ON THE EXISTENCE AND OPTIMALITY OF EQUILIBRIA IN LOBBYING ECONOMIES

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To my parents,

BRYCE *and* PATRICIA COGGINS,

for their continued love and support.

ABSTRACT

Suppose that members of a society are accorded status as both economic and political agents. If the polity responds to the same actors for whom economic policy matters, a simultaneity of political and economic determination is introduced. The first goal of the research presented here concerns the microeconomic question, How does an agent choose economic and political behavior simultaneously when his or her political activity has an explicit effect on the economic environment? The second is to determine conditions under which an equilibrium will exist in a generic strategic lobbying situation. The third is to discover the effect of lobbying behavior on society's welfare position.

The model of the thesis begins with an exchange economy in which agents are asked to make economic and political decisions simultaneously. A government mechanism sets relative prices by law in response to donations by political interests whose stakes in the price level are diametrically opposed, trading with a larger world economy to clear the distorted domestic markets.

The first result is the demonstration that under certain conditions on the economy, the model possesses a lobbying equilibrium. The welfare properties of this equilibrium are evaluated by comparing agents' utility levels at the lobbying outcome with those which would obtain in corresponding competitive economies. Both agents may be worse off with lobbying than without it, but one agent is sometimes better off with the lobbying program.

Using cooperative game theoretic techniques, the potential for agents to gain by forming a coalition and overturning the intervention policy is studied. The potential for the government to achieve an analogous improvement by implementing a tax/transfer scheme is also evaluated. It is shown that cases in which agents could help themselves by cooperating coincide precisely with cases in which the government could help them by replacing the price policy of the model with a tax/transfer policy.

In order to investigate these welfare properties, and to draw conclusions about the relationships between agents' characteristics and the pricing rule, numerical experiments are carried out in which equilibrium outcomes are calculated for example economies.

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Finally, I thank my parents, to whom this thesis is dedicated. They gave me all that I have that matters, and from them I learned nearly all of the really important things.

"There cannot be many things in man's political history more ancient than the endeavor of governments to direct economic affairs."

G. Stigler, 1975, p. *ix*

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GLOSSARY

This is a glossary of mathematical and economic symbols and abbreviations. Terms which require a more elaborate definition appear in italics in the text, and are accompanied by their definition. This is not a complete list, since many mathematical operators and symbols are omitted.

X_i	consumption set of agent i; assumed to be $\mathbb{R}^2_+$; see section 2.2
$\succsim_i$	preference relation of agent i; see 2.2
$\succ_i$, $\sim_i$	strict preference and indifference relations derived from $\succsim_i$; see 2.2
ω_i	endowment of agent i; see 2.2
$\mathcal{R}$	set of admissible characteristics; see 2.2
U_i	utility function of agent; derived from $\succsim_i$; see 2.2
p, q	normalized prices of goods 1 and 2, respectively; $p + q = 1$; see 2.2
$\mathbf{P}$; $\mathbf{P}^*$	two-vector of normalized prices and competitive equilibrium price, respectively; $\mathbf{P} \in \Delta$; see 2.2
Δ	one-dimensional simplex in $\mathbb{R}^2_{++}$; see 2.2
$\hat{\mathcal{E}}$	competitive economy; see 2.2
$\beta_i(\mathbf{P};\mathbf{P}\cdot\omega)$	budget set of agent i at price $\mathbf{P}$ and income $\mathbf{P}\cdot\omega$; see 2.2
$x_i(\mathbf{P};\mathbf{P}\cdot\omega)$	demand function of agent i; see 2.2
$z(\mathbf{P})$	aggregate excess demand; see 2.2
η_i, η	lobbying donation of agent i and vector of donations; see 2.2
$p(\eta)$	government pricing function; see 2.2
$\mathcal{P}$	set of admissible pricing functions; see 2.2

$y_i(\eta)$	after-lobbying income of agent i; see 2.2
$\mathcal{E}$	lobbying economy; see 2.2
$\mathfrak{E}$	set of lobbying economies; see 2.2
$\hat{\eta}_i(\eta_{-i})$	exhaustive level of lobbying for agent i; see 2.2
$\hat{\hat{\eta}}_i$	maximum of $\hat{\eta}_i(\eta_{-i})$ in η_{-i}; see 2.3
$\psi_i(\eta_{-i})$	choice set of agent i; see 2.2
$M_i(\eta_{-i})$	lobbying decision problem of agent i; see 2.2
$\tilde{\omega}_i$, $\tilde{\beta}_i$	after-lobbying endowment and budget set, respectively, of agent i; see 2.2
$\tilde{x}_i$, $\tilde{z}$	after-lobbying demand of agent i and excess demand, respectively; see 2.2
$\pi(\eta)$	net government income; see 2.2
H_i	strategy set of agent i; see 2.3
$\varphi_i(\eta_{-i})$	constraint correspondence of agent i; see 2.3
$V_i(\eta)$	indirect utility function of agent i; see 2.3
$\Gamma_{\mathcal{E}}$	the lobbying game for $\mathcal{E}$; see 2.3
$\mu_i(\eta)$	best response correspondence for agent i; see 2.3
$LGE(\Gamma_{\mathcal{E}})$	a lobbying game equilibrium for $\Gamma_{\mathcal{E}}$; see 2.3
η^*	generic element of $LGE(\Gamma_{\mathcal{E}})$; see 2.3
$p(\eta^*)$	lobbying equilibrium price; see 2.5
$LE(\mathcal{E})$	a lobbying equilibrium for $\mathcal{E}$; see 2.2
K_i	graph of φ_i; see 2.5
Δ^c	c-simplex; see 2.5
T	zero set of $P^* - P(\eta)$; see 2.5
$SALE(\mathcal{E})$	a strong active lobbying equilibrium for $\mathcal{E}$; see 3.2
F, $\tilde{F}$	feasible sets without and with lobbying, respectively; see 3.2

$L_i(x_i)$	upper level set for agent i at x_i; see 3.2
G_1, G_2, G	better than lobbying sets; see 3.2
$PO(\mathcal{E})$	Pareto optimal allocations for $\mathcal{E}$; see 3.2
$\mathbb{U}$	utility possibility set; see 3.3
$\mathring{U}$	Pareto set of utility pairs; see 3.3
U^*, $U(\eta^*)$	competitive and lobbying utility outcomes, respectively; see 3.3
Z	set of utility pairs dominated by U^*; see 3.3
$N_{\mathcal{E}}$	fixed threat bargaining game; see 3.3
$\hat{U}$	Nash solution to the bargaining game; see 3.3
$LC(\mathcal{E})$	the lobbying core for $\mathcal{E}$; see 3.3

CHAPTER ONE

INTRODUCTION

> *"If you make a law that I shall be obliged to sell my grain, my cattle, or any commodity, at a certain price, you not only do what is unjust and impolitic, but with all respect be it said, you speak nonsense; for I do not sell them at all: you take them from me. . . . I cannot help observing that laws of this kind have an inherent weakness in themselves, [in that] they are an attempt to apply authority to that which is not its proper object, and to extend it beyond its natural bounds. . . ."*
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> John Witherspoon, "An Essay On Money," 1802

Economic theory typically regards agents' economic decisions and their concurrent political decisions as distinct and separable. This is not to say that political decisions are either uninteresting or unimportant to economists. The simple truth is that accounting simultaneously for the economic and political features of individuals' and society's decision problems is difficult. Economics most often chooses, as a discipline, to place emphasis on the former.

Nevertheless, governments display an enduring willingness to intervene in economic markets. The world is rich with examples where political decision-makers alter economic conditions. One such example, to which analysts have often pointed, is that of the American sugar industry. Domestic producers constitute a politically active interest group which chooses to devote resources to a collective effort to alter the Federal policy governing sugar imports. They have been dramatically successful over the last several years, and by nearly all accounts sugar policy has significantly changed the face of the industry in this country (see, e.g., Krueger, 1988).

In this thesis, interest will focus on the intertwined economic and political features of situations such as this. The example is political because of the essential *authoritative* nature of the sugar policy. All agents are required by law to abide by that policy. It is economic because agents are permitted to make private consumption and production decisions in a decentralized fashion. These features are intertwined because the political outcome feeds back on the economic problem within which agents choose political behavior.

Economic theory, including the theory of general economic equilibrium, has not treated joint political and economic decision problems as successfully as it has treated purely economic problems. A current enterprise in economics, whose spirit animates the present study, moves beyond general equilibrium theory by formally combining economic and political considerations in modeling social situations. This literature is distinguished by its attempt to place economizing agents in models which call upon them to choose politically.

There are, however, some gaps in the literature. The microeconomic decision problem of politically active agents is not often treated carefully. Rather, most studies have been macroeconomic in focus. The strategic or conflictual nature which opposing political interests bring to such problems has also proven troublesome, as pointed out below. In the research reported here, the tools of general equilibrium theory are brought to bear, in a novel way, upon a model whose members are both political and economic. As shall be seen, the surprising complexities which were encountered in this effort help to explain why advances in the theoretical literature have not come quickly.

The model of the thesis is an exchange economy model of political economic agents which attempts to capture the microeconomic problem of political activity. Agents are asked to choose how to enter the economic market, while at the same time determining their willingness to expend resources to influence the economic environment through the political market. A government mechanism sets relative prices by law in response to donations by political interests whose stakes in the price level are diametrically opposed. The strategic equilibrium question is also treated in the model. After reformulating the economy as a generalized game, a theorem is proved which establishes the existence of equilibria in the strategic political economic model. Calculations are also carried out which verify this result numerically for some familiar functional specifications.

The goals of this research, then, simply stated, are to help fill in these gaps in the political economic literature. The first goal concerns a formal treatment of the microeconomic question, How does an agent choose economic and political behavior simultaneously when his or her political activity has an explicit effect on the economic environment? Whether one wishes to understand trade policy or domestic economic regulation, whenever economic agents are permitted to devote resources to altering the course of economic policy, this question arises.

The second question is one of strategic political equilibrium. In political conflicts, opposing interests attempt to achieve policies favorable to them, countervailing their opponents' political activity. If the essence of individuals' economic and political problems consists of balancing the tension between the economic costs and benefits of political activity, then the analogous political problem of society is one of opposing interests achieving some sort of strategic equilibrium. For a generic specification of the political economy, a question which remains unanswered in the extant literature is whether rational economic agents will or should, in equilibrium, use resources to affect economic policy. Indeed, the more basic question of when or whether such an equilibrium will exist remains open.[1] The existence of equilibrium is investigated in chapter two of this work.

The third major theme of the thesis is a study of the welfare consequences of political behavior. Currently the question of whether political behavior on the part of economic agents is good or bad for the aggregate lies at the center of a lively controversy. In chapter three a series of results is presented regarding how endogenously determined government intervention in economies affects groups' and aggregate welfare.

The remainder of this chapter will briefly review a selected subset of the existing literature on political economy; this will be undertaken with an eye to providing background relevant to this study.

1.1 GOVERNMENT ECONOMIC INTERVENTION

At the least, a society's governing body, in order to carry out its most fundamental tasks (e.g., national defense, police protection, and

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[1]For evidence that the existence of such equilibria is neither guaranteed nor, indeed, very well understood, see, for example, Applebaum and Katz (1986), Findlay and Wellisz (1982), or Tullock (1980). In the lobbying games of the latter, in fact, Nash equilibria are known *not* to exist. See also Tollison (1982).

the provision of infrastructure) must raise financial resources. Only rarely have governments been content with this rudimentary function. Rather, economic conditions attract the attention of governments everywhere.

One might first ask how economic theory recommends political intervention in economies. This question is a central theme of much of scholarly economic investigation. Adam Smith, for example, directed his scorn toward the trade restrictions prevailing in the eighteenth century. Economists have, for the most part, spoken against interventionist trade policy. The doctrine of the discipline has usually been to let the market work.

In the 1950's, economists used models of ideal, competitive economies to show that efficient outcomes are achieved in the absence of state intervention (see, e.g., Debreu, 1959). To the extent that the real economy departs from the competitive specifications of these abstract models, one may argue that the potential exists for costless, fully informed government intervention to improve upon the competitive allocation. Several such arguments have been offered over the years (see, e.g., Shepsle and Weingast (1984) for a brief review).[2]

How do interventionist policies affect the welfare of agents? This question motivates the field of welfare economics. Harberger (1954) made an

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[2]For example, a state's economic viability may require protection from foreign interests to preserve national security (Timmer, 1986). Public goods provision is sometimes seen as a legitimate economic role for the state (see Starrett, 1988). Producers of some goods, it is argued, must be protected from the income and price variation inherent in their industries (Just, *et. al.*, 1978). As Geanakoplos and Polemarchakis (1986) point out, incomplete asset markets prevent the market from making efficient use of existing assets, and state control may correct this inefficiency.

early attempt to measure the effect of political intervention on aggregate welfare by constructing measures of the welfare losses resulting from monopoly. Johnson (1960) improved the technique, incorporating surplus measures in a trade model. The following years witnessed more studies of this kind, but the measured inefficiency losses were very small, a finding which caused some consternation among economists. Mundell (1962, p. 622) was moved to suggest that unless the tools employed there were overhauled, such results might recommend a conclusion that "economics has ceased to be important."

Partly in response to this criticism, Tullock (1967) proposed that the Harberger-like studies underestimated the true losses to monopoly and tariff policies. Interventionist policies, Tullock wrote, create rents which some groups will struggle to obtain. The resources expended in this struggle represent an additional misallocation whose associated deadweight losses should add significantly to the pure welfare effects estimated by Harberger. Krueger (1974) labelled this activity "rent-seeking" and developed a model to estimate the full social costs of trade policies. She found that the losses due to rent-seeking behavior in Turkey reached 15 percent of that country's gross national product.

Krueger's label, rent-seeking, is still widely used in the literature (see Buchanan, *et.al.* (1980); Tollison (1982)), but other terms have been used as well. Rent-seeking as Krueger used it meant the pursuit of rents arising from pre-existing distortionary policy. Buchanan (1980, p. 4), defines the term as those activities where "individual efforts to maximize value generate social waste rather than social surplus." Revenue-seeking and tariff-seeking (Bhagwati, 1980) may denote the pursuit of pre-existing rents; they may also refer to the pursuit of the distortions themselves.

Bhagwati (1982) coined the phrase directly unproductive profit-seeking (DUP) and differentiated it from rent-seeking by virtue of its greater generality. In a comment on Tollison's (1982) survey, Bhagwati (1983) offered a detailed set of definitions and differentiated the various terms in a number of ways. The term *lobbying* will be used throughout this thesis to denote any activity in which agents or groups purposefully expend resources to exert self-interested political pressure on a central authority.[3]

Now, how is lobbying supposed to affect welfare in an economy? Krueger's early result has been given; other early works supposed that welfare is unambiguously reduced by lobbying behavior. In 1980, Bhagwati showed that under some circumstances, where distortionary policies are pre-existing, lobbying to change them might actually improve aggregate welfare (see also Bhagwati and Srinivasan (1982); Feenstra and Bhagwati (1982); and Bhagwati, Brecher, and Srinivasan (1984)). It remains very much an open question whether or when the usual outcome of political activity to achieve distortionary policies is welfare enhancing or reducing.

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[3]The generality of this label is most valuable here in that it does not distinguish between efforts to install a distortionary policy and efforts to collect the benefits of a policy already in place. The model of this thesis is a hybrid of these two in the sense that the policy outcome is endogenous, while the policy instrument is not. The term lobbying is certainly not new in this context; see the titles of Bhagwati (1980), Dinopoulos (1983), and Wellisz and Wilson (1986).

1.2 MICROECONOMIC LOBBYING BEHAVIOR

This investigation is motivated both by an unresolved controversy among experts and by an observed phenomenon imperfectly understood. A political and economic reality is that whenever they are afforded the opportunity, interest groups *do* lobby. Evidently, the system of rules and social institutions supported by democratic political arrangements admit this activity. When individuals and interest groups choose to lobby, they must have decided it would be worthwhile to do so. They may be mistaken, but the ubiquity of this phenomenon compels one to entertain the notion that it may be optimal.

Our conceptual understanding of this problem is weak relative to the state of the theory of economic behavior. It is the purpose of this thesis to improve the current level of understanding. Following a long tradition in the literature, our model will be simple, abstract, and stylized. It will, at some cost, ignore many interesting things. The purpose for this, however, is that our approach permits a rigorous treatment of the primary subject.

A consumer in the usual general equilibrium model takes prices as given and maximizes his or her utility by choosing a consumption bundle among those which are available under a price-dependent income constraint (Debreu (1959), chapter 4). A consumer who acts politically, perhaps to alter the price, does not fit this specification well. When agents may influence prices and income by becoming politically active, balancing the cost of this activity against the potential gains from achieved favorable price policies, they face the "essential microeconomic lobbying problem." When realism is sacrificed in the sequel, it will most often be to preserve this driving feature of the model.

What if the "agent" is actually a group of agents who share some political interest? Before choosing a lobbying strategy, members of such a group must achieve a solution to their collective action problem. The study of how they do so, the theory of interest group behavior, is a distinct field of inquiry in political science. While this inquiry is important, it will not be featured in these pages.[4] The point is that individuals or groups who act politically solve a joint economic and political problem, and the natural interpretation of the solution is a demand for political intervention.

1.3 STRATEGIC LOBBYING AND EQUILIBRIUM

Suppose, as is usually true, that a proposed economic policy has beneficial and deleterious effects, respectively, on two opposing interest groups. Each group, in choosing whether to expend resources in affecting the policy, and also in choosing a level of activity, must solve a lobbying problem of the kind described above. If it is a large group, it will also face a collective action problem. But the conflicting interests bring to

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[4]Until relatively recently, pluralist notions of the nature of interest groups, namely that their members share a common interest which maintains the group's identity and that directs group behavior, ruled the day in political interest group theory. Olson's (1965) revolutionary *The Logic of Collective Action*, however, dealt a severe blow to such notions, ascribing rational and self-interested motives to political actors which led to conclusions in direct opposition to the pluralist tradition. More recent developments, including the role of the hypothetical political entrepreneur, are found in Moe (1980). See also Hardin (1982), Hansen (1985), and Wellisz and Wilson (1986).

the situation an important *strategic* component. Presumably, if the two groups are economically rational, they will try to take some account of their opponent's political activity in choosing a course of action for themselves.[5]

A source of tension in this social situation is embodied in the non-cooperative strategic conflict; in some sense this subsumes the groups' microeconomic lobbying problems. It also constitutes the second theme of the thesis. Economists, including the rent seeking theorists, care about questions of equilibrium existence. Tullock (1980) devised a model of a lobbying game which captures the strategic lobbying problem. However, his model does not resemble equilibrium economic models, as it proceeds in the absence of prices, good markets, and preferences. Applebaum and Katz (1986) extend Tullock's work, but there are still only shadowy ties to economies in their paper. Findlay and Wellisz (1982) build an economic model of trade which incorporates politically active interest groups, and a well-specified equilibrium notion, but do not demonstrate the existence of equilibrium in the model. In short, the existence of equilibrium in lobbying situations is a matter of continuing interest. It will receive considerable attention below.

Implicit in any formulation of a lobbying economy is the set of institutional arrangements giving rise to intervention. As there is a

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[5]Consider, for example, the situation faced by U.S. auto manufacturers and domestic auto importers. These groups are natural adversaries in the battle to set a tariff rate on car and truck imports. Knowing full well that their opponent is also acting to influence government policy, each group still chooses to lobby.

demand for political regulation of economic conditions, so there must be a corresponding supply. Who supplies political output, and why? Becker (1983) supposes that political output is supplied by politicians in response to campaign contributions by interest groups. Their effect on the policy is generated by pressure functions which map political pressure into economic policies. Lindbeck (1976) and Peltzman (1976) provide two influential works which also incorporate active political intermediaries who supply political output.[6] Once given an objective, the government may be endowed with one of a number of policy tools with which to achieve it. These include, for example, tariffs and quotas in trade models (for a comparison between these, see, e.g., Mayer and Reizman (1987); Dinopoulas (1983); and Cassing and Hillman (1985)).

The modeling of institutional arrangements leading to political output is an interesting matter. It is, however, not the focus of this study. Following Findlay and Wellisz (1982, 1984), our model employs a policy function which maps lobbying donations directly into policy outcomes. This function may be thought of generally as a political production function;[7] it will be called a pricing function here.

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[6]Alternative specifications for the objective of government in supplying political output may be found, for example, in Brock and Magee (1975, 1978) and in Mayer (1984), where self-interested politicians act to maximize their re-election chances. Benevolent social planners direct government behavior in Roe and Yeldan (1988) and in Becker (1983). Rausser, *et. al.* (1980) review several additional specifications for government objectives. These matters are not addressed further in this study.

[7]For one political scientist's criticism of this choice of representation of political institutions and the source of political supply, see Nelson (1988, p. 817).

Of course, the real mapping from groups' lobbying donations to government policy outcomes is elaborate and complex. For example, after sugar growers choose a level of lobbying donations, their organization must decide how to spend this money. Some may be paid directly to politicians as campaign contributions , and some may go to hire lobbyists who present the group's pleas to members of Congress. The route from this activity to actual sugar policy is still a circuitous one, and includes the drafting of legislation, a vote by the Congress, and the ensuing political battles. If a bill becomes law, then it must be implemented by the bureaucracy of the Federal government. These steps in the process are ignored here at some cost, but not without careful reflection. One return to such a simplification is that the model is tractable analytically. A more important one is that the results are not obscured by complicating forces whose influence is ambiguous.

Underlying the lobbying model is a standard two-agent exchange economy. As such, it consists of a pair of traders with preferences over two goods, with which they are asymmetrically endowed, and a government[8] which establishes a relative price in the economy in response to lobbying contributions. Each agent in the economy takes the government's pricing rule and the level of his or her opponent's lobbying expenditures as given and chooses a lobbying level and a consumption bundle. He or she must balance the loss in income due to lobbying payments against the potential

[8]While the term "government" may be somewhat misleading in that there is no active role for the central authority, it will be used here for expositional ease.

gain in wealth from an advantageous price movement.

The model is inherently disequilibrium in nature on the economic side, as once the government has set a price in response to lobbying, markets needn't clear. To sustain the mandated price in the face of this disequilibrium, we introduce a world market with which the government may trade, at some cost, in order to clear the domestic markets. While there are alternative means of handling disequilibrium situations (e.g., quantity rationing schemes (Benassy, 1982)), the choice here of a trade mechanism is motivated by observed phenomena and by the literature cited above. In order to avoid the free resource problem of unlimited trading in the world market, a feasibility restriction is imposed. The government has "revenue" equal to lobbying donations. Its "costs" are those incurred in its trading operation. Feasibility requires that these costs do not exceed government's revenue.

1.4 OBJECTIVES, PROCEDURES, AND OVERVIEW OF THE STUDY

As stated in the opening paragraphs of this chapter, the objectives of this study are three-fold. First, how does an agent choose economic and political behavior simultaneously when his or her political activity has an explicit effect on the economic environment? Second, what are the conditions under which an equilibrium will exist in a strategic lobbying situation, and will the equilibrium be such that agents will choose to lobby? Finally, how is the welfare position of society altered when lobbying behavior occurs?

The procedures to be followed in achieving our objectives may be summarized as follows.

a) Build a general equilibrium model of lobbying behavior in which agents face a microeconomic lobbying problem and in which opposing interests seek equilibrium lobbying levels;

b) Define a coherent notion of equilibrium in the lobbying model and demonstrate conditions under which the model possesses a lobbying equilibrium; and

c) Carry out a welfare comparison between competitive equilibria and the lobbying equilibria obtained from the associated lobbying economies. This welfare analysis will include an investigation of conditions under which lobbying behavior might be beneficial to political agents and also when either agents or the government may wish to override the political process.

The first two objectives will be fulfilled in the second chapter, where the model is laid out and shown to possess an equilibrium. In chapter three the welfare comparison of c) is carried out. Chapter four includes material which pertains to both b) and c). There, numerical calculations serve to demonstrate that lobbying equilibria do exist and also to point out some welfare implications of the lobbying process.

CHAPTER TWO

EXISTENCE OF EQUILIBRIA IN LOBBYING ECONOMIES

"The final political equilibrium will be not that one [group] or the other 'wins' according to whose power is larger, but rather that an intermediate solution will be attained, where—at the margin—the strengths of the two [groups] are equal."

G. Stigler, 1975, p. 139

2.1 INTRODUCTION

In games of strategy, an equilibrium is a set of strategies for players at which no one player has an incentive to deviate unilaterally. A perfectly competitive economy, though, offers its members no opportunity to behave strategically. There, individuals may neglect the effect of their opponents' behavior upon their own decisions. Economic equilibrium, in that case, is a benign sort of thing: individuals probably regard the fact that markets clear with a detached curiosity. As has been noted, the model of this thesis is distinctly not perfectly competitive. Instead, it may be called *truly* competitive in the sense of oligopoly models. Agents in the model struggle against one another to influence an economic policy. Their interests in the policy choice are diametrically opposed. The equilibrium notion which will be adopted here is that of strategic equilibrium, one at which opposing interests have no incentive to deviate unilaterally.

The goals of this chapter are twofold. The first is to construct an economic model of political behavior which captures the strategic feature of opposing interests' interaction. The model will incorporate a government policy which may distort markets in Tullock's (1967) sense in response to political activity by agents. The second goal is to show that the model has an equilibrium. Interest in the demonstration of this result lies mainly in the fact that it is more general than comparable results in the existing literature.

The model has two agents and two goods.[9] Each of the agents owns only one of the goods; this fact means they have opposing interests in the relative price level. The government is represented by a pricing function which maps agents' lobbying contributions into a relative goods price. There are many ways, including supply controls and deficiency payments, by which governments may distort economic markets. There are also many ways by which members of a society may apply political pressure to a governing body. Here, the price-setting policy is supposed to be a representative distortionary policy. This, and political activity in the form of direct contribution of "dollars" to the government, are the simplest formulations which capture the two fundamental notions. These are 1) that the political outcome is distortionary (lump-sum taxes, for example, are regarded as non-distortionary policies), and 2) that agents must give up economic resources to apply political pressure. Given 1), point 2) here implies that price and income will be determined simultaneously, a fact which will present some technical difficulties later on.

Certain features of the results of this chapter are of independent

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[9]Economics principles textbooks often employ models of this sort, representing them in the "Edgeworth box," to provide an intuitive explanation of the general equilibrium perfectly competitive economy. As two agents should not be expected to regard their affect on the price system as negligible, this model is not, strictly speaking, appropriate for that purpose. In the formulation of this thesis, however, agents are not assumed to behave as though they cannot affect prices.

technical interest. In particular, first, the choice sets of agents, when they are asked to choose political and economic behavior simultaneously, are inherently non-convex (see Appendix 2). This surprising result has interest in that it renders the usual approach to equilibrium analysis—which relies heavily upon the convexity of choice sets—inapplicable. However, and this is second, the decision problems of agents are reformulated so that economic choices are implicit and mathematically dependent, through the government's pricing policy, upon political behavior. In effect, agents' optimization programs are turned into two-stage problems, dependent upon the political choices of both agents, and the resulting strategic political conflict is shown to possess an equilibrium.

This approach allows the non-convexity problem to be avoided, but it introduces a new difficulty in showing payoff functions to be quasiconcave. This difficulty is resolved by a restriction on preferences which delivers the required quasiconcavity.

The remainder of the chapter is organized as follows. In section 2.2 the lobbying economy $\mathcal{E}$ is specified and developed, and the concept of economic equilibrium is defined. A generalized game $\Gamma_{\mathcal{E}}$ is derived from the economic model and a lobbying game equilibrium is defined in section 2.3. The existence theorem for game equilibria is stated and proved in section 2.4. In section 2.5 we prove the existence of a lobbying equilibrium by proving that the game equilibrium of section 2.4 is feasible for the government. Concluding comments appear in section 2.6.

2.2 THE LOBBYING ECONOMY $\mathcal{E}$

The economy under examination is a two-agent exchange economy with two traded goods. Goods are labelled 1 and 2; agents are indexed by $i \in I = \{1,2\}$. Throughout, subscripts denote traders, while superscripts denote commodities. Consumption sets X_i are taken to be $\mathbb{R}^2_+$, the non-negative quadrant in Euclidean space. Agent i possesses a preference ordering $\succsim_i$ over elements x of X_i. Ordered pairs of elements (x,y) of X_i are members of the relation $\succsim_i$ if x is *preferred by* i to y. This is denoted $x \succsim_i y$. When $x \succsim_i y$ holds but $y \succsim_i x$ does not, we write $x \succ_i y$, and say that x is *strictly preferred* by i to y. If the relation holds in both directions, then i is *indifferent* between x and y. This is denoted $x \sim_i y$. In the sequel, agent i's preference relation $\succsim_i$ over bundles in X_i is assumed to admit representation by a continuous utility function $U_i : X_i \longrightarrow \mathbb{R}$ that is twice differentiable and that is strictly quasiconcave and monotone increasing on $\text{int}(X_i)$. For x,y in X_i, we say $U_i(x) \geq U_i(y)$ if and only if $x \succsim_i y$.

Agent i is assumed to be endowed only with the ith good. Henceforth, $\omega^i > 0$ will denote the finite scalar value of i's endowment, while ω_i will be used to denote the pair in $\mathbb{R}^2_+$ satisfying $\omega_i^i = \omega^i$ and $\omega_i^{-i} = 0$.[2] A *price vector* is a pair $\mathbf{P} = (p^1,p^2) \in \mathbb{R}^2_{++}$. This exchange economy, denoted $\hat{\mathcal{E}}$ and consisting of two agents along with their preferences and endowments, in which consumers treat prices parametrically, underlies the lobbying economy.

Given a price vector $\mathbf{P}$, let agent i's income be defined as $\mathbf{P}\cdot\omega_i$.[3] The

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[2]This notation is interpreted as $y^{-i} = (y^1,\ldots,y^{i-1},y^{i+1},\ldots,y^n)$ for any n-dimensional vector y.

[3]Throughout, inner products will be denoted in this manner. It will be understood that the first vector is a row vector and the second is oriented as a column.

budget set of agent i is given by $\beta_i(P,P\cdot\omega_i) = \{x \in X_i : P\cdot x \leq P\cdot\omega_i\}$. A *competitive equilibrium* is a pair of allocations $(x_i^{1*},x_i^{2*})_{i=1,2}$ and a price vector $P^* = (p^{1*},p^{2*})$ such that i) agents' chosen bundles x_i^* maximize utility on $\beta_i(P,P\cdot\omega_i)$; and ii) markets clear. A standard result from general equilibrium theory guarantees that economies satisfying the conditions specified above, and also the condition $P^* \in \mathbb{R}^2_{++}$, have non-empty equilibrium sets (Debreu, 1959).

Agent i's *demand*, which maximizes the function U_i over $\beta_i(P,P\cdot\omega_i)$, and which maps a price-income pair into a subset of $\beta_i(P,P\cdot\omega_i)$, is given by

$$x_i(P,P\cdot\omega_i) = \{x \in \beta_i(P,P\cdot\omega_i) : \text{for each } k \in \beta_i(P,P\cdot\omega_i),\ x \succsim_i k\}.$$

Since the utility functions are continuous and strictly quasiconcave, and since prices and ω^1 are strictly positive, demand functions are continuous. Because the U_i are also twice differentiable, demand x_i will fail to be differentiable only when U_i is maximized on a boundary of X_i. Such points are commonly called "irregular points" of demand (see, e.g., Mas-Colell, p. 84). It shall be assumed that irregular demand points do not occur in our agents' consumption spaces; thus, demand functions are differentiable. Agent i's *excess demand* is given by $z_i(P,P\cdot\omega_i) = x_i(P,P\cdot\omega_i) - \omega_i$. *Aggregate excess demand* is simply the sum of individuals' excess demands:

$$z(P) = \Sigma_i\ z_i(P,P\cdot\omega_i).$$

For our purposes, it is important that the equilibrium price vector P^* be unique. This is assured for exchange economies whenever $z(P)$ is such that for all prices P, all goods are gross substitutes (see, e.g., Arrow and Hahn, 1971, p. 223).

Definition: Two goods, i and j, are *gross substitutes* (GS) at **P** if $\frac{\partial z^i}{\partial p^j}(\mathbf{P}) > 0$ for all $i \neq j$.

It will be assumed that (GS) holds for every price **P**. Let $\mathbf{P}^*$ denote the unique competitive equilibrium price for the undistorted exchange economy.

Let an agent's *characteristic* be given by the pair $a_i = (\succsim_i, \omega_i)$. Let $\mathcal{R}$ be the set of all pairs a_i satisfying our convention on endowments and such that the resulting $z(\mathbf{P})$ satisfies (GS). The set $\mathcal{R}$ will be called the set of *admissible characteristics*.

Demand functions are easily shown to be homogeneous of degree zero in prices and income. Prices are normalized to the one-dimensional simplex $\Delta \subset \mathbb{R}^2_{++}$ by dividing each p^1 by the sum $(p^1 + p^2)$. In what follows, let $p \in (0,1)$ denote the normalized price of good 1, and let $q = (1 - p)$ denote the price of good 2. A price system for the economy is thus fully specified by the scalar parameter p.

There is, in the background of the economy, a "government" which stands prepared to alter the price in the economy in response to lobbying on the part of consumers. This price-setting government exists as a social institution as a result of the society's history and its norms, and it embodies these characteristics as well as any goals or objectives which the central authority incorporates in governing. Each consumer may choose to donate a part, η_i, of his or her income to the government to influence the government's price policy; η_i is agent i's *lobbying donation.* The government is fully specified by the function $p : \mathbb{R}^2 \longrightarrow (0,1)$, given by

$p = p(\eta_1,\eta_2)$, by which it sets the price.[12] Hereafter, the symbol $\mathbf{P}$ will always denote a price pair $(p,q) \in \Delta$; when it refers to the government's mandated price, we will write $\mathbf{P}(\eta) = (p(\eta),(1-p(\eta)))$, where $\eta = (\eta_1,\eta_2)$.

The pricing function $p(\eta)$ will be assumed to satisfy a collection of conditions. The first of these is differentiability.

(A1) The function $p(\eta)$ is $\mathbb{C}^1$.

