}/w_{t1})[1/(q_3)^2] & 0 \\ 0 & 0 & 0 & -(w_{t4}/w_{t1})[1/(q_4)^2] \end{bmatrix}$$

or

$$(18') \quad [T] = \hat{p}_{t1}\lambda[T]$$



Using (15), it turns out that there is already almost enough information to fill in [T]. To repeat (15) for the first food item:

$$(19) \quad p_1 = \hat{p}_b \bar{b}_1 + \hat{p}_v T_1 + \hat{p}_{t1} T_1 \\ = \hat{p}_b (b_2 z_1 + 2b_3 \bar{z}_1) + \hat{p}_v (-M/T^2) + w_{t1} (-1/q_1)$$

Apart from what it is already necessary to know to fill in [B] and [V] (specifically  $\hat{a}_2$  and  $\hat{p}_v$ ), the only additional unknown necessary for solving (19) for  $w_{t1}$ , is  $\hat{p}_b b_2$  (which is  $\hat{a}_1$  in (15')).

However, an alternative method is to exogenously specify the ratio  $b_2/2b_3$ , the point at which marginal utility from bulk becomes negative, where there is actual physical discomfort from eating more food. Letting  $b_2/2b_3 = S$  ( $S < 0$ ;  $S$  may be obtained from  $\hat{a}_1/\hat{a}_2$  from estimation of (15')),  $\hat{p}_b b_2$  may be solved for using  $\hat{a}_2$ . This, in turn, allows solution of (19) for  $w_{t1}$ . Moreover, by re-specifying (19) for foods 2, 3, and 4, all three unknown ratios inside of [T] may be solved for, specifically  $w_{t2}/w_{t1}$ ,  $w_{t3}/w_{t1}$ , and  $w_{t4}/w_{t1}$ .

Briefly summarizing, given estimates of  $\hat{a}_2$ ,  $\hat{p}_v$ , and  $S$ , all the elements in [B], [V], and [T] may be filled in. The only remaining unknown element in the bordered Hessian matrix of second derivatives is NF55.

To develop an estimate for NF55, (20) and (21) below give expressions for the income elasticity for the non-food good and for  $\lambda$ , which can be derived from the bordered Hessian of second derivatives as shown below:

$$(20) \quad \frac{Y}{q_{nf}} \frac{\partial q_{nf}}{\partial Y} = \frac{Y}{q_{nf}} (-1) \frac{D_{65}}{D}$$

$$(21) \quad \frac{Y}{\lambda} \frac{\partial \lambda}{\partial Y} = \frac{Y}{\lambda} (-1) \frac{D_{66}}{D}$$

where:

$D_{65}$  = the cofactor of the element in the sixth row and fifth column of the bordered Hessian (the negative of the determinant of this element),

$D_{66}$  = the cofactor of the element in the sixth row and sixth column of the bordered Hessian (the determinant of this element),

$D$  = the determinant of the entire bordered Hessian.

Let  $|D_{44}|$  designate the determinant of the matrix formed by the first four rows and four columns in the bordered Hessian. Dividing (20) by (21), expanding  $D_{65}$  by the elements in the fifth row, and expanding  $D_{66}$  by the elements in the fifth column gives (22):

$$(22) \quad \frac{\eta_{nf}}{w} = \frac{\lambda}{q_{nf}} \frac{-(-p_{nf}|D_{44}|)}{NF_{55}|D_{44}|}$$

where:

$\eta_{nf}$  = the income elasticity for non-foods,

$w$  = Frisch's flexibility of the marginal utility of income, or money flexibility,

$D_{65} = -(-p_{nf}|D_{44}|)$

$D_{66} = NF_{55}|D_{44}| = \frac{Y}{\lambda} \frac{\partial \lambda}{\partial Y}$

Solving (22) for  $NF_{55}$  gives (23):

$$(23) \quad NF_{55} = \lambda \frac{p_{nf}}{q_{nf}} \frac{w}{\eta_{nf}}$$

By exogenously specifying a value for the ratio,  $w/\eta_{nf}$ ,  $NF_{55}$  is determined and the entire bordered Hessian matrix of second derivatives can be filled in (again recall that the choice of  $\lambda$  is arbitrary; see Section II.3).

## II.2. Relationship to Frisch Linear Expenditure System

It is instructive at this point to use (23) to relate the particular demand system as represented in (1) to a demand system which is well known in the literature, specifically where utility of all individual goods is strongly separable, or additive, an assumption which the Frisch technique employs. (1) essentially reduces to this system for  $w_p=0$ ,  $w_y=0$ , and where no explicit function for  $U_{xi}$  is specified as in (6). Thus, all diagonal elements in the bordered Hessian of second derivatives are analogous to (23) and all other elements are zero (except of course for the price borders along the last row and last column).

Under the assumption of strong separability, from (23), the complete bordered Hessian can be filled in given income elasticities for all goods, and estimates of  $w$  and  $\lambda$ .  $\lambda$  may be identified from (24) [see Henderson and Quandt, p.36]:

$$(24) \quad \frac{\partial q_i}{\partial q_j} = \frac{D_{ji}\lambda}{D} + q_j \frac{D_{ni}}{D}$$

A computational algorithm for identifying  $\lambda$  would be to arbitrarily specify a range of values for  $\lambda$ , to compute a series of demand elasticity matrices implied by this range of values for  $\lambda$ , and then to select that particular demand matrix which contains the particular own-price or cross-price elasticity which is exogenously specified. This yields the familiar result that given income elasticities for all goods, one price elasticity, and an estimate of  $w$ , all demand elasticities can be identified.

The demand system (for food) proposed in this paper, then, differs from the Frisch assumptions in that (1) expressions for bulk and variety are added to the utility function and (2) explicit functions are proposed for  $U_b$ ,  $U_v$ , and  $U_{ti}$ . What advantage is lost in the sense of generality by proposing explicit functions for  $U_b$ ,  $U_v$ , and  $U_{ti}$ , is compensated for by the ability to derive an explicit function for (15). This not only gives an estimating equation (15') for deriving parameter estimates to identify the demand system, but more importantly (1) greatly reduces the number of parameters which need to be estimated using (19), and (2) drops the assumption of strong separability.<sup>2</sup>

### II.3. The Effect of $\lambda$

Income and price elasticity estimates do depend on  $w$ , but do not depend on  $\lambda$ . To show this,  $\lambda$  has been factored out of (16'), (17'), (18'), and (23), indicating that all terms in the bordered Hessian, with the exception of the price borders in the last row and the last column, are multiplied by  $\lambda$ . Multiplying  $\lambda$  by an arbitrary constant  $k$ , then, has the effect of changing the value of the determinant of the entire bordered Hessian,  $D$ , by a factor of  $k$  to the power of the number of foods in the demand system.

To show this for our system of four foods, expanding by the fifth column and then fifth row,  $D$  may be written as in (25):

$$(25) \quad D = NF_{55} \begin{vmatrix} x & x & x & x & -p_1 \\ x & x & x & x & -p_2 \\ x & x & x & x & -p_3 \\ x & x & x & x & -p_4 \\ -p_1 & -p_2 & -p_3 & -p_4 & 0 \end{vmatrix} - p_5^2 [D_{44}]$$

where:

$x$  = designates any element in the first four rows or columns of the bordered Hessian,

[D44] = the determinant of the elements in the first four rows and columns of the bordered Hessian (from which a  $\lambda^4$  term may be factored out).

Further expansion of the determinant in the first term in (25) by the last row and last column gives the sum of sixteen terms. Each of these terms involves the determinant of a three by three matrix of "x's" multiplied by price terms (from which a  $\lambda^3$  term may be factored out). Since  $NF_{55}$  is a linear function of  $\lambda$ , multiplying  $\lambda$  by an arbitrary factor  $k$ , multiplies both the first and second term in (25) by a factor of  $k$  raised to the fourth power. This means that the denominator of both terms in (24) is raised by  $k$  to the fourth power.

The numerator in the second term of (24) would also be raised by  $k$  to the fourth power, as shown already by the numerator of (22) which was derived from the expression in (20). Turning to the numerator of the first term in (24):

$$(26) \quad D_{ij} = \begin{vmatrix} x & x & x & 0 & -p_x \\ x & x & x & 0 & -p_x \\ x & x & x & 0 & -p_x \\ 0 & 0 & 0 & NF_{55} & -p_5 \\ -p_x & -p_x & -p_x & -p_5 & 0 \end{vmatrix} \quad i, j < 5$$

$$= NF_{55} \begin{vmatrix} x & x & x & -p_x \\ x & x & x & -p_x \\ x & x & x & -p_x \\ -p_x & -p_x & -p_x & 0 \end{vmatrix} - p_5^2 \begin{vmatrix} x & x & x \\ x & x & x \\ x & x & x \end{vmatrix}$$

Further expansion of the determinant in the first term in (26) by the last row and last column gives the sum of nine terms. Each of these terms involves the determinant of a two by two matrix of "x's" multiplied by price terms (from which a  $\lambda^2$  term may be factored out. Since  $NF_{55}$  is a linear function of  $\lambda$ , multiplying  $\lambda$  by an arbitrary factor  $k$ , multiplies both the first and second term in (26) by a factor of  $k$  raised to the third power. This, in turn, means that the numerator and denominators of both terms in (24) are raised by a factor of  $k$  to the fourth power, which cancel each other out. The result is invariant with the number of foods in the demand matrix and with the value of  $k$ . Thus, demand elasticities are not a function of  $\lambda$ .

To summarize, given only four values,  $\hat{a}_2$  (a linear function of the shadow price for bulk, specifically  $\hat{p}b_2b_3$ ),  $\hat{p}_y$  (the shadow price for variety),  $S$  (the point at which bulk has disutility, or  $b_2/2b_3 = \hat{a}_1/\hat{a}_2$ ), and  $w/\eta_{nf}$ , the entire matrix of demand elasticities may be estimated for  $n$  foods. Three of these values can be estimated from (15'). The fourth value,  $w/\eta_{nf}$ , may be obtained from direct estimates of  $w$  and  $\eta_{nf}$ .

## II.4. An Algorithm for Solving the Model

Since none of these four values are commonly estimated parameters (to say the least), a preferable alternative may be to substitute from one to four income or price elasticities for any or all of the four values just listed. That is, any combination of four out of a total of  $(n+1)$  times  $(n+2)$  demand elasticities plus the four values,  $\hat{a}_2$ ,  $\hat{p}_v$ ,  $S$ , and  $w/n_{nf}$  may be selected to identify not only the entire demand system, but these four additional utility function parameter values as well.

Various combinations of arbitrary values may be specified for  $\hat{a}_2$ ,  $\hat{p}_v$ ,  $S$ , and  $w/n_{nf}$ , and various demand matrices computed, until the unique demand matrix is found which contains all of the four exogenously specified price and income elasticities. To assist in an orderly search for the particular combination of  $\hat{a}_2$ ,  $\hat{p}_v$ ,  $S$ , and  $w/n_{nf}$ , which will give the four exogenously specified elasticities, (30) is developed below. First, rewrite (25) (which gives an expression for the determinant of the bordered Hessian) as (26) using a different notation.

$$(27) \quad D = NF_{55}|D_{55}| - p_5^2|D_{44}|$$

where  $D_{55}$  and  $D_{44}$  are values of a five by five and a four by four determinant, respectively.

To compute the income elasticity for any food, say for the first food, a value needs to be obtained for the co-factor for the last element in the first row of the bordered Hessian:

$$(28) \quad D_{61} = \begin{vmatrix} x_{12} & x_{13} & x_{14} & 0 & -p_1 \\ x_{22} & x_{23} & x_{24} & 0 & -p_2 \\ x_{32} & x_{33} & x_{34} & 0 & -p_3 \\ x_{42} & x_{43} & x_{44} & 0 & -p_4 \\ 0 & 0 & 0 & NF_{55} & 0 \end{vmatrix}$$

$$= NF_{55} \begin{vmatrix} x_{12} & x_{13} & x_{14} & -p_1 \\ x_{22} & x_{23} & x_{24} & -p_2 \\ x_{32} & x_{33} & x_{34} & -p_3 \\ x_{42} & x_{43} & x_{44} & -p_4 \end{vmatrix} = NF_{55}|D_{14}|$$

By analogy, for the next or second food, (29) may be written:

$$(29) \quad D_{62} = -NF_{55}|D_{24}|$$

The equivalent expression for the non-food income elasticity has already been derived and is given in the numerator of (22) above. Note that the ratio of any two food income elasticities does not involve  $w/n_{nf}$ ; that is, the  $NF_{55}$  term is absent, as shown in (30):

$$(30) \quad \frac{\eta_1}{\eta_2} = - \frac{q_2}{q_1} \frac{|D_{14}|}{|D_{24}|}$$

The first step in the computational algorithm would be to arbitrarily select the lowest value for  $S$  ( $b_2/2b_3$ ) within a feasible range (which then can then be incremented in discrete steps as the process to be described below is repeated several times). Second, three food income elasticities out of the total of four elasticities are selected which give two ratios, as expressed on the left-hand side of (30).

Third, a "sub-algorithm," so to speak, is set up which, for a given  $S$ , determines the unique values for  $\hat{a}_2$  and  $\hat{p}_v$ , which give the two ratios selected in step two. This sub-algorithm would involve arbitrarily specifying a value say for  $\hat{a}_2$ , and then stepping through a series of values for  $\hat{p}_v$  (the three values  $S$ ,  $\hat{a}_2$ , and  $\hat{p}_v$  are substituted for the three unknowns on the right-hand side of (30)) until one ratio is obtained exactly. This also gives a computed value for the second ratio, which is most likely not the second ratio value being searched for. However, the difference between the value computed in the first iteration and the target value (for the second ratio) provides an indication of how to adjust  $\hat{a}_2$  for a second iteration and so on, until the correct values of  $\hat{a}_2$  and  $\hat{p}_v$  are found which give the desired ratios.

Finally, in step four, the three values just identified for  $\hat{a}_2$ ,  $\hat{p}_v$ , and  $S$ , are substituted into the bordered Hessian. Values for the remaining unknown in the bordered Hessian, specifically  $w/n_{ff}$ , are stepped through and matrices of elasticities computed until the matrix containing the desired fourth elasticity found. This completes the first iteration of the overall algorithm and gives estimates of the four pre-specified elasticities, most likely only one of which (the fourth) matches the four elasticities exogenously specified.

Return to step one. Increment  $S$  and repeat all steps. Continuing to increment  $S$  within a feasible range will give several combinations of estimates of the four elasticities, which overall (if the proposed demand structure has any validity and assuming realistic estimates of the four elasticities) should "bracket" the first three elasticities above and below, with one combination being a close estimate of all four exogenously specified elasticities.

The practical implication of the above analysis, which may be generalized to a demand system of any number of foods and one non-food, is that given:

1. household-level data on prices, quantities, and calorie conversion rates for  $n$  foods, which include all sources of calories for the household;

2. data on total non-food expenditures, number of household members, and the ages and sex of these household members;
3. a priori knowledge of any combination of four of the following (n+1) times (n+2) plus four parameters:
  - (a)  $\hat{a}_2$  (a linear function of the shadow price for bulk, specifically  $\hat{p}_b 2b_3$ ),
  - (b)  $\hat{p}_v$  (the shadow price for variety),
  - (c) S (the point at which bulk has disutility, or  $b_2/2b_3 = \hat{a}_1/\hat{a}_2$ ),
  - (d)  $w/\eta_{nf}$  (the ratio of the money flexibility over the income elasticity for non-foods),
  - (e) the (n+1) by (n+2) elasticities in the demand matrix,

an entire matrix of demand elasticities may be constructed which does not depend on the assumption of additivity of utility between foods in the utility function.