What's more, if neither agent chooses to lobby, then it is assumed that the government selects the competitive equilibrium price.

(A2) $p(0,0) = p^*$.

Because of the asymmetry of agents' endowments, and under the monotonicity of U_1, Mr. 1 is made better off by an exogenous price increase, while Ms. 2 is made worse off. This divergent interest lends to the model its non-cooperative nature. The following assumption ensures that agents' lobbying donations have the effect on government policy which they expect, and also that lobbying expenditures do not become more productive at the margin as the level of lobbying increases.

(A3) (*Productive Lobbying*). $p(\eta_1,\eta_2)$ is strictly increasing and concave (resp. strictly decreasing and convex) in η_1 (resp. η_2).[13]

The final restriction which will be placed on the function $p(\eta_1,\eta_2)$

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[12] This function is very much like the political "production function" of Findlay and Wellisz (1982, 1983) and of Wellisz and Wilson (1986).

[13] The concavity restrictions on p prevent the price from "blowing up" in either variable. There are arguments in favor of allowing more general curvature properties. Here, the technical difficulties such non-convexities introduce are avoided in order to focus on more fundamental issues.

delivers an upper bound for agents' lobbying activity.

(A4) (*Bounded Lobbying*). For each agent i, for every η_{-i}, there exists an $\hat{\eta}_i(\eta_{-i}) < +\infty$, depending on η_{-i}, sufficiently large so that $P(\hat{\eta}_i(\eta_{-i}),\eta_{-i})\cdot\omega_i = \hat{\eta}_i(\eta_{-i})$.

That is, given an η_{-i}, if i chooses to devote $\hat{\eta}_i(\eta_{-i})$ to the government in lobbying expenditures, then none of his or her wealth is left over for purchasing goods (see Figure 2-1). Formally, $\hat{\eta}_i(\eta_{-i}) = \{x \in \mathbb{R}_+ : P(x,\eta_{-i})\cdot\omega_i = x\}$. By our assumptions on $p(\eta_i,\eta_{-i})$, $\hat{\eta}_i(\eta_{-i})$ is single-valued; that it is a continuous function of η_{-i} follows directly from the continuity of $p(\eta)$.

Let $\mathcal{P} = \left\{ p : \mathbb{R}^2 \longrightarrow (0,1) : p(\eta) \text{ satisfies (A1) — (A4)} \right\}$. A generic element $p(\eta)$ of $\mathcal{P}$ is called an *admissible pricing function*. In the remainder of the chapter, attention will be restricted to pricing functions defined over $\mathbb{R}^2_+$; let $\mathcal{P}_+$ denote the subset of $\mathcal{P}$ with elements so defined. Allowing $\eta_i < 0$ for some $i \in I$ permits an interesting investigation of tax/transfer schemes, a topic which shall be revisited in Chapter 4.

Let the set of all lobbying economies be given by the Cartesian product $\mathfrak{E} = \mathcal{R}^2 \times \mathcal{P}$ of admissible characteristics and admissible pricing functions. Let $\mathfrak{E}_+ = \mathcal{R}^2 \times \mathcal{P}_+$ be similarly defined. Henceforth, a lobbying economy, assumed to lie in $\mathfrak{E}_+$, will be denoted $\mathcal{E} = ((\succsim_i,\omega_i)_{i=1,2};\ p(\eta))$.

The optimization program of consumers may now be spelled out. Given an η_{-i}, the set of triples (x_i^1,x_i^2,η_i) in $\mathbb{R}^3_+$ from which agent i may choose is given by

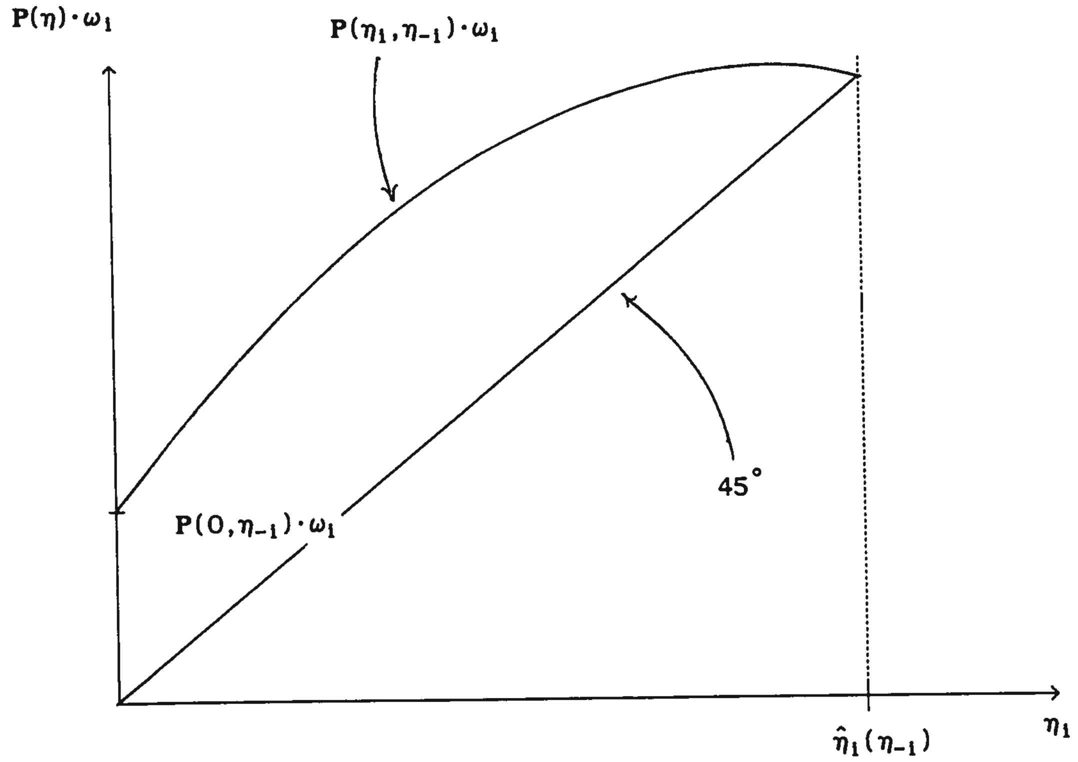

Figure 2-1a. *Value of endowment ω_1 at the lobbying price with η_{-1} given. At $\eta_1 = \hat{\eta}_1(\eta_{-1})$, $P(\eta)\cdot\omega_1 = \eta_1$.*

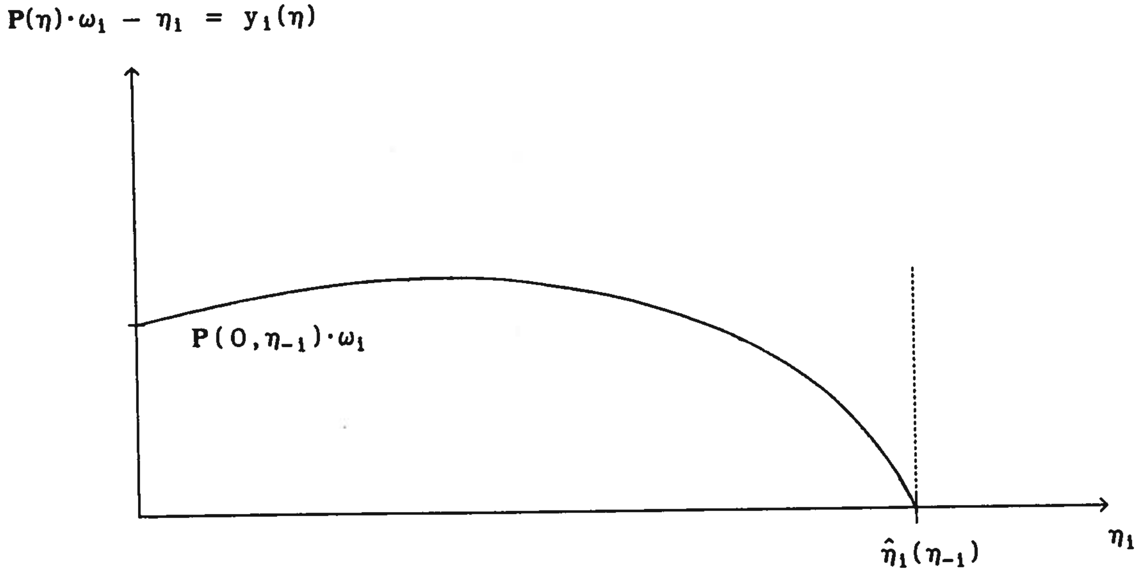

Figure 2-1b. *After-lobbying income $y_1(\eta)$; η_{-1} given.*

$$\psi_i(\eta_{-i}) = \left\{(x_i^1, x_i^2, \eta_i) \in \mathbb{R}_+^3 : P(\eta_i, \eta_{-i})\cdot(x_i^1, x_i^2) \le P(\eta_i, \eta_{-i})\cdot\omega_i - \eta_i\right\}.$$

Given η_{-i}, agent i solves the problem

$$M_i(\eta_{-i}) \qquad \max_{(x_i^1, x_i^2, \eta_i)\in\psi_i(\eta_{-i})} U_i(x_i^1, x_i^2).$$

Associated with this program is a demand relation different from our $x_i(p, p\cdot\omega_i)$. Given a pair (η_1, η_2), let $\tilde{\omega}^1 = \omega^1 - \eta_1/p(\eta)$, and let $\tilde{\omega}^2 = \omega^2 - \eta_2/(1-p(\eta))$. Let the (after-lobbying) budget set of agent i be given as $\tilde{\beta}_i(p(\eta);P(\eta)\cdot\tilde{\omega}_i)$. The demand relation of agent i arising from program $M_i(\eta_{-i})$ may now be defined as $\tilde{x}_i(p(\eta);P(\eta)\cdot\tilde{\omega}_i)$. After-lobbying excess demand $\tilde{z}_i$ is the difference between $\tilde{x}_i$ and $\tilde{\omega}_i$; $\tilde{z}$ is the sum of $\tilde{z}_i$ over $i \in I$. By our assumptions on preferences and $p(\eta)$, the relations $\tilde{x}_i$, $\tilde{z}_i$, and $\tilde{z}$ are all differentiable functions.

The function $p(\eta)$ is common knowledge; i.e. both agents know $p(\eta)$ with certainty, and they both know that their opponent knows p, etc. Once the rule $p(\eta)$ is announced, the government does nothing further to influence agents' choices. It simply carries through on its promise to enforce the price $p(\eta)$. This is also common knowledge. It does not optimize; neither does it choose the function $p(\eta)$ based upon any influence from agents.

Once the price is determined, markets may not clear as a result of trade between agents. An alternative mechanism is employed to deliver a reasonable notion of a feasible equilibrium. It is assumed that the two-agent economy is small relative to a world economy in the two goods. The world price is assumed to equal p^*, and the government carries out trade with the rest of the world without transport cost in order to sustain the prices determined by $p(\eta)$.

We must restrict the model to ensure that the government's market-clearing activity is feasible. The quantity $(\eta_1 + \eta_2)$ is the government's "revenue" in terms of a (non-existent) domestic currency. The "cost" of supporting $p(\eta)$ is $(P^* - P(\eta))\cdot\tilde{z}(p(\eta))$.[14] The following definition of feasibility will be employed in our equilibrium definition:

Definition: Given a lobbying economy $\mathcal{E}$, the 6-tuple $(x_i^1, x_i^2, \eta_i)_{i=1,2}$ is *government feasible* if $\pi(\eta) = (\eta_1 + \eta_2) - (P^* - P(\eta))\cdot\tilde{z}(p(\eta)) \geq 0$.

Note that if $\eta_1 = \eta_2 = 0$, then $p^* = p(\eta_1,\eta_2)$, so that $\pi(0,0) \equiv 0$.

We are now in a position to define our equilibrium concept for the lobbying economy.

Definition: Given a lobbying economy $\mathcal{E}$, a *lobbying equilibrium*, denoted LE($\mathcal{E}$), is a 6-tuple $(x_i^{1*}, x_i^{2*}, \eta_i^*)_{i=1,2}$ satisfying:

i) for each i, $(x_i^{1*}, x_i^{2*}, \eta_i^*)$ solves $M_i(\eta_{-i}^*)$; and

ii) $(x_i^{1*}, x_i^{2*}, \eta_i^*)_{i=1,2}$ is government feasible.

We may now proceed to a specification of the game which derives from this economy. In the following section we first formulate the economic model as a generalized game. Then, we study the equilibrium characteristics of the game and relate its equilibria to equilibria in the underlying economy.

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[14]It may readily be verified that this expression is equal to $P^*\cdot\tilde{z}(p(\eta))$; adding the budget constraints of the two agents together yields the equality $P(\eta)\cdot\tilde{z}(p(\eta)) = 0$. The version used in the text will be adopted here and elsewhere, though, as it is more suggestive of the cost of trade, and because it lends itself more easily to an intuitive explanation of Proposition 2.1 below.

2.3 THE LOBBYING GAME Γ_g

The central defining characteristic of a game is the dependence of individual players' payoffs on the strategies of all players. A generalized game displays the additional property that players' strategy *sets* are affected by their opponents' strategies. The game which emerges naturally from the lobbying economy is a generalized game.

Suppose that a game Γ is to be played by n players. Let their strategy sets be given by $H_i \subset \mathbb{R}^m$, with generic element η_i, where i takes integral values from 1 to n. Given a vector η_{-i} of his or her opponents' strategies, player i's choice is restricted to a subset $\varphi_i(\eta_{-i})$ of H_i. The correspondence[15] $\varphi_i(\eta_{-i})$ is called player i's *constraint correspondence.* The *payoff* or utility of the ith player resulting from a play $\eta \in H = \times_{i=1,\ldots,n} H_i$ is given by the function $V_i(\eta)$.

Suppose that Nash behavior characterizes interaction between agents. That is, for any vector $\eta \in H$, player i takes η_{-i} as given and chooses an action or strategy t to maximize $V_i(t,\eta_{-i})$ on $\varphi_i(\eta_{-i})$. A *generalized game* is denoted $\Gamma = (H_i, V_i, \varphi_i)_{i=1,\ldots,n}$. An element η^* of H is an *equilibrium* for Γ if, for each i, η_i^* maximizes $V_i(t,\eta_{-i}^*)$ on $\varphi_i(\eta_{-i}^*)$.

The following theorem, which is a special case of Debreu's (1952) generalization of Nash's (1950) theorem, lists conditions sufficient for the existence of an equilibrium in Γ. This version of the theorem is used in

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[15]For $X \subset \mathbb{R}^m$, $Y \subset \mathbb{R}^n$, a *correspondence* $\varphi : X \twoheadrightarrow Y$ is a rule which associates with every element x of X a non-empty subset $\varphi(x)$ of Y. φ is *convex-valued* if for every x in X, $\varphi(x)$ is a convex subset of Y. Its *graph* is the set $G(\varphi(x)) = \{(x,y) \in X \times Y : y \in \varphi(x)\}$.

Arrow and Debreu (1954) to prove the existence of an equilibrium for a competitive economy; its statement here follows that of Debreu (1982).

Theorem 2.1 (Debreu). If, for every player i, the set H_i is a non-empty, compact, convex subset of a Euclidean space, V_i is a continuous real-valued function on $H = \times H_i$ that is quasiconcave in its ith variable, and φ_i is a continuous, convex-valued correspondence from H to H_i, then the game $\Gamma = (H_i, V_i, \varphi_i)_{i=1,\ldots,n}$ has an equilibrium.

The task at hand is to reformulate the lobbying economy as a generalized game, and to exhibit conditions under which Theorem 2.1 can be applied to prove the existence of an equilibrium in the game. A natural approach to this problem is to focus on program $M_i(\eta_{-i})$. Then, the constraint correspondence of player i would coincide with the feasible set $\psi_i(\eta_{-i})$ defined in section 2.2 above. Unfortunately, this correspondence is not convex-valued in general. In fact, it may be shown that when $p(\eta)$ satisfies (A3) and (A4), for each $i \in I$ the set $\psi_i(\eta_{-i})$ above is not convex for *any* η_{-i}. (In Figure 2-2 the set $\psi_1(\eta_2)$ is presented to motivate the result geometrically.) This fact, which necessitates a non-standard approach to the equilibrium existence theorem, is proved in Appendix 2 (at the end of this chapter) to preserve the flow of the chapter. Rather than focusing on conditions on $\mathcal{E}$ under which the relevant subset of $\psi_i(\eta_{-i})$ is convex, we take an alternative approach, based on a two-stage maximization formulation of $M_i(\eta_{-i})$.

Note that for any η_{-i}, once agent i has selected an η_i, p is uniquely determined and i's optimization program over goods is well-defined. It is assumed that agents choose consumption bundles optimally given a price and

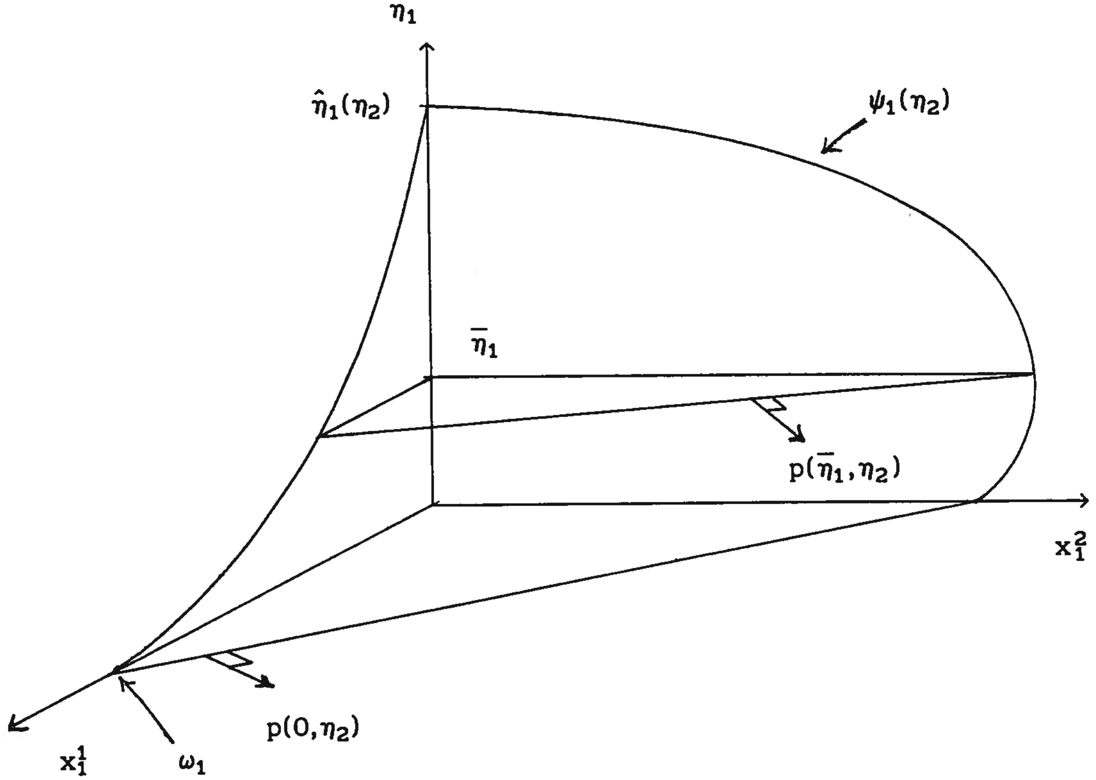

Figure 2-2. *Mr. 1's choice set $\psi_1(\eta_2)$ for program $M_1(\eta_2)$ with η_2 given.*

income vector, and the indirect utility functions are used as payoff functions in the game. With optimal consumption choices assumed, the only strategy open to agent i is a choice of η_i from $[0,\hat{\eta}_i(\eta_{-i})]$, a set which is obviously convex. The problem, given an η_{-i}, is to solve

$$M'_i(\eta_{-i}) \qquad \max_{\eta_i \in [0,\eta_i(\eta_{-i})]} V_i(p(\eta),y_i(\eta)),$$

where $y_i(\eta) = P(\eta)\cdot\omega_i - \eta_i$ is i's "after-lobbying income," (see Figure 2-1b) and $V_i(p(\eta),y_i(\eta)) = \max_{x_i \in \tilde{\beta}_i(p(\eta);P(\eta)\cdot\tilde{\omega}_i)} U_i(x_i)$. If no ambiguity results, the function $V_i(p(\eta),y_i(\eta))$ will be denoted $V_i(\eta_i,\eta_{-i})$, which makes clear the connection to payoff functions in the generalized game. The programs $M_i(\eta_{-i})$ and $M'_i(\eta_{-i})$ are equivalent. The generalized game which will be used to represent the economy $\mathcal{E}$ may now be specified.

The set of players is the set of agents $I = \{1,2\}$.[16] Players' *strategy sets* are given by $H_i = [0,\hat{\eta}_i]$, where $\hat{\eta}_i = \max_{\eta_{-i}} \hat{\eta}_i(\eta_{-i})$.[17] *Payoff functions* are given by $V_i = V_i(p(\eta),y_i(\eta))$. Player i's *constraint correspondence*, mapping η_{-i} into a subset of H_i, is given by $\varphi_i(\eta_{-i}) = [0,\hat{\eta}_i(\eta_{-i})]$. The lobbying game for economy $\mathcal{E}$ may now be defined.

Definition: Given a lobbying economy $\mathcal{E}$, its corresponding *lobbying game*, $\Gamma_{\mathcal{E}}$, is given by the collection $\Gamma_{\mathcal{E}} = (H_i,V_i,\varphi_i)_{i\in I}$

In this game, player i takes η_{-i} as given and optimizes by choosing a

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[16]Our government may be thought of as drawing a pricing function $p(\eta)$ from $\mathcal{P}$. Thus, we do not include the imaginary player whose role is analogous to the Walrasian auctioneer or "market player" in the abstract economy model of Arrow and Debreu.

[17]The existence of this maximum is demonstrated in the proof of Lemma 2.1 below.

strategy from the set[18]

$$\mu_i(\eta_i,\eta_{-i}) = \left\{ x \in \varphi_i(\eta_{-i}) \;:\; V_i(x,\eta_{-i}) = \max_{t\in\varphi_i(\eta_{-i})} V_i(t,\eta_{-i}) \right\}.$$

Let $H = \times_{i\in I} H_i$, with generic element $\eta = (\eta_1,\eta_2)$. An equilibrium in the lobbying game $\Gamma_{\mathcal{E}}$ is defined as follows.

Definition: The vector $\eta^* \in H$ is a *lobbying game equilibrium* of $\Gamma_{\mathcal{E}}$, denoted LGE($\Gamma_{\mathcal{E}}$), if for each $i \in I$, $\eta_i^* \in \mu_i(\eta^*)$.

Equivalently, $\eta^* \in$ LGE($\Gamma_{\mathcal{E}}$) if for each $i \in I$, η_i^* solves $M'_i(\eta_{-i}^*)$. Defining $\mu(\eta) = \times_{i\in I}\mu_i(\eta_i,\eta_{-i})$, $\eta^* \in$ LGE($\Gamma_{\mathcal{E}}$) if $\eta^* \in \mu(\eta^*)$, or if η^* is a fixed point of the correspondence μ.

Notice that a LGE($\Gamma_{\mathcal{E}}$) differs from a lobbying equilibrium LE($\mathcal{E}$) only by the absence of a feasibility restriction in the game equilibrium. In the following section, it is shown under what conditions on $\mathcal{E}$ the set LGE($\Gamma_{\mathcal{E}}$) is non-empty. Section 2.5 goes on to state conditions under which at any $\eta^* \in$ LGE($\Gamma_{\mathcal{E}}$), the government feasibility condition is satisfied in $\mathcal{E}$. Attention is now turned to the first main result of the thesis—the existence of a lobbying game equilibrium in $\Gamma_{\mathcal{E}}$.

2.4 EXISTENCE OF A LOBBYING GAME EQUILIBRIUM

The objective in this section is to show that the lobbying game $\Gamma_{\mathcal{E}}$ associated with $\mathcal{E}$ has an equilibrium. This will be accomplished by showing that under certain restrictions on $\mathcal{E}$, $\Gamma_{\mathcal{E}}$ satisfies the conditions of Theorem 2.1. In applying Theorem 2.1 to the game $\Gamma_{\mathcal{E}}$, three sets of

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[18]While μ_i does not depend upon η_i, expressing μ in this manner eases exposition.

restrictions must be met: those on H_i, on V_i, and on φ_i. By reformulating agents' optimization programs as $M'_i(\eta_{-i})$, we manage to evade the difficulty related to non-convex-valued constraint correspondences. The reformulation introduces a difficulty in guaranteeing that V_i is quasiconcave in η_i.[11] This difficulty, however, has proven to be more readily surmounted than that concerning φ_i.

The restriction which guarantees that the V_i are quasiconcave requires that agents prefer to consume their own good. We assume that this preference is sufficiently strong. Formally, we have the following definition.

Definition: Consumer i's preference relation $\succsim_i$ is said to satisfy *own good bias* (OGB) if for every $\eta \in H$, $x_i^i(p(\eta), y_i(\eta)) \geq y_i(\eta)$.[12]

The technical content of this definition will become apparent in the proof (see also Figure 2-3, where the restriction is displayed graphically). Its economic content is that our agents always choose to consume more of

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[11]Dasgupta and Maskin (1986a,b) study a class of economic games which fail to posses an equilibrium. This failure stems from the failure of payoff functions in these games either to be quasi-concave or to be continuous. Dasgupta and Maskin show that, with non-quasi-concave utility, mixed strategies may correct the non-existence problem. In our model quasi-concavity of the payoff function V_i is shown to follow from more primitive conditions. Absent these, a mixed strategies approach could perhaps be fruitfully employed, an issue needing further research.

[12]This condition and the others required by the model are all satisfied by an appropriately restricted homogeneous of degree one Cobb-Douglas (C-D) utility function. For a lobbying economy $\mathcal{E}$, let $\bar{p} = p(\hat{\eta}_1, 0) < 1$. This denotes the maximum value achieved by the function $p(\eta)$ on H. Now suppose, in the case of Mr. 1, that the C-D parameter is α. The slope of indifference curves at the 45°-line is $-\alpha/(1-\alpha)$, while the budget line has slope $-\bar{p}/(1-\bar{p}) > -\infty$. It is straightforward to verify that own good bias is satisfied by this function whenever $\alpha > \bar{p}$.

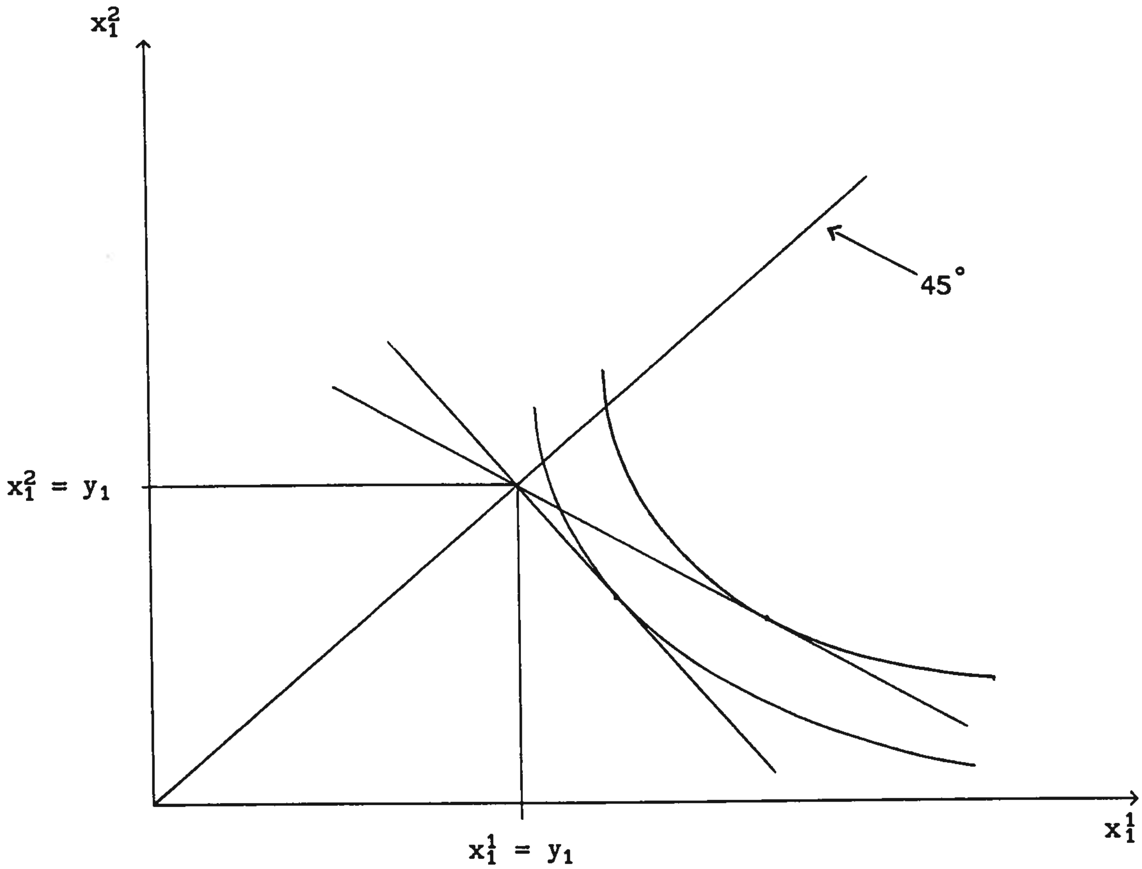

Figure 2-3. *Own good bias: income for agent 1 is fixed at* y_1.

the good they enter the world with than of the other good. With this definition, the groundwork is now laid for a statement of the game equilibrium existence theorem.

Theorem 2.2 (Existence of a Lobbying Game Equilibrium). Suppose that in the lobbying economy $\mathcal{E} = ((\succsim_i, \omega_i)_{i=1,2};\ p(\eta))$, for every i, preferences are strictly quasiconcave, monotone increasing, twice differentiable, and satisfy own good bias; and the function $p(\eta)$ satisfies (A1)—(A4). Then the associated lobbying game $\Gamma_{\mathcal{E}} = (H_i, V_i, \varphi_i)_{i=1,2}$ has an equilibrium.[21]

This theorem will be proved via a series of lemmas. The strategy sets H_i, constraint correspondences φ_i, and payoff functions V_i are shown in these lemmas to meet the conditions of Theorem 2.1; the proof of Theorem 2.2 follows from them immediately. Theorem 2.1 requires that for each i in I, H_i be a non-empty, compact, convex subset of a Euclidean space. This is established for the lobbying game $\Gamma_{\mathcal{E}}$ in the first lemma.

Lemma 2.1. Suppose that $p(\eta)$ satisfies (A3) and (A4). Then for each i, H_i is a non-empty, compact, convex subset of $\mathbb{R}$.

Proof. i.) (Non-emptiness). Clearly, $0 \in H_i$. Thus, $H_i \neq \varnothing$.

ii.) (Compactness). Since $H_i \subset \mathbb{R}$, it is compact precisely when it

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[21]In the language of abstract lobbying economies developed above, this theorem may be concisely restated. Let $\mathfrak{E}_+^{OPT} \subset \mathfrak{E}_+$ denote the set of all lobbying economies for which the corresponding lobbying game $\Gamma_{\mathcal{E}}$ has an equilibrium. Let $\mathcal{R}^{OGB}$ denote the set of admissible characteristics a_i such that $\succsim_i$ satisfies own good bias. Theorem 2.2 may be restated as follows: If, for each $i \in I$, $a_i \in \mathcal{R}^{OGB}$, then $\mathcal{E} \in \mathfrak{E}_+^{OPT}$. Thus, if preferences are appropriately restricted, the game $\Gamma_{\mathcal{E}}$ will have an equilibrium for any pricing rule in $\mathcal{P}_+$.

is closed and bounded. Closedness is immediate from the definition of H_1. To show boundedness, it is enough to show that $\hat{\hat{\eta}}_1 < +\infty$. Note that $\hat{\eta}_1(\eta_{-1})$ is strictly decreasing in η_{-1}. To see this, consider the case of Ms. 2, and suppose not. Then there is a pair t, t′ ∈ H_1 such that $\hat{\eta}_2(t) < \hat{\eta}_2(t')$ and $t' > t$. This implies that $P(\hat{\eta}_2(t),t')\cdot\omega_2 > P(\hat{\eta}_2(t),t)\cdot\omega_2$, contradicting (A3). We conclude that $\hat{\eta}_2(\eta_1)$ is strictly decreasing in η_1. The argument for Mr. 1 is similar. From this, it follows that $\hat{\hat{\eta}}_1 = \hat{\eta}_1(0)$, which is finite by assumption (A4). Therefore, H_1 is bounded. It follows that it is compact.

iii.) (Convexity). The convexity of H_1 follows immediately from the definition. This completes the proof of Lemma 2.1. ■

Theorem 2.1 requires that for each i in I, the constraint correspondence $\varphi_i(\eta_{-i})$ be convex-valued and continuous. The following definitions will be useful. For purposes of the definitions, let X and Y be subsets of arbitrary Euclidean spaces, and let ρ be a correspondence mapping X into Y.

Definition: The correspondence $\rho : X \twoheadrightarrow Y$ is *upper hemi-continuous* (*u.h.c.*) *at a point x^o of X* if for every sequence $\{x^n\}$ in X converging to x^o and every sequence $\{y^n\}$ in Y with $y^n \in \rho(x^n)$ for each n, we have that $y^o \in \rho(x^o)$. ρ is upper hemi-continuous on X if it is upper hemi-continuous at every $x \in X$.

Definition: The correspondence $\rho : X \twoheadrightarrow Y$ is *lower hemi-continuous* (*l.h.c.*) *at a point x^o of X* if for every sequence $\{x^n\}$ in X converging to x^o, and every $y^o \in \rho(x^o)$, there is a sequence $\{y^n\}$ in Y with $y^n \longrightarrow y^o$ such that for each n, $y^n \in \rho(x^n)$. ρ is lower hemi-continuous on X if it is lower hemi-continuous at every $x \in X$.