### III. A HYPOTHETICAL APPLICATION OF THE CHARACTERISTIC FOOD DEMAND SYSTEM: VARYING PARAMETER ESTIMATES TO INVESTIGATE SOME PROPERTIES OF THE SYSTEM

To investigate some properties of the characteristic food demand system proposed in this paper and to gain some intuitive insight into how various assumptions affect parameter estimates, a "base" set of demand parameter estimates will be derived by assuming hypothetical values for  $\hat{a}_2$ ,  $\hat{p}_v$ , S, and  $w/\eta_{nf}$ , and then varying these values one at a time to see the direction of and extent to which the base elasticities change.

Table 7 gives the values assumed for  $\hat{a}_2$ ,  $\hat{p}_v$ , S, and  $w/\eta_{nf}$  for the base demand elasticity matrix. These combined with the data given in Table 6 generate the three sets of demand matrices given in Tables 8, 9, 10 which show the observed (uncompensated) demand matrix elasticity matrix, the compensated elasticity (utility constant; income elasticities are not applicable) demand matrix, and the compensated  $\partial q/\partial p$  (parameters not expressed in terms of percentage changes) demand matrix.

Before proceeding, note that the four common restrictions derived from demand theory -- adding up, homogeneity, symmetry and negativity -- hold for these estimates. The calculations are not shown here, but the bordered principal minors of the bordered Hessian alternate in



Table 7--Base assumed values

$$\begin{aligned}\hat{\alpha}_2 &= -0.1000 \\ \hat{p}_y &= 0.650 \\ S &= -3.0^* \\ w/\eta_{mf} &= -1.0\end{aligned}$$

\* Calories expressed in units of 1,000 per adult equivalent per day.

Table 8--Base observed demand elasticities

Dependent Variable	Corn Price	Rice Price	Fruit/ Vegetable Price	Meat Price	Non-Food Price	Income
Corn	-.393	.157	.062	.088	.000	.086
Rice	.539	-1.136	.002	.023	.000	.572
Frt./veg.	.057	.006	-.645	.086	.000	.496
Meat	-.259	-.045	-.052	-.996	.000	1.351
Non-foods	-.499	-.083	-.138	-.121	-1.000	1.842

Table 9--Base compensated demand elasticities

Dependent Variable	Corn Price	Rice Price	Fruit/ Vegetable Price	Meat Price	Non-Food Price
Corn	-.369	.162	.071	.109	.026
Rice	.702	-1.098	.060	.165	.171
Fruits/vegetables	.198	.039	-.594	.209	.149
Meat	.126	.044	.086	-.661	.405
Non-foods	.024	.038	.051	.335	-.447

Table 10--Base compensated  $\partial q/\partial p$  demand parameters

Dependent Variable	Corn Price	Rice Price	Fruit/ Vegetable Price	Meat Price	Non-Food Price
Corn	-.767	.260	.099	.031	.024
Rice	.260	-.313	.015	.008	.029
Fruits/vegetables	.099	.015	-.198	.014	.034
Meat	.031	.008	.014	-.023	.046
Non-foods	.024	.029	.034	.046	-.204



sign with the first principal minor positive. This ensures a constrained maximization (see Chiang [1967], p. 360). Other values could have been chosen for the variables in Table 8 which would not have satisfied negativity (the bordered principal minors of the bordered Hessian do not alternate in sign or the first principal minor is not positive). It was already pointed out above that the bordered Hessian matrix is symmetric. Adding up is assured by derivation of the bordered Hessian under a budget constraint.

In Tables 11 through 14 below values for  $\hat{a}_2$ ,  $\hat{p}_v$ ,  $S$ , and  $w/\eta_{nf}$  are varied one at a time to see how changes in these values are related to the magnitudes of the estimated demand elasticities for various foods. In reviewing these tables, keep in mind that any combination of four income or own-price elasticities (in any particular row) could be switched with the assumed values for the parameters of the utility function to give identical elasticity and parameter estimates.

Table 11 shows the effect of varying the shadow price for bulk on the parameter estimates. In general, the higher is the shadow price for bulk, the lower are the income elasticities for staples and the higher are the income elasticities for meats (or non-staples). Table 11 also shows that in general price-responsiveness goes down as the bulk shadow price increases, as hypothesized earlier in the paper, although price responsive increases for rice.

Table 12 shows that as the point of bulk disutility is increased (for example for populations which are physically active because of their occupations), income elasticities tend to increase for bulk-intensive foods such as rice and corn. Income elasticities decline for foods which provide variety in the diet.

Table 13 shows that increasing the variety shadow price has the opposite effect from increasing the bulk shadow price on income and price elasticities for corn -- elasticities increase. For foods that add variety to the diet at low cost, income and price responsiveness go down. The strong desire for variety constrains the reactions to changes in the price of the low-variety-cost good.

Table 14 shows that price-responsiveness of foods is little effected by variation in  $w/\eta_{nf}$ . However, income elasticities for all foods, and income and price-responsive for the non-food good are extremely sensitive to this parameter.

Reviewing the overall pattern of elasticity estimates in Tables 11 through 14, elasticity estimates in general, but especially for corn, are sensitive to assumed values for  $\hat{a}_2$ ,  $\hat{p}_v$ ,  $S$ , and  $w/\eta_{nf}$ . This is in one sense encouraging, and in another sense discouraging. It is encouraging in that the proposed demand system can accommodate a wide range of parameter estimates which are consistent with the four general restrictions imposed by demand theory. Such generality, however, comes with a price. Apparently, fairly precise estimates of

Table 11--Effect of varying the shadow price for bulk ( $\hat{p}_b 2b_3$ )\*

Assumption	Income Elasticities				
	Corn	Rice	Fruits/ Vegetables	Meat	Non-Food
$\hat{p}_b 2b_3 = -0.050$	0.47	0.73	0.17	1.20	1.68
$\hat{p}_b 2b_3 = -0.075$	0.21	0.59	0.40	1.32	1.78
$\hat{p}_b 2b_3 = -0.100(B)$	0.09	0.57	0.50	1.35	1.84
$\hat{p}_b 2b_3 = -0.125$	0.02	0.60	0.55	1.34	1.89
$\hat{p}_b 2b_3 = -0.150$	-0.02	0.66	0.58	1.26	1.97

  

Assumption	Own-Price Elasticities				
	Corn	Rice	Fruits/ Vegetables	Meat	Non-Food
$\hat{p}_b 2b_3 = -0.050$	-0.75	-0.97	-0.70	-1.04	-1.00
$\hat{p}_b 2b_3 = -0.075$	-0.49	-1.03	-0.67	-1.02	-1.00
$\hat{p}_b 2b_3 = -0.100(B)$	-0.39	-1.14	-0.65	-1.00	-1.00
$\hat{p}_b 2b_3 = -0.125$	-0.35	-1.27	-0.62	-0.96	-1.00
$\hat{p}_b 2b_3 = -0.150$	-0.33	-1.41	-0.58	-0.89	-1.00

\*  $\hat{p}_b b_2$  varies directly with  $\hat{p}_b 2b_3$  (S is constant), so that the first derivative of bulk increases as the absolute value of  $-\hat{p}_b 2b_3$  increases, in the range where the marginal utility of bulk is positive.

Table 12--Effect of varying the point of bulk disutility ( $b_2/2b_3$ )

Assumption	Income Elasticities				
	Corn	Rice	Fruits/ Vegetables	Meat	Non-Food
$b_2/2b_3 = -2.0$	0.08	0.42	0.51	1.42	1.82
$b_2/2b_3 = -2.5$	0.08	0.48	0.50	1.40	1.82
$b_2/2b_3 = -3.0(B)$	0.09	0.57	0.50	1.35	1.84
$b_2/2b_3 = -3.5$	0.11	0.70	0.48	1.22	1.91
$b_2/2b_3 = -4.0$	0.36	0.54	0.32	0.56	2.30

  

Assumption	Own-Price Elasticities				
	Corn	Rice	Fruits/ Vegetables	Meat	Non-Food
$b_2/2b_3 = -2.0$	-0.32	-0.79	-0.06	-1.03	-1.00
$b_2/2b_3 = -2.5$	-0.35	-0.94	-0.06	-1.03	-1.00
$b_2/2b_3 = -3.0(B)$	-0.39	-1.14	-0.65	-1.00	-1.00
$b_2/2b_3 = -3.5$	-0.46	-1.39	-0.60	-0.90	-1.00
$b_2/2b_3 = -4.0$	-0.48	-0.93	-0.29	-0.40	-1.00

Table 13--Effect of varying the shadow price for variety ( $\hat{p}_v$ )

Assumption	Income Elasticities				
	Corn	Rice	Fruits/ Vegetables	Meat	Non-Food
$\hat{p}_v = 0.000$	0.02	0.61	1.43	1.44	1.50
$\hat{p}_v = 0.300$	0.05	0.60	0.88	1.40	1.71
$\hat{p}_v = 0.650(B)$	0.09	0.57	0.50	1.35	1.84
$\hat{p}_v = 1.000$	0.15	0.57	0.21	1.31	1.92
$\hat{p}_v = 1.300$	0.25	0.60	0.04	1.25	1.95

  

Assumption	Own-Price Elasticities				
	Corn	Rice	Fruits/ Vegetables	Meat	Non-Food
$\hat{p}_v = 0.000$	-0.37	-1.59	-1.06	-1.06	-1.00
$\hat{p}_v = 0.300$	-0.37	-1.34	-0.80	-1.02	-1.00
$\hat{p}_v = 0.650(B)$	-0.39	-1.14	-0.65	-1.00	-1.00
$\hat{p}_v = 1.000$	-0.47	-1.00	-0.57	-1.00	-1.00
$\hat{p}_v = 1.300$	-0.63	-0.93	-0.57	-1.04	-1.00

Table 14--Effect of varying the ratio of the money flexibility over the income elasticity for non-foods ( $w/\eta_{nf}$ )

Assumption	Income Elasticities				
	Corn	Rice	Fruits/ Vegetables	Meat	Non-Food
$w/\eta_{nf} = -0.20$	0.15	1.02	0.89	2.42	0.66
$w/\eta_{nf} = -0.60$	0.11	0.73	0.64	1.74	1.42
$w/\eta_{nf} = -1.00(B)$	0.09	0.57	0.50	1.35	1.84
$w/\eta_{nf} = -2.00$	0.06	0.37	0.32	0.87	2.37
$w/\eta_{nf} = -3.00$	0.04	0.27	0.24	0.64	2.63

  

Assumption	Own-Price Elasticities				
	Corn	Rice	Fruits/ Vegetables	Meat	Non-Food
$w/\eta_{nf} = -0.20$	-0.41	-1.16	-0.67	-1.07	-0.36
$w/\eta_{nf} = -0.60$	-0.40	-1.14	-0.66	-1.02	-0.77
$w/\eta_{nf} = -1.00(B)$	-0.39	-1.14	-0.65	-1.00	-1.00
$w/\eta_{nf} = -2.00$	-0.39	-1.13	-0.63	-0.96	-1.29
$w/\eta_{nf} = -3.00$	-0.38	-1.12	-0.63	-0.95	-1.43

$\hat{\alpha}_2$ ,  $\hat{p}_y$ ,  $s$ , and  $w/\eta_{nf}$  are needed to obtain precise estimates for all elements in the demand matrix.

In selecting which income or price elasticities to specify (estimate) exogenously, the best approach would appear to be to choose parameters for goods with high budget shares (corn and non-foods using our hypothetical data set) and which are sensitive to the utility function parameter which needs to be identified, those which "drive" the consumption decision. For example, one would want to identify  $\hat{p}_y$  by a priori specification of the income elasticity of fruits and vegetables, rather than the income elasticities of meat or rice. The methodology is flexible enough that depending on data availability and estimates available from other studies, one may choose to exogenously specify a certain parameter which one has some confidence in, say the non-food income elasticity, and not to choose to estimate another parameter, say the money flexibility.

Finally, the wide range of elasticities produced in Tables 11 through 14 and the expectation that shadow prices for the various characteristics will vary by income group, demonstrate that the proposed framework can account both for a pattern of declining elasticities (in absolute value) as income increases, and a pattern of first increasing and then decreasing elasticities as income increases.

#### IV. AN APPLICATION USING PUBLISHED INFORMATION FROM A NATIONWIDE NUTRITION SURVEY

##### IV.1. Deriving the Food Demand Matrices

This methodology (a food demand model based on demand for characteristics) is applied here to published data from the 1978 Philippine nationwide nutrition survey, to derive food demand elasticity estimates for urban and rural populations.<sup>3</sup> Table 15 presents data on per capita consumption for 17 food groups, average price paid per kilo for each of these 17 food groups, and average household size and number of adult equivalents, disaggregated by urban and rural populations. Also provided in Table 15 are kilogram-to-calorie conversion rates (which are available for the surveywide data, but not broken down by urban and rural groups), and nonfood budget shares which are not taken from the FNRI surveys, but are suggested by other nationwide expenditure surveys (NEDA 1985).

A final requirement for application of the methodology is pre-specification of four price and income elasticities, one set of elasticities for the urban food demand matrix and a separate set of elasticities for the rural food demand matrix. Unfortunately, no direct estimates are available from other studies, specifically for urban and rural populations. However, several demand estimates are available for rice and corn (see Bennagen 1982 for a survey of food demand elasticities for the Philippines), which are disaggregated by region and

Table 15--Per capita consumption levels, calorie conversion rates, prices paid for various food groups, household size, adult equivalents, and food budget shares, by urban and rural populations for the Philippines, 1978

BEFORE AGGREGATION										
Food Group	Aggregation Category	Calories Per Kilogram	Urban			Rural				
			Grams Per Day	Price Per Kilo		Grams Per Day	Price Per Kilo			
Corn grits	Corn	3,608	18	1.77		49	1.60			
Milled rice	Rice	3,452	255	2.12		323	2.11			
Other cereal products	Other cereals	3,350	39	5.79		12	7.01			
Fish	Fish	672	116	6.44		95	5.81			
Meat and poultry	Meat	2,153	53	12.32		21	11.64			
Green leafy/yellow vegetables	Fruits/vegetables	265	28	2.49		38	1.75			
Vitamin C rich foods	Fruits/vegetables	307	54	2.34		44	1.98			
Other fruits and vegetables	Fruits/vegetables	332	174	2.80		166	2.46			
Rice products	All others	1,804	11	6.19		6	7.16			
Corn products	All others	1,875	0	7.99		0	6.89			
Starchy roots/tubers	All others	1,073	20	2.33		46	1.30			
Sugars and syrups	All others	2,472	43	2.82		19	2.84			
Fats and oils	All others	6,800	20	6.54		10	5.91			
Eggs	All others	1,353	14	9.76		5	10.50			
Milk and milk products	All others	2,843	55	8.61		22	6.58			
Dried beans/nuts/seeds	All others	2,481	9	5.31		8	5.20			
Miscellaneous	All others	601	23	12.02		19	8.35			
AFTER AGGREGATION										
Food Group	Urban					Rural				
	Kilos Per Week	Price Per Kilo*	Calories Per Kilo	Food Budget Share	Price Per 1000 Calories	Kilos Per Week	Price Per Kilo*	Calories Per Kilo	Food Budget Share	Price Per 1000 Calories
Corn	0.13	1.04	3,608	0.01	0.29	0.34	0.94	3,608	0.03	0.26
Rice	1.79	1.25	3,452	0.13	0.36	2.26	1.24	3,452	0.24	0.36
Other cereals	0.27	3.41	3,350	0.05	1.02	0.08	4.12	3,350	0.03	1.23
Fish	0.81	3.79	672	0.18	5.64	0.67	3.42	672	0.20	5.09
Meat	0.37	7.25	2,153	0.16	3.37	0.15	6.85	2,153	0.09	3.18
Fruits/vegetables	1.79	1.57	319	0.16	4.92	1.74	1.33	317	0.20	4.20
All others	1.37	3.93	2,539	0.31	1.55	0.95	2.67	2,042	0.22	1.31
Food budget share out of total expenditures†			0.45					0.55		
Household size			6.4					6.3		
Adult equivalents per household			5.09					4.92		

\* Prices are normalized on a corn price of P1.70 per kilo.