Definition: The correspondence $\rho : X \twoheadrightarrow Y$ is *continuous at a point x in X* (resp. *continuous on X*) if it is both l.h.c. and u.h.c. at x (resp. l.h.c. and u.h.c. on X).

Lemma 2.2 establishes that the constraint correspondence is indeed convex-valued and continuous.

Lemma 2.2. Under the conditions of Theorem 2.2, for each i in I, the constraint correspondence $\varphi_i(\eta_{-i})$ is convex-valued and continuous.

Proof. i.) (Convex-valued). That $\varphi_i(\eta_{-i})$ is convex-valued follows from the definition of $\varphi_i(\eta_{-i})$.

ii.) (Continuity). It suffices to show that $\varphi_i(\eta_{-i})$ is upper and lower hemi-continuous. We have noted that $\hat{\eta}_i(\eta_{-i})$ is continuous on $[0,\hat{\eta}_{-i}]$. Thus, the graph of $\varphi_i(\eta_{-i})$ is closed. As $\varphi_i(\eta_{-i})$ is also compact-valued by Lemma 2.1, it is upper hemi-continuous (see, e.g., Border, 1985, p. 57). To show lower hemi-continuity, consider a sequence $\{\eta^n_{-i}\}$ in H_{-i} converging to η^o_{-i}, and take an arbitrary $\eta^o_i \in \varphi_i(\eta^o_{-i})$. If $\eta^o_i < \hat{\eta}_i(\eta^o_{-i})$, then for N large, we may set $\eta^n_i = \eta^o_i$ for $n \geq N$. Then clearly $\eta^n_i \longrightarrow \eta^o_i$, and the conditions for lower hemi-continuity are satisfied. If $\eta^o_i = \hat{\eta}_i(\eta^o_{-i})$, then let $\eta^n_i = \hat{\eta}_i(\eta^n_{-i})$. As $\hat{\eta}_i(\eta_{-i})$ is continuous, the conditions for lower hemi-continuity are again satisfied. We conclude that $\varphi_i(\eta_{-i})$ is lower hemi-continuous. Thus, it is continuous. This completes the proof of Lemma 2.2. ■

Theorem 2.1 requires that for each i in I, V_i be continuous and quasiconcave in η_1.[22] To demonstrate that V_1 is quasiconcave in η_1 we will need to prove some intermediate results. As a first step, quasiconcavity is defined.

Definition: Let $S \subset \mathbb{R}^m$, $T \subset \mathbb{R}^n$, and $g : S \longrightarrow T$ be a function mapping elements of S into T. g is *quasiconcave* if for every s^1 and s^2 in S, for each $v \in [s^1,s^2]$, $g(v) \geq \min\{g(s^1),g(s^2)\}$.

Several alternative characterizations of the quasiconcavity of differentiable functions may be found in the literature (see, e.g., Diewert, *et.al.* (1981)). The following result is yet another. This criterion is applicable when a differentiable function g maps $\mathbb{R}$ into $\mathbb{R}$. While similar to Theorem 2 in Diewert, *et.al.*, it is more specialized and simpler for our purposes.

Lemma 2.3. Let $X \subset \mathbb{R}$, $Y \subset \mathbb{R}$, and let $g : X \longrightarrow Y$ be differentiable. Then g is quasiconcave if and only if for every pair of elements x, x′ of X, [$g'(x) < 0$ and $x' > x$] imply $g'(x') \leq 0$.

Proof. i.) (Necessity). Suppose g is quasiconcave, and that for x,x′ in X, $g'(x) < 0$ and $x' > x$. By way of contradiction, suppose that $g'(x') > 0$. Let $y = \text{argmin}_{z \in \{x,x'\}} g(z)$. Then for any $\varepsilon > 0$, there exists a $y' \in \mathbb{B}(y;\varepsilon)$, the open ε-ball around y, with $y' \neq y$, such that $g(y') < g(y)$ and $y' \in (x,x')$. But this contradicts that g is quasiconcave. We conclude $g'(x') \leq 0$.

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[22]The function V_1 is quasiconcave in η_1 precisely when it is "single-peaked." The connection between our requirement and the assumption used to rule out Arrovian impossibilities in social choice problems or to guarantee the existence of equilibria in pure voting models will not be explored here.

ii.) (Sufficiency). Suppose that for any x, x′ in X, [$g'(x) < 0$ and $x' > x$] imply $g'(x') \leq 0$. We must show that for $y \in [x,x']$, $g(y) \geq \min\{g(x),g(x')\}$. Otherwise, suppose not: there exists a $z \in (x,x')$ with $g(z) < \min\{g(x),g(x')\}$. We then have

$$\frac{g(z) - g(x')}{z - x'} > 0.$$

By the Mean Value Theorem, there is a $w \in (z,x')$ such that

$$g'(w) = \frac{g(z) - g(x')}{z - x'} > 0,$$

a contradiction. We conclude that g is quasiconcave. This completes the proof of Lemma 2.3. ∎

The condition defined in Lemma 2.3 requires that once g begins declining in x, it may never increase as x increases further. It shall now be demonstrated that for each i, the payoff function V_i satisfies this condition as a function of η_i.

Recall that agent i's indirect utility function is $V_i = V_i(p(\eta),y_i(\eta))$. That V_i is differentiable is immediate from the 2-differentiability of U_i and the differentiability of $p(\eta)$. The derivative of V_i with respect to η_i is given by

$$\partial V_i = \frac{\partial V_i}{\partial \eta_i}(p,y_i) = \frac{\partial V_i}{\partial p}\cdot\frac{\partial p}{\partial \eta_i} + \frac{\partial V_i}{\partial y_i}\cdot\frac{\partial y_i}{\partial \eta_i}.$$

Let $\partial_p V_i = \frac{\partial V_i}{\partial p}\cdot\frac{\partial p}{\partial \eta_i}$ and let $\partial_y V_i = \frac{\partial V_i}{\partial y_i}\cdot\frac{\partial y_i}{\partial \eta_i}$. These expressions will hereafter be referred to as the price and income effect, respectively, of a change in η_i on V_i. They refer to the effect, for a given η_{-i}, of an incremental change in η_i on indirect utility through the price (with y_i held constant) and through income (with p held constant). In showing that V_i is quasiconcave, we may treat these two terms separately. First, consider $\partial_y V_i$.

Lemma 2.4. Suppose that $p(\eta)$ satisfies (A3) and (A4), and that preferences are monotone for every i. If $\partial_y V_i(x) < 0$ for some $x \in [0,\hat{\eta}_i(\eta_{-i})]$, then for $x' > x$ with $x' \in [0,\hat{\eta}_i(\eta_{-i})]$, $\partial_y V_i(x') \le 0$.

Proof. Consider $y_i(\eta) = P(\eta)\cdot\omega_i - \eta_i$. Under assumption (A3), $y_i(\eta)$ is concave in η_i for each i in I. Thus, it is quasiconcave in η_i. By Lemma 2.3, if $\partial y_i/\partial\eta_i(\eta_i) < 0$ for some $x \in [0,\hat{\eta}_i(\eta_{-i})]$, then for $x' > x$ with $x' \in [0,\hat{\eta}_i(\eta_{-i})]$, $\partial y_i/\partial\eta_i(x') \le 0$.

Under the monotonicity of U_i, $\partial V_i/\partial y_i > 0$. Thus, $\partial_y V_i$ agrees in sign with $\partial y_i/\partial\eta_i$ at every η_i. We conclude that if $\partial_y V_i(x) < 0$ for some $x \in [0,\hat{\eta}_i(\eta_{-i})]$, then for $x' > x$ with $x' \in [0,\hat{\eta}_i(\eta_{-i})]$, $\partial_y V_i(x') \le 0$. This completes the proof of Lemma 2.4. ∎

Now, it remains only to show that $\partial_p V_i$ doesn't increase in η_i "too much." By too much is meant that, while $\partial_y V_i$ is always negative in η_i once it becomes negative, the sum ∂V_i goes positive after once having been negative. In Lemma 2.5 it is shown that as η_i increases, the affect on V_i through the price does not offset the eventually negative income effect. In fact, this Lemma shows something stronger: that for a fixed income y_i, under the OGB assumption, $\partial_p V_i$ is non-positive.

Lemma 2.5. Suppose that for i in I, $\succsim_i$ satisfies own good bias and monotonicity. Then for every $\eta \in H$, $\partial_p V_i(\eta) \le 0$.

Proof. i.) For an $i \in I$, fix $y_i > 0$. Let $E_i = \{(x^1,x^2) \in \mathbb{R}^2_+ : x^1 \ge y_i\}$. For any $p \in (0,1)$, let $\beta_i(p,y_i) = \{(x^1,x^2) \in \mathbb{R}^2_+ : P\cdot(x^1,x^2) \le y_i\}$. By OGB, we have that the demanded bundle $x_i(p,y_i) \in \beta_i(p,y_i) \cap E_i = \beta_i^+(p,y_i)$.

ii.) Now, for $p' > p$, we have that $\beta_1^+(p',y_1) \subset \beta_1^+(p,y_1)$ (resp. $\beta_2^+(p,y_2) \subset \beta_2^+(p',y_2)$). By monotonicity of $\succsim_i$, then, $x_1(p,y_1) \succsim_1 x_1(p',y_1)$ (resp. $x_2(p',y_2) \succsim_2 x_2(p,y_2)$).

Combining i.) and ii.), for any pair p,p' with $p' > p$, $V_1(p,y_1) \geq V_1(p',y_1)$ and $V_2(p,y_2) \leq V_2(p',y_2)$. Since p and p′ were arbitrary, and since $\partial p/\partial\eta_1 > 0$ and $\partial p/\partial\eta_2 < 0$, the preceding argument is sufficient to demonstrate that $\partial_p V_i(\eta) \leq 0$, which was to be shown. This completes the proof of Lemma 2.5.[23] ∎

That V_i is continuous and quasiconcave in η_i is now easily established.

Lemma 2.6. Under the conditions of Theorem 2.2, for each $i \in I$, V_i is continuous and quasiconcave in η_i.

Proof. i.) (Continuity). Since U_i is 2-differentiable and $p(\eta)$ is differentiable, V_i is a composition of continuous functions. It is therefore continuous.

ii.) (Quasiconcavity). The quasiconcavity of V_i in η_i is immediate from Lemmas 2.3, 2.4, and 2.5 and the definition of $\partial V_i/\partial\eta_i$. This completes the proof of Lemma 2.6. ∎

Before proceeding to the proof of Theorem 2.2, some comments upon the results of Lemmas 2.3 to 2.6 are in order. First, the requirement of quasiconcavity of payoff functions V_i is more than just a technical

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[23]It may be shown that if preferences satisfy OGB, monotonicity, and if they are differentiable, then demands will be such that $x_i^i(p(\eta),y_i(\eta)) > y_i$ on $[0,\hat{\eta}_i(\eta_{-i}))$. From this strict inequality it follows that $\partial_p V_i < 0$, a result which is stronger than is required for the Lemma or for Theorem 2.2.

restriction. V_1 fails to be quasiconcave when it declines in η_1 at some point and then rises as η_1 continues to increase. There is an intuitive appeal to the idea that such a circumstance may lead to a lobbying game without an equilibrium. Also, the assumption of own good bias is stronger than is needed. Since $\partial_y V_1$ is eventually negative, $\partial_p V_1$ can become positive as η_1 increases, as long as the *sum* ∂V_1 remains non-positive. There is room, then, for weakening the restriction on preferences required to guarantee the existence of an equilibrium in the lobbying game. Let us now turn to the proof of Theorem 2.2.

Proof of Theorem 2.2.

Combining Lemma 2.1, Lemma 2.2, and Lemma 2.6, it is immediate that Γ_g satisfies the conditions of Theorem 2.1. Thus, Γ_g has an equilibrium. That is, there is an $\eta^* \in H$ such that $\eta^* \in \mu(\eta^*)$. This completes the proof of Theorem 2.2. ■

It has now been established that for economies satisfying the conditions of Theorem 2.2, there exists a pair η^* of strategies at which each agent is responding optimally to his or her opponent's strategy. In the following section, we specify conditions on the pricing function $p(\eta)$ which guarantee that η^* is also government feasible. Together, Theorem 2.2 and the feasibility result establish the existence of a lobbying equilibrium for the lobbying economy.

2.5 EXISTENCE OF A LOBBYING EQUILIBRIUM

The objective of the remainder of the chapter is to show that any lobbying game equilibrium η^* for the game Γ_g, along with the associated consumption bundles $\tilde{x}_i(\eta^*)$, is also an economic equilibrium, i.e., it is a lobbying equilibrium in $\mathcal{E}$. The approach which we will follow involves restricting the pricing function $p(\eta)$ without further restricting preferences. From Theorem 2.2, we know that if $a_i \in \mathcal{R}^{OGB}$ for each i, then $\mathcal{E} \in \mathfrak{E}_+^{OPT}$ (see footnote 21). To obtain feasibility, a condition on $p(\eta)$, together with OGB, is used to show that $\pi(\eta^*) \geq 0$, and thus that $LE(\mathcal{E}) \neq \emptyset$.

While Theorem 2.2 ensures that $LGE(\Gamma_{\mathcal{E}}) \neq \emptyset$, it has nothing to say about the location of η^* in H, except that each η_i^* must lie in the set $\varphi_i(\eta_{-i}^*)$. Thus, the feasibility condition $\pi(\eta^*) \geq 0$ must be shown to hold for *every* possible feasible pair η. First, however, a portion of H may be eliminated from consideration; along the way a bit of intuitive motivation for the feasibility proposition is offered. For i = 1,2, let

$$K_i = \left\{\eta \in \mathbb{R}_+^2 : \eta_i \leq \hat{\eta}_i(\eta_{-i}) \text{ and } \eta_{-i} \in [0,\mathring{\eta}_{-i}]\right\}.^{24}$$

Let $K = K_1 \cap K_2 \subset H$. Clearly, if $\eta \notin K$, then for at least one i, η_i is not individually feasible. What's more, by definition $\tilde{z}_i(\hat{\eta}_i(\eta_{-i}),\eta_{-i}) = 0$ for any $\eta_{-i} \in [0,\mathring{\eta}_{-i}]$, since any agent who chooses to devote all of his or her resources to lobbying must have no individual excess demand. Thus, on $(K_1 \cup K_2)^c$, we may define $\tilde{z} \equiv 0$. Recalling the condition for government feasibility, $\pi(\eta) = \eta_1 + \eta_2 - (P^* - P(\eta))\cdot\tilde{z}(p(\eta)) \geq 0$, we have that $\pi(\eta) = \eta_1 + \eta_2 \geq 0$ on $(K_1 \cup K_2)^c$.

Let $\Delta^c = \{\eta \in \mathbb{R}_+^2 : \eta_1 + \eta_2 = c\}$ for an arbitrary $c \geq 0$, and let $\eta_2 = c - \eta_1$. Then the function $p(\eta)$ may be expressed as $p(\eta_1;c)$, where the

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[24] In fact, K_i corresponds exactly to the graph of the constraint correspondence $\varphi_i(\eta_{-i})$ of the previous section.

intervention price depends only on η_1 given c. A feature of the pricing function $p(\eta)$ which offers some intuition for the feasibility argument is that for any $c \geq o$, there is an $\bar{\eta}_1 \in [0,c]$ such that $p(\bar{\eta}_1;c) = p^*$,[25] whence $\pi(\bar{\eta}) = \bar{\eta}_1 + \bar{\eta}_2 \geq 0$. The set of all such points in H is a continuous curve, and along this curve $\pi(\eta) = \eta_1 + \eta_2 \geq 0$ holds; not because excess demand is necessarily zero, but because $(P^* - P(\eta)) = 0$. (See Figure 2-4. There, K and Δ^c are as defined above, and T is the zero set of $P^* - P(\eta)$.)

For a given $c \geq 0$, we may write $\pi(\eta_1;c) = c - (P^* - P(\eta_1;c))\cdot\tilde{z}(p(\eta_1;c))$. The government feasibility condition is satisfied either when p^* is "close" to $p(\eta_1;c)$ or when $\tilde{z}(p(\eta_1;c))$ is "small." One possibility, that $p(\eta_1;c) \equiv p^*$, the constant function, is ruled out by the productive lobbying assumption (A3). However, this suggests that if the function $p(\eta_1;c)$ is "flat enough," even if $\tilde{z}(p(\eta_1;c)$ is relatively large, the feasibility condition will be satisfied. Only when $p(\eta)$ is far from p^* with $\eta_1 + \eta_2$ small will the feasibility condition be violated.

To obtain a bound on the degree of flatness required, we exploit the fact that own good bias limits the amount by which demanded bundles (or the tangency points of indifference curves) may separate along the price line.[26]

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[25]This follows immediately from (A3), the productive lobbying assumption, which implies $p(c,0) > p^* > p(0,c)$. As $p(\eta)$ is also continuous, there must be an $\bar{\eta}_1$ in [0,c] such that $p(\bar{\eta}_1,c-\bar{\eta}_1) = p.^*$

[26]For exchange economies, Geller (1986) provides a bound on per capita excess demand which is independent of preferences. This bound is essentially the product of the norm of the average endowment and the square root of the ratio of the number of commodities to the number of traders. Unfortunately, the result ensures only that the bound is satisfied for *some* price vector. Thus, Geller's bound is not helpful here; our interest is in the size of excess demand at the specified price $p(\eta)$.

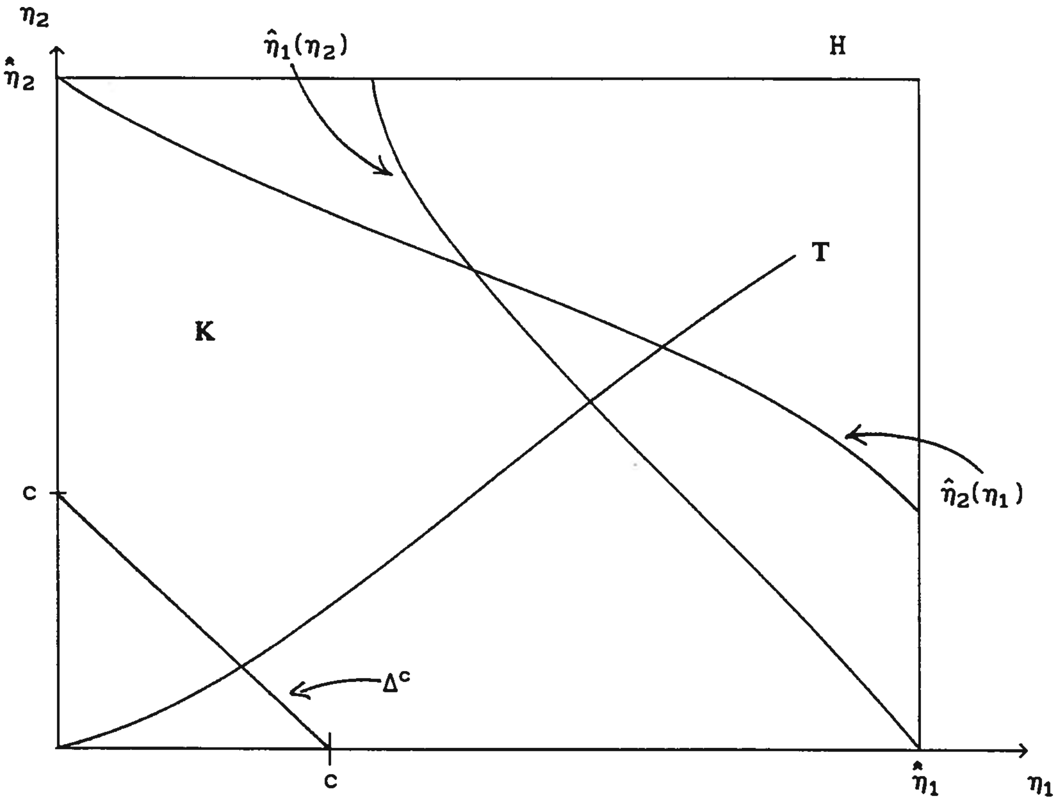

Figure 2–4. *The sets* K, T, H *and* Δ^c *in* η_1,η_2*-space.*

What follows is designed to achieve a steepness restriction on $p(\eta)$ which, together with OGB, ensures $\pi(\eta) \geq 0$.

Proposition 2.1 (Feasibility of the Lobbying Game Equilibrium)

Suppose that in a lobbying economy $\mathcal{E}$ the conditions of Theorem 2.2 are satisfied, and take an $\eta^* \in LGE(\Gamma_{\mathcal{E}})$. Suppose further that for each $i \in I$, $\eta_i^* < \hat{\eta}_i(\eta_{-i}^*)$, so that neither agent devotes his or her entire resource endowment to lobbying activity. If at η^*, $\tilde{\omega}^1 \geq \tilde{\omega}^2$, (resp. $\tilde{\omega}^2 > \tilde{\omega}^1$), then $\pi(\eta^*) \geq 0$ whenever

$$1 - \frac{p^*}{p(\eta^*)} \leq \frac{\eta_1^*}{y_1(p(\eta^*))} \tag{2.1}$$

(resp. whenever $1 - \dfrac{1-p^*}{1-p(\eta^*)} \leq \dfrac{\eta_2^*}{y_2(p(\eta^*))}$).[27]

Proof: Take a pair $\eta^* \in LGE(\Gamma_{\mathcal{E}})$. We know that $\eta_1^* + \eta_2^* = c \geq 0$. If $c = 0$, then $\pi(\eta_1^*;c) = 0$ by definition. Now, consider a $c > 0$. Suppose that $\tilde{\omega}^1 \geq \tilde{\omega}^2$. Condition (2.1) may be written

$$1 - \frac{p^*}{p(\eta_1^*;c)} \leq \frac{\eta_1^*}{y_1(p(\eta_1^*;c))}, \tag{2.2}$$

where $y_1(p(\eta_1^*;c)) > 0$ is guaranteed by our assumption that $\eta_1^* < \hat{\eta}_1(\eta_{-1}^*)$. Rearranging, equation (2.2) becomes

$$\left(p(\eta_1^*;c) - p^* \right) \cdot \left(p(\eta_1^*;c) - \frac{\eta_1^*}{\omega^1} \right) \leq p(\eta_1^*;c) \cdot \left(\frac{\eta_1^*}{\omega^1} \right),$$

or, multiplying by $\omega^1/p(\eta_1^*;c)$,

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[27]Here, p^* is the competitive equilibrium (lobbying-free) price; $p(\eta^*)$ corresponds to the politically dictated price which results when η^* is an equilibrium outcome in the lobbying game.

$$\left(p(\eta_1^*;c) - p^* \right) \cdot \left(\omega^1 - \frac{\eta_1^*}{p(\eta_1^*;c)} \right) \leq \eta_1^* \leq c. \tag{2.3}$$

It suffices to show that when (2.3) is satisfied, $\pi(\eta_1^*;c) \geq 0$. Note that with $\tilde{\omega}^1 \geq \tilde{\omega}^2$ and with the lobbying price determined as $p(\eta^*)$, the condition OGB places a bound on the magnitude of $\pi(\eta_1^*;c)$. Under OGB, $\pi(\eta_1^*;c)$ achieves a maximum if agent 1 consumes on the 45-degree line, where $x_1^1 = x_1^2 = y_1(p(\eta_1^*;c))$, and agent 2 simply consumes her "after-lobbying" endowment $\tilde{\omega}^2$. We have that the aggregate "after-lobbying" excess demand corresponding to this maximum level of π is the pair

$$\tilde{z}^1(p(\eta^*)) = y_1(p(\eta^*)) - \tilde{\omega}^1, \tag{2.4a}$$

$$\tilde{z}^2(p(\eta^*)) = y_1(p(\eta^*)) + \tilde{\omega}^2 - \tilde{\omega}^2 = y_1(p(\eta^*)). \tag{2.4b}$$

Now, we claim that (2.3) holds if and only if $\pi(\eta) \geq 0$. From (2.3), we have that

$$c - \left(p(\eta_1^*;c) - p^* \right) \cdot \left(\omega^1 - \frac{\eta_1^*}{p(\eta_1^*;c)} \right) \geq 0.$$

Adding and subtracting $y_1(p(\eta_1^*;c))$ in the second bracketed term on the left, and noting that $\left(\omega^1 - \frac{\eta_1}{p(\eta_1^*;c)} \right) = \tilde{\omega}^1$, we may write this as

$$c - \left(p(\eta_1^*;c) - p^* \right) \cdot \left(y_1(p(\eta_1^*;c)) - \tilde{\omega}^1 - y_1(p(\eta_1^*;c)) \right) \geq 0.$$

Upon rearranging, and using the fact that $q = (1 - p)$, we obtain

$$c - \left(P^* - P(\eta_1^*;c) \right) \cdot \left(y_1(p(\eta_1^*;c)) - \tilde{\omega}^1,\ y_1(p(\eta_1^*;c)) \right) \geq 0, \tag{2.5}$$

where each of the bracketed terms in (2.5) is a two-dimensional vector. But (2.5), together with equations (2.4), yields the condition

$$\pi(\eta_1^*;c) = \eta_1^* + \eta_2^* - \left(P^* - P(\eta_1^*;c) \right)\cdot \left(\tilde{z}(p(\eta_1^*;c)) \right) \geq 0,$$

which was to be obtained. All of the steps in the proof are reversible, so that the claim is established: (2.3) holds if and only if $\pi(\eta^*) \geq 0$.

For the case with $\tilde{\omega}^2 > \tilde{\omega}^1$, due to the symmetry of our formulation and the price normalization employed, the same argument applies if we let $q(\eta_2^*;c) = 1 - p(\eta_2^*;c)$, and note that $y_2(p(\eta_2^*;c)) = \omega^2\cdot q(\eta_2^*;c) - \eta_2^*$. This completes the proof of Proposition 2.1. ∎

Before turning to the last theorem, we offer a few remarks on condition (2.1). With $\eta_1 + \eta_2 = c \geq 0$ fixed, recall that for some $\bar{\eta}_1 \in [0,c]$, $p(\bar{\eta}_1;c) = p^*$. If $\eta_1^* < \bar{\eta}_1$, then $p(\eta_1^*;c) < p^*$, and the left hand side of (2.1) is negative. As the right hand side is non-negative, the required condition is always satisfied. An intuitive interpretation of the inequality in (2.1) is that it provides a bound on the degree to which $p(\eta_1^*;c)$ may move away from p^* in response to lobbying donations. The expression $\eta_1^*/y_1(p(\eta_1^*;c))$ is, in some sense, a measure of 1's political involvement. This is the ratio of his lobbying donations to his goods consumption expenditures; it takes values on $[0,+\infty)$. When it is near zero, the resources of the government are also relatively small. Then, if feasibility is to be satisfied, the lobbying price $p(\eta_1^*;c)$ must be "close to" p^*. When the ratio $\eta_1^*/y(p(\eta_1^*;c))$ is large, $p(\eta_1^*;c)$ is allowed to be larger than *and* to be "far from" p^*. Hence, (2.1) is precisely the required bound on the maximum steepness of $p(\eta^*)$.

We now combine Theorem 2.2 with Proposition 2.1 to establish the existence of a lobbying equilibrium in the lobbying economy $\mathcal{E}$.

Theorem 2.3 (Existence of a Lobbying Equilibrium)

Consider a lobbying economy $\mathcal{E} \in \mathfrak{E}_{+}$. If $\mathcal{E}$ meets the conditions of Theorem 2.2 and of Proposition 2.1, then $LE(\mathcal{E}) \neq \emptyset$.

Proof: From Theorem 2.2, we know that $LGE(\Gamma_{\mathcal{E}}) \neq \emptyset$. Take an $\eta^{*} \in LGE(\Gamma_{\mathcal{E}})$. Proposition 2.1 guarantees that $\pi(\eta^{*}) \geq 0$. Therefore, η^{*} is government feasible. Thus, the set of lobbying equilibria $LE(\mathcal{E})$ is non-empty. This completes the proof of Theorem 2.3. ∎

2.6 CONCLUSIONS

Economic behavior is often dependent upon political circumstance. In democratically organized societies, political institutions usually permit individuals and interest groups to influence the economic policy of their government. In this event, the neoclassical economic model of agents as price-takers is not very instructive. In this chapter, an economic model has been constructed in which economizing agents make political decisions to influence price policy. These agents, in a strategic struggle over a contested interventionist price level, can discover equilibrium strategies. The level of generality of this result and of the underlying economic model give it interest in the context of extant literature.

In the sense that it does not resolve a real controversy in the field (no one argues that the world is devoid of political equilibria at which agents and groups lobby), the existence theorem is not of primary interest. Had this result not been achieved, what follows would have less merit. The fact that equilibria exist in the lobbying economy, though, helps to legitimize the welfare comparisons to which attention is now turned.

In the following chapter, the outcome of the lobbying program is compared welfare-wise to the competitive outcome which would have obtained in the underlying economy without lobbying and political behavior. Early works in the rent seeking literature supposed that lobbying would reduce aggregate welfare (suitably defined) unambiguously. Bhagwati (1980) first suggested that if distortions are already present, then lobbying to affect them may actually increase aggregate welfare. The results of chapter three suggest another possibility. Under certain conditions, one agent may be so much better off at the lobbying equilibrium that the other agent cannot arrange a bribe which is improving for him or her, and acceptable for the first.

APPENDIX 2

In this appendix we show that the choice set $\psi_1(\eta_{-1})$ is not convex in $\mathbb{R}^3_+$. The claim shall be demonstrated for the case of Mr. 1; the argument for Ms. 2 is similar.

Take an arbitrary $\eta_2 \in [0,\hat{\eta}_2]$. Note that Mr. 1 may simply eat his endowment, so that $z_1 = (\omega^1,0,0) \in \psi_1$ for any η_2. What's more, while it will never be an optimal decision under the monotonicity of U_1, he may give all of his endowment to the government in lobbying donations, so that $z_2 = (0,0,\hat{\eta}_1(\eta_2)) \in \psi_1$. The non-convexity of ψ_1 is guaranteed if there is a scalar $t \in (0,1)$ such that $z_t = t\cdot z_1 + (1-t)\cdot z_2 \notin \psi_1$.

Attention has been restricted here to elements in the intersection of ψ_1 with the x_1^1,η_1-plane. The "upper" boundary of this intersection is all pairs (x_1^1,η_1) satisfying $p(\eta)\cdot x_1^1 = p(\eta)\cdot\omega^1 - \eta_1$. Take an arbitrary $t \in (0,1)$. Let $\tilde{\omega}^1(t) = \omega^1 - \frac{(1-t)\cdot\hat{\eta}_1}{p((1-t)\cdot\hat{\eta}_1,\eta_2)}$. We are through if it can be shown that $t\omega^1 > \tilde{\omega}^1(t)$ (see Figure A2-1). But this inequality may be rewritten as

$$p((1-t)\hat{\eta}_1,\eta_2)\cdot t\omega^1 > p((1-t)\hat{\eta}_1,\eta_2)\cdot\omega^1 - (1-t)\cdot\hat{\eta}_1.$$

Combining terms,

$$p((1-t)\hat{\eta}_1,\eta_2)\cdot(t-1)\cdot\omega^1 > (t-1)\cdot\hat{\eta}_1.$$

Dividing by $(t-1) < 0$, which reverses the inequality,

$$p((1-t)\hat{\eta}_1,\eta_2)\cdot\omega^1 < \hat{\eta}_1.$$

Since $p(\eta)$ is strictly increasing in η_1, we have that

$$p((1-t)\hat{\eta}_1,\eta_2) < p(\hat{\eta}_1,\eta_2).$$

Multiplying by $\omega^1 > 0$,

$$p((1-t)\hat{\eta}_1,\eta_2)\cdot\omega^1 < p(\hat{\eta}_1,\eta_2)\cdot\omega^1.$$

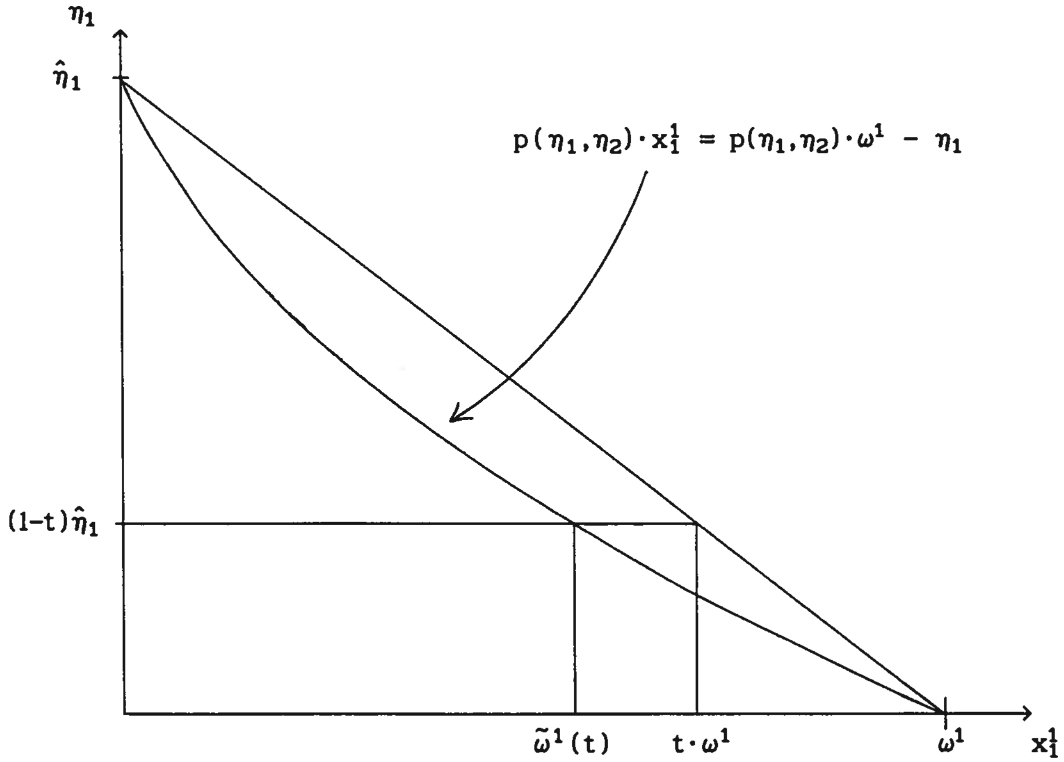

Figure A2-1. *Nonconvexity of $\psi_1(\eta_2)$.*

But by definition,

$$p(\hat{\eta}_1,\eta_2)\cdot\omega^1 = \hat{\eta}_1.$$

Combining the last two expressions, we have

$$p((1-t)\hat{\eta}_1,\eta_2)\cdot\omega^1 < \hat{\eta}_1,$$

which must hold by the definition of $\hat{\eta}_1$ and by the assumption that $p(\eta)$ is increasing in η_1. Since all of the steps in this derivation are reversible, it has been shown that $t\omega^1 > \tilde{\omega}^1(t)$, which completes the appendix.