† Food budget share out of total expenditures are taken from NEDA (1984); otherwise all other data are found in FNR1 (1981).

income group and which suggest that low-income groups are more price-responsive than high-income groups.

National aggregate estimates of -0.6, 0.1, and -0.4 for the own-price elasticity of rice, the income elasticity of rice, and the income elasticity of corn were obtained by Bouis (1982), using quarterly household food expenditure surveys conducted by the Ministry of Agriculture. Urban and rural elasticity estimates are weighted averages of these national aggregate estimates, and it can be presumed that the lower income rural households are more price responsive than urban households. Thus, rural elasticities are greater (in absolute value) than the national aggregate estimate, and urban elasticities are smaller.

Experiments were run with various sets of urban and rural estimates, which bracketed these national base estimates.<sup>4</sup> Table 16 gives the set of estimates which were eventually selected for simulation. Nonfood expenditure elasticities of 1.49 and 1.61, for urban and rural groups, respectively, shown in Table 16, are consistent with an estimate of 1.50 obtained by Canlas (undated) for the country as a whole, and correspond to food expenditure elasticities of 0.40 and 0.50, for urban and rural groups, respectively.

Application of the characteristic food demand methodology using the data contained in Tables 15 and 16 give the demand elasticity matrices shown in Table 17 for an aggregation of seven food groups and one nonfood.

Examining the two demand matrices, note the tendency for the higher-priced foods such as meats (for nonstaples) and other cereals (for staples) to have the highest income elasticities, and for these income elasticities to be higher for the (lower income) rural group than for the (higher income) urban group. The model structure ensures that the estimates conform to the four standard restrictions of demand theory: (1) adding up, (2) homogeneity, (3) symmetry, and (4) negativity, which, among other things, means that the cross-price elasticities of a high budget proportion item such as rice will tend to be higher (in absolute value) than the cross-price elasticities of items with a lower budget proportion.

## IV.2. Simulating Reactions to Changes in Prices and Income

### IV.2.1. Urban price changes

How well do these elasticity estimates "predict" actual consumption levels which were surveyed in 1982? To answer this, changes in prices and incomes for the period 1978-1982 were applied to the derived demand matrices. The published summary of the 1982 nutrition survey does not report prices paid for various foods, as does the 1978

Table 16--Price and income elasticities specified a priori for application of characteristic demand methodology

Group	Rice Own-Price Elasticity	Rice Income Elasticity	Corn Income Elasticity	Nonfood Expenditure Elasticity
Urban	-0.40	-0.20	-0.45	1.49
Rural	-0.65	0.20	-0.45	1.61

Table 17--Demand elasticity matrices derived using characteristic demand methodology

URBAN									
	Corn	Rice	Other Cereals	Fish	Meat	Fruits/ Vegetables	All Others	Non- Food	Incom.
Corn	-0.794	0.603	0.093	0.011	0.120	-0.102	0.672	-0.152	-0.450
Rice	0.036	-0.400	0.070	0.001	0.101	-0.108	0.569	-0.068	-0.200
Other cereals	0.010	0.131	-0.902	-0.016	0.031	-0.078	0.188	0.161	0.475
Fish	-0.003	-0.046	-0.008	-0.994	0.027	0.098	0.120	0.201	0.595
Meat	0.002	0.031	0.005	0.021	-1.096	0.053	0.031	0.241	0.712
Fruits/vegetables	-0.007	-0.109	-0.018	0.140	0.088	-0.761	0.420	0.063	0.186
All others	0.014	0.196	0.032	0.083	0.031	0.197	-1.205	0.165	0.487
Nonfood	-0.008	-0.106	-0.018	-0.043	-0.024	-0.088	-0.093	-1.109	1.495
RURAL									
	Corn	Rice	Other Cereals	Fish	Meat	Fruits/ Vegetables	All Others	Non- Food	Income
Corn	-1.429	1.383	0.035	0.039	0.061	-0.038	0.474	-0.076	-0.449
Rice	0.148	-0.650	0.017	0.007	0.034	-0.067	0.276	0.034	0.200
Other cereals	0.013	0.058	-1.007	-0.028	-0.000	-0.082	0.015	0.150	0.881
Fish	-0.013	-0.070	-0.003	-0.978	0.005	0.073	0.062	0.134	0.790
Meat	-0.002	-0.004	-0.001	-0.005	-1.073	0.007	-0.015	0.159	0.934
Fruits/vegetables	-0.017	-0.098	-0.003	0.123	0.033	-0.703	0.283	0.056	0.327
All others	0.044	0.250	0.006	0.075	0.010	0.227	-1.330	0.104	0.615
Nonfood	-0.034	-0.177	-0.026	-0.057	-0.016	-0.127	-0.092	-1.104	1.613



report, so that consumer price indices compiled by the Bureau of Census and reported in Table 17, were used instead.<sup>5</sup>

Table 18 shows that real rice and corn prices fell substantially, by about 20 percent over the four-year period. Real nonfood prices increased by 9 percent, while real meat prices declined by 9 percent. These price changes were applied to the urban demand elasticities. Derivation of price changes for the rural population and of income changes for both the urban and rural populations is somewhat involved and is discussed in Appendix B. It is concluded that urban incomes fall by 7.5% over the period from 1978 to 1982 while rural incomes increased by 10%.

#### IV.2.2. Comparing Predicted and Observed Changes in Food Consumption

Given the derived demand matrices and the changes in prices and incomes just discussed, it is now possible to predict 1982 consumption levels and to compare these predicted values with the consumption levels that were actually surveyed, as a way of testing the validity of the methodology used to derive the demand matrices, in conjunction with the assumptions made as to the changes in prices and incomes facing urban and rural consumers. A comparison of the predicted and observed values is presented in Table 19.

The model distinguishes quite well between an increase in rice consumption for the urban group and a decrease in rice consumption for the rural group. Similarly for "other cereals" (primarily wheat-based products), the model correctly predicts no change for the rural group (despite a hefty price increase) and a decrease in consumption for the urban group. The only instance where the model performs poorly is that it fails to predict a small decrease in corn consumption for the urban group, which results in a small overestimate in the overall change in calories consumed from cereals for the urban group.

Despite a substantial fall in cereal prices, urban consumers do not increase overall cereal consumption, but reveal a strong preference for noncereal items in the diet which tend to be more expensive calorie sources. Rural consumers decrease cereal consumption as cereal prices fall, not because incomes go down (incomes are assumed to increase), but apparently because of decreased subsistence cereal production, the consequent rise in cost implicit in greater dependence on food purchases in the market (paying retail instead of wholesale prices in terms of the characterization presented in Appendix B), and higher input costs.

The model successfully distinguishes between the large increase in calories for the rural group for the catch-all category of "all other noncereals," and the much smaller decrease for that category for the urban group. In general, the model does well in predicting rather



Table 18--Price and income changes for urban and rural population in the Philippines, 1978-1982

Category	Price Index For 1982 (1978=100)*	Urban		Rural	
		Percentage Change	Recomputed Price Index for 1982 Assuming a 5 Percent Increase in Cereal Prices	Percentage Change	
All items	168.4		176.0		
Nonfoods	183.1	+8.7	183.1	+4.0	
Corn	137.5	-18.4	184.8	+5.0	
Rice	132.0	+21.7	184.8	+5.0	
Other cereals	198.1	+17.6	198.1	+12.6	
Fish	176.0	+4.5	176.0	0.0	
Meat	156.3	-9.3	156.3	-11.2	
Fruits/vegetables	170.0	+1.0	170.0	-3.5	
Miscellaneous foods	157.3	-6.5	157.3	-11.7	

  

	Year					Growth	
	1975	1976	1978	1980	1982	Period	Percentage Change
Gross national product†	68.5	73.0	83.1	92.6	98.7	1978-82	18.8
Agriculture	18.2	19.7	21.6	23.7	25.4	1978-82	17.4
Industry	22.7	24.9	29.6	33.5	35.8	1978-82	21.0
Services	27.5	28.4	31.6	35.5	37.9	1978-82	20.0
Total population#	42.1	43.4	45.8	48.1	-	1976-80	10.8
Urban	14.0	14.9	16.4	17.9	-	1976-80	20.7
Rural	28.0	28.5	29.4	30.1	-	1976-80	5.7
Wages							
Urban (nominal)§	12.5	13.2	14.4	15.8	-		
Rural (nominal)	7.5	9.0	10.1	11.7	13.9		
Urban (real)¶	72.9	72.3	68.3	53.4	-	1978-80	-21.2
Rural (real)¶	119.8	135.7	131.1	109.2	103.2	1978-82	-21.3

\* Consumer price indices for February 1982, national level, Bureau of Census and Statistics.

† Billions of constant 1972 pesos, 1984 Philippine Statistical Yearbook, Table 3.10, NEDA (1984).

# Millions of people, 1984 Philippine Statistical Yearbook, Table 1.3, NEDA (1984).

§ Pesos per day, Central Bank of the Philippines.

Pesos per day, Table 5, p. 111.17, Agenda for Action for the Philippine Rural Sector, PIDS (1987).

¶ Index where 1972=100, money wage rate deflated by consumer price index.

Table 19--Observed and predicted changes in calorie intakes, by aggregate food groups and for all foods for urban and rural populations, in the Philippines, 1978-1982

Simulation	Changes in Calories From Consumption of Cereals*			
	Rice	Corn	Other Cereals	All Cereals
Urban-observed	48.3	-25.3	-26.8	-3.7
Urban-simulated	47.8	-0.2	-29.4	18.2
Rural-observed	-44.9	-14.4	0.0	-59.3
Rural-simulated	-39.4	-15.3	-1.1	-55.8

  

Simulation	Changes in Calories From Consumption of Noncereals*			
	Fish	Meat	Fruits/Vegetables	All Others
Urban-observed	-2.7	36.6	1.0	-18.8
Urban-simulated	-5.7	7.1	-1.9	9.7
Rural-observed	11.4	15.1	-8.6	56.3
Rural-simulated	4.5	10.2	1.3	63.1

  

Group	Total Calorie Consumption	
	1978	1982
Philippines-reported	1,804	1,808
Philippines-using 1978 calorie conversion rates	1,804	1,837
Urban-reported	1,872	1,831
Urban-using 1978 calorie conversion rates	1,845	1,857
Rural-reported	1,769	1,797
Rural-using 1978 calorie conversion rates	1,795	1,820

  

Simulation	Changes in Total Calorie Consumption*	
	Estimated Observed Income Change	Constant Income
Urban-observed	+12	
Urban-simulated	27.5	45.9
Rural-observed	+35	
Rural-simulated	23.3	-24.9

\* All changes in calorie consumption are calculated using 1978 calorie conversion rates as reported for the total national sample.

small calorie changes for the remaining three food groups, with the glaring exception of the meat group for urban residents.

There is some tendency for the model to understate preferences for fish and meat and to overstate preferences for fruits and vegetables and for all other noncereals. Demand behavior for meats in particular is driven by demand behavior of higher income groups so that the uneven distribution of the change in income (discussed in Appendix B) can account for this discrepancy. Table 20 indicates that the households in the highest 20 percent of the income distribution eat about half of the meat consumed in the Philippines, and that about three-fourths of the increase in meat consumption between 1978 and 1982 was accounted for by the upper one-third of the income distribution. This only serves to underscore the potential pitfalls of examining demand changes and their nutritional consequences at such an aggregate level as has been undertaken here, thus emphasizing the desirability of disaggregating by occupation group (especially for the rural population) and by income group.

The model does quite well in predicting the relatively small changes in total calorie intakes that were observed between 1978 and 1982 for both the urban and rural groups. In the case of the rural group, the model did a reasonably good job of predicting changes in the consumption levels of each of the seven food groups, so that it was a foregone conclusion that predicted and observed change in total calories would be similar. In the case of the urban group, however, overestimates of calories consumed from corn and all other noncereals and an underestimate of calories consumed from meat netted out to give an accurate projection of total change in calories. This is not just coincidence, but because total calorie intake (bulk) is included as an explicit argument in the utility function used to derive the demand matrices.

To obtain estimates of the elasticities of calorie intakes with respect to income implicit in the demand matrices presented in Table 15, total calorie intakes were re-estimated assuming no change in income. For the urban group, for a 7.5 percent increase in income, the change in calorie intakes went from +27.5 to +45.9 (see Table 19), or percentage increase of 1.0 percent (18 divided by 1,857). Dividing 1.0 percent by 7.5 percent gives an elasticity of 0.13. For the rural group, for a 10 percent decrease in income, the change in calorie intakes went from +23.3 to -21.0, or a percentage decrease of 2.5 percent (-48 divided by 1,820). Dividing -2.6 percent by -10.0 percent gives an elasticity of 0.26.

The elasticity is higher for rural groups for two reasons. First, ceteris paribus, rural incomes are lower and calorie intakes lower so that the marginal utility from increases in bulk is higher. Second, activity levels of rural households can be expected to be higher than those of urban households. Even if calorie intakes for

Table 20--Heat consumption levels, by approximate income quintile, for the Philippines, 1978 and 1982

Approximate Income Quintile	Absolute Percentage of Total Sample		Cumulative Percentage Of Total Sample		Heat Consumption In Grams Per Day		Change
	1978	1982	1978	1982	1978	1982	
1	21.2	15.1	21.2	15.1	9	14	+5
2	23.3	21.7	44.5	36.8	19	20	+1
3	24.1	28.0	68.6	64.8	27	35	+8
4	13.4	19.3	82.0	84.1	38	55	+17
5	18.0	15.9	100.0	100.0	76	103	+27

Source: FNRI (1981 and 1984).

urban and rural populations were equal, marginal utility from increases in bulk would still be higher for rural groups.

## V. CONCLUSION

A demand for characteristics approach has been used in this paper to explain food acquisition behavior, in particular demand for bulk to alleviate hunger, demand for variety in the diet, and demand for tastes inherent in particular goods. The analysis has shown that such a framework can be used to explain why, in most empirical studies in the literature, low income groups have demonstrated a greater price-responsiveness to changes in food prices than high income groups in less developed countries. This framework can also account for the less frequently observed phenomenon of highest price elasticities for middle income groups for some foods.

Bulk and variety enter the utility function in such a way that utility from consumption of any one food depends on the level of consumption of all other foods. This avoids an assumption of strong or weak separability among food groups, which underlies existing linear expenditure systems and which is inappropriate for estimating a highly disaggregate food demand matrix necessary for many types of policy analysis.