CHAPTER THREE

OPTIMALITY OF EQUILIBRIA IN LOBBYING ECONOMIES

> *"Formal economic analysis rarely can tell a policy maker just what he should do, but it has been far more successful in telling him what he should seek to avoid if he is not to regret the consequences."*
>
> W. Baumol, 1986, p. 12

3.1 INTRODUCTION

Economic research in the rent seeking genre seeks to capture the full effect upon economic outcomes of populating a model with political actors and allowing the model itself to select an economic policy. This contrasts with more traditional studies in which economic policies are chosen exogenously and their effect on the economy is a separate issue. There is some controversy surrounding the question of the welfare effects of endogenously determined government intervention in economies. The public choice literature on rent seeking begins, for the most part, with an assertion that the seeking of rents is unavoidably and undeniably bad for society (Buchanan, 1980; see also Samuels and Mercuro, 1984, p. 56). This view seems to stem from Tullock (1967), whose claim that the seeking of rents created by government economic policy is inherently welfare-reducing, has survived for over two decades in the work of his colleagues.

Bhagwati's (1980, 1982) work on the second best suggests that rent seeking may be welfare-enhancing in the presence of certain kinds of pre-existing market imperfections. However, he also asserts that lobbying is unequivocally bad for open economies in which tariff revenues are sought by politically active interest groups (Bhagwati, *et.al.* (1984)). In the trade and, especially, in the public choice literatures, then, there appears a bias against the rent seeking activities of economic agents. And yet, modern democracies establish the required institutional arrangements for such rent seeking to take place, and we observe individuals and groups expending enormous amounts of money on such activities (Frey, *et.al.*, (1984)).

This chapter is concerned with the question of whether or not rent seeking behavior, even in a relatively well-ordered (perfect market) world, might be good for society in some sense. Using the model and set-up of the previous chapter, our purpose here is to investigate the welfare properties of a lobbying equilibrium.

The model is designed to capture the essence of the individuals' lobbying problem—choosing political activity while also making economic decisions which rely upon the collective's political activity. We shall draw welfare comparisons between the lobbying economy under investigation and its competitive counterpart, which is identical in all respects except for the absence of distortions in the latter. The competitive economy is familiar, but it affords agents no opportunity for acting politically. Comparisons of outcomes arising from conditions in which they do and do not have this opportunity are valuable because they shed light on the difference between *what is* and *what might have been*.

We find that the lobbying outcome may be dominated by the competitively determined allocation, but that the converse never occurs. There may not be a Pareto rank ordering of these two outcomes. In this case, we use the lobbying equilibrium as a benchmark and assess whether any other feasible outcome might dominate the lobbying outcome. We also show exactly what it means here to have the lobbying outcome improve upon the distortion-free allocation, and while this possibility has neither been shown to exist nor to be impossible, we illustrate when this sort of outcome *might* occur.

As candidate alternative allocation mechanisms, we consider two possibilities. First, we suppose that the government may implement lump-sum transfer schemes in order to achieve allocations on the Pareto optimal set, asking when such policies could improve upon the lobbying outcome. Then, we suppose that the agents may cooperate by entering into binding agreements, asking when such cooperation could yield allocations which improve upon the lobbying outcome. The last result of the chapter establishes a tight correspondence between cases in which the government and the agents may achieve such an improvement and thereby bypass the lobbying institution altogether.

3.2 WALRAS AND TRANSFER EQUILIBRIA

The model of this chapter is the same as the one introduced in chapter two, except that the utility functions are assumed to be concave. Once again, $\mathcal{E}$ denotes the lobbying economy and $\hat{\mathcal{E}}$ denotes the underlying competitive lobby-free economy. Having established that $\mathcal{E}$, under a certain set of assumptions, has a non-empty equilibrium set LE($\mathcal{E}$), the task at hand is to evaluate the optimality of such equilibria, and to compare agents' welfare levels there to the corresponding competitive allocations.[28]

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[28] Bhagwati, *et.al.* (1984) argue that comparisons between politically determined economic outcomes and related outcomes which arise from policies determined exogenously are "not compelling." By way of explanation, it

Sometimes, the lobbying equilibrium leaves all agents worse off than they were at the competitive allocation x^*. In this case, we might naturally ask whether any "participant" in the model (either the agents or the government, where the latter is best regarded as a mechanism which draws a pricing function $p(\eta)$ from $\mathcal{P}$) could behave so as to improve his or her individual well-being or the aggregate welfare. It is natural to wonder how the capacity for cooperation between Mr. 1 and Ms. 2 could affect their behavior and the outcome. Would they ever, given that such binding agreements were made available, collude and ignore the price-setting policy embodied in $p(\eta)$? This matter is taken up in section 3.3.

In this section, we suppose that consumers are passive and ask, Under what conditions on $\mathcal{E}$ will agents wish that a different $p(\eta)$ had been drawn from $\mathcal{P}$? That is, suppose that the constant function $p(\eta) \equiv p^*$ on H is added to $\mathcal{P}$; had this pricing function been drawn, lobbying would be effectively ruled out. If the government were also endowed with the authority to transfer income between agents via lump-sum redistribution schemes, then in $\mathcal{E}$, for a given $p(\eta)$ selection and the resulting outcome $(x_i^1, x_i^2, \eta_i)_{i=1,2}$, we may assess the optimality of the lobbying outcome.

As a first step, a few definitions must be introduced. Recall the definition of a lobbying equilibrium LE($\mathcal{E}$) for economy $\mathcal{E}$. In order to ease the notational complexity, η^* will henceforth denote the equilibrium

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should be noted that this is not quite the comparison being made here. Rather, this chapter is built around comparisons between outcomes resulting from a particular kind of political process and the corresponding outcomes of economies shorn of the political sector. The competitive economy $\mathcal{E}$ is a benchmark against which the lobbying economy $\mathcal{E}$ is measured. No more natural comparison comes to mind for our purposes.

lobbying outcome; we consider only economies for which $LE(\mathcal{E}) \neq \emptyset$. Further, $x_i(\eta^*)$ will denote consumer i's chosen consumption bundle under the price and income pair $(p(\eta^*), y_i(p(\eta^*)))$. That is, $x_i(\eta^*)$ coincides with the demand function $\tilde{x}_i(\eta)$ of chapter two, evaluated at η^*. If $\eta^* = 0$, then we have $p(\eta^*) = p^*$, the competitive equilibrium price, and $x_i(\eta^*) = x_i^*$ for every consumer i. Standard general economic equilibrium results ensure that this allocation is feasible in the competitive economy, that it is market-clearing, and that it is Pareto optimal. Our interest here, then, is in lobbying equilibria at which $\eta^* \neq 0$. What follows is an equilibrium definition which includes this condition.

Definition: Given a lobbying economy $\mathcal{E}$, a *strong active lobbying equilibrium*, denoted SALE($\mathcal{E}$), is a collection $(x_i(\eta^*), \eta_i^*)_{i=1,2}$ satisfying:

i) $(x_i(\eta^*), \eta_i^*)_{i=1,2}$ is a lobbying equilibrium, and

ii) $\eta^* \in \mathbb{R}^2_{++}$.

Throughout the remainder of the chapter, we consider only economies for which $\eta^* \in$ SALE($\mathcal{E}$).[29] However, note that with strictly increasing preferences $\succsim_i$, all of the results of this section hold even if $\eta_i^* > 0$ for one agent, but $\eta_{-i}^* = 0$. In any neighborhood of such an economy is another economy for which $\eta_i^* > 0$ for every agent, and for which allocations and utilities are in the neighborhoods of the original levels. For simplicity we choose to assume that $\eta_i^* > 0$ for each agent. Recall the after-lobbying endowment vector $(\tilde{\omega}^1, \tilde{\omega}^2) = \tilde{\omega} \in \mathbb{R}^2_{++}$. This vector provides an upper bound on

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[29]Where no ambiguity results, a strong active lobbying equilibrium for $\mathcal{E}$ will be denoted η^*, it being understood that the associated consumption bundles are given by $x(\eta^*)$ as defined above.

the resources available for consumption in the after-lobbying economy. Let $\tilde{F} = \{(x^1,x^2) \in \mathbb{R}^2_+ : (x^1,x^2) \leq \tilde{\omega}\} \subset \mathbb{R}^2_+$. This is the set of all feasible bundles in the after-lobbying economy; without trade, consumers face this resource constraint. Similarly, let $F = \{(x^1,x^2) \in \mathbb{R}^2_+ : (x^1,x^2) \leq \omega\}$. Clearly, F is convex. Since $\omega \in \mathbb{R}^2_{++}$, F is also compact. In the sequel, for the pair of allocations $x = (x_1,x_2) \in \mathbb{R}^4_+$, we will sometimes abuse notation and write $x \in F$. Strictly speaking, of course, x cannot be a member of F; here it is to be understood that the pair (x_1,x_2) is such that $x_1 + x_2 \in F$. The feasibility condition may now be defined.

Definition: The pair of consumption bundles $(x_1,x_2) \in \mathbb{R}^4_+$ is *feasible* (resp. *tilde-feasible*) if $x_1 + x_2 \in F$ (resp. if $x_1 + x_2 \in \tilde{F}$), where $x_1 + x_2$ denotes vector addition in the plane.

Using these, our notions of domination and optimality may be defined.

Definition: For the lobbying economy $\mathcal{E}$ and a corresponding $\eta^* \in \text{SALE}(\mathcal{E})$, let $\tilde{\omega}$ be as defined above. We say that the pair of vectors $x = (x_1,x_2) \in X_1 \times X_2$ is *dominated* if there is a pair $y = (y_1,y_2) \in X_1 \times X_2$ such that $y_i >_i x_i$ for each $i \in I$ and such that $y_1 + y_2 \in F$. x is *tilde-dominated* if such a y may be found with $y_1 + y_2 \in \tilde{F}$.

Definition: For the lobbying economy $\mathcal{E}$ and a corresponding $\eta^* \in \text{SALE}(\mathcal{E})$, let $\tilde{\omega}$ be as defined above. We say that the feasible pair of vectors $x = (x_1,x_2) \in X_1 \times X_2$ is *optimal* if it is not dominated. The set of optimal pairs is denoted $\text{PO}(\mathcal{E})$. x is *tilde-optimal* if it is not tilde-dominated. Since $F \supset \tilde{F}$, x is tilde-optimal whenever it is optimal.

A last definition and some additional notation complete the preparatory discussion. For each i, for any $x_i \in X_i$, let $L_i(x_i) = \{y \in X_i : y \succsim_i x_i\}$. $L_i(x_i)$ is the *upper level set* of x_i for agent i. It consists of all bundles in X_i which stand in relation $\succsim_i$ to x_i. For convenience, let $G_1 = L_1(x_1(\eta^*))$. Let $\tilde{G}_2 = (\tilde{\omega} - L_2(x_2(\eta^*))) \cap \mathbb{R}^2_+$, and let $G_2 = (\omega - L_2(x_2(\eta^*))) \cap \mathbb{R}^2_+$. These sets are the intersections of reflections of $L_2(x_2(\eta^*))$ about the endowment points $\tilde{\omega}$ and ω, respectively, with $\mathbb{R}^2_+$.[30] By the concavity and the continuity of the U_i, the G_i are convex and closed, respectively. We will adopt the notational convention that $x \in G$ whenever both $x_1 \in G_1$ and $x_2 \in G_2$. The vector x in this case is restricted to its projection on X_1. Finally, let $G = G_1 \cap G_2 \subset \mathbb{R}^2_+$, and let the interior of a set $A \subset \mathbb{R}^m$ be denoted int(A). We note that $\text{int}(G) = \text{int}(G_1 \cap G_2) = \text{int}(G_1) \cap \text{int}(G_2)$, being the intersection of convex sets, is also convex (see Figure 3-1).

Now, we will very soon devote attention to a welfare-wise comparison between $x(\eta^*)$ and x^*. The fact that x^* lives in F, while $x(\eta^*)$ lives in $\tilde{F}$, is fundamental since the outcomes x^* and $x(\eta^*)$ must be measured against a common yardstick. As Figure 3-1 suggests, there is no allocation in $\tilde{F}$ which tilde-dominates the lobbying allocation $x(\eta^*)$; nothing which is feasible without trade after lobbying makes both agents better off than they are at $x(\eta^*)$. This result is established in the following proposition.[31]

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[30]Specifically, for any set $A \subset \mathbb{R}^m$, by $-A$ we mean the set $-A = \{a \in \mathbb{R}^m : -a \in A\}$. If $c \in \mathbb{R}^m$, then $(c - A) = \{a \in \mathbb{R}^m : (c - a) \in A\}$.

[31]In order not to disrupt the discussion, the proofs of all of the propositions of this chapter appear in Appendix 3, which is found at the end of the chapter.

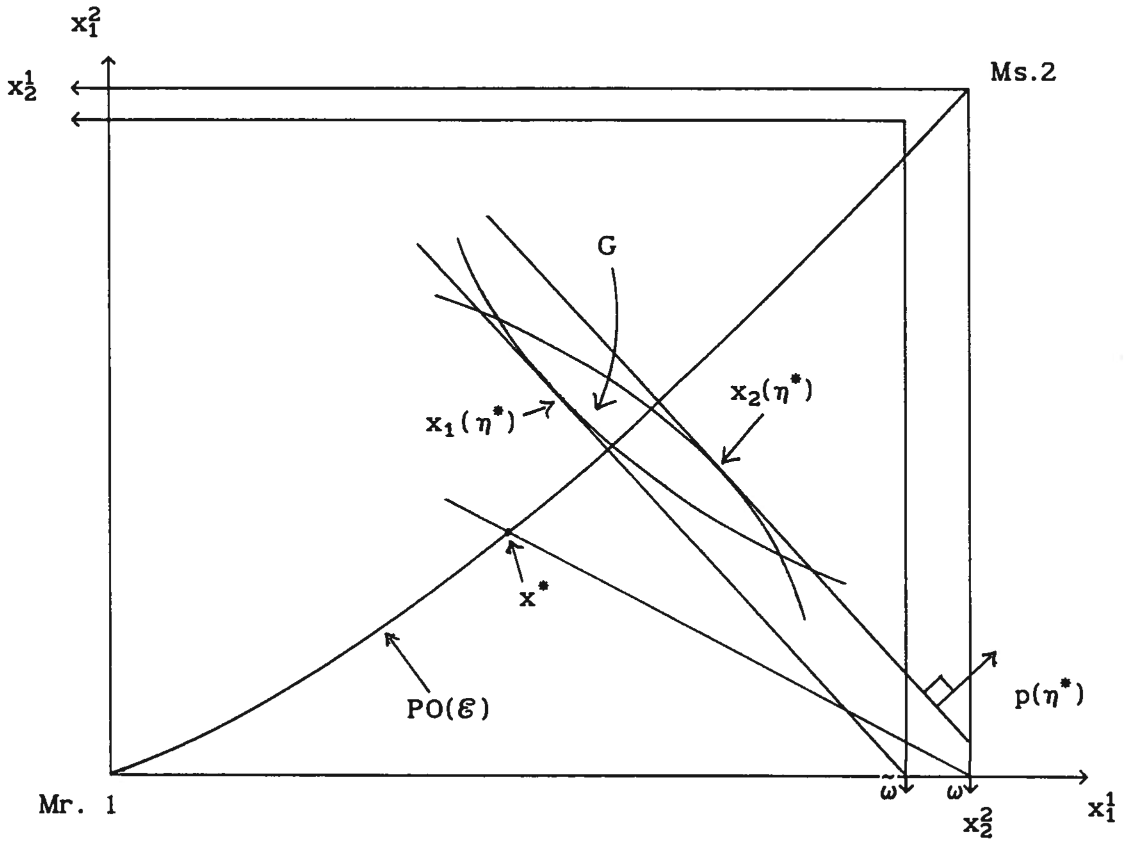

Figure 3-1. *The lobbying economy with the* **better than lobbying set** G.

Proposition 3.1. For the lobbying economy $\mathcal{E}$, if $(x_i(\eta^*),\eta_i^*)_{i=1,2} \in \text{SALE}(\mathcal{E})$, then $x(\eta^*)$ is not tilde-dominated.

It is now clear that we may safely abandon the hope of finding an allocation in $\tilde{F}$ which compares favorably in Pareto's sense with the lobbying equilibrium outcome. The following result makes equally clear the fact that the lobbying outcome $x(\eta^*)$ does not dominate the competitive outcome x^*. While x^* must be Pareto optimal in the non-distorted economy, the absence in the after-lobbying economy of a dominating outcome in our sense is less obvious.

Proposition 3.2. For the lobbying economy $\mathcal{E}$, suppose that $(x_i(\eta^*),\eta_i^*)_{i=1,2} \in \text{SALE}(\mathcal{E})$, and consider the competitive equilibrium outcome (x^*,P^*) in $\hat{\mathcal{E}}$. $x(\eta^*)$ does not dominate x^*. That is, if $x_i(\eta^*) \succsim_i x_i^*$ for some i, then $x_{-i}^* \succ_{-i} x_{-i}(\eta^*)$.

Let us now embark upon an investigation of the optimality of the chosen function $p(\eta)$. The next result offers conditions under which the lobbying outcome is inefficient in that the distortion-free competitive equilibrium x^* dominates $x(\eta^*)$. It establishes the set G as a useful instrument for assessing the efficiency properties of allocations in the economy.

Proposition 3.3. Consider a lobbying economy $\mathcal{E}$, with associated competitive equilibrium outcome (x^*,P^*) in the underlying competitive economy. x^* dominates $x(\eta^*)$ if and only if $x^* \in \text{int}(G)$.

More than anything, this proposition provides a convenient shorthand for the situation in which everyone is worse off at the lobbying outcome than at the competitive outcome. While $x^* \in \text{int}(G)$, one might argue that the government should simply have stayed out of things. The agents, behaving optimally, find themselves in a suboptimal prisoners' dilemma outcome.

An intuitive argument for the plausibility of such an outcome is easily constructed. Suppose that the two agents are approximately equal in their political power, and that their endowments and preferences are also approximately symmetric about the 45° line in the Edgeworth box. If the pricing function is very steep at the origin in both directions, then each trader knows that the first increment of lobbying donation will pay off a lot. Both will choose to lobby at some (perhaps small) positive level. If the pricing function $p(\eta)$ is also approximately symmetric, then we may have $p(\eta^*)$ close to p^* (see section 2.5 above); then it makes sense for both sides to lose. In the world, this represents two powerful political interest groups pitted against each other. It is the classic inefficient rent seeking situation.

It has been established that whenever $x^* \in \text{int}(G)$, $x(\eta^*)$ is suboptimal. Suppose now that $x^* \notin G$. We show that ranking x^* and $x(\eta^*)$ as potential social outcomes already involves, in this case, normative judgments about who should be better off. In short, if $x^* \notin G$, then proposing one of the two outcomes as better involves interpersonal comparisons of utility of the sort economics is usually reluctant to make.[5] The essence of this intuitive

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[5]See, for example, Harsanyi (1977) for a discussion of the ethical considerations required in assessing which is better for society. North (1984), in advocating a transactions cost approach to the measurement of economic performance, argues that the Pareto efficiency standard is an inappropriate criterion.

observation is formalized in

Proposition 3.4. Consider a lobbying economy $\mathcal{E}$, with associated competitive equilibrium outcome (x^*,P^*) in the underlying competitive economy. Suppose that $x^* \notin G$. Then there is an i in I such that $x_i(\eta^*) >_i x_i^*$.

Because we know that $x_i(\eta^*) \gtrsim_i x_i^*$ for some i implies that $x_{-i}^* >_{-i} x_{-i}(\eta^*)$, this result guarantees that if $x^* \notin G$, then one agent is better off, and the other is worse off, at the lobbying allocation than at the competitive allocation. Earlier, it was noted that $x^* \in \text{int}(G)$ implies that both agents would prefer to go back to the competitive equilibrium and ignore the lobbying game. If $x^* \notin G$, then one agent will be unwilling to do so. He or she is able to achieve at least $U_i(x_i(\eta^*))$ by lobbying at the level η_i^*; agent $-i$ in that instance can do no better than choosing η_{-i}^*. Perhaps the richest intuition suggestive of this outcome is the case of a powerful interest group, whose opposition in the political struggle is relatively weak, successfully pressing for a favorable price policy $p(\eta)$. The implications of the condition $x^* \in \text{int}(G)$ for cooperative outcomes is treated in the following section.

Attention may now be turned more directly to the lobbying equilibrium allocation $x(\eta^*)$. The relevance of x^* for welfare comparisons has been established, but the usefulness of such comparisons is exhausted for our purposes. Instead, we now turn to the relationship between the lobbying outcome $x(\eta^*)$ and arbitrary elements of the feasible set F. In the following proposition, we show that when $\text{int}(G) \neq \emptyset$, there are feasible allocations which dominate $x(\eta^*)$.

Proposition 3.5. Consider a lobbying economy $\mathcal{E}$. The lobbying allocation $x(\eta^*)$ is dominated if and only if $\text{int}(G) \neq \emptyset$.

Now, as was stated earlier, $x(\eta^*)$ is our benchmark or *status quo*. If $x^* \in \text{int}(G)$, it is easy to argue that the government should have stayed out of things. If the lobbying program were simply eradicated, leaving the economy at rest with x^*, all agents would be better off. By Proposition 3.5, more generally, if $\text{int}(G) \neq \emptyset$, then there are feasible allocations which make both agents better off than they were at $x(\eta^*)$. One might ask the question, Could the government ever achieve one of these outcomes by an alternative instrument choice?

In the next two propositions, we show that it could indeed. Proposition 3.6 establishes first that whenever the interior of G is non-empty, it contains an optimal allocation. This result is preliminary in nature. We will later show that any such optimal allocation may be achieved under an appropriate "non-distortionary" income transfer scheme.

Proposition 3.6. Consider a lobbying economy $\mathcal{E}$. Whenever $\text{int}(G) \neq \emptyset$, $\text{int}(G) \cap PO(\mathcal{E}) \neq \emptyset$. That is, if int(G) is non-empty, then it contains an optimal point.

Suppose now that the government is endowed with a new policy instrument for regulating the economy. It may now select the function $p(\eta) \equiv p^*$, announce this pricing function to agents,[33] and it may also impose an income

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[33]If the lobbying price identically equals p^* over H, and if the preferences of agents are strictly monotone, it is apparent intuitively (and also easily demonstrated mathematically) that no agent will ever choose $\eta_i > 0$.

transfer scheme on the economy. This comparison is purely hypothetical, for the economy's underlying institutional, political, and societal conditions are embodied in the function $p(\eta)$. The outcome $x(\eta^*)$ is the standard, but we wish to discover whether this allocation could be improved upon were it possible to ignore existing social norms.

A last definition is required. A competitive equilibrium has been defined as a pair of allocations and a price vector such that agents optimize under the resource constraint defined by the property rights system (ω_1, ω_2), and such that markets clear. A more general notion of equilibrium, to be employed here, is that of an equilibrium relative to a price system, in which only the aggregate endowment ω matters.

Definition. Take a competitive economy $\hat{\mathcal{E}} = (\succsim_i, \omega_i)_{i=1,2}$. An allocation $\hat{x} \in F$ is a *price equilibrium relative to the price* $\hat{P} \in \Delta$ if for every i in I, $y \in X_i$ and $y \succ_i \hat{x}_i$ together imply that $\hat{P} \cdot y > \hat{P} \cdot \hat{x}_i$ (preference maximization).

It shall be shown that every allocation $\hat{x}$ in $\text{int}(G) \cap \text{PO}(\mathcal{E})$ is a price equilibrium relative to some price. This result also establishes that for any such allocation there is available an income transfer scheme, depending on $\hat{P}$, under which $\hat{x}$ is supported at the equilibrium price.

Proposition 3.7. Consider a lobbying economy $\mathcal{E}$, and suppose that $\text{int}(G) \cap \text{PO}(\mathcal{E}) \neq \emptyset$. Then

i. Any $\hat{x} \in \text{int}(G) \cap \text{PO}(\mathcal{E})$ is an equilibrium relative to some price $\hat{P} \in \Delta$;

ii. The allocation $\hat{x}$ may be supported by an income transfer of

$\hat{P}\cdot(\hat{x}_i - \omega_i)$ to each agent i; and

iii. $\hat{x}$ dominates $x(\eta^*)$.

With this result, we have shown that the lobbying outcome is inefficient by the optimality criterion employed here whenever $\text{int}(G) \neq \emptyset$. The last result of this section involves the case with $G = \emptyset$ (see Figure 3-2). This possibility has not been shown to be impossible; neither has an example been discovered in which $G = \emptyset$. The question of whether this can ever occur is a topic for further research. For completeness, we record a result whose proof follows from Proposition 3.5.

Proposition 3.8. Consider a lobbying economy $\mathcal{E}$. If $\text{int}(G) = \emptyset$, then $x(\eta^*)$ is not dominated.

To conclude this section, we discuss briefly just what would have to be true of $\mathcal{E}$ in order for $G = \emptyset$ to obtain at $\eta^* \in \text{SALE}(\mathcal{E})$. Three characteristics of the economy seem important. First, all else equal, $G = \emptyset$ is more likely if the indifference curves "bend sharply" at $x(\eta^*)$. This corresponds to a relatively low substitution elasticity between x^1 and x^2 for both agents. Second, if a small lobbying contribution by both agents moves the price a great deal in one direction, then the chosen bundles $x_1(\eta^*)$ and $x_2(\eta^*)$ may be quite distant along the price line; this makes $G = \emptyset$ more likely. Finally, if a given movement in prices induces a relatively large shift in the ratio of x^1 to x^2 at the chosen bundles, then we are more likely than otherwise to find $G = \emptyset$ at η^*.

3.3 EDGEWORTH AND COOPERATION

In the previous section, we examined the efficiency properties of the

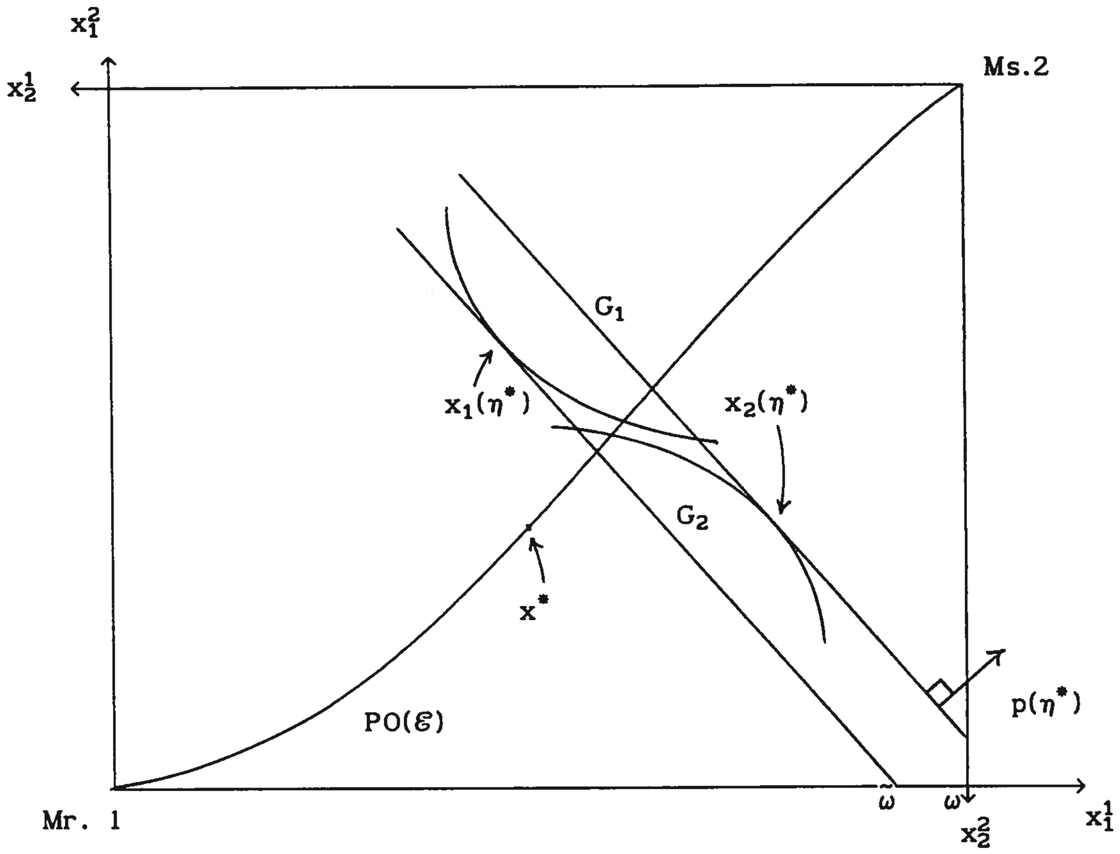

Figure 3-2. *The lobbying economy with* G = ∅.

lobbying equilibrium outcome $x(\eta^*)$. The approach there was to retain the assumption of no cooperation between agents, but to entertain notions of intervention which were unavailable in $\mathcal{E}$ as it was originally formulated. The payoff to this alteration was a collection of results providing insights into the question, Is lobbying activity "good or bad" for society?

This section also alters the original model of lobbying economies; once again the aim is to assess when or whether $x(\eta^*)$ will improve agents' welfare over x^*. In particular, the focus here is upon the effect of allowing agents to enter into binding agreements with one another. Indeed, the primary distinction between cooperative and noncooperative games is that such agreements are possible in the former, and not in the latter. Our objective is to first devise a framework within which such agreements are meaningful and well-defined; then we evaluate their potential for improving agents' welfare over $x(\eta^*)$.

The solution concept to be employed amounts to an *arbitration scheme* because it arises from a prearranged mechanism which lies outside the game itself. In this sense, the agents' opportunities for actively choosing among alternatives is quite limited. The Nash cooperative solution concept or the Nash fixed threat bargaining game, which we adopt, effectively precludes individual action. Once the game is fully specified, the Nash outcome is unique and inevitable from the viewpoint of agents inside the model. Fortunately, the analyst is still free to legitimately evaluate the results against various alternatives.

One of the two basic building blocks of a two-person Nash cooperative game is the set of attainable utility pairs $\mathbb{U} \subset \mathbb{R}^2_+$.[34] This set, to be referred to as the utility set, has a deep and fundamental connection to the allocation space and, in particular, to the feasible set F (see Figure 3-3).

Definition. Given a lobbying economy $\mathcal{E}$, the *utility set* $\mathbb{U} \subset \mathbb{R}^2_+$ is the set of utility vectors (U_1,U_2) achievable by feasible allocations. That is, $\mathbb{U} = \{(U_1,U_2) \in \mathbb{R}^2_+ : \text{For some } x \in F,\ U_i = U_i(x_i) \text{ for each } i \in I\}$.

An important feature of the bargaining game formulation employed here is that we must use von Neumann and Morgenstern expected utility functions. This implies, among other things, that while the solution concept for the cooperative game is invariant under affine transformations of the U_i, it is not so under arbitrary monotone transformations.

The set $\mathbb{U}$ is the image under the function $U = (U_1,U_2)$ of the set F of feasible allocations. Since the U_i are assumed concave for every agent i, $\mathbb{U}$ is a convex set.[35] Elements of $\mathbb{U}$ will be denoted variously, and without confusion, as U(x) or as $U(\eta)$, where in the latter case the intermediate variable $x_i(\eta)$ is understood to be the argument of U_i. Note that by definition $U(x) \geq U(y)$ if and only if $x_i \succsim_i y_i$ for every i. Let $\mathring{\mathbb{U}}$ denote the set of elements on the northeast boundary of $\mathbb{U}$. That is,

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[34]Being the image under a continuous function of the compact set F, $\mathbb{U}$ is also compact. As the utility functions representing preferences $\succsim_i$ are invariant to location shifts, we may take the range of U_i to be the non-negative portion of $\mathbb{R}$. In this case, $\min_{x \in F} U_i(x_i) = 0$.

[35]This fact is apparent when one notes that for any pair x, x′ of feasible allocations, for any $\lambda \in [0,1]$, $x'' = \lambda x + (1-\lambda)x'$, is also feasible by the convexity of F. Thus, $U(x'') \in \mathbb{U}$, and by the concavity of U_i, $U(x'') \geq \lambda U(x) + (1-\lambda)U(x')$, which implies that $\mathbb{U}$ is convex.

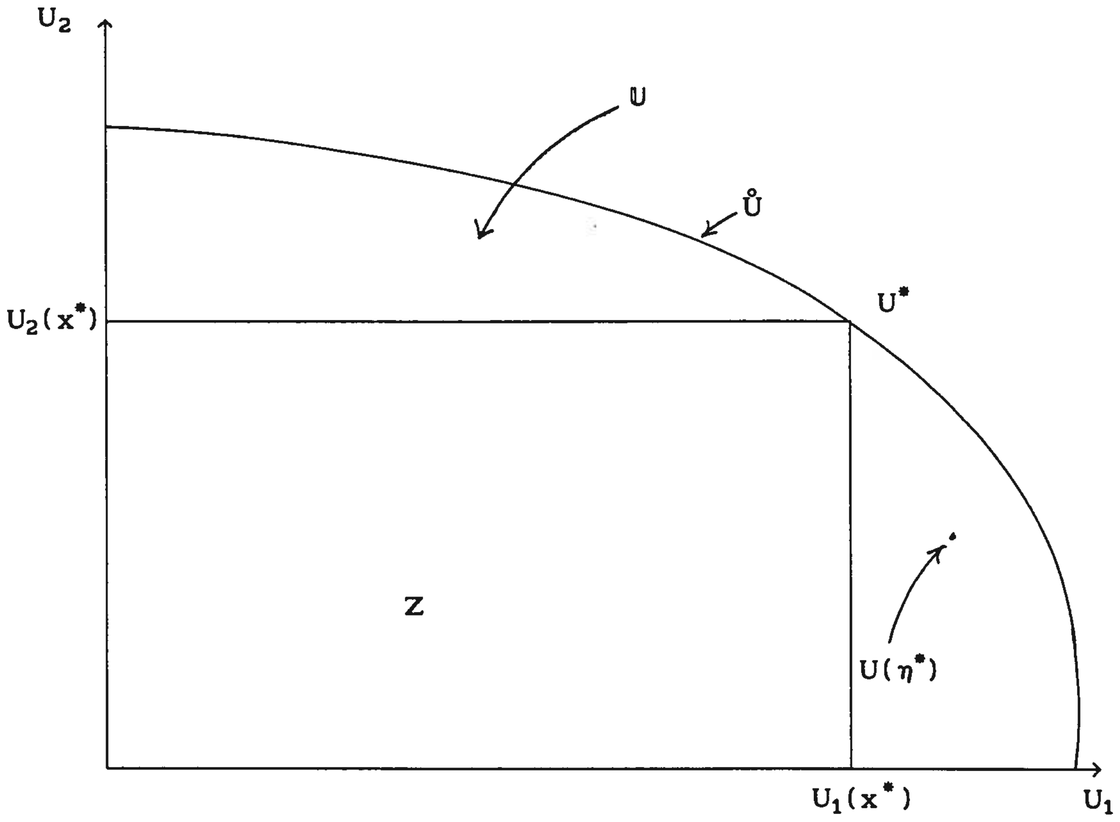

Figure 3-3. *The utility possibility set* $\mathbb{U}$.

$\mathring{U} = \{(U_1,U_2) \in \mathbb{U} : U' \geq U \text{ and } U' \neq U \text{ imply } U' \notin \mathbb{U}\}$. Clearly, elements of $\mathring{U}$ are images of optimal allocations in F. While the function U need *not* be one-to-one on all of F, it turns out that when preferences $\succsim_i$ are strictly convex and monotone on X_i, U is one-to-one on the set of optimal allocations. That is, the pre-image of an arbitrary $U \in \mathring{U}$ is unique in F.[36]

The required definition of a cooperative game may now be built up from the utility set $\mathbb{U}$. Let U^* denote the pair $U(x^*) \in \mathbb{U}$ which obtains at the competitive equilibrium allocation. This vector is optimal; thus, it is on the boundary of $\mathbb{U}$. Let $Z = \{y \in \mathbb{U} : y \leq U^*\}$ denote the set of elements of $\mathbb{U}$ less than U^*. Let $U(\eta^*)$ represent the utility pair arising from the equilibrium lobbying vector $\eta^* \in \mathbb{R}^2_+$. In addition to $\mathbb{U}$, the bargaining model consists of a utility pair which is designated as the *threat point*; these utility levels will fall to agents in the absence of an agreement. Note that either agent, finding the potential bargaining agreement unacceptable, may choose to lobby at the level η_i^*. In this case, his or her opponent can do no better than to lobby η_{-i}^*. Thus, we will always take the threat point to be $U(\eta^*)$. This assumption is made explicit in the following definition.

Definition. Given a lobbying economy $\mathcal{E}$, its corresponding *fixed threat bargaining game*, denoted $N_{\mathcal{E}}$, is given by the pair $N_{\mathcal{E}} = (\mathbb{U}, U(\eta^*))$.

As a solution to the game $N_{\mathcal{E}}$, we seek a unique element $\hat{U}$ of $\mathbb{U}$ which is supportable as a reasonable outcome of the bargaining process. While other

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[36] In fact, the connection between $\mathring{U}$ and $PO(\mathcal{E})$ is stronger than this. When preferences are strictly convex and monotone (which together imply that they are strictly monotone), U is a bijection on $PO(\mathcal{E})$ (see, e.g., Mas-Colell (1985, p. 155, Proposition 4.6.2)). We are not interested in the fact that U is onto $\mathring{U}$ over $PO(\mathcal{E})$.

choices are available, the Nash solution will be adopted here. The Nash cooperative solution to $N_{\mathcal{E}}$ is defined by the following conditions.

(C1) $\hat{U} \geq U(\eta^*)$ for every player i;

(C2) If $N' = (\mathbb{U}', d')$ is related to $N_{\mathcal{E}}$ by $\mathbb{U}' = \{y \in \mathbb{R}_+ : x_i = a_i z_i + b_i, i = 1,2; y \in \mathbb{U}\}$ and $d_i' = a_i U_i(\eta^*) + b_i$ for every i, where $a_i \in \mathbb{R}_{++}$, $b_i \in \mathbb{R}$, then $\hat{U}_i' = a_i U_i(\eta^*) + b_i$, $i = 1,2$;

(C3) If, for N', $U_1(\eta^*) = U_2(\eta^*)$ and $(x_1,x_2) \in \mathbb{U}$ whenever $(x_2,x_1) \in \mathbb{U}$, then $\hat{U}' = \hat{U}$; and

(C4) If, for N', $d' = U(\eta^*)$ and $\hat{U}' \in \mathbb{U}$, then $\hat{U}' = \hat{U}$.

In short, the Nash solution to a game $N_{\mathcal{E}}$ may be characterized as follows. It selects the unique element of $\mathbb{U}$ which maximizes the product of gains from agreement $(U_1 - U_1(\eta^*))\cdot(U_2 - U_2(\eta^*))$. If the point $U(\eta^*)$ is regarded as the origin of a translated coordinate system in $\mathbb{R}^2$, then $\hat{U}$ will be the point on the boundary of $\mathbb{U}$ which is tangent to the highest rectangular hyperbola touching $\mathbb{U}$ (see Figure 3-4).

This brief overview of bargaining games is sufficient for the present discussion. In what follows, the possibility for improvement over $x(\eta^*)$ by cooperation is investigated in the context of $N_{\mathcal{E}}$. Much of the remainder of the section is linked closely to results of section 3.2. Where this is the case, we leave the intuitive justification alone and refer to the earlier discussion.

To begin, suppose that x^*, the competitive equilibrium allocation pair, is in the interior of the set G. Earlier, this outcome was interpreted as one which agents would look back upon with some disappointment. If $x^* \in int(G)$, then both agents prefer x^* to $x(\eta^*)$. Proposition 3.9 shows

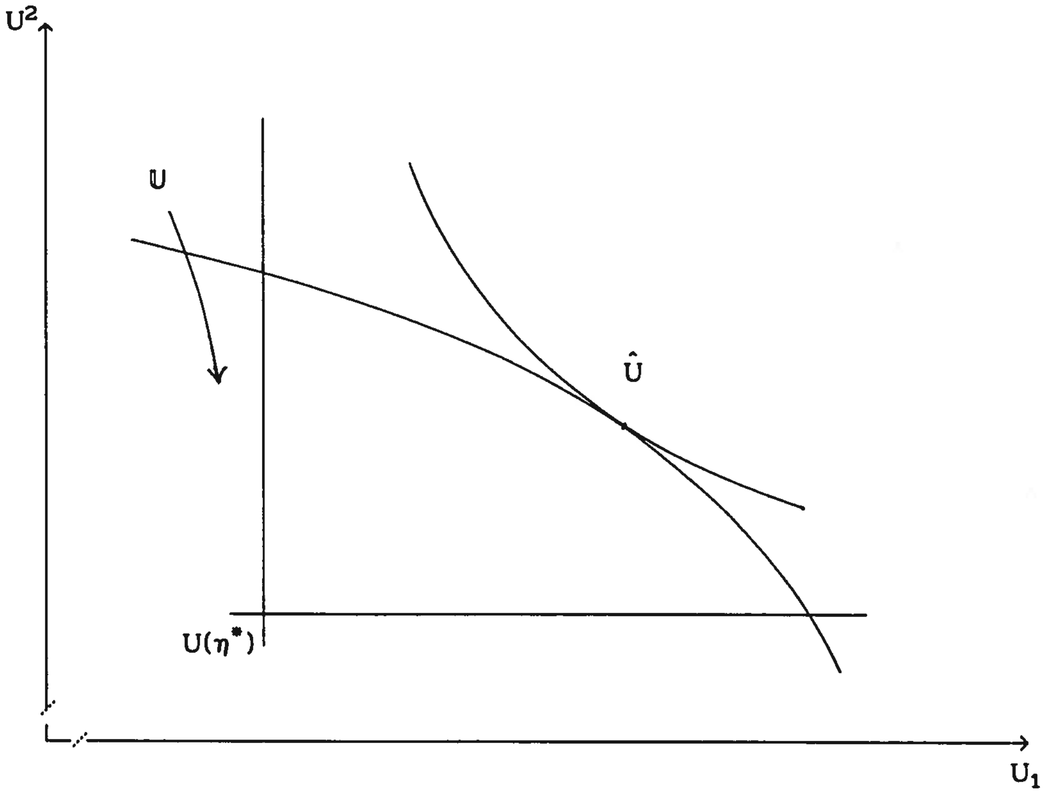

Figure 3-4. *The cooperative solution* $\hat{U}$.

that in such an economy $\mathcal{E}$, a bargaining agreement will also be available in which both players achieve allocations which they prefer to the lobbying outcome.

Proposition 3.9. Consider a lobbying economy $\mathcal{E}$. The following statements are true:

i.) $U(\eta^*) \in Z$ if and only if $x^* \in G$; and

ii.) $U_i(\eta^*) \in \mathbb{U}\backslash\mathring{U}$ for each i whenever $x^* \in int(G)$.

Now, following the discussion of Propositions 3.6 and 3.7, suppose that $G \neq \emptyset$, but that $x^* \notin G$. In this case, for one player i, $x_i(\eta^*) >_i x_i^*$, which implies that $x_{-i}^* >_{-i} x_{-i}(\eta^*)$. That is, the lobbying outcome leaves one agent better and the other worse off than at x^*. In the utility set $\mathbb{U}$, this corresponds to pairs U which are not comparable by the optimality criterion to U^*.

Proposition 3.10. Consider a lobbying economy $\mathcal{E}$. We have that $[x^* \notin G$ and $G \neq \emptyset]$ if and only if $U(\eta^*) \in \mathbb{U}\backslash Z$.

A point has now been reached where the bargaining game outcome may be brought into the discussion. The objective in this section has been to show how the possibility of cooperation might change things. When would agents prefer to collude, trade goods with each other, and thereby ignore or overrule the lobbying rule proposed by the government? The following result provides a link between this section and the previous one, establishing that the Nash bargaining outcome improves upon or dominates the lobbying outcome $x(\eta^*)$ whenever G has non-empty interior.

Proposition 3.11. Consider a lobbying economy $\mathcal{E}$. If $int(G) \neq \emptyset$, then

$\hat{U}_i > U_i(\eta^*)$ for each i.

Except for Proposition 3.6, each of the main results in section 3.2 has now been given an analog for the cooperative formulation. We have seen that the bargaining outcome will make both better off than $x(\eta^*)$ whenever $\text{int}(G) \neq \emptyset$, and that if $x^* \in G$, U^* and $\hat{U}$ both improve upon $U(\eta^*)$. However, what are the implications for bargaining outcomes of the condition $G = \emptyset$? Whenever $G = \emptyset$, the utility pair $U(\eta^*)$ was not attainable in the competitive economy; it lies outside of $\mathbb{U}$. How may this outcome be interpreted for the bargaining game and the $\mathbb{U}$ set? A fundamental agreement between the cooperative approach and the alternative pricing mechanism approach of the last section is established in

Proposition 3.12. Consider a lobbying economy $\mathcal{E}$. Suppose that for some i, $L_i(x_i(\eta^*)) \cap F \neq \emptyset$. If and only if $G = \emptyset$, then $U(\eta^*) \notin \mathbb{U}$.

Thus, precisely when there is no opportunity for a transfer scheme to improve upon the lobbying outcome, agents cannot reach a cooperative agreement which both prefer to the lobbying outcome. While a game in which $U(\eta^*) \notin \mathbb{U}$ is not, strictly speaking, a Nash fixed threat bargaining game, it does have a place in the cooperative game theory literature. Harsanyi (1977) calls a game with threat point outside of $\mathbb{U}$ a *negative embedded bargaining game*. Agents will never agree to cooperate in this instance, and the concept is usually reserved for the analysis of multi-play or repeated cooperative games. Here it is also interesting in a much different way. It turns out that when $G = \emptyset$, the *lobbying core*, suitably defined, is also empty, so that no opportunity exists for coalition formation or recontracting to improve matters for agents.

We now turn our attention to this analysis. The core of an economy consists of all allocations which are rational for agents and for groups, in the sense that no coalition of any size may assure itself of more, acting alone, than it is given at the core allocation. Put another way, no coalition can unilaterally adopt an alternative strategy that is better for all of its members. Edgeworth, in 1881, proposed that an equilibrium for exchange economies may be achieved through unrestricted trade between agents and groups rather than through market transactions. In Edgeworth's formulation of economic equilibrium, any collection of traders may agree to redistribute its collective endowment among its members. An equilibrium for Edgeworth, then, is any set of trades which delivers to each trader at least as much utility as he or she would achieve by consuming his or her endowment, and to each possible coalition at least as much as it could achieve by trades only among its own members.

A thorough treatment of the theory of the core of an economy and a review of the related literature may be found in Hildenbrand (1982). In this thesis, only two agents populate the economy. Opportunities for coalition formation in this case are quite limited; the formal definition of the core is quite easily formulated as a result.

Definition. For a lobbying economy $\mathcal{E} = ((\succsim_i, \omega_i)_{i=1,2};\ p(\eta))$, with $\eta^* \in \text{SALE}(\mathcal{E})$, the allocation $x \in X_1 \times X_2$ is *individually rational for agent i* if $x_i \succsim_i \omega_i$. x is *individually η^*-rational for agent i* if $x_i \succsim_i x_i(\eta^*)$.

Definition. For a lobbying economy $\mathcal{E} = ((\succsim_i, \omega_i)_{i=1,2};\ p(\eta))$, with $\eta^* \in \text{SALE}(\mathcal{E})$, the *lobbying core*, denoted $\text{LC}(\mathcal{E})$, is the set of allocations which are optimal and which are individually η^*-rational for each agent i.

Implicit in this definition is the assumption that perfect information is available to agents and that transactions costs are zero. The core does not rely upon a specification of how agents find each other; the *process* of transactions is not spelled out. Edgeworth's concept of the contract curve and recontracting do not address the means of transaction either. If there are many outcomes in the core, the theory is indeterminate on the trading outcome. In this sense, the cooperative Nash solution had some advantages in that it did specify a particular outcome $\hat{U}$ (See Harsanyi (1977), p. 142).

Our objective here is to show that the lobbying core is empty (so that no possible improving exchange from $x(\eta^*)$ is possible) precisely when the set G is empty. The core, it must be noted, is a concept entirely free of prices. Thus, the main result here provides a fundamental link between the possibility for competing political groups to reach an agreement, and for government to achieve an outcome by transfers, even in an ideal world, which would be improving for society in our sense.

The lobbying core is depicted in two ways in Figures 3-5a and 3-5b. In 3.5a, the core is all allocations on the intersection of $PO(\mathcal{E})$ and G. In 3.5b, the corresponding core utility pairs are seen to be those which lie on the northeast boundary of $\mathbb{U}$ and also northeast of $U(\eta^*)$. As mentioned earlier, the correspondence between these two sets is well-defined. In particular, it may be shown that the pre-image under the function $U(x) = (U_1(x_1), U_2(x_2))$ of such utility pairs is single-valued. Without providing a proof, we note here that $LC(\mathcal{E}) \neq \emptyset$ whenever $U(\eta^*) \in \mathbb{U}$. In particular, if $U(\eta^*) \in int(\mathbb{U})$, then $\hat{U} \in LC(\mathcal{E})$. These two results are

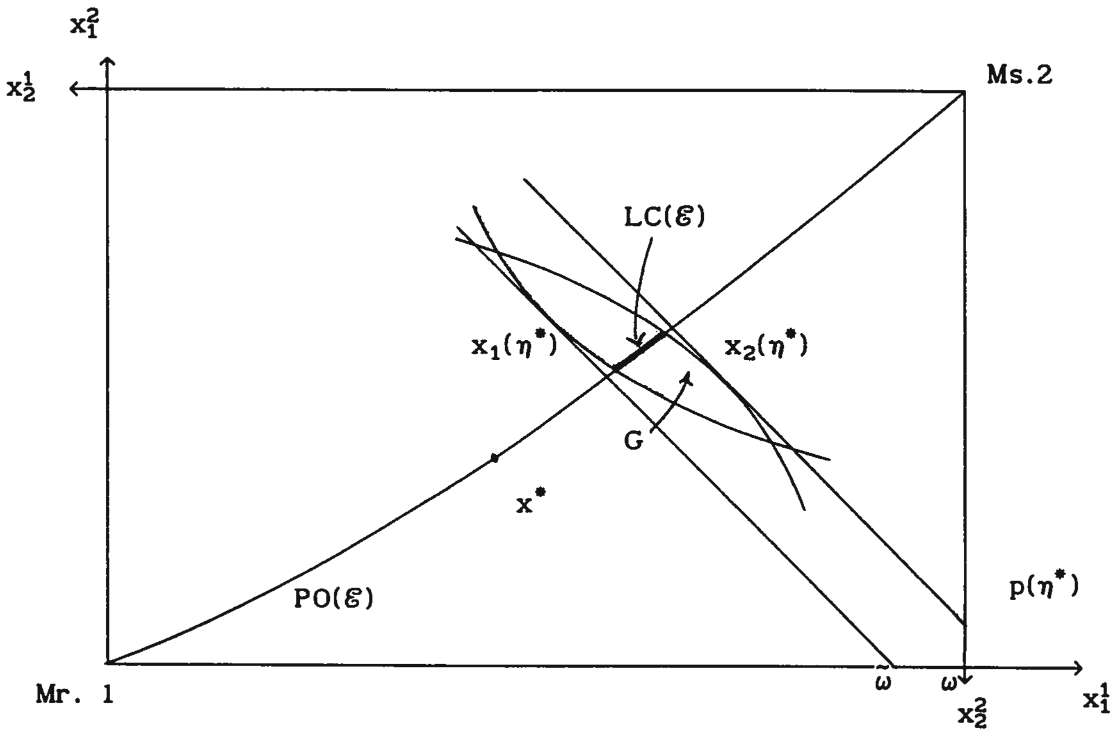

Figure 3-5a. *The lobbying core in commodity space.*

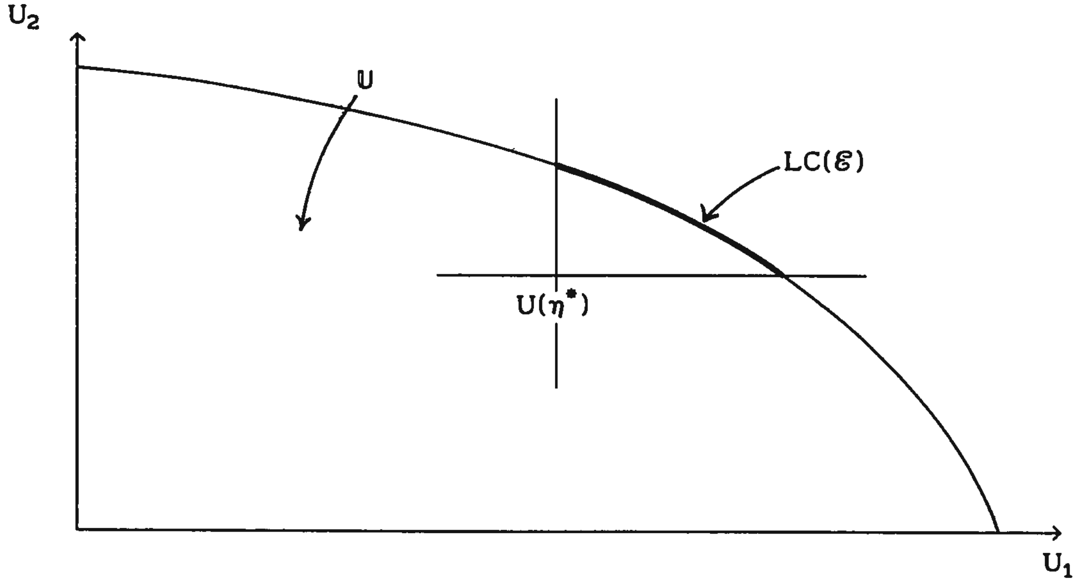

Figure 3-5b. *The lobbying core in utility space.*

immediate from the following proposition, which is our primary result upon the lobbying core.

Proposition 3.13. Consider a lobbying economy $\mathcal{E}$. $LC(\mathcal{E}) = \emptyset$ if and only if $G = \emptyset$.

Because this result provides a necessary and sufficient condition for $LC(\mathcal{E}) = \emptyset$, we may conclude directly from it that if $G \neq \emptyset$, then $LC(\mathcal{E}) \neq \emptyset$. From Proposition 3.7, we know that any $x \in int(G) \cap PO(\mathcal{E})$ may be achieved, when the proper price system prevails, by lump-sum income transfers between agents. The flip side of this notion may be summarized as follows. Given a lobbying economy $\mathcal{E}$, whenever the lobbying outcome delivers a utility pair which was unavailable in the competitive economy, two equivalent conditions hold. First, the individuals in the economy, our agents, will not be able to collude or agree to override the lobbying game. One agent will resist any campaign by his or her opponent to join such a coalition. Second, there will be no transfer of income by which government could achieve for agents, through a price mechanism, an outcome which society prefers in any sense to $x(\eta^*)$.

3.4 CONCLUSIONS

This chapter has attempted to shed some light on the question of the efficiency properties of lobbying behavior, using the lobbying model of chapter two to analyze the welfare implications of a generic lobbying institution. We have provided some evidence that lobbying behavior, even in nicely formulated economies, may not be unequivocally suboptimal. Moreover, rent seeking behavior in small economies which may trade with a larger world economy might be good for society in the sense that utility levels after lobbying are unachievable by a corresponding perfectly competitive economy.

In the next chapter, functional forms are chosen for the utility functions and the pricing function of the model. Using this mathematical representation of the economy, equilibria are calculated numerically for a large number of examples, and these equilibria are evaluated for their welfare properties. The exercise reported there provides insights into the analytical results of this chapter.

APPENDIX 3

This appendix contains the proofs of the propositions stated in this chapter.

Proof of Proposition 3.1.

Take such a $(x_i(\eta^*),\eta_i^*)_{i=1,2} \in \mathrm{SALE}(\mathcal{E})$. By preference maximization, for every $y \in \mathrm{int}(G_1)$, we claim that $P(\eta^*)\cdot y > P(\eta^*)\cdot x_1(\eta^*)$. To see this, suppose not: There exists $z \in \mathrm{int}(G_1)$ with $P(\eta^*)\cdot z \leq P(\eta^*)\cdot x_1(\eta^*)$. Then by monotonicity and continuity of preferences, there is an $\varepsilon > 0$ sufficiently small so that $z' = (z - (\varepsilon,\varepsilon)) \in \mathrm{int}(G_1)$. But then $P(\eta^*)\cdot z' < P(\eta^*)\cdot z \leq P(\eta^*)\cdot x_1(\eta^*)$. Although $z' >_1 x_1(\eta^*)$, z' was available when $x_1(\eta^*)$ chosen, violating the preference maximization of $x_1(\eta^*)$, and establishing the claim. Similarly, for every $y \in \mathrm{int}(L_2(x_2(\eta^*)))$, $P(\eta^*)\cdot y > P(\eta^*)\cdot x_2(\eta^*)$. Thus, by the definition of $\tilde{G}_2$, for every $y \in \mathrm{int}(\tilde{G}_2)$, $P(\eta^*)\cdot y < P(\eta^*)\cdot x_1(\eta^*)$. It follows immediately that

$$\mathrm{int}(G_1) \cap \mathrm{int}(\tilde{G}_2) = \emptyset. \quad \text{(A3.1)}$$

Now, suppose that there exists $x^o = (x_1^o,x_2^o)$ which dominates $(x_1(\eta^*),x_2(\eta^*))$. We derive a contradiction to (A3.1). By the strict convexity of preferences and the definition, $x_1^o \in \mathrm{int}(G_1)$. Also, $x_2^o \in \mathrm{int}(L_2(x_2(\eta^*)))$, and by the definition of $\tilde{G}_2$, $x_1^o \leq \tilde{\omega} - x_2^o$, so that $x_1^o \in \mathrm{int}(\tilde{G}_2)$. Finally, $x_1^o \in \mathrm{int}(G_1) \cap \mathrm{int}(\tilde{G}_2)$, contradicting (A3.1). We conclude that $x(\eta^*)$ is not tilde-dominated, which completes the proof of Proposition 3.1. ■

Proof of Proposition 3.2.

The proof is carried out for $i = 1$; the case $i = 2$ is largely the same.

We proceed in two steps. First, it is shown that $p(\eta^*) > p^*$ whenever $x_1(\eta^*) \succsim_1 x_1^*$. Then we show it follows that $x_2^* \succ_2 x_2(\eta^*)$.

i) Suppose that $x_1(\eta^*) \succsim_1 x_1^*$. We have that

$$P^* \cdot x_1(\eta^*) \geq P^* \cdot x^* = P^* \cdot \omega_1 > P^* \cdot \tilde{\omega}_1. \tag{A3.2}$$

The first inequality follows from preference maximization and the last from the definition of $\tilde{\omega}_1$ and the assumption $\eta_1^* > 0$. The equality follows from monotonicity of preferences. Monotonicity also implies that

$$P(\eta^*) \cdot x_1(\eta^*) = P(\eta^*) \cdot \tilde{\omega}_1 \tag{A3.3}$$

Subtracting (A3.3) from (A3.2) and rearranging,

$$(P^* - P(\eta^*)) \cdot x_1(\eta^*) > (P^* - P(\eta^*)) \cdot \tilde{\omega}_1,$$

which may be rewritten as

$$(P^* - P(\eta^*)) \cdot (x_1(\eta^*) - \tilde{\omega}_1) > 0. \tag{A3.4}$$

The second vector in this inner product has first element $x_1^1(\eta^*) - \tilde{\omega}_1^1 \leq 0$; its second element is $x_1^2(\eta^*) - 0 > 0$. Thus, the first element of $P^* - P(\eta^*)$, namely $p^* - p(\eta^*)$, must be strictly negative; otherwise we would have $p^* - p(\eta^*) \geq 0$, which would violate (A3.4). We conclude $p(\eta)^* > p^*$, which was to be shown.

ii) Suppose now that $p(\eta)^* > p^*$. We must show that $x_2^* \succ_2 x_2(\eta^*)$. Let the unique scalar c be such that $P^* \cdot x_2(\eta^*) = c(1-p^*)$. Because $p(\eta^*) > p^*$, we know that $c(1-p^*) < P^* \cdot \tilde{\omega}_2$. Thus, we have

$$P^* \cdot x_2(\eta^*) = c(1-p^*) < P^* \cdot \tilde{\omega}_2 < P^* \cdot \omega_2 = P^* \cdot x_2^*. \tag{A3.5}$$

The last equality holds by monotonicity, while the last inequality is due to $\eta_2^* > 0$. From eqn. (A3.5), $x_2(\eta^*)$ was available at the price vector P^*, when x_2^* was chosen. By strict convexity of preferences, then, $x_2^* \succ_2 x_2(\eta^*)$. This completes the proof of Proposition 3.2. ∎

Proof of Proposition 3.3.

To show necessity, suppose that x^* dominates $x(\eta^*)$. It follows immediately that $x_1^* >_1 x_1(\eta^*)$; thus $x_1^* \in \text{int}(G_1)$. What's more, $x_2^* \in L(x_2(\eta^*))$, so that $\omega - x_2^* \in \text{int}(G_2)$. Thus, $x^* \in \text{int}(G)$. Sufficiency follows from the definition of G. ■

Proof of Proposition 3.4.

Suppose, to the contrary, that $x_i^* \gtrsim_i x(\eta^*)$ for every i. By the definition, this implies that $x^* \in G$, contradicting the hypotheses of the proposition, and completing its proof. ■

Proof of Proposition 3.5.

To show sufficiency, suppose that x^o dominates $x(\eta^*)$. Then $x_1^o >_1 x_1(\eta^*)$, which implies $x_1^o \in \text{int}(G_1)$. Similarly, $x_2^o \in \text{int}(L_2(x_2(\eta^*)))$. Since x^o was assumed feasible, $x_1^o = \omega - x_2^o$. It follows immediately by this construction that $x_1^o \in \text{int}(G_2)$. Thus, $x_1^o \in \text{int}(G)$. Necessity follows as in Proposition 3.3 above if we take an element z of int(G). This completes the proof of Proposition 3.5. ■

Proof of Proposition 3.6.

Suppose, under the hypotheses of the proposition, that $z \in \text{int}(G)$. Let $G_2^z = (\omega - L_2(\omega - z)) \cap \mathbb{R}_+^2$. By continuity of preferences, G_2^z is closed; it is also convex. G_2^z is contained in the closed ball in $\mathbb{R}^2$ given by $\mathbb{B}(0,r) = \{x \in \mathbb{R}^2 : \|x\| \leq r\}$ with $r = \|\omega\|$. Thus, G_2^z is bounded, and also compact. Since it is continuous, by the Weierstrass theorem the function U_1 achieves a maximum x_1^z on the compact, convex set G_2^z. That $x_1^z \in \text{bd}(G_2^z)$ follows directly from the monotonicity of U_1. We claim that x_1^z is also unique. To see this, suppose to the contrary that there is an $x_1^o \in G_2^z$, $x_1^o \neq x_1^z$ with $U_1(x_1^o) \geq U_1(x_1^z)$. Then for $\lambda \in [0,1]$, $x^\lambda = \lambda \cdot x_1^o + (1-\lambda) \cdot x_1^z \in G_2^z$. But by the strict quasiconcavity of U_1, $U_1(x^\lambda) > U_1(x_1^z)$, violating that x_1^z maximizes U_1 on G_2^z. Thus, x_1^z is unique; by construction $x_1^z \succsim_1 y$ for every $y \in G_2^z$. Let $x_2^z = \omega - x_1^z$, and let $x^z = (x_1^z, x_2^z)$. Also by construction, $x^z \in PO(\mathcal{E})$.

It remains only to show that $x^z \in int(G)$. We have, first, that $x_2^z \sim_2 (\omega - z) \succ_2 x_2(\eta^*)$, so that $x^z \in int(G_2)$. Further, as x_1^z maximizes $\succsim_1$ on G_2^z, and since by definition $x_1^z + x_2^z = \omega$, $x_1^z \succsim_1 z \succ_1 x_1(\eta^*)$ since $z \in int(G_1)$. Thus, $x^z \in int(G)$, as was to be shown. This completes the proof of Proposition 3.6. ∎

Proof of Proposition 3.7.

In carrying out the proof of this result, we will need an additional equilibrium definition. If one $\hat{P}^j$ is allowed to equal zero, then the allocation $\hat{x}$ may not maximize preferences for all agents. A weaker equilibrium condition allows zero prices, requiring that agents who are not maximizing preferences are at least minimizing expenditures at $\hat{x}$.

Definition. Take a competitive economy $\hat{\mathcal{E}} = (\succsim_i, \omega_i)_{i=1,2}$. An allocation $\hat{x} \in F$ is a *price quasi-equilibrium relative to the price* $\hat{P} \in \Delta$ if for every i in I, $y \in X_i$ and $y \succsim_i \hat{x}_i$ together imply that $\hat{P} \cdot y \geq \hat{P} \cdot \hat{x}_i$. (expenditure minimization).