By specifying an explicit functional form for these characteristics in the utility function, it turns out that the entire matrix of price and income elasticities can be derived for a system of  $n$  foods and one nonfood good from prior knowledge of just four elasticities in the  $(n+1)$  by  $(n+2)$  matrix of price and income elasticities. This methodology, then, provides a means for computing food demand matrices with data which is often available in published form, which has the potential for substantially lowering the costs of policy analysis.

The demand system was applied to two Philippine data sets, published summaries of food consumption information from a nationwide nutrition survey and a household food expenditure survey conducted in a southern rural province. The model was found to generate demand estimates which are similar to estimates obtained by other authors for the Philippines using direct estimation techniques, and did a reasonably good job of "predicting" observed changes in food consumption between 1978 and 1982 for two nationwide samples.

## APPENDIX A: DEFINING VARIETY

The first two issues to be resolved are (1) is variety a relative or an absolute concept? and (2) is variety monotonous and so subject to diminishing marginal returns to utility? The first question is concerned with constructing an objective index of variety ( $V$  in (1)) from household food expenditure data. The second question is concerned with the function into which the index of variety is to be substituted ( $U_V(V)$  in (1)). A simple example will help to understand these two issues.

Suppose there are only two foods, cereals and meat, each a homogeneous good without quality differences. Person A consumes one kilo of cereals and one-half kilo of meat and person B consumes two kilos of cereals and one kilo of meat. Should A and B be assigned the same index of variety (variety is relative) or should B be assigned a higher index (variety is absolute)? Many would perhaps say that A and B have the same variety, but consider the example below.

Let A and B sit down together and each eat a meal of one kilo of cereals and one-half kilo of meat. However one measures variety, A and B both enjoyed the same, objectively measured amount of variety at that meal (even if they happen to value it differently in their personal utility functions). Now let B eat his/her second kilo of cereals and one-half kilo of meat while A watches. B enjoyed some equal measure of variety from this second plateful while A had none. Therefore diet B not only has more variety, but twice as much variety as diet A.

Now what about the marginal utility of the variety from this second plateful? As economists, we know that the total marginal utility from all three characteristics is less from the second plateful than from the first plateful. The extra bulk has a declining marginal utility as outlined in the paper. The intrinsic tastes of cereals and meat also have declining marginal utilities. These two characteristics alone can account for the declining marginal utility of the additional quantities of food (so that the marginal utility from variety need not necessarily decline to preserve declining total marginal utility in the sense that it is usually perceived). Does the utility from variety also decline?

It is possible to argue just the opposite, that enjoyment of a particular characteristic may actually rise with increased consumption. For example, in most instances watching the same movie the second time is less enjoyable than the first viewing. However, certain aspects of the movie, such as the photography or the directing, may be appreciated more the second time when one is not so involved in the plot.

Consider how much more enjoyable even a simple, usually tasteless meal can be when one is unusually hungry, perhaps as enjoyable as a normally delicious meal when, for whatever reason, we sit down to the table with our hunger already satiated. It is at least plausible, then, that the marginal utility from the first "food units" consumed is relatively "bulk-intensive" (the proportion of total marginal utility accounted for by bulk) and that the marginal utility from the last "food units" consumed is relatively more "variety-intensive".

Without claiming to have resolved this issue in any final way, in order to proceed it is assumed in the paper that variety is a relative concept, and that there is a neutral, linear relationship between  $U_v(V)$  and  $V$ , that there is not declining marginal utility in variety (that variety is not monotonous).

## APPENDIX B

### DERIVATION OF INDICES FOR PRICE CHANGES FOR RURAL POPULATION AND FOR INCOME CHANGES FOR URBAN AND RURAL POPULATIONS

#### I. SUBSISTENCE CONSUMPTION AND JOINT PRODUCTION-CONSUMPTION DECISIONS

The derivation of appropriate price indices for rural areas (that is, changes in real prices as viewed from the demand side) is much more problematic than for urban areas, since so much of what is consumed in rural areas is own-farm production and never marketed. Production and consumption decisions, especially for the main staples, rice and corn, are joint decisions, so that consumption levels depend, for example, on production input prices which do not appear in the matrices in Table 15, but affect how rural households evaluate the relative costs of growing and consuming cereals from one's own farm, or buying staples in the market [see Singh, Squire, and Strauss 1986]. In this sense, it is unrealistic to attempt to separate out demand side and supply side effects of food price changes, which is implicit in the exercise being undertaken. Nevertheless, the goal here is to determine the extent to which consumption behavior can be understood using simplifying assumptions.

A substantial number of rural households grow rice and corn both for subsistence consumption and for sale in the market.<sup>6</sup> To the extent that rural households can be characterized as semisubsistence producers of cereals, the price changes for rice and corn which would be appropriate for evaluating demand-side substitution effects should reflect changes in the relative cost of growing cereals for subsistence consumption versus (say) growing export crops and buying staples in the market. A powerful incentive to own-production is that farmers do not have to pay average marketing margins of about 25 percent for cereals (see Bouis and Haddad 1988, Chapter 4), which comprise a high proportion of the total cost of food and nonfood items consumed by the household. Thus, for example, even if a (nonedible) cash crop were 20 percent more profitable to produce than rice, it would still pay a farmer to grow rice up to a maximum of the expected level of consumption of his family. Because of production variability and a desire for food security, the household may produce more than its subsistence needs. For various reasons including the need for cash, even surplus producers may buy and sell rice in the same crop year, and consume rice out of own consumption.

As the price of rice falls and the attractiveness of rice production declines, farmers will, of course, take some land out of rice



production, which (to continue with the above example) will mean more frequent market purchases of rice for consumption needs. The rice price decline leads to an unambiguous fall in income on the supply side. But how does the farmer evaluate the relative change in price on the demand side? The retail price was higher before, but the farmer grew more rice for own consumption, which was purchased at what might be thought of as wholesale prices. Retail prices are lower now, but more is purchased at retail prices. The net effect at the margin is that the rice price has not changed at all for the rice producer qua consumer. However, if the the rice price had remained constant and input prices had risen instead, there would have been an unambiguous rise in the relative cost of rice consumption on the demand side.

The conclusion is that a 20 percent decline in the real cost of cereals may well seriously misrepresent the actual relative price change for rural producers as consumers. Instead a modest five percent increase in real cereal prices is assumed for rural residents, which is reflective of marginally increasing input costs for cereal producers.

## II. INCOME CHANGES FOR URBAN AND RURAL POPULATIONS

Turning now to income changes for urban and rural populations, no directly surveyed measure is available, so that it will be necessary to derive approximations. Table 18 shows that real GNP increased by slightly less than 19 percent over the four years from 1978 to 1982 and that this growth was more or less uniform across the three aggregate sectors--agriculture, industry, and services. Agricultural sector growth can serve as proxy for rural income growth, and growth in industry and services as a proxy for urban income growth. The fact that growth was so uniform across sectors makes this assumption less restrictive.

These figures need to be corrected for population growth. Population censuses were taken in 1975 and 1980 which show that the overall population increased by just over 14 percent during this five-year period, but that the urban population grew much more rapidly than the rural population (again, see Table 18). Taking the 1976-1980 percentage increases in urban and rural populations as a proxies for 1978-1982, gives a 21 percent increase in urban population and a 6 percent increase rural population, or a weighted average of just under 11 percent for the whole population. This would indicate that on a per capita basis, real urban incomes remained about constant, while per capita real rural incomes increased by 12 percent, with the weighted average being an 8 percent increase in real per capita incomes.