If there is a good j with $\hat{P}^j = 0$, then there may be one or more agents who are not maximizing preferences at $\hat{x}$. Because we wish to promote the price $\hat{P}$ as a plausible alternative policy instrument, it must be shown that for any

$\hat{x} \in \text{int}(G) \cap PO(\mathcal{E})$, every agent is maximizing preferences under $\hat{P}$. Let us now proceed to prove the proposition.

i.) The first part of the proposition will require showing (a) that $\hat{x}$ is a price quasi-equilibrium with respect to $\hat{P}$; then (b) that $\hat{P} \in \mathbb{R}^2_{++}$; and finally (c) that $\hat{x}$ is thus an equilibrium relative to $\hat{P}$.

i.) a. Let $W = L_1(\hat{x}_1) + L_2(\hat{x}_2)$. Clearly W, being the sum of convex sets, is convex in $\mathbb{R}^2$. We have that $\Sigma_{i=1}^2 \hat{x}_i = \omega \in W \cap F$ (see Figure A3-1). Also, $W \cap \text{int}(F) = \emptyset$, since otherwise there is a y with $y_i \succ_i x_i$ for all i in I by the monotonicity of $\succsim_i$, violating that $\hat{x} \in PO(\mathcal{E})$. By the separating hyperplane theorem, there is $\hat{P} \in \mathbb{R}^2$ with $\hat{P} \neq 0$ such that $\hat{P} \cdot (\Sigma_i \hat{x}_i) \geq \hat{P} \cdot z$ for every $z \in F$, and such that $\hat{P} \cdot z \geq \hat{P} \cdot (\Sigma \hat{x}_i)$ for every $z \in W$. Clearly, $\hat{P} \geq 0$, since for any $z \in F$, $\omega \geq z$. Now, suppose that there is x^o with $x_i^o \succsim_i \hat{x}_i$ for some agent i. Then $x_i^o + \hat{x}_{-i} \in W$, which implies that $\hat{P} \cdot x_i^o \geq \hat{P} \cdot \hat{x}_i$ (expenditure minimization). Thus, as i was arbitrary, $\hat{x}$ is a price quasi-equilibrium.

i.) b. Note that we have assumed $\omega \in \mathbb{R}^2_{++}$. Since $\hat{P} \neq 0$ and by the monotonicity of preferences, $\hat{P} \cdot (\Sigma_i \hat{x}_i) = \hat{P} \cdot \omega > 0$. Thus, there is an i such that $\hat{P} \cdot \hat{x}_i > 0$. We claim that for this i, $\hat{P} \cdot x_i^o > \hat{P} \cdot \hat{x}_i$ whenever $x_i^o \succ_i \hat{x}_i$. To see this, take such an x_i^o, and note that $(1-\varepsilon) \cdot x_i^o \succsim_i \hat{x}_i$ for ε sufficiently small. Since $\hat{x}$ is a price quasi-equilibrium, $(1-\varepsilon) \cdot \hat{P} \cdot x_i^o \geq \hat{P} \cdot \hat{x}_i > 0$. Thus, $\hat{P} \cdot x_i^o > 0$, from which $\hat{P} \cdot x_i^o > (1-\varepsilon) \cdot \hat{P} \cdot \hat{x}_i \geq \hat{P} \cdot \hat{x}_i$, which establishes the claim. For a good j, define y by $y^{-j} = \hat{x}_i^{-j}$ and $y^j = \hat{x}_i^j + 1$. Then $y \succ_i \hat{x}_i$ by strict monotonicity, from which it follows that $\hat{P} \cdot y > \hat{P} \cdot \hat{x}_i$. This last expression yields $\hat{P}^j > 0$, and because j was arbitrary we have that $\hat{P} \in \mathbb{R}^2_{++}$.

i.) c. It remains to show that the price quasi-equilibrium $\hat{x}$ is a

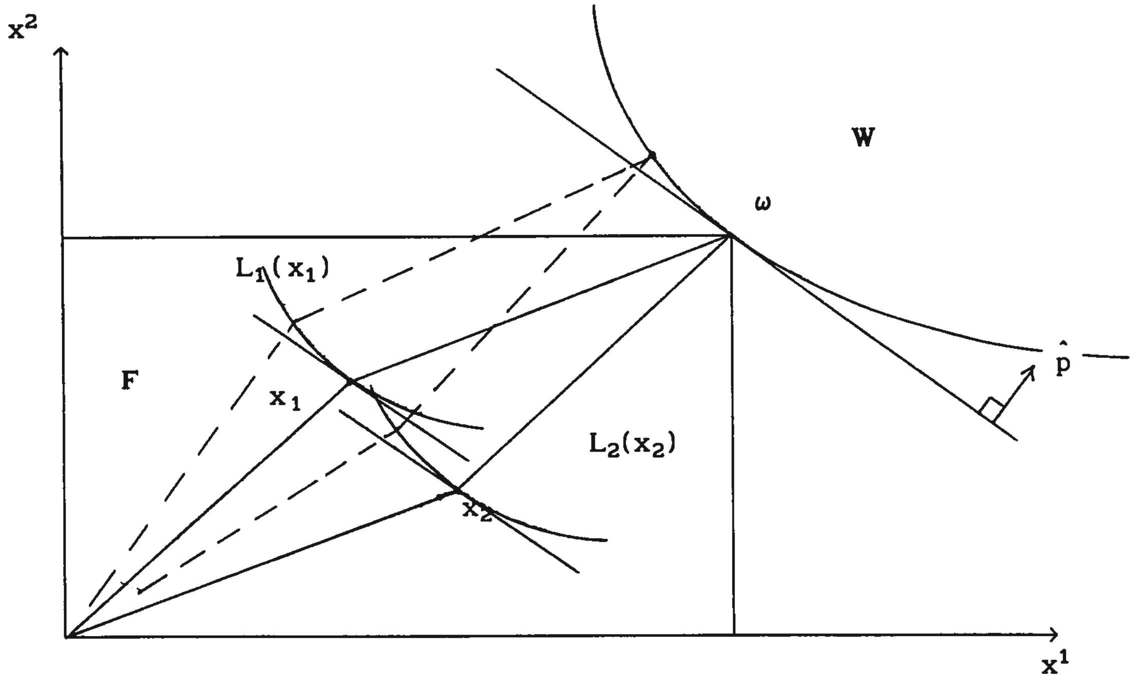

Figure A3-1. *Existence of an equilibrium relative to a price.*

price equilibrium relative to $\hat{P}$ whenever $\hat{P} \in \mathbb{R}^2_{++}$. Take such a $\hat{P}$, and suppose that $\hat{x}_i \neq 0$ for agent i. By the argument of i.) b. above, agent i is maximizing preferences, since $\hat{P} \cdot \hat{x} > 0$. If $\hat{x}_i = 0$, then agent i obviously maximizes preferences at $\hat{x}_i$. Finally, at $\hat{P} \in \mathbb{R}^2_{++}$, each agent is maximizing preferences at $\hat{x}_i$, so that the definition of a price equilibrium relative to $\hat{P}$ is satisfied by the allocation $\hat{x}$.

ii.) It must be shown that if each agent i receives transfer $t_i = \hat{P} \cdot (\hat{x}_i - \omega_i)$ (where $t_i < 0$ simply implies that i pays a tax), then $\hat{x}_i$ is supported as a price equilibrium relative to $\hat{P}$. Agent i holds goods, before the transfer, of value $\hat{P} \cdot \omega_i$. Immediately, we see that $\hat{P} \cdot \omega_i + t_i = \hat{P} \cdot \hat{x}_i$, which is the condition required.

iii.) That $\hat{x} \in \text{int}(G)$ dominates $x(\eta^*)$ follows directly from the definition of G. This completes the proof of Proposition 3.7. ∎

Proof of Proposition 3.8.

See proof of Proposition 3.5 above. ∎

Proof of Proposition 3.9.

i.) Under the hypotheses of the proposition, take $x^* \in G$. Then by the definition of G, $U_i(x_i^*) \geq U_i(x_i(\eta^*))$ for every i. To show sufficiency, suppose that $U(\eta^*) \in Z$. By the definitions of Z and of $\mathbb{U}$, there is $(x_1,x_2) \in F$ with $U_i(x_i) = U_i{}^*\eta^*)$. Since $U_i(\eta^*) \leq U_i^*$, we have that $x_i^* \succsim_i x_i(\eta^*)$ for each i. Thus, $x_1^* \in G_1$, and $x_2^* \in G_2$. Finally, it follows that $x^* \in G$.

ii.) Suppose that $x^* \in \text{int}(G)$. Then $x_i^* \succ_i x_i(\eta^*)$ for every i in I; from this it follows that, in particular, $U_i(x_i^*) > U_i(\eta^*)$ for every i, so that $U(\eta^*) \notin \mathring{U}$. Since $x^* \in F$, we know that $U(x^*) \in \mathbb{U}$. This completes the proof of Proposition 3.9. ∎

Proof of Proposition 3.10.

(Sufficiency). Suppose that $G \neq \emptyset$, and that $x^* \notin G$. By Proposition 3.6, there is an i with $U_i(x_i(\eta^*)) > U_i^*$, from which we conclude $U(\eta^*) \notin Z$. In showing that $U(\eta^*) \in \mathbb{U}$, two cases must be considered. If $\text{int}(G) = \emptyset$, then $G = \{x(\eta^*)\}$ is a singleton set. In this case, $x(\eta^*)$ is in F; therefore $U(\eta^*) \in \mathbb{U}$. If $\text{int}(G) \neq \emptyset$, take an arbitrary $x^o \in \text{int}(G)$. By definition, $x^o \in F$, so that $U(x^o) \in \mathbb{U}$. Since for each i, $x_i^o \succsim_i x_i(\eta^*)$, $U(x_i^o) \geq U(\eta^*)$. Thus, $U(\eta^*) \in \mathbb{U}$.

(Necessity). Suppose that $U(\eta^*) \in \mathbb{U}\backslash Z$. To show that $x^* \notin G$, it suffices to note that for some i in I, by the definition of Z, $U_i(x_i^*) < U_i(x_i(\eta^*))$. Thus, $x_i(\eta^*) \succ_i x_i^*$, from which we have $x^* \notin G$. It remains to show that $G \neq \emptyset$. We consider two possible cases. If $U(\eta^*) \in \mathbb{U}\backslash\mathring{U}$, then there is a scalar $\alpha > 0$ such that $U' = U(\eta^*) + e\cdot\alpha \in \mathring{U}$, where e is a 2-vector of ones. Clearly, by monotonicity, $U'_i > U_i(\eta^*)$ for each i in I. Since U is one-to-one on $PO(\mathcal{E})$, there is a unique feasible vector $x' \in F$ with $U_i(x') = U'_i$. By construction, $x'_i \succ_i x_i(\eta^*)$; therefore $x' \in G$. Finally, if $U(\eta^*) \in \mathring{U}$, then by the strict convexity of preferences and by the definition of G, $\{x(\eta^*)\} = G$. Since the two cases considered are exhaustive, we have shown that $G \neq \emptyset$. This completes the proof of Proposition 3.10. ∎

Proof of Proposition 3.11.

By Proposition 3.10, $U(\eta^*) \in \mathbb{U}\backslash\mathring{U}$. Thus, there is $\varepsilon > 0$ small enough so that $\mathbb{B}(U(\eta^*);\varepsilon) \cap \mathbb{R}^2_+ \subset \mathbb{U}\backslash\mathring{U}$ (see Figure A3-2). Now, let

$k = \max_{U \in \mathbb{B}(U(\eta^*);\varepsilon) \cap \mathbb{R}^2_+} (U_1 - U_1(\eta^*))\cdot(U_2 - U_2(\eta^*)) = (1/2)\cdot\varepsilon^2 > 0$. But

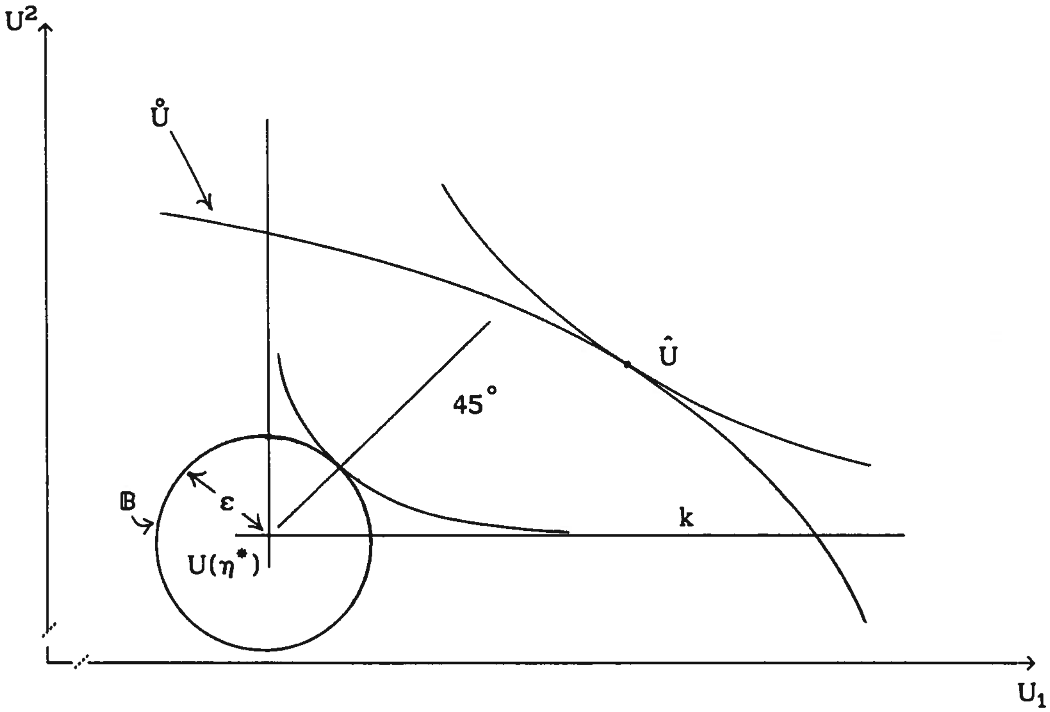

Figure A3-2. *Cooperative outcome* Û *improves on the lobbying outcome* U(η*).

since $\mathbb{B}(U(\eta^*);\varepsilon) \cap \mathbb{R}^2_+ \subset \mathbb{U}\backslash\mathring{\mathbb{U}}$, $\max_{U \in \mathbb{U}} (U_1 - U_1(\eta^*))\cdot(U_2 - U_2(\eta^*)) \equiv$ $(\hat{U}_1 - U_1(\eta^*))\cdot(\hat{U}_2 - U_2(\eta^*)) \geq k > 0$. Thus, since $\hat{U}_i < U_i(\eta^*)$ is impossible, each term in this last product must be strictly positive. We conclude that $\hat{U}_i > U_i(\eta^*)$ for each agent i in I. This completes the proof of the proposition. ∎

Proof of Proposition 3.12.

(Sufficiency). Suppose that $G = \emptyset$, and further, under the hypotheses of the proposition, that $L_2(x_2(\eta^*)) \cap F \neq \emptyset$. Consider the convex set $G_2 \subset \mathbb{R}^2_+$. As was shown in the proof of Proposition 3.6, there is a unique $x_1^o \in G_2$ such that for all $y \in G_2$, $x_1^o \gtrsim_1 y$. Clearly, setting $x_2^o = \omega - x_1^o$, $x^o \in PO(\mathcal{E})$. Thus, $U(x^o) \in \mathring{\mathbb{U}}$. We know $x_1^o \notin G_1$, for otherwise we would have $(x_1^o, x_2^o) \in G$, a contradiction. Thus, $x_1(\eta^*) \in \text{int}(L_1(x_1^o))$, so that $U_1(x_1(\eta^*)) > U_1(x_1^o)$. Since $x_2^o \sim_2 x_2(\eta^*)$, $U_2(x_2^o) = U_2(x_2(\eta^*))$; from this we have $U(x(\eta^*)) \geq U(x^o)$, proving that $U(\eta^*) \notin \mathbb{U}$.

(Necessity). Suppose that $U(\eta^*) \notin \mathbb{U}$ and, by way of contradiction, that there is an x with $x \in G$. By Proposition 3.6, there is an $x' \in G \cap PO(\mathcal{E})$. We have that $U(x') \in \mathring{\mathbb{U}}$, and since $x'_i \gtrsim_i x_i(\eta^*)$ for every i in I, we have $U(x(\eta^*)) \leq U(x') \in \mathbb{U}$. This contradicts that $U(x(\eta^*)) \notin \mathbb{U}$, and we conclude that $G = \emptyset$. This completes the proof of Proposition 3.12. ∎

Proof of Proposition 3.13.

(Sufficiency). Suppose that $LC(\mathcal{E}) = \emptyset$, but by way of contradiction suppose that there is x^o with $x^o \in G$. If $x^o \in \text{int}(G)$, then by Proposition 3.6, there is $y \in G$ with $y \in PO(\mathcal{E})$. This y is in the lobbying core by definition, contradicting that $LC(\mathcal{E}) = \emptyset$. In this case, we conclude that $G = \emptyset$. If $\{x^o\} = G$, then clearly $\{x^o\} = LC(\mathcal{E})$, another contradiction.

Again, we conclude that $G = \emptyset$. Finally, if $x^o \in bd(G)$, but there is an $x^1 \in G$ with $x^o \neq x^1$, then by the strict concavity of U_1, there is a $y \in int(G)$. Proposition 3.6 again guarantees that there is $z \in int(G)$ with $z \in LC(\mathcal{E})$ as above. This is a contradiction, allowing us to conclude that $G = \emptyset$.

(Necessity). Suppose now that $G = \emptyset$. This condition implies that $(\omega - x_1) \notin L_2(x_2(\eta^*))$ whenever $x_1 \in L_1(x_1(\eta^*))$. But by definition elements of the lobbying core must satisfy the two conditions $x_1 \in L_1(x_1(\eta^*))$ and $(\omega - x_1) \in L_2(x_2(\eta^*))$, an impossibility in light of the preceding. We conclude that there can be no allocation in the lobbying core. This completes the proof of Proposition 3.13. ∎

CHAPTER FOUR

LOBBYING EQUILIBRIUM: RESULTS FROM SOME NUMERICAL EXPERIMENTS

> *"It would seem to be a blatant injustice if someone should be forced to contribute toward the costs of some activity which does not further his interests or may even be diametrically opposed to them."*
>
> Knut Wicksell, "A New Principle of Just Taxation," 1896

4.1 INTRODUCTION

What does a lobbying equilibrium look like, how does it change as the parameters of the economy change, and especially how are agents affected by the lobbying game? In chapter two, the lobbying economy model was shown to possess an equilibrium under certain conditions. In chapter three, lobbying equilibria were compared to the outcome achieved in the underlying lobby-free economies. In this chapter, specific example economies will be constructed and solved numerically for equilibrium points. Using a microcomputer, a large number of lobbying equilibria are calculated. Their properties, as well as the relationships between the parameters and variables of the economy, will be reported upon here.

The first objective of the numerical experiments is a successful demonstration of the existence of lobbying equilibria. Using Cobb-Douglas utility functions and a simple pricing rule, agents' optimization problems are solved analytically for systems of equations in the lobbying variables η_i whose joint solution is an equilibrium in the lobbying game Γ_g. We find that agents will lobby under a variety of circumstances. Second, most of the results of chapter three are verified. In particular, many examples will be shown in which the lobbying outcome leaves one agent better off than he or she would have been without lobbying: there is i with $x_i(\eta^*) >_i x_i^*$. The third objective is to draw comparisons between outcomes for related economies in order to uncover relationships between various of the parameters and variables. As will become apparent here, an exhaustive analytical comparative statics analysis of the equilibrium system would be much more difficult than instructive. In some sense, this exercise serves as a substitute for comparative statics investigation. In fact, we find nonlinear relationships between variables in equilibria, which implies that first order derivative systems will have zeros in some variables on both sides of the origin, depending on other variables or parameters.

Each agent in each of the example economies owns a certain level of wealth and of political influence or power. By wealth is meant a level of endowment of one of the goods. Agent i's level of influence is captured by a parameter of the pricing function which determines, in effect, the steepness of the pricing function in the lobbying level η_i at the origin. Parameters of the utility functions also help determine the lobbying outcome, as agents' relative desires for the two goods affect their willingness to lobby. In developing countries, groups' preference for home and imported goods are important determinants of their political activity and of their country's price policies.

The formulation employed here also allows both agents to be given equal wealth and influence. Under some conditions on endowments, preferences, and pricing function parameters, the lobbying economy is symmetric in the sense that p^* and $p(\eta^*)$ are equal and $\eta_1^* = \eta_1^*$. Outcomes in these cases do not favor either agent, as $U_1(\eta^*) = U_2(\eta^*)$. What's more, markets clear after lobbying without need for trade with the world, so that $\pi(\eta^*)$ is always non-negative. Whenever agents lobby, they are both made worse off; in this sense symmetric economies mimic the classic prisoners' dilemma of game theory. They permit a study of the effect of variable wealth and influence on government income and on welfare levels.

4.2 THE EQUILIBRIUM LOBBYING SYSTEM

From chapter two, the maximization program of agent 1 is known to be

$$M_1(\eta_2) \qquad \max_{(x_1^1, x_1^2, \eta_1) \in \psi_1(\eta_2)} U_1(x_1^1, x_1^2),$$

where the choice set $\psi_1(\eta_2)$ is given by

$$\psi_1(\eta_2) = \left\{ (x_1^1, x_1^2, \eta_1) \in \mathbb{R}_+^3 : P(\eta)\cdot(x_1^1, x_1^2) \le P(\eta)\cdot\omega_1 - \eta_1 \right\}.$$

Our first task is to extract from this program an implicit response function in the lobbying variable η_1 which depends only on η_2. Once this function and its counterpart for agent 2 are discovered, their joint solution yields a fixed point to the best response system. This fixed point will be an equilibrium in the lobbying game $\Gamma_{\mathcal{E}}$. Using the government feasibility definition, each $\eta^* \in LGE(\Gamma_{\mathcal{E}})$ may be tested to see whether $\pi(\eta^*) \ge 0$. In those economies for which the pair η^* passes this test η^* is a lobbying equilibrium. All other economies will be eliminated from consideration; their game equilibria are not lobbying equilibria.

Recall that in the lobbying game of chapter two, due to the non-convexity of the set $\psi_1(\eta_{-1})$, agents' optimization problems were reformulated as two-stage programs. In the first stage, the pair η was treated exogenously, and demand functions (price-dependent optimal consumption bundles) were derived. These functions were then inserted into the $U_1(x_1)$ and the resulting indirect utility function was maximized in η_1. Agents' choices were thus reduced to choices over the political variable η_1; their market decisions were made implicit. The present development will be in that spirit.

We assume first that the optimal consumption bundle x_1^* lies in $\mathbb{R}^2_{++}$. Because the utility functions we use are of the Cobb-Douglas form, this will hold while prices are non-zero.[37] Further, the restriction that $\eta_1 \geq 0$ is dispensed with in this chapter. Note that we do *not* assume that $\eta_1 > 0$. Rather, each η_1 is allowed to take values anywhere in $\mathbb{R}$.

With $\eta = (\eta_1, \eta_2)$ taken as given, Mr. 1's optimization program is

$$\max_{x_1 \in \mathbb{R}^2_{++}} U_1(x_1^1, x_1^2),$$

$$\text{subject to} \qquad P(\eta)\cdot(x_1^1, x_1^2) \leq p(\eta)\cdot\tilde{\omega}_1^1$$

where $\tilde{\omega}^1 = \omega^1 - \eta_1/p(\eta)$ is Mr. 1's "after-lobbying" endowment. Now, suppose that Mr. 1 has preferences representable by a homogeneous of degree one Cobb-Douglas utility function

$$U_1(x_1^1, x_1^2) = (x_1^1)^{\alpha}\cdot(x_1^2)^{(1-\alpha)}, \tag{4.1}$$

where $\alpha \in (0,1)$. The η-dependent demand functions which obtain from this maximization program are given by[38]

$$x_1^1(p(\eta)) = \alpha\cdot\tilde{\omega}^1. \tag{4.2}$$

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[37]In the language of chapter two, there are no irregular points of demand when utility functions are Cobb-Douglas.

[38]To keep the exposition as brief as possible, the details of the derivation of the response functions for agents 1 and 2 have been placed in Appendix 4A.

$$x_1^2(p(\eta)) = (1-\alpha)\cdot\left[\frac{p(\eta)}{1-p(\eta)}\right]\cdot\tilde{\omega}^1. \tag{4.3}$$

Suppose further that Mr. 1 regards η_2 as exogenous when selecting η_1. Inserting equations (4.2) and (4.3) into (4.1), we obtain indirect utility as a function only of η.

$$V_1(\eta_1;\eta_2) = \left(\alpha\cdot\tilde{\omega}^1\right)^{\alpha}\cdot\left[(1-\alpha)\cdot\frac{p(\eta)}{1-p(\eta)}\cdot\tilde{\omega}^1\right]^{(1-\alpha)}. \tag{4.4}$$

By differentiating this expression partially with respect to η_1 and setting the derivative equal to zero, the following implicit function in η_1 and η_2 is obtained.

$$\eta_1 = \frac{p(\eta)}{\alpha}\cdot\left[\frac{1}{\partial p(\eta)/\partial\eta_1} - \frac{(1-\alpha)\cdot(\omega^1-\eta_1)}{(1-p(\eta))}\right] \tag{4.5}$$

For any given η_2, the η_1 value which solves (4.5) is Mr. 1's best response level of lobbying donations. It is his demand for political output from the government.

The corresponding optimization program for Ms. 2, from chapter two, is

$$M_2(\eta_1) \qquad \max_{(x_2^1,x_2^2,\eta_2)\,\in\,\psi_2(\eta_1)} U_2(x_2^1,x_2^2),$$

where the choice set $\psi_2(\eta_1)$ is given by

$$\psi_2(\eta_1) = \left\{(x_2^1,x_2^2,\eta_2) \in \mathbb{R}_+^3 : P(\eta)\cdot(x_2^1,x_2^2) \le P(\eta)\cdot\omega_2 - \eta_2\right\}.$$

Again assuming that the optimal consumption bundle is chosen after lobbying levels have been determined, we may solve for Ms. 2's demand functions. With $\eta = (\eta_1,\eta_2)$ taken as given, Ms. 2's optimization program is

$$\max_{x_2\,\in\,\mathbb{R}_{++}^2} U_2(x_2^1,x_2^2),$$

subject to $\quad P(\eta)\cdot(x_2^1,x_2^2) \le (1-p(\eta))\cdot\tilde{\omega}^2,$

where $\tilde{\omega}^2 = \omega^2 - \eta_2/(1-p(\eta))$. Assuming that Ms. 2's utility function is homogeneous of degree one Cobb-Douglas,

$$U_2(x_2^1,x_2^2) = (x_2^1)^{\beta}\cdot(x_2^2)^{(1-\beta)}, \tag{4.6}$$

where $\beta \in (0,1)$. The η-dependent demand functions which obtain from this maximization program are

$$x_2^1(p(\eta)) = \beta\cdot\left[\frac{1-p(\eta)}{p(\eta)}\right]\cdot\tilde{\omega}^2 \tag{4.7}$$

$$x_2^2(p(\eta)) = (1-\beta)\cdot\tilde{\omega}^2. \tag{4.8}$$

Inserting equations (4.7) and (4.8) into (4.6), and following the steps used above for Mr. 1's problem, indirect utility V_2 is found to equal

$$V_2(\eta_2;\eta_1) = \left[\beta\cdot\frac{1-p(\eta)}{p(\eta)}\cdot\tilde{\omega}^2\right]^{\beta}\cdot\left((1-\beta)\cdot\tilde{\omega}^2\right)^{(1-\beta)}. \tag{4.9}$$

Differentiating this expression partially with respect to η_2 and setting the result equal to zero, the implicit response function in η_2 and η_2 is found to be

$$\eta_2 = \frac{1-p(\eta)}{\beta - 1}\cdot\left[\frac{1}{\partial p(\eta)/\partial\eta_2} + \frac{\beta\cdot(\omega^2 - \eta_2)}{p(\eta)}\right] \tag{4.10}$$

Equations (4.5) and (4.10) form the system of implicit best response functions in the lobbying decision variables for our two agents. Any pair η^* which solves these two equations simultaneously constitutes precisely the lobbying game equilibrium LGE(Γ_g) discussed in chapter two. For each agent, η_i^* solves the relevant maximization problem, given η_{-i}^*.

In order to solve the system, all that remains is to specify the form for the pricing function $p(\eta)$. Recalling assumptions (A1) to (A4) of chapter two, one may easily verify that they are satisfied by the following function:

$$p(\eta_1,\eta_2) = p^{*}\cdot[1 - \exp(-\delta_1\cdot\eta_1) + \exp(-\delta_2\cdot\eta_2)],$$

where $\delta_i > 0$ for each i are the "influence" parameters mentioned above. As δ_i grows, the marginal effect of η_i on $p(\eta)$ near the origin also grows.

However, this function is in need of a slight modification. In its present form, this expression maps pairs η into $(0,2p^{*})$. Thus, if $p^{*} > 0.5$, the lobbying price may exceed one for $\eta \in \mathbb{R}^2_+$. What is needed is a scaling procedure to press $p(\eta)$ down below one for large p^{*}. The following function, to be used hereafter, includes the required scaling.

$$p(\eta) = \begin{cases} p^{*}[1 - \exp(-\delta_1\eta_1) + \exp(-\delta_2\eta_2)] & \text{if } p^{*} \le 1/2 \\ p^{*}[1 - \left(\frac{1-p^{*}}{p^{*}}\right)\cdot(\exp(-\delta_1\eta_1) - \exp(-\delta_2\eta_2))] & \text{if } p^{*} > 1/2 \end{cases} \quad (4.11)$$

Finally, we must derive the competitive equilibrium price p^{*} from the lobbying-free economy $\hat{\mathcal{E}}$. First, as in Appendix 4A, the price-dependent demand functions which arise in the absence of lobbying are

$$x_1(p) = (x_1^1(p),x_1^2(p)) = \left(\alpha\cdot\omega^1 \; , \; (1-\alpha)\cdot\frac{1}{1-p}\cdot\omega^1\right), \text{ and}$$

$$x_2(p) = (x_2^1(p),x_2^2(p)) = \left(\beta\cdot\frac{1-p}{p}\cdot\omega^2 \; , \; (1-\beta)\cdot\omega^2\right).$$

Aggregate excess demand $z_1(p)$ for good 1 may, by Walras' law, be solved for the competitive equilibrium price p^{*}. At the market-clearing price p^{*}, we have

$$z_1(p) = \alpha\cdot\omega^1 + \beta\cdot\frac{1-p^*}{p^*}\cdot\omega^2 - \omega^1 = 0,$$

from which

$$p^* = \left(\frac{1}{1+\phi}\right),$$

where $\phi = ((1-\alpha)/\beta)/(\omega^2/\omega^1)$. This price, which does not depend on lobbying activity, clears goods markets in its absence.

Now, given the chosen forms for utility and pricing functions, and given our convention on endowments and individual behavior, a lobbying economy is fully specified by the parameter vector $\gamma = (\omega^1, \omega^2, \alpha, \beta, \delta_1, \delta_2)$. On occasion, the symbol γ will be used to denote a lobbying economy $\mathcal{E}$.

The trio of equations, (4.5), (4.10), and (4.11) together constitute the system of equations whose solution $(\eta^*, p(\eta^*))$ is a lobbying game equilibrium. If in addition $\pi(\eta^*) \geq 0$, then the pair η^* is a lobbying equilibrium. For several thousand example economies, a microcomputer was used to find zeros of the system. The remainder of this chapter will be concerned with presenting and interpreting these solutions.

The system was solved using a non-linear equation system solver written in the GAUSS programming language. An example of the program which selected parameter vectors, calculated p^*, solved for η^* and $p(\eta^*)$ and reported these three numbers for each γ is reproduced, with a more detailed explanation, in Appendix 4B. We may now proceed to a discussion and interpretation of the results.

4.3 LOBBYING EQUILIBRIUM

As suggested by the parameter vector γ, each agent in the economy possesses three essential characteristics. They include his or her level of *wealth* (ω_i), of *political influence* or power (captured by δ_i), and *degree of preference* for good i (captured by α and β,). For each agent, these parameters may be varied to discover their individual and joint effects. Because they are suggestive of the characteristics which make real interest groups successful, the wealth and influence parameters will be emphasized.

Most of the time, only relative wealth and influence matter. Cases with $\omega_1 = \omega_2$ and $\delta_1 = \delta_2$ are special because of their symmetry; these cases receive treatment in the next section. Otherwise, we may have $\omega_1 < \omega_2$ (scalar values for endowments are indexed with a subscript hereafter) or $\omega_1 > \omega_2$, and either $\delta_1 < \delta_2$ or $\delta_1 > \delta_2$. Aside from choices about α and β, there are four possible combinations of the four parameters. Mr. 1 may be more or less wealthy and more or less powerful than Ms. 2. Because $\omega_1 < \omega_2$ and $\delta_1 > \delta_2$ is a mirror image of the case with $\omega_1 > \omega_2$ and $\delta_1 < \delta_2$, we may treat only two of the four. A relabelling of the agents and goods would lead to largely the same results for the remaining two.

As there are three essential characteristics of agents, so there are three features of lobbying outcomes which are of interest. These are the lobbying levels, the utility levels, and the value of the government's net income $\pi(\eta^*)$. Easily the most interesting implications for equilibria of moving γ are the resultant movements in these five values. Interpreting the outcome of lobbying economies will amount to investigating the relationship between η_i^*, $U_i(\eta^*)$, and $\pi(\eta^*)$ on the one hand, and γ on the other.

Recall that the whole of chapter two represented an effort to demonstrate the existence of a lobbying equilibrium for $\mathcal{E}$. This was in fact accomplished analytically, but one cost of achieving the general existence result was a high degree of abstraction. This chapter is much less abstract. Therefore, before proceeding we pause to note that nearly all of the numerical results results presented below are elements of SALE($\mathcal{E}$) for the respective economies $\mathcal{E}$. That is, members of a large subset of $\mathfrak{E}$ have non-empty equilibrium sets SALE($\mathcal{E}$) = $\emptyset$. From the few outcomes at which $\pi(\eta^*) < 0$, it is possible to discover some of the important determinants of government feasibility.

Most of the calculated equilibria are also such that $U_i(\eta^*) < U_i^*$ for each agent i. That is, both lose from lobbying in most cases. However, in chapter three the possibility for $U_i(\eta^*) > U_i^*$ at a SALE($\mathcal{E}$) was mentioned. In this case, we have $x^* \notin G$, or $U_i^* \notin Z$, using the notation of chapter three. In the subsequent discussion, we will come upon other of these cases, and the conditions giving rise to them will be unveiled. For now, a collection of such cases is presented in Table 4-1.[39] From this table it appears that two requisite conditions for $x^* \notin G$ are that one agent (in this case, Mr. 1) be both richer and more influential than the other.

4.3.1 Mr. 1 Rich and Influential.

Suppose that $\omega_1 \geq \omega_2$ and $\delta_1 \geq \delta_2$. Here, Mr. 1 is both more wealthy and more powerful politically (that is, more influential) than Ms. 2.[40] How do

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[39]In each row of this and the other tables, a γ vector occupies the first six positions, while the p^* associated with γ is in the seventh. The remainder of the numbers are the solution to equations (4.5), (4.10), and (4.11) along with the utility levels before and after lobbying and $\pi(\eta^*)$.