There is strong evidence that these changes in income within the urban and rural sectors was not equitably distributed. Table 18 shows the nominal and real wage rates for urban and rural workers over

time.<sup>8</sup> Note, first, that nominal wages are much higher for the urban sector over the entire series than for the rural sector, which is consistent with rapid migration to urban areas indicated by the population figures given in Table 18. Second, and most importantly, real wages in both sectors decline quite precipitously, by 22 percent for urban workers from 1978 to 1980, and by 21 percent for rural workers from 1978 to 1982. Third, real wage rates appear to have fallen faster in urban areas over the four year period from 1978 to 1982 than in rural areas, which is consistent with the lower per capita real income growth rate for urban areas suggested above.

If, as appears to be the case, incomes of upper-income groups increased, while incomes of low-income groups declined (or at best increased substantially less than the incomes of upper-income groups), applying the average income change to the demand matrices derived previously will result in an overestimation of total calories consumed. This is because marginal propensities to buy food out of increments in income and to buy calories out of increments in food expenditures, are higher for low-income groups than for high-income groups. Average per capita changes in income were thus adjusted downward to -7.5 percent for urban areas and to 10 percent for rural areas.<sup>9</sup>

## NOTES

<sup>1</sup> At an intuitive level, the demand system being presented appeals to some notion of an individual's preference structure. An interesting and important but difficult extension of the model presented here would be to incorporate intra-household distribution of food into the utility function. What is being implicitly assumed with respect to intra-household distribution, is that foods are being allocated in an egalitarian fashion. Nevertheless, with respect to demand for bulk, or household energy requirements, some account is taken of the age/sex structure of the household by expressing  $B$  in per adult equivalent terms rather than in per capita terms. What this means mathematically, among other things, is that when derivatives are taken (the change in per capita  $q_i$ ), the  $z_i$  terms (which convert kilograms to calories) in (4) and (5) need to be corrected for the ratio of the number of household members divided by the number of adult equivalents in the household. Household-level data expressed on a per capita basis are typically what are available for empirical analysis, and the estimations presented later in this paper are no exception.

<sup>2</sup> Weak separability is likewise an unnecessary assumption. The marginal rate of substitution between rice and corn, for example, depends on level of consumption of non-staples.

<sup>3</sup> See Quisumbing (1987 and 1988) for demand estimates derived from the FHRI survey data through direct econometric estimation using the household-level observations.

<sup>4</sup> The aggregate own-price elasticity of  $-0.6$ , which is used as a base here, is higher than estimates obtained by other authors, which tend to range from  $-0.2$  to  $-0.4$ . Note also the negative income elasticity for rice for the urban group. Application of the characteristic demand system does not guarantee that any set of four prespecified elasticities will be consistent with such restrictions as positive shadow prices for bulk and variety, or utility maximization. A positive income elasticity for rice for the urban data tended to violate such restrictions.

<sup>5</sup> No breakdown is available by urban and rural groups so that the national aggregate consumer price index is used. Unfortunately, these national aggregate figures are available only for a limited number of aggregate food groups so that, although it would have been technically feasible to further disaggregate the demand matrices shown in Table 17 into several other food groups, no information is available on price changes for these more disaggregate groups.

6 While landlessness is becoming an increasingly important problem in the Philippines, a large majority of rural households have access to some land, so that the characterization which is described in the text is not untypical. Data cited in Philippine Institute for Development Studies (1987; Table 11, p. 11.19) for the bottom 30 percent of the national income distribution (where representation of persons without access to land would be disproportionately high) show that for rural occupations (farmer owners; farmer part-owners; farmer tenants; fishermen, laborers, and related workers; loggers and other forestry workers; miners, quarrymen, and related workers), only 17 percent do not have access to land (that is, are not farm owners, part-owners, or tenants).

A random sample of households in a predominantly rural southern Philippine province (see Bouis and Haddad 1988, Chapter 3), shows that only 7 percent of households had no access to land, using these same occupation categories. Using all occupation categories (including transportation related jobs, skilled workers, professionals, and so on), only 26 percent of households could not be categorized as either farm owner or farm tenant. Moreover, detailed surveys of a stratified sample of farm owner and tenant households, showed that all households produced some rice or corn for subsistence consumption, even those that were predominantly engaged in non-cereal, cash crop production. Some households that characterized themselves as landless laborers or as engaged in nonagricultural occupations (included in the 26 percent landless cited above), had small plots of land on which they produced either rice or corn.

7 Nominal fertilizer prices rose faster than nominal rice prices, and interest rates rose [PIOS 1987, Table 17, p. V.87].

8 The urban series was unfortunately discontinued after 1980 when the indicated trend became an embarrassment to the Marcos administration.

9 Alternative simulations were run, which assumed zero income change for both urban and rural groups, so that the sensitivity of predicted calorie consumption levels to these income growth assumptions could be checked. These results are reported in Table 19.

## REFERENCES

- Alderman, Harold. 1986. The effect of food price and income changes on the acquisition of food by low-income households. Washington, D.C.: International Food Policy Research Institute.
- Bennagen, Ma. Eugenia C. 1982. Staple food consumption in the Philippines. Rice policies in Southeast Asia project working paper no. 5. Washington, D.C.: International Food Policy Research Institute.
- Bouis, Howarth. Forthcoming. Evaluating demand for calories for urban and rural populations in the Philippines: Consequences for nutrition policy under economic recovery. World Development.
- \_\_\_\_\_. 1982. Rice policy in the Philippines. Ph.D. dissertation, Stanford University.
- Bouis, Howarth and Lawrence Haddad. Forthcoming. A case study of the commercialization of agriculture in the southern Philippines: The income, consumption, and nutrition effects of a switch from corn to sugar production in Bukidnon. Research report. Washington, D.C.: International Food Policy Research Institute.
- Canlas, Dante B. Undated. Estimating price and income elasticities of the demand for food: The Philippines, 1965. Quezon City: University of the Philippines, School of Economics (mimeographed).
- Chiang, Alpha. 1967. Fundamental methods of mathematical economics. Tokyo: McGraw-Hill.
- Food and Nutrition Research Institute. 1981. First nationwide nutrition survey, Philippines, 1978, 2nd revision. Manila: National Science and Technology Authority.
- \_\_\_\_\_. 1984. Second nationwide nutrition survey, Philippines, 1982. Manila: National Science and Technology Authority.
- Henderson, James, and Quandt, Richard. 1971. Microeconomic theory, a mathematical approach, 2nd ed. New York: McGraw-Hill.
- Ladd, George, and Veraphol Suvannant. 1976. A model of consumer goods characteristics. American Journal of Agricultural Economics 58 (no. 3, August).
- National Economic and Development Authority. 1984. Philippine statistical yearbook, 1984. Manila: NEDA.

Philippine Institute for Development Studies. 1987. Agenda for action for the Philippine rural sector. Manila: PIDS.

Pinstrup-Andersen, Per, Norha Ruis de Londoño, and Edward Hoover. 1976. The impact of increasing food supply on human nutrition: Implications for commodity priorities in agricultural research and policy. American Journal of Agricultural Economics 58 (no. 2, May): 131-142.

Quisumbing, Ma. Agnes R. 1985. The Philippine food and nutrition situation. Trends and policy issues. Philippine Journal of Nutrition 38 (nos. 3 and 4, July-December).

\_\_\_\_\_. 1987. The effects of food price and income policies on the nutrition of low-income groups: A Philippine case study. Food and Nutrition Bulletin 2 (no. 2).

\_\_\_\_\_. 1988. Regional consumption differences, marginal consumers, and nutrition policy in the Philippines. Quezon City: University of the Philippines, School of Economics (mimeographed).

Singh, Inderjit, Lyn Squire, and John Strauss, eds. 1986. Agricultural household models: Extensions, applications, and policy. Baltimore, MD: Johns Hopkins University Press.

Timmer, Peter. 1981. Is there curvature in the Slutsky matrix? Review of Economics and Statistics LXIII (no. 3, August).