[40]Agent i is "more powerful" than agent −1, whenever $p(\eta)$ is steeper *at the origin* in η_i than in η_{-i}. The function must eventually get flat in both

TABLE 4-1 *Lobbying Outcomes Beneficial to One Agent*

ω_1	ω_2	α	β	δ_1	δ_2	p^*	η_1^*	η_2^*	p^η	U_1^*	U_1^η	U_2^*	U_2^η
5.2	4.6	.68	.32	4.0	1.50	.531[1]	0.596	0.619	.673[2]	2.899[3]	2.903[4]	2.363	1.149
5.2	4.6	.68	.32	5.0	1.75	.531	0.512	0.615	.654	2.889	2.895	2.363	1.229
5.2	4.6	.68	.30	4.0	1.50	.515	0.591	0.561	.678	2.830	2.935	2.454	1.240
5.2	4.6	.68	.30	5.0	1.75	.515	0.506	0.568	.656	2.830	2.907	2.454	1.321
5.2	4.6	.68	.28	4.0	1.50	.497	0.589	0.493	.688	2.768	2.986	2.550	0.470
5.2	4.6	.68	.28	5.0	1.50	.497	0.520	0.498	.696	2.768	3.101	2.550	1.298
5.2	4.6	.68	.28	5.0	2.00	.497	0.488	0.515	.631	2.768	2.810	2.550	1.522
5.2	4.6	.70	.32	5.0	1.50	.547	0.522	0.654	.683	2.986	3.033	2.315	1.059
5.2	4.6	.70	.30	4.0	1.50	.531	0.586	0.594	.678	2.929	2.943	2.407	1.196
5.2	4.6	.70	.30	5.0	1.50	.531	0.517	0.599	.688	2.929	3.053	2.407	1.154
5.2	4.6	.70	.30	5.0	1.75	.531	0.503	0.593	.659	2.929	2.934	2.407	1.275
5.2	5.2	.68	.32	5.0	1.50	.500	0.498	0.642	.649	2.778	2.886	2.778	1.478
5.2	5.2	.68	.30	4.0	1.50	.483	0.549	0.582	.632	2.721	2.752	2.878	1.670
5.2	5.2	.68	.30	5.0	1.50	.483	0.488	0.581	.644	2.721	2.869	2.878	1.621
5.2	5.2	.68	.30	5.0	1.75	.483	0.473	0.592	.610	2.721	2.728	2.878	1.747

[1] $p^* = 1/(1+\varphi)$, where $\varphi = \frac{(1-\alpha)}{\beta} \cdot \frac{\omega_2}{\omega_1}$.

[2] $p^\eta = p(\eta^*) = p^*[1 - \exp(-\delta_1\eta_1^*) + \exp(-\delta_2\eta_2^*)]$.

[3] $U_i^* = U_i(x_i^*)$, where $x_i^* = x_i(p^*)$, i's demanded bundle under price p^* and without lobbying.

[4] $U_i^\eta = U_i(x_i(\eta^*))$, where $x_i(\eta^*)$ is i's demanded bundle after lobbying.

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directions. Political power is associated with steepness at the origin. Further, we choose to let $\omega_1 \geq \omega_2$ represent agent 1's wealth advantage. Clearly, whenever $p^* < 0.5$, the *value* of 1's endowment may still fall short of $\omega_2 \cdot (1-p^*)$, which is the value of 2's endowment.

changes in γ affect η_1^*, $U_1(\eta^*)$, and $\pi(\eta^*)$ while these two inequalities hold? One expects that Mr. 1 will be most successful in such instances. Table 4-2 reveals that this is indeed true; it also helps explain why.

The first row of Table 4-2 is a symmetric economy: $\omega_1 = \omega_2$, $\alpha + \beta = 1$, and $p^* = 0.5$. As is always true of such examples, $\eta_1^* = \eta_2^*$, $p(\eta^*) = 0.5$, and $U_1(\eta^*) = U_2(\eta^*)$. Now, as δ_1 grows, making Mr. 1 more powerful, η_1^* declines while $p(\eta^*)$ increases. These two changes both help Mr. 1, the second at Ms. 2's expense. $U_1(\eta^*)$ grows from 2.1 at $\delta_1 = 2$ to 2.8 at $\delta_1 = 7$. In this last example, $U_1(\eta^*) > U_1^*$; the lobbying outcome leaves Mr. 1 better off than he would have been without lobbying. Note that he is quite influential here, but that $\omega_1 = \omega_2$. Thus, at $p^* = 0.5$, pre-lobbying *income* is identical for the two agents.

As ω_1 increases to 9 and then to 14, the same responses to changing δ_1 are observed, with one important exception. When $\omega_1 = 5$, an increase in δ_1 leads 2 to lobby less: η_2^* *decreases*. However, for $\omega_1 = 9$, the same increase in δ_1 leads 2 to respond with more lobbying: η_2^* *increases*. Thus, the optimal response to an opponent's increased political influence differs according to whether he or she is much more or about the same level of resources to devote to lobbying. In every row of the table, $U_1(\eta^*) \geq U_2(\eta^*)$; Mr. 1 is best off after lobbying. Also, $U_2^* > U_2(\eta^*)$ in each case, which means that Ms. 2 is always hurt by the lobbying program here.

Table 4-3 presents the results of increasing ω_1 for three selected levels of δ_1. Again, the first row is a symmetric economy. As ω_1 increases with $\delta_1 = 2$, p^*, η_1^*, η_2^*, $p(\eta^*)$, U_1^*, and $U_1(\eta^*)$ all increase monotonically. However, U_2^* and $U_2(\eta^*)$ both decrease, the latter dramatically so. When δ_1

TABLE 4-2 *Effect on U_1^η of increasing δ_1 for various levels of ω_1. Mr. 1 wealthy and influential.*

ω_1	ω_2	α	β	δ_1	δ_2	p^*	η_1^*	η_2^*	p^η	U_1^*	U_1^η	U_2^*	U_2^η	$\pi(\eta)$
5[1]	5	.72	.28	2	2	.50	0.60	0.601	.50	2.76	2.099	2.76	2.099	1.203[2]
				3	2		0.56	0.572	.57		2.388		1.889	1.081
				4	2		0.50	0.564	.59		2.557		1.792	0.955
				5	2		0.45	0.562	.61		2.670		1.736	0.855
				6	2		0.41	0.560	.62		2.752		1.700	0.776
				7	2		0.37	0.560	.63		2.813		1.674	0.713
9	5	.72	.28	2	2	.64	1.18	0.746	.69	5.86	5.035	2.34	1.147	1.671
				3	2		0.93	0.753	.70		5.375		1.084	1.331
				4	2		0.77	0.757	.71		5.577		1.053	1.122
				5	2		0.67	0.759	.71		5.711		1.034	0.981
				6	2		0.59	0.761	.71		5.808		1.021	0.878
				7	2		0.53	0.762	.71		5.881		1.012	0.799
14	5	.72	.28	2	2	.74	1.59	0.930	.77	10.3	9.198	2.07	0.400	2.103
				3	2		1.20	0.936	.77		9.645		0.365	1.644
				4	2		0.98	0.939	.77		9.900		0.348	1.380
				5	2		0.83	0.940	.77		10.07		0.338	1.207
				6	2		0.72	0.942	.77		10.19		0.331	1.083
				7	2		0.64	0.942	.77		10.27		0.326	0.991

[1] Blank entries in this and all following tables take the value of the last element appearing in the column above them.

[2] $\pi(\eta) = \pi(\eta^*) = \eta_1^* + \eta_2^* - P(\eta^*)\cdot\tilde{z}(p(\eta^*))$.

TABLE 4-3 *Effect on U_1^η of increasing ω_1 for various levels of δ_1. Mr. 1 wealthy and influential.*

ω_1	ω_2	α	β	δ_1	δ_2	p^*	η_1^*	η_2^*	p^η	U_1^*	U_1^η	U_2^*	U_2^η	$\pi(\eta)$
5	5	.72	.28	2	2	.50	0.60	0.60	.50	2.76	2.099	2.76	2.099	1.20
6						.55	0.79	0.62	.58	3.49	2.820	2.62	1.764	1.35
7						.58	0.94	0.61	.63	4.25	3.537	2.51	1.524	1.47
8						.62	1.07	0.70	.66	5.04	4.273	2.42	1.325	1.57
9						.64	1.18	0.74	.69	5.86	5.034	2.34	1.147	1.67
10						.67	1.18	0.78	.71	6.71	5.821	2.27	0.984	1.76
11						.69	1.36	0.82	.73	7.58	6.632	2.21	0.829	1.85
12						.71	1.45	0.86	.74	8.47	7.466	2.16	0.682	1.93
13						.72	1.52	0.89	.76	9.38	8.322	2.11	0.539	2.02
14						.74	1.59	0.93	.77	10.3	9.198	2.07	0.400	2.10
5	5	.72	.28	5	2	.50	0.45	0.56	.61	2.76	2.670	2.76	1.736	0.85
6						.55	0.51	0.61	.64	3.49	3.387	2.62	1.531	0.87
7						.58	0.57	0.66	.67	4.25	4.134	2.51	1.352	0.90
8						.62	0.62	0.71	.69	5.04	4.909	2.42	1.188	0.94
9						.64	0.66	0.75	.71	5.86	5.711	2.34	1.034	0.98
10						.67	0.70	0.80	.72	6.71	6.538	2.27	0.886	1.02
11						.69	0.73	0.83	.74	7.58	7.388	2.21	0.744	1.06
12						.71	0.77	0.87	.75	8.47	8.261	2.16	0.606	1.11
13						.72	0.80	0.90	.76	9.38	9.154	2.11	0.471	1.16
14						.74	0.82	0.94	.77	10.3	10.07	2.07	0.338	1.20
5	5	.72	.28	7	2	.50	0.37	0.56	.63	2.76	2.814	2.76	1.764	0.71
6						.55	0.42	0.61	.65	3.49	3.535	2.62	1.488	0.72
7						.58	0.46	0.67	.68	4.25	4.288	2.51	1.319	0.74
8						.62	0.49	0.71	.69	5.04	5.071	2.42	1.162	0.76
9						.64	0.52	0.76	.71	5.86	5.881	2.34	1.012	0.79
10						.67	0.55	0.80	.73	6.71	6.716	2.27	0.868	0.83
11						.69	0.57	0.84	.74	7.58	7.574	2.21	0.728	0.87
12						.71	0.60	0.87	.75	8.47	8.454	2.16	0.592	0.91
13						.72	0.62	0.91	.76	9.38	9.354	2.11	0.458	0.95
14						.74	0.64	0.94	.77	10.3	10.27	2.07	0.326	0.99

increases to 7, the situation is different in one important way. Here, at $\omega_1 = 5$, Mr. 1 prefers the lobbying outcome by 2.81 to 2.76. As ω_1 increases to 14, *cet. par.*, this result is reversed and he prefers the competitive outcome. Since Ms. 2 always prefers x_2^* to $x_2(\eta^*)$, we see that $x(\eta^*)$ moves from $\mathbb{Z}$ to $\mathbb{U}\setminus\mathbb{Z}$ in U_1,U_2-space as ω_1 increases and for specific values of $\gamma\setminus\omega_1$. Evidently, if Mr. 1 is both very powerful and very rich relative to Ms. 2, a compensation scheme of the sort discussed in chapter three is available by which the lobbying program may be overruled.

Note also, from Table 4-4, that Ms. 2's best response to an increase in ω_1, for any δ_1, seems to be an increase in η_2^*. Thus, she loses twice as the underdog facing a progressively more wealthy opponent. Her endowment loses value because $p(\eta^*)$ rises, and the cost of countervailing Mr. 1's lobbying also goes up.

How do changes in the α and β values affect lobbying donations and agents' utility levels? From Table 4-5, it is apparent that an increase in α leads to an increase in U_1^* and in $U_1(\eta^*)$. An increase in α means that Mr. 1 has a stronger preference for good 1 over good 2. Likewise, a decrease in β leads to the same kind of increase in U_2^* and in $U_2(\eta^*)$. This is true regardless of the levels of δ_1 and of ω_1.

4.3.2 Ms. 2 Rich and Mr. 1 Influential.

We turn now to cases in which one agent has more political influence, but his or her opponent has greater initial endowment. In Table 4-6, there are listed the results of calculating equilibria for economies in which Mr. 1 becomes progressively more influential than Ms. 2 for various levels of ω_2. How do these changes affect lobbying outcomes? The first six rows

TABLE 4-4 *Effect on η_1 of increasing ω_1 for various levels δ_1. Mr. 1 wealthy and influential.*

ω_1	ω_2	α	β	δ_1	δ_2	p^*	η_1^*	η_2^*	p^η	U_1^*	U_1^η	U_2^*	U_2^η	$\pi(\eta)$
5	5	.68	.32	2	2	.50	0.65	0.65	.50	2.67	1.97	2.67	1.97	1.30
6	5					.55	0.84	0.66	.58	3.39	2.70	2.52	1.64	1.44
7	5					.58	0.99	0.69	.63	4.16	3.43	2.40	1.40	1.55
8	5					.62	1.12	0.73	.66	4.96	4.18	2.29	1.21	1.64
9	5					.64	1.23	0.77	.69	5.80	4.96	2.21	1.04	1.72
10	5					.67	1.33	0.80	.71	6.66	5.77	2.14	0.88	1.80
11	5					.69	1.42	0.84	.73	7.56	6.61	2.07	0.74	1.88
12	5					.71	1.50	0.87	.74	8.48	7.47	2.08	0.60	1.95
13	5					.72	1.58	0.90	.76	9.42	8.36	1.96	0.47	2.03
14	5					.74	1.65	0.93	.77	10.4	9.27	1.92	0.35	2.10
5	5	.68	.32	4	2	.50	0.52	0.61	.59	2.67	2.44	2.67	1.68	1.03
6	5					.55	0.61	0.65	.63	3.39	3.17	2.52	1.46	1.04
7	5					.58	0.68	0.70	.66	4.16	3.93	2.40	1.27	1.07
8	5					.62	0.74	0.74	.68	4.96	4.71	2.29	1.11	1.11
9	5					.64	0.80	0.78	.70	5.80	5.53	2.21	0.96	1.13
10	5					.67	0.85	0.81	.72	6.66	6.38	2.14	0.81	1.17
11	5					.69	0.89	0.85	.74	7.56	7.25	2.07	0.68	1.21
12	5					.71	0.93	0.88	.75	8.48	8.15	2.01	0.55	1.25
13	5					.72	0.97	0.91	.76	9.42	9.07	1.96	0.43	1.29
14	5					.74	1.00	0.94	.77	10.4	10.1	1.92	0.30	1.34
5	5	.68	.32	7	2	.50	0.38	0.60	.62	2.67	2.71	2.67	1.57	0.77
6	5					.55	0.43	0.65	.65	3.39	3.44	2.52	1.38	0.76
7	5					.58	0.47	0.70	.67	4.16	4.21	2.40	1.22	0.77
8	5					.62	0.51	0.74	.69	4.96	5.02	2.29	1.06	0.77
9	5					.64	0.54	0.78	.71	5.80	5.85	2.21	0.92	0.79
10	5					.67	0.56	0.82	.72	6.66	6.71	2.14	0.78	0.81
11	5					.69	0.59	0.85	.74	7.56	7.60	2.07	0.65	0.84
12	5					.71	0.61	0.88	.75	8.48	8.51	2.01	0.53	0.87
13	5					.72	0.68	0.91	.76	9.42	9.45	1.96	0.41	0.90
14	5					.74	0.65	0.94	.77	10.4	10.4	1.92	0.29	0.93

TABLE 4-5 *Effect on U_1^η, η_1 of changing α and β for various levels of ω_1 and δ_1. Mr. 1 wealthy and influential.*

ω_1	ω_2	α	β	δ_1	δ_2	p^*	η_1^*	η_2^*	p^η	U_1^*	U_1^η	U_2^*	U_2^η	$\pi(\eta)$
5	5	.68	.28	2	2	.47	0.60	0.58	.47	2.56	1.92	2.86	2.22	1.17
			.28	5	2	.47	0.44	0.52	.58	2.56	2.50	2.86	1.89	0.81
			.32	2	2	.50	0.65	0.65	.50	2.67	1.97	2.67	1.97	1.30
			.32	5	2	.50	0.46	0.60	.60	2.67	2.56	2.67	1.63	0.92
5	5	.72	.28	2	2	.50	0.60	0.60	.50	2.76	2.09	2.76	2.09	1.20
			.28	5	2	.50	0.45	0.56	.61	2.76	2.67	2.76	1.73	0.85
			.32	2	2	.53	0.65	0.76	.53	2.86	2.14	2.56	1.83	1.32
			.32	5	2	.53	0.46	0.64	.62	2.86	2.76	2.56	1.52	0.99
9	5	.68	.28	2	2	.61	1.18	0.70	.67	5.56	4.85	2.43	1.29	1.54
			.28	5	2	.61	0.67	0.71	.69	5.56	5.55	2.43	1.18	0.80
			.32	2	2	.64	1.23	0.77	.69	5.03	4.96	2.21	1.04	1.72
			.32	5	2	.64	0.68	0.78	.71	5.80	5.67	2.21	0.94	0.98
9	5	.72	.28	2	2	.64	1.18	0.74	.69	5.86	5.03	2.34	1.14	1.76
			.28	5	2	.64	0.66	0.75	.71	5.86	5.71	2.34	1.03	0.98
			.32	2	2	.67	1.23	0.81	.71	6.08	5.15	2.12	0.88	1.83
			.32	5	2	.67	0.68	0.82	.72	6.08	5.83	2.12	0.78	1.13
13	5	.68	.28	2	2	.69	1.53	0.84	.74	9.03	8.11	2.19	0.74	1.84
			.28	5	2	.69	0.80	0.85	.74	9.03	8.96	2.19	0.68	0.94
			.32	2	2	.72	1.58	0.90	.76	9.42	8.36	1.96	0.47	2.03
			.32	5	2	.72	0.82	0.91	.76	9.42	9.24	1.96	0.42	1.22
13	5	.72	.28	2	2	.72	1.52	0.89	.76	9.38	8.32	2.11	0.53	2.02
			.28	5	2	.72	0.80	0.90	.76	9.38	9.15	2.11	0.47	1.16
			.32	2	2	.75	1.57	0.96	.77	9.74	8.55	1.88	0.26	2.19
			.32	5	2	.75	0.82	0.97	.78	9.74	9.41	1.88	0.20	1.31

TABLE 4-6 *Effect on U_1^{η}, η_1 of changing δ_1 for various levels of ω_2. Mr. 1 influential and Ms. 2 wealthy.*

ω_1	ω_2	α	β	δ_1	δ_2	p^*	η_1^*	η_2^*	p^{η}	U_1^*	U_1^{η}	U_2^*	U_2^{η}	$\pi(\eta)$
5	5	.68	.32	2	2	.50	0.65	0.65	.50	2.67	1.97	2.67	1.97	1.30
				3	2		0.59	0.62	.56	2.67	2.27	2.67	1.77	1.16
				4	2		0.52	0.61	.59	2.67	2.44	2.67	1.68	1.03
				5	2		0.46	0.60	.60	2.67	2.56	2.67	1.63	0.92
				6	2		0.42	0.60	.61	2.67	2.67	2.67	1.59	0.84
				7	2		0.38	0.60	.62	2.67	2.71	2.67	1.57	0.77
5	8	.68	.32	2	2	.38	0.43	0.96	.28	2.29	1.35	4.96	4.83	0.74
				3	2		0.42	0.88	.34	2.29	1.63	4.96	4.38	1.11
				4	2		0.38	0.84	.37	2.29	1.79	4.96	4.19	1.19
				5	2		0.35	0.82	.39	2.29	1.90	4.96	4.07	1.20
				6	2		0.32	0.81	.41	2.29	1.98	4.96	4.00	1.20
				7	2		0.30	0.80	.42	2.29	2.04	4.96	3.95	1.18
5	10	.68	.32	3	2	.33	0.36	1.03	.26	2.14	1.39	6.66	6.39	0.76
				4	2		0.33	0.98	.29	2.14	1.55	6.66	6.09	1.10
				5	2		0.31	0.95	.31	2.14	1.66	6.66	5.91	1.12
				6	2		0.29	0.94	.33	2.14	1.78	6.66	5.80	1.17
				7	2		0.26	0.92	.33	2.14	1.79	6.66	5.72	1.20

of this table, with $\omega_2 = 5$, are very much like the first six rows of Table 4-2. For $\omega_2 = 8$, the same trends in p^*, η_1^*, and $p(\eta^*)$ are all observed: all increase with δ_1. The same is true when $\omega_2 = 10$, although Ms. 2's utility after lobbying is naturally quite large in this instance.

Table 4-7 records the responses of η_1^* and $U_1(\eta^*)$ to increases in ω_2 for fixed δ_1 levels of 2, 4, and 6. In each case, η_1^* declines monotonically with increases in ω_2 while η_2^* increases. That η_2^* may achieve a maximum and decline is apparent from the last row with $\delta_1 = 4$, where η_2^* declines from 1.14 to 1.12 as ω_2 increases from 13 to 14. Mr 1's utility falls while Ms. 2's utility increases in ω_2. Note that in this table there are four cases with $\pi(\eta^*) < 0$; these will be revisited shortly.

The effects of changing preferences are to be found in Table 4-8. As α increases, η_1^* and η_2^* both increase slightly, as do $p(\eta^*)$ and $U_1(\eta^*)$. $U_2(\eta^*)$, on the other hand, decreases with increasing α. The opposite effect is observed to flow from declining β. Thus, as either agent increases a preference for his or her own good, he or she is made better off at the expense of the opponent.

4.3.2 The Government Budget $\pi(\eta^*)$.

One of the two conditions which define a lobbying equilibrium is that the lobbying donations exceed the cost to the government of trade with the rest of the world. That is, along with the optimal response condition, equilibrium requires $\pi(\eta^*) \geq 0$. We have seen that this need not hold in every economy. Here a more general question is asked: How does $\pi(\eta^*)$ move with changes in γ?

Table 4-9 shows that the effect of increasing δ_1 on $\pi(\eta^*)$ varies

TABLE 4-7 *Effect on U_1^η, η_1 of changing ω_2 for various levels of δ_1. Mr. 1 influential and Ms. 2 wealthy.*

ω_1	ω_2	α	β	δ_1	δ_2	p^*	η_1^*	η_2^*	p^η	U_1^*	U_1^η	U_2^*	U_2^η	$\pi(\eta)$
5	5	.70	.30	2	2	.50	0.62	0.62	.50	2.71	2.03	2.71	2.03	1.25
5	6					.45	0.52	0.73	.40	2.57	1.77	3.44	2.92	1.17
5	7					.42	0.45	0.84	.33	2.45	1.57	4.20	3.87	0.97
5	8					.38	0.40	0.94	.27	2.35	1.41	5.00	4.88	0.69
5	9					.36	0.36	1.04	.23	2.27	1.28	5.82	5.95	0.32
5	10					.33	0.33	1.13	.20	2.20	1.17	6.68	7.10	−0.13
5	11					.31	0.30	1.21	.17	2.14	1.08	7.56	8.31	−0.68
5	12					.29	0.28	1.29	.15	2.08	0.99	8.47	9.59	−1.32
5	13					.28	0.26	1.36	.13	2.03	0.92	9.40	10.9	−2.07
5	5	.70	.30	4	2	.50	0.51	0.58	.59	2.71	2.50	2.71	1.73	0.99
5	6					.45	0.45	0.66	.50	2.57	2.22	3.44	2.53	1.12
5	7					.42	0.40	0.74	.43	2.45	2.02	4.20	3.36	1.17
5	8					.38	0.37	0.81	.37	2.35	1.85	5.00	4.24	1.56
5	9					.36	0.34	0.89	.33	2.27	1.72	5.82	5.16	1.09
5	10					.33	0.32	0.96	.29	2.20	1.61	6.68	6.12	0.98
5	11					.31	0.30	1.02	.26	2.14	1.52	7.56	7.13	0.84
5	12					.29	0.29	1.08	.24	2.08	1.43	8.47	8.17	0.67
5	13					.28	0.27	*1.14*	.21	2.03	1.36	9.40	9.26	0.46
5	14					.26	0.26	*1.12*	.20	1.99	1.29	10.3	10.4	0.23
5	5	.70	.30	6	2	.50	0.41	0.58	.61	2.71	2.69	2.71	1.64	0.81
5	6					.45	0.37	0.64	.53	2.57	2.42	3.44	2.41	1.00
5	7					.42	0.34	0.71	.46	2.45	2.21	4.20	3.21	1.11
5	8					.38	0.31	0.78	.41	2.35	2.04	5.00	4.05	1.16
5	9					.36	0.29	0.85	.36	2.27	1.91	5.82	4.92	1.17
5	10					.33	0.28	0.91	.33	2.20	1.80	6.61	5.84	1.14
5	11					.31	0.26	0.97	.29	2.14	1.70	7.56	6.79	1.09
5	12					.29	0.25	1.02	.27	2.08	1.62	8.47	7.77	1.01
5	13					.28	0.24	1.07	.25	2.03	1.55	9.40	8.79	0.91
5	14					.26	0.23	1.12	.23	1.99	1.48	10.3	9.85	0.79

TABLE 4-8 (a) *Effect on U_1^{η}, η_1 of changing α. Mr. 1 influential and Ms. 2 wealthy.*

ω_1	ω_2	α	β	δ_1	δ_2	p^*	η_1^*	η_2^*	p^{η}	U_1^*	U_1^{η}	U_2^*	U_2^{η}	$\pi(\eta)$
5	5	.68	.32	3	2	.50	0.59	0.62	.56	2.67	2.27	2.67	1.77	1.16
		.70				.52	0.59	0.63	.57	2.76	2.34	2.61	1.72	1.18
		.72				.53	0.59	0.65	.58	2.86	2.41	2.56	1.65	1.20
5	8	.68	.32	3	2	.38	0.42	0.88	.34	2.29	1.63	4.96	4.38	1.11
		.70				.40	0.42	0.88	.36	2.40	1.73	4.86	4.28	1.12
		.72				.42	0.42	0.88	.37	2.51	1.83	4.76	4.17	1.13
5	11	.68	.32	3	2	.31	0.33	1.10	.23	2.07	1.29	7.56	7.47	0.51
		.70				.33	0.33	1.10	.24	2.18	1.39	7.40	7.31	0.53
		.72				.34	0.34	1.10	.26	2.30	1.50	7.24	7.14	0.55

Table 4-8 (b) *Effect on U_1^{η}, η_1 of changing β. Mr 1 influential and Ms. 2 influential.*

ω_1	ω_2	α	β	δ_1	δ_2	p^*	η_1^*	η_2^*	p^{η}	U_1^*	U_1^{η}	U_2^*	U_2^{η}	$\pi(\eta)$
5	5	.70	.32	3	2	.52	0.59	0.63	.57	2.76	2.34	2.61	1.72	1.18
			.30			.50	0.57	0.59	.56	2.71	2.32	2.71	1.83	1.12
			.28			.48	0.55	0.55	.55	2.65	2.30	2.81	1.96	1.06
5	8	.70	.32	3	2	.40	0.42	0.88	.36	2.40	1.73	1.86	4.28	1.12
			.30			.38	0.40	0.85	.34	2.35	1.69	5.00	4.44	1.07
			.28			.37	0.38	0.82	.32	2.30	1.65	5.14	4.60	1.02
5	11	.70	.32	3	2	.33	0.33	1.10	.24	2.18	1.39	7.40	7.31	0.53
			.30			.31	0.32	1.08	.23	2.14	1.35	7.56	7.49	0.48
			.28			.30	0.30	1.06	.21	2.10	1.31	7.72	7.67	0.42

TABLE 4-9 *Effect on $\pi(\eta)$ of changing δ_1 as ω_1 move.*

ω_1	ω_2	α	β	δ_1	δ_2	p^*	η_1^*	η_2^*	p^η	U_1^*	U_1^η	U_2^*	U_2^η	$\pi(\eta)$
5	5	.68	.32	2	2	.50	0.65	0.65	.50	2.67	1.97	2.67	1.97	1.300
				3		.50	0.59	0.62	.56	2.67	2.27	2.67	1.77	1.164
				4		.50	0.52	0.61	.59	2.67	2.44	2.67	1.68	1.030
				5		.50	0.46	0.60	.60	2.67	2.56	2.67	1.63	0.925
				6		.50	0.42	0.60	.61	2.67	2.67	2.67	1.59	0.842
				7		.50	0.38	0.60	.62	2.67	2.71	2.67	1.57	0.777
5	9	.68	.32	2	2	.36	0.39	1.06	.24	2.21	1.22	5.80	5.92	0.374
				3		.36	0.38	0.96	.30	2.21	1.50	5.80	5.36	0.962
				4		.36	0.36	0.91	.33	2.21	1.66	5.80	5.11	1.125
				5		.36	0.33	0.89	.35	2.21	1.77	5.80	4.97	1.184
				6		.36	0.30	0.87	.36	2.21	1.85	5.80	4.88	1.207
				7		.36	0.28	0.86	.37	2.21	1.91	5.80	4.81	1.214
9	5	.68	.32	2	2	.64	1.23	0.77	.69	5.80	4.97	2.21	1.04	1.723
				3		.64	0.96	0.77	.70	5.80	5.32	2.21	0.98	1.358
				4		.64	0.80	0.78	.70	5.80	5.53	2.21	0.96	1.136
				5		.64	0.68	0.78	.71	5.80	5.67	2.21	0.94	0.986
				6		.64	0.60	0.78	.71	5.80	5.77	2.21	0.93	0.877
				7		.64	0.54	0.78	.71	5.80	5.85	2.21	0.92	0.794

according to whether ω_1 is greater than or less than ω_2. When $\omega_1 = \omega_2 = 5$, $\pi(\eta^*)$ declines with increases in δ_1; the same is true when Mr. 1 is relatively rich (that is, when $\omega_1 = 9$). However, for $\omega_2 = 9$ and $\omega_1 = 5$, $\pi(\eta^*)$ grows as Mr. 1 becomes more powerful. Mr. 1 decreases his lobbying by more than one-half, from 1.2 to .54, as δ_1 increases with $\omega_1 = 9$, but η_2^* increases slightly. This is accompanied by an increase in the absolute value of the cost of trade $\mathbf{P}\cdot\tilde{z}(p(\eta^*))$, which is negative here. With $\omega_2 = 9$, $\eta_1^* + \eta_2^*$ declines modestly, while $\mathbf{P}\cdot\tilde{z}(p(\eta^*))$ goes from -1.1 to +.06. The difference, $\pi(\eta^*)$, increases markedly as a result. We note that when one agent is rich, and the other becomes more influential, the value of trade operations changes sign because $p(\eta^*)$ goes from less to greater than p^*.

Table 4-10 highlights the effect of increasing ω_1 on $\pi(\eta^*)$ for various levels of δ_1. There, $\pi(\eta^*)$ generally increases as ω_1 increases. This is due to an increase in $\eta_1^* + \eta_2^*$. However, $\pi(\eta^*)$ may also decline for a bit as ω_1 increases. This occurs for δ_1 very large, and at ω_1 near ω_2. In Table 4-11, δ_1 again takes three values, but ω_2 is increased. In this case, $\pi(\eta^*)$ achieves a maximum in ω_2 before declining; this is the opposite of the previous example.

The five examples with $\pi(\eta^*) < 0$ are quite instructive. They show that the combination of relative wealth and influence are critical in determining whether $\pi(\eta^*)$ is greater than or less than zero. When Mr. 1 is both rich and influential, the budget surplus is positive. When Ms. 2 is rich Mr. 1 is influential, π is again positive. However, when $\omega_2 > \omega_1$ and the δ_1 are both equal to 2, the lobbying price becomes much smaller than p^*, while the excess demand for good 1 is large and positive. This brings $\pi(\eta^*)$ below

TABLE 4-10 *Effect on $\pi(\eta)$ of increasing ω_1 as δ_1 moves.*

ω_1	ω_2	α	β	δ_1	δ_2	p^*	η_1^*	η_2^*	p^η	U_1^*	U_1^η	U_2^*	U_2^η	$\pi(\eta)$
5	5	.68	.30	2	2	.48	0.62	0.61	.49	2.61	1.95	2.76	2.09	1.240
6						.53	0.81	0.62	.57	3.32	2.67	2.62	1.75	1.391
7						.57	0.97	0.66	.62	4.08	3.39	2.50	1.52	1.476
8						.60	1.10	0.70	.65	4.86	4.13	2.40	1.33	1.560
9						.63	1.21	0.74	.68	5.68	4.90	2.32	1.16	1.640
10						.65	1.31	0.77	.70	6.53	2.69	2.24	1.01	1.718
11						.67	1.40	0.81	.72	7.40	6.52	2.18	0.87	1.794
12						.69	1.48	0.84	.73	8.31	7.36	2.12	0.73	1.869
13						.71	1.55	0.87	.75	9.23	8.23	2.07	0.60	1.923
14						.72	1.62	0.90	.76	10.1	9.13	2.03	0.47	2.015
5	5	.68	.30	6	2	.48	0.41	0.56	.60	2.61	2.61	2.76	1.72	0.791
6						.53	0.47	0.61	.64	3.32	3.37	2.62	1.50	0.754
7						.57	0.52	0.66	.66	4.08	4.11	2.50	1.34	0.759
8						.60	0.56	0.71	.68	4.86	4.90	2.40	1.19	0.773
9						.63	0.59	0.75	.70	5.68	5.71	2.32	1.04	0.791
10						.65	0.63	0.78	.72	6.53	6.55	2.24	0.91	0.819
11						.67	0.65	0.82	.73	7.40	7.41	2.18	0.78	0.849
12						.69	0.68	0.85	.74	8.31	8.30	2.12	0.66	0.880
13						.71	0.71	0.88	.75	9.23	9.22	2.07	0.53	0.914
14						.72	0.73	0.91	.77	10.1	10.1	2.03	0.41	0.949
5	5	.68	.30	7	2	.48	0.37	0.56	.61	2.61	2.68	2.76	1.70	0.725
6						.53	0.42	0.61	.64	3.32	3.43	2.62	1.48	0.681
7						.57	0.46	0.66	.67	4.08	4.18	2.50	1.32	0.683
8						.60	0.50	0.71	.68	4.86	4.97	2.40	1.18	0.694
9						.63	0.53	0.75	.70	5.68	5.78	2.32	1.04	0.711
10						.65	0.56	0.78	.72	6.53	6.63	2.24	0.90	0.733
11						.67	0.58	0.82	.73	7.40	7.50	2.18	0.77	0.760
12						.69	0.61	0.85	.74	8.31	8.39	2.12	0.65	0.789
13						.71	0.63	0.88	.75	9.23	9.31	2.07	0.53	0.830
14						.72	0.65	0.91	.77	10.1	10.2	2.03	0.41	0.853

TABLE 4-11 *Effect on $\pi(\eta)$ of increasing ω_2 as δ_1 moves.*

ω_1	ω_2	α	β	δ_1	δ_2	p^*	η_1^*	η_2^*	p^η	U_1^*	U_1^η	U_2^*	U_2^η	$\pi(\eta)$
5	5	.68	.30	2	2	.48	0.62	0.61	.47	2.61	1.95	2.76	2.09	1.240
	6					.44	0.52	0.73	.39	2.46	1.68	3.50	2.99	1.162
	7					.40	0.45	0.84	.31	2.35	1.47	4.28	3.95	0.967
	8					.37	0.40	0.94	.26	2.25	1.31	5.09	4.98	0.676
	9					.34	0.36	1.04	.22	2.16	1.19	5.94	6.08	0.296
	10					.32	0.33	1.13	.18	2.09	1.08	6.81	7.24	−0.175
	11					.30	0.30	1.21	.16	2.03	0.98	7.71	8.48	−0.740
	12					.28	0.27	1.29	.14	1.97	0.90	8.63	9.80	−1.407
	13					.27	0.25	1.37	.12	1.92	0.83	9.58	11.1	−2.185
	14					.25	0.23	1.44	.11	1.88	0.76	10.5	12.6	−3.087
5	5	.68	.30	5	2	.48	0.45	0.56	.59	2.62	2.53	2.76	1.75	0.872
	6					.44	0.40	0.63	.50	2.46	2.24	3.50	2.54	1.054
	7					.40	0.37	0.71	.43	2.35	2.03	4.28	3.37	1.142
	8					.37	0.34	0.79	.38	2.25	1.86	5.09	4.23	1.166
	9					.34	0.32	0.86	.33	2.16	1.73	5.94	5.14	1.144
	10					.32	0.30	0.92	.30	2.09	1.62	6.81	6.08	1.086
	11					.30	0.28	0.99	.27	2.03	1.52	7.71	7.07	0.988
	12					.28	0.27	1.08	.24	1.97	1.44	8.63	8.10	0.883
	13					.27	0.25	1.10	.22	1.92	1.36	9.58	9.17	0.743
	14					.25	0.24	1.15	.20	1.88	1.30	10.5	10.2	0.578
5	5	.68	.30	7	2	.48	0.37	0.56	.61	2.61	2.68	2.76	1.70	0.725
	6					.44	0.34	0.62	.52	2.46	2.39	3.50	2.47	0.950
	7					.40	0.31	0.69	.46	2.35	2.17	4.28	3.27	1.081
	8					.37	0.29	0.76	.40	2.25	2.00	5.09	4.11	1.150
	9					.34	0.27	0.83	.36	2.16	1.87	5.94	4.98	1.176
	10					.32	0.26	0.89	.32	2.09	1.75	6.81	5.90	1.170
	11					.30	0.24	0.95	.29	2.03	1.66	7.71	6.85	1.138

zero.

The first ten rows of Tables 4-10 and 4-11 show the importance of our chapter two assumption of own good bias, but in a new way. Since for each i we have assumed that $\omega_i^{-i} = 0$, and that agents prefer their own good (here, this means, roughly, that $\alpha > 0.5$ and $\beta < 0.5$), if $p(\eta^*)$ goes far below p^*, then the value of excess demand at the competitive price p^*, which is a negative term in the expression for $\pi(\eta^*)$, becomes large and positive. If $p(\eta^*)$ goes much above p^* with the same Cobb-Douglas utility functions, then the value of excess demand is negative, and it becomes a positive term in $\pi(\eta^*)$.

4.4 SYMMETRIC ECONOMIES

As was mentioned in the introduction, if $\omega_1 = \omega_2$ and $\delta_1 = \delta_2$, and $\alpha + \beta = 1$, then the economy is symmetric in the following sense. The corresponding Edgeworth box is a square, and the competitive equilibrium price p^* equals 0.5. It is also true that $U_1^* = U_2^*$. Because the influence parameters are equal, agents choose identical equilibrium lobbying levels: $\eta_1^* = \eta_2^*$. Due to the pricing function's form, this implies that $p(\eta^*) = 0.5$. In addition, we have $U_1(\eta^*) = U_2(\eta^*)$, and finally that $\tilde{z}(p(\eta^*)) = 0$, or that domestic markets clear after lobbying without trade. This clearly implies that $\mathbf{P}^* \cdot \tilde{z}(p(\eta^*)) = 0$, so that $\pi(\eta^*) = \eta_1^* + \eta_2^*$.

The intuitive interpretation of this situation is that two groups with divergent interest in a government's choice of economic policy are both willing to devote resources to lobbying efforts to change the policy, but they are equally effective. As a result, the government is pressed equally from both of the interests, and, not being swayed in either direction, leaves the policy unchanged. In the process, however, both sides have used some resources in lobbying. This leaves them worse off than they would have been if no political activity had taken place. Their decisions to participate, though, were optimal for each of them because of the prisoners' dilemma nature of the lobbying game.

In Table 4-12, we present the results of solving the lobbying equilibrium system for 24 symmetric example economies. There, α and β remain fixed, but the ω_i and δ_i are varied. Three things are apparent from this table. First, and not surprisingly, whenever the η_i^* increase, after-lobbying utilities $U_i(\eta^*)$ decrease. With no price effects present, additional lobbying donation affect utilities only through a negative income effect. Second, the optimal lobbying levels η_i^* are non-linear in the influence parameters. That is, they increase and then decline as the δ_i increase. Finally, while there are non-linear relationships between some variables, it turns out that the relationship between the η_i^* and the *decrease* in utility resulting from lobbying, $U_i(\eta^*) - U_i^*$, is linear.

Table 4-13 includes the results of experiments manipulating the preference parameters α and β along with the δ_i. We see there that as the strength of agents' preference for their own good increases, they lobby less and their utility levels after lobbying increase. This is true because agents know that they will spend a certain share of their after-lobbying wealth on good i. As that share increases, which is the same as α increasing, the gain in utility resulting from a favorable price movement declines because their utility derives less from purchases of their

TABLE 4-12 *Effect on* U_1^{η}, η_1, *and* $\pi(\eta)$ *of changing* δ_1, ω_1. *Symmetric economies.*

ω_1	α	β	δ_1	p^*	η_1^*	η_2^*	$p(\eta)$	$U_1^* = U_2^*$	$U_1^{\eta} = U_2^{\eta}$	$\pi(\eta)$
4	.60	.40	1.0	0.5	0.535	0.535	0.5	2.04	1.495	1.069
4			1.5	0.5	0.635	0.635	0.5	2.04	1.393	1.269
4			2.0	0.5	0.619	0.619	0.5	2.04	1.409	1.238
4			4.0	0.5	0.479	0.479	0.5	2.04	1.552	0.957
4			6.0	0.5	0.385	0.385	0.5	2.04	1.648	0.770
4			8.0	0.5	0.324	0.324	0.5	2.04	1.710	0.647
5	.60	.40	1.0	0.5	0.767	0.767	0.5	2.55	1.768	1.534
5			1.5	0.5	0.783	0.783	0.5	2.55	1.752	1.565
5			2.0	0.5	0.728	0.728	0.5	2.55	1.808	1.457
5			4.0	0.5	0.533	0.533	0.5	2.55	2.007	1.066
5			6.0	0.5	0.421	0.421	0.5	2.55	2.121	0.842
5			8.0	0.5	0.351	0.351	0.5	2.55	2.193	0.702
6	.60	.40	1.0	0.5	0.952	0.952	0.5	3.06	2.090	1.904
6			1.5	0.5	0.902	0.902	0.5	3.06	2.140	1.805
6			2.0	0.5	0.817	0.817	0.5	3.06	2.227	1.685
6			4.0	0.5	0.577	0.577	0.5	3.06	2.472	1.154
6			6.0	0.5	0.451	0.451	0.5	3.06	2.601	0.901
6			8.0	0.5	0.373	0.373	0.5	3.06	2.680	0.746
8	.60	.40	1.0	0.5	1.238	1.238	0.5	4.08	2.818	2.475
8			1.5	0.5	1.090	1.090	0.5	4.08	2.969	2.179
8			2.0	0.5	0.957	0.957	0.5	4.08	3.105	1.914
8			4.0	0.5	0.647	0.647	0.5	4.08	3.421	1.295
8			6.0	0.5	0.498	0.498	0.5	4.08	3.574	0.995
8			8.0	0.5	0.409	0.409	0.5	4.08	3.665	0.817

TABLE 4-13 *Effect on* U_1^{η}, η_1, *and* $\pi(\eta)$ *of changing* α, β. *Symmetric economies.*

ω_1	α	β	δ_1	p^*	η_1^*	η_2^*	$p(\eta)$	$U_1^* = U_2^*$	$U_1^{\eta} = U_2^{\eta}$	$\pi(\eta)$
4	.60	.40	1.0	0.5	0.53	0.53	0.5	2.04	1.495	1.069
	.60	.40	1.5	0.5	0.63	0.63	0.5	2.04	1.393	1.269
	.60	.40	2.0	0.5	0.61	0.61	0.5	2.04	1.409	1.238
	.60	.40	4.0	0.5	0.47	0.47	0.5	2.04	1.552	0.957
	.60	.40	6.0	0.5	0.38	0.38	0.5	2.04	1.648	0.770
	.60	.40	8.0	0.5	0.32	0.32	0.5	2.04	1.710	0.647
4	.65	.35	1.0	0.5	0.42	0.42	0.5	2.09	1.650	0.847
	.65	.35	1.5	0.5	0.57	0.57	0.5	2.09	1.495	1.143
	.65	.35	2.0	0.5	0.57	0.57	0.5	2.09	1.494	1.145
	.65	.35	4.0	0.5	0.45	0.45	0.5	2.09	1.618	0.908
	.65	.35	6.0	0.5	0.36	0.36	0.5	2.09	1.709	0.735
	.65	.35	8.0	0.5	0.31	0.31	0.5	2.09	1.769	0.620
4	.70	.30	1.0	0.5	0.26	0.26	0.5	2.17	1.881	0.536
	.70	.30	1.5	0.5	0.49	0.49	0.5	2.17	1.636	0.987
	.70	.30	2.0	0.5	0.51	0.51	0.5	2.17	1.610	1.035
	.70	.30	4.0	0.5	0.42	0.42	0.5	2.17	1.710	0.851
	.70	.30	6.0	0.5	0.34	0.34	0.5	2.17	1.794	0.695
	.70	.30	8.0	0.5	0.29	0.29	0.5	2.17	1.852	0.589
4	.75	.25	1.0	0.5	0.00	0.00	0.5	2.28	2.280	0.000
	.75	.25	1.5	0.5	0.38	0.38	0.5	2.28	1.836	0.777
	.75	.25	2.0	0.5	0.44	0.44	0.5	2.28	1.769	0.895
	.75	.25	4.0	0.5	0.39	0.39	0.5	2.28	1.834	0.782
	.75	.25	6.0	0.5	0.32	0.32	0.5	2.28	1.911	0.647
	.75	.25	8.0	0.5	0.27	0.27	0.5	2.28	1.965	0.552

opponent's good, which requires outlays of wealth. The relationship between δ_i and $U_i(\eta^*)$ for each i is the same as above for a fixed α, β pair. The same relationships hold for larger endowments ω_i.

Note that when γ = (4, 4, .75, .25, 1, 1), lobbying contributions are effectively zero (they are positive in the fifth decimal place, as is $\pi(\eta^*)$. When δ_i increase from 1.0, lobbying increases fairly quickly, as it does when α moves to .70 or when ω_i increase from 4.0. This example, then, lies on the edge of the set of economies at which SALE($\mathcal{E}$) $\neq \emptyset$. We may conclude from it that the combination of a high degree of preference by each i for good i, a low influence parameter, and relatively low wealth leads people to not lobby very much. When the δ_i both grow, lobbying first increases while the agents beat upon each other and then declines as they find that later contributions are not very productive.

4.5 CONCLUSIONS

In this chapter we have endeavored to show how lobbying behavior affects equilibrium economic outcomes, and how agents' characteristics interact to determine who is aided and who hindered by the lobbying program. The existence of strong active lobbying equilibria for concrete, straightforward examples lends some credence to the viability of the lobbying model.

In the experimental data we saw that wealth and influence have almost everything to do with who is successful in lobbying and who benefits from it. When one agent is both most influential and most wealthy, that individual gains mightily from the lobbying program over the competitive outcome. However, when one agent is most influential and the other most wealthy, the outcome is more ambiguous. The rich trader generally achieves the higher level of absolute utility, although the power of his or her opponent dictates the degree of loss he or she suffers in lobbying.

The net income $\pi(\eta^*)$ of the government is likely, for the formulation employed here, to meet the non-negativity requirement. It might fail to do so when Ms. 2 is very rich and the influence parameters are about equal. This is true because the homothetic utility functions leave the value of excess demand at price $\mathbf{P}^*$ large and positive for this case.

When the economy is symmetric in the sense noted above, the prisoners' dilemma nature of the lobbying game is emphasized. There, both agents are drawn into a strategic conflict which damages both of them. Were they able to agree to cooperate, as suggested in chapter three, they would both benefit, as they would if the lobbying game were replaced with an efficient tax/transfer scheme.

APPENDIX 4A

In this appendix the details of the derivation of response functions (4.5) and (4.10) are provided. As above, with $\eta = (\eta_1,\eta_2)$ taken as given, Mr. 1's optimization program is

$$\max_{x_1 \in \mathbb{R}^2_{++}} \quad U_1(x_1^1,x_1^2),$$

$$\text{subject to} \qquad P(\eta)\cdot(x_1^1,x_1^2) \le p(\eta)\cdot\tilde{\omega}_1,$$

where $\tilde{\omega}_1 = \omega_1 - \eta_1/p(\eta)$ is Mr. 1's "after-lobbying" endowment. The first step is to derive the η-dependent demand functions from this maximization program. The Lagrangian function associated with this constrained maximization program is given by

$$\mathcal{L}_1(x_1;\eta,\omega_1) = U_1(x_1) + \lambda\cdot[p(\eta)\cdot\tilde{\omega}^1 - P(\eta)\cdot(x_1^1,x_1^2)]. \tag{4A.1}$$

Differentiating this function with respect to the choice variables x_1^1 and x_1^2, and setting the resulting expressions equal to zero,

$$\partial\mathcal{L}_1/\partial x_1^1 = \partial U_1/\partial x_1^1 - \lambda\cdot p(\eta) = 0.$$

$$\partial\mathcal{L}_1/\partial x_1^2 = \partial U_1/\partial x_1^2 - \lambda\cdot(1-p(\eta)) = 0.$$

By the monotonicity of U_1, and under the assumption that the optimal bundle $x_1^* \in \mathbb{R}^2_{++}$, these derivatives must equal zero at the optimal pair x_1^*. What's more, these equalities constitute both a necessary and a sufficient set of conditions for (4A.1) to be maximized. They also yield

$$\frac{\partial U_1/\partial x_1^1}{\partial U_1/\partial x_1^2} = \frac{p(\eta)}{(1-p(\eta))}.$$

The utility function of Mr. 1 has been assumed to take the Cobb-Douglas form, which is given by:

$$U_1(x_1^1,x_1^2) = (x_1^1)^{\alpha}\cdot(x_1^2)^{(1-\alpha)}, \qquad (4.1)$$

where $\alpha \in (0,1)$. This expression easily yields

$$\frac{\partial U_1/\partial x_1^1}{\partial U_1/\partial x_1^2} = \left(\frac{\alpha}{1-\alpha}\cdot\frac{x_1^2}{x_1^1}\right) = \frac{p(\eta)}{(1-p(\eta))},$$

from which

$$x_1^1 = \frac{\alpha}{1-\alpha}\cdot\frac{(1-p(\eta))}{p(\eta)}\cdot x_1^2. \qquad (4A.2)$$

Inserting (4A.2) into the budget constraint,

$$(1-p(\eta))\cdot\left(\frac{\alpha}{1-\alpha}+1\right)\cdot x_1^2 = p(\eta)\cdot\tilde{\omega}^1.$$

Rearranging, we obtain

$$x_1^2(p(\eta)) = (1-\alpha)\cdot\left[\frac{p(\eta)}{1-p(\eta)}\right]\cdot\tilde{\omega}^1, \qquad (4.3)$$

which may in turn be inserted into (4A.2), yielding

$$x_1^1(p(\eta)) = \alpha\cdot\tilde{\omega}_1. \qquad (4.2)$$

Equations (4.2) and (4.3) were reported in the text of the chapter as the η-dependent demand functions of agent 1 for the two market goods. They may be inserted into his utility function, as they were above, to yield the following indirect utility function over η

$$V_1(\eta_1;\eta_2) = \left(\alpha\cdot\tilde{\omega}^1\right)^{\alpha}\cdot\left[(1-\alpha)\cdot\frac{p(\eta)}{1-p(\eta)}\cdot\tilde{\omega}^1\right]^{(1-\alpha)}. \qquad (4.4)$$

Our task is now to solve the second stage of Mr. 1's optimization problem by maximizing V_1 in η_1. First, let

$$f_1(\eta) = \left[\alpha\cdot\left(\omega^1 - \frac{\eta_1}{p(\eta)}\right)\right]^{\alpha}, \text{ and}$$

$$f_2(\eta) = \left[(1-\alpha)\cdot\frac{p(\eta)}{1-p(\eta)}\cdot\left(\omega_1 - \frac{\eta_1}{p(\eta)}\right)\right]^{(1-\alpha)}.$$

Clearly, $V_1(\eta_1;\eta_2) = f_1(\eta)\cdot f_2(\eta)$; thus, by the Product Rule, $\partial V_1(\eta_1;\eta_2)/\partial\eta_1 = f_1\cdot(\partial f_2/\partial\eta_1) + f_2\cdot(\partial f_1/\partial\eta_1)$. We wish to calculate this derivative and set it equal to zero, solving the resulting expression for the optimal level of η_1. We have that

$$\frac{\partial f_1}{\partial\eta_1} = \alpha^2\cdot\left[\alpha\cdot\left(\omega^1 - \frac{\eta_1}{p(\eta)}\right)\right]^{(\alpha-1)}\cdot\left[\frac{\eta_1\cdot(\partial p/\partial\eta_1) - p(\eta)}{(p(\eta))^2}\right],$$

$$\frac{\partial f_2}{\partial\eta_1} = (1-\alpha)^2\cdot\left[(1-\alpha)\cdot\frac{p(\eta)}{1-p(\eta)}\cdot\left(\omega^1 - \frac{\eta_1}{p(\eta)}\right)\right]^{-\alpha}\cdot\left[\frac{(\omega^1-\eta_1)\cdot(\partial p/\partial\eta_1)-(1-p(\eta))}{(1-p(\eta))^2}\right].$$

The partial derivative of V_1 with respect to η_1, then, is given by

$$\frac{\partial V_1}{\partial\eta_1} = (\alpha\cdot\tilde{\omega}^1)^{\alpha}\cdot(1-\alpha)^2\cdot\left[(1-\alpha)\cdot\frac{p(\eta)}{1-p(\eta)}\cdot\tilde{\omega}^1\right]^{-\alpha}\cdot\left[\frac{(\omega^1-\eta_1)\cdot(\partial p/\partial\eta_1)-(1-p(\eta))}{(1-p(\eta))^2}\right].$$

$$+\left[(1-\alpha)\cdot\frac{p(\eta)}{1-p(\eta)}\cdot\tilde{\omega}^1\right]^{(1-\alpha)}\cdot\alpha^2\cdot(\alpha\cdot\tilde{\omega}^1)^{(\alpha-1)}\cdot\left[\frac{\eta_1\cdot(\partial p/\partial\eta_1) - p(\eta)}{(p(\eta))^2}\right].$$

Setting this expression equal to zero, canceling terms, and rearranging, we find that

$$\left(\frac{1-\alpha}{\alpha}\right)\cdot\left(\frac{p(\eta)}{1-p(\eta)}\right)\cdot\left[(\omega^1-\eta_1)\cdot\frac{\partial p}{\partial\eta_1} - (1-p(\eta))\right] = p(\eta) - \eta_1\cdot\frac{\partial p}{\partial\eta_1}. \qquad (4A.2)$$

Letting $\varphi = ((1-\alpha)/\alpha)\cdot(p(\eta)/(1-p(\eta)))$, this equation may be expressed as

$$\eta_1\cdot(1-\varphi)\cdot\frac{\partial p}{\partial\eta_1} + \omega^1\cdot\varphi\cdot\frac{\partial p}{\partial\eta_1} = \left[1 + \left(\frac{1-\alpha}{\alpha}\right)\right]\cdot p(\eta),$$

which becomes, after dividing by $\partial p/\partial\eta_1$ and rearranging,

$$\eta_1 = \frac{1}{\partial p/\partial\eta_1}\cdot\left(\frac{p(\eta)}{\alpha}\right) - \left(\frac{(1-\alpha)}{\alpha}\cdot\frac{p(\eta)}{1-p(\eta)}\right)\cdot(\omega^1 - \eta_1).$$

This last may be manipulated to yield

$$\eta_1 = \frac{p(\eta)}{\alpha}\cdot\left[\frac{1}{\partial p(\eta)/\partial\eta_1} - \frac{(1-\alpha)\cdot(\omega^1-\eta_1)}{(1-p(\eta))}\right], \qquad (4.5)$$

which was to be found.

Similarly, with $\eta = (\eta_1, \eta_2)$ taken as given, Ms. 2's optimization program is

$$\max_{x_2 \in \mathbb{R}^2_{++}} \quad U_2(x_2^1, x_2^2),$$

$$\text{subject to} \qquad \mathbf{P}(\eta)\cdot(x_2^1, x_2^2) \le (1-p(\eta))\cdot\tilde{\omega}^2,$$

where $\tilde{\omega}^2 = \omega^2 - \eta_2/(1-p(\eta))$. The Lagrangian function arising from this problem is

$$\mathcal{L}_2(x_2;\eta,\omega^2) = U_2(x_2) + \lambda\cdot[(1-p(\eta))\cdot\tilde{\omega}^2 - \mathbf{P}(\eta)\cdot(x_2^1,x_2^2)]. \tag{4A.3}$$

Differentiating (4A.3) with respect to η_2, and setting the results to zero,

$$\partial\mathcal{L}_2/\partial x_2^1 = \partial U_2/\partial x_2^1 - \lambda\cdot p(\eta) = 0.$$

$$\partial\mathcal{L}_2/\partial x_2^2 = \partial U_2/\partial x_2^2 - \lambda\cdot(1-p(\eta)) = 0.$$

By the monotonicity of U_2, and under the assumption that the optimal bundle $x_2^* \in \mathbb{R}^2_{++}$, these derivatives must equal zero at the optimal pair x_2^*. What's more, these equalities constitute both a necessary and a sufficient set of conditions for (4A.3) to be maximized. They also yield

$$\frac{\partial U_2/\partial x_2^1}{\partial U_2/\partial x_2^2} = \frac{p(\eta)}{(1-p(\eta))}.$$

The utility function of Ms. 2 has been assumed to take the Cobb-Douglas form, which is given by:

$$U_2(x_2^1,x_2^2) = (x_2^1)^{\beta}\cdot(x_2^2)^{(1-\beta)}, \tag{4.6}$$

where $\beta \in (0,1)$. This expression easily yields

$$\frac{\partial U_2/\partial x_2^1}{\partial U_2/\partial x_2^2} = \left(\frac{\beta}{1-\beta}\cdot\frac{x_2^2}{x_2^1}\right) = \frac{p(\eta)}{(1-p(\eta))},$$

from which

$$x_2^1 = \frac{\beta}{1-\beta}\cdot\frac{(1-p(\eta))}{p(\eta)}\cdot x_2^2. \tag{4A.4}$$

Again using the budget constraint, this may be manipulated to achieve

$$x_2^1(p(\eta)) = \beta\cdot\left[\frac{1-p(\eta)}{p(\eta)}\right]\cdot\tilde{\omega}^2 \tag{4.7}$$

$$x_2^2(p(\eta)) = (1-\beta)\cdot\tilde{\omega}^2. \tag{4.8}$$

Now, inserting these demand expressions into U_2, we find the indirect demand function as follows.

$$V_2(\eta_2;\eta_1) = \left[\beta\cdot\frac{1-p(\eta)}{p(\eta)}\cdot\tilde{\omega}^2\right]^{\beta}\cdot\left((1-\beta)\cdot\tilde{\omega}^2\right)^{(1-\beta)}. \tag{4.9}$$

As above, our task is now to solve the second stage of Ms. 2's optimization problem by maximizing V_2 in η_2. First, let

$$g_1(\eta) = \left[\beta\cdot\frac{1-p(\eta)}{p(\eta)}\cdot\left(\omega^2 - \frac{\eta_1}{1-p(\eta)}\right)\right]^{\beta}, \text{ and}$$

$$g_2(\eta) = \left[(1-\beta)\cdot\left(\omega^2 - \frac{\eta_1}{1-p(\eta)}\right)\right]^{(1-\beta)}.$$

Again using the Product Rule, we have that $\partial V_2(\eta_2;\eta_1)/\partial\eta_2 = g_1\cdot(\partial g_2/\partial\eta_2) + g_2\cdot(\partial g_1/\partial\eta_2)$. We wish to calculate this derivative and set it equal to zero, solving the resulting expression for the optimal level of η_1. In a manner much like that used above for agent 1, we find that

$$\frac{\partial V_2}{\partial\eta_2} = -\left[\beta\left(\frac{1-p(\eta)}{p(\eta)}\right)\tilde{\omega}^2\right]^{\beta}\cdot(1-\beta)^2\left((1-\beta)\cdot\tilde{\omega}^2\right)^{-\beta}\cdot\left[\frac{\eta_2\cdot(\partial p/\partial\eta_2) + (1-p(\eta)}{(1-p(\eta))^2}\right] -$$

$$- \left((1-\beta)\cdot\tilde{\omega}^2\right)^{1-\beta}\cdot\beta^2\cdot\left[\beta\left(\frac{1-p(\eta)}{p(\eta)}\right)\tilde{\omega}^2\right]^{\beta-1}\cdot\left[\frac{(\omega^2 + \eta_2)\cdot(\partial p/\partial\eta_2) + p(\eta)}{(p(\eta))^2}\right].$$

Setting this derivative equal to zero, and solving for η_2, the result is indeed

$$\eta_2 = \frac{1 - p(\eta)}{\beta - 1} \cdot \left[\frac{1}{\partial p(\eta)/\partial \eta_2} + \frac{\beta \cdot (\omega^2 - \eta_2)}{p(\eta)}\right], \qquad (4.10)$$

which was to be found.

APPENDIX 4B

```
/*-----------------------------------------------------------------
LOBLOOP.G -- This program accomplishes three tasks:
     i.)   it loops through all possible parameter vectors;
    ii.)   it calculates n1star, n2star, and p(nstar) for each
           parameter vector using NLSYS; and
   iii.)   it stores these variables, along with the parameters, for
           each example economy.

       It also loads the required procs.

       Run NLSYS.SET before using this program.                    */
/*----------------------------------------------------------------*/
/* STEP 1:  Establish the loop which will carry the parameter vector
            through all possible values.
       ----------------------------------------------------------*/

clear w1, w2, d1, d2, a, b, pstar, phi;

output file = c:\gauss\pricsol1.out on;

let w1 = 4;
do while w1 le 4.7;                     /* Increment is 0.6        */

  let w2 = 4;
  do while w2 le 4.7;                   /* Increment is 0.6        */

    let a = 0.68;
    do while a le 0.73;                 /* Increment is 0.02       */

      let b = 0.32;
      do while b ge 0.27;               /* Increment is 0.02       */

        let d1 = 1.5;
        do while d1 le 5.0;             /* Increment varies        */

          let d2 = 1.5;
          do while d2 le 5.0;           /* Increment varies        */
/*----------------------------------------------------------------*/

zed = zeros(10,1);                 /* Name the output vector zed  */

zed[1:6,1] = w1 | w2 | a | b | d1 | d2;

/*  STEP 2: Calculate the equilibrium price pstar.
    ------------------------------------------------------------*/

    phi = ((1-a)/b)/(w1/w2);
    pstar = 1/(1+phi);
    zed[7,1] = pstar;
```

```
/*-----------------------------------------------------------------*/
/*  The following if statement selects the correct scaling factor
    for the pricing function, depending on whether pstar is
    le or gt 1/2.
    -------------------------------------------------------------*/

    if pstar le 0.5; const = 1;
    else; const = (1-pstar)/pstar; endif;

clear x1, x0;          /* These globals must be clear on each pass     */
let x0 = 1 1 0.5;      /* Name of starting values vector must be x0    */

/*-----------------------------------------------------------------*/
  vf = zeros(rows(x0),1);
  proc f(x);
/*-----------------------------------------------------------------*/
/*  STEP 3: In this proc we specify the equations to be solved as a
            function of the arguments.
    -------------------------------------------------------------*/

     local n1, n2, p;
     n1 = x[1,1]; n2 = x[2,1]; p = x[3,1];

vf[1,1] = (p/a)*(1/(pstar*d1*exp(-d1*n1))-((1-a)*(w1-n1))/(1-p))-n1;
vf[2,1] = ((1-p)/(b-1))*(1/(-pstar*d2*exp(-d2*n2))+(b*(w2-n2))/p)-n2;
vf[3,1] = pstar * (1 - const*(exp(-d1*n1) - exp(-d2*n2))) - p;
/*-----------------------------------------------------------------*/

     retp( vf );
     endp;

/*-----------------------------------------------------------------*/
/* SPECIFY OPTIONS
   --------------------------------------------------------------*/

   convtol   =  1e-6;     /* convergence tolerance.                   */
    prntit   =  0;        /* if 1, print on each iteration            */
   prntout   =  0;        /* if 1, print final output                 */
     fname   =  &f;       /* names proc containing functions          */
  gradname   =  &grad1;   /* specifies proc to compute the Jacobian   */
       jc0   =  0;        /* uses default initial values of Jacobian  */

/*-----------------------------------------------------------------*/
/* The following statement calls the solver     */

      x1 = nlsys(fname,x0,jc0,convtol,prntit,prntout);

      zed[8:10,1] = x1;
```

```
/*  The result vector zed is now complete.  It is ready to be sent
    to disk by the subroutine DATOUT.
    ---------------------------------------------------------------*/

gosub datout;

          if d2 < 2; d2 = d2 + 0.25;
          elseif d2 ge 2 and d2 < 3; d2 = d2 + 0.5;
          elseif d2 ge 3; d2 = d2 + 1; else; endif;
          endo;

        if d1 < 2; d1 = d1 + 0.25;
        elseif d1 ge 2 and d1 < 3; d1 = d1 + 0.5;
        elseif d1 ge 3; d1 = d1 + 1; else; endif;
        endo;

      b = b - 0.02;
      endo;

    a = a + 0.02;
    endo;

  w2 = w2 + 0.6;
  endo;

w1 = w1 + 0.6;
endo;

end;

/* --------------------- SUBROUTINE FOLLOWS -------------------- */

datout:

  zedt = zed';
  format 1,5;
  zedt;

return;
```

CHAPTER FIVE

SUMMARY AND CONCLUSIONS

> *"'I wish,' said I, 'you would let us throw the whole lot of these dollars down to them and leave them to fight it out amongst themselves, while we get a rest'*
>
> *"'Now you talk wild, Jukes,' says he, looking up in his slow way that makes you ache all over, somehow. 'We must plan out something that would be fair to all parties.'"*
>
> Joseph Conrad, *Typhoon*, 1902

Governments often establish economic policy in response to political pressure by interest groups. Unlike ships' captains, democratically chosen political authorities can seldom dictate "something fair" in distributing society's resources. Neither do they often, at their pleasure, leave individuals to "fight it out amongst themselves." Instead, governments whose mission it is to be responsive to the popular will may completely ignore that will only at some risk to themselves. When the means by which agents may influence government's economic policy are institutionalized, these agents face joint economic and political decision problems which differ crucially from the usual economizing decisions. Because resources are diverted from productive endeavors to marshal and to apply political pressure, the outcome may not be economically optimal from society's viewpoint.

The first goal of this study has been to devise a coherent equilibrium-based economic model of political behavior in which opposing interests might choose to devote resources to lobbying activity in competition over distortionary government price policy. The microeconomic decision problems faced by the two agents involve simultaneous economic and political choices. Taking his or her opponent's lobbying activity as given, each agent was asked to choose a lobbying level which determined the price and income levels entering his or her economic optimization program.

The individual decision problems of our agents were joined in a non-cooperative lobbying game which featured the strategic interaction between agents. A second goal was to show that the lobbying model possesses an equilibrium where agents respond optimally to each other, and the lobbying outcome meets an appropriately defined feasibility requirement. Finally, the third objective of the study was to compare the utility levels of agents after lobbying to the utility they would have achieved in the underlying economy devoid of any lobbying program.

In chapter two the lobbying model was devised, and equilibria for the lobbying game and the lobbying economy were defined. Using Debreu's (1952) theorem on the existence of equilibria in generalized games, the lobbying game was shown to possess an equilibrium. Because of a non-convexity which arose in the choice sets of agents' utility maximization problems, these were reformulated as "two-stage" indirect utility maximization problems. Then, employing the theorem required showing that indirect utility functions were quasiconcave in the political choice variables, a technical difficulty which was overcome in section 2.4. The equilibrium of the game was shown also to be an economic equilibrium in section 2.5. This result, based upon a generic political economic specification, adds to the extant literature, where the question of equilibrium existence in models of this style has been open to a certain degree.

In chapter three, the welfare properties of the lobbying equilibrium were compared analytically to equilibria in the corresponding perfectly competitive economies. It was shown how the lobbying outcome may be dominated, according to the Pareto criterion, by the competitive outcome. However, the possibility that a rank ordering among these two outcomes might not exist was also pointed out. A more interesting possibility, that the utility pair after lobbying was not available in the underlying economy, was mentioned and explained, although the possibility or impossibility of this result have not been established.

Also in chapter three, the possibility that government in the model should, if it wishes to enhance agents' utility levels, choose a (non-distortionary) transfer policy rather than the price policy, was investigated. It was shown that whenever the after-lobbying allocation is dominated by the competitive equilibrium, the government could, by instituting the appropriate tax/transfer scheme, have achieved an outcome which Pareto dominates the lobbying outcome. Also, the possibility for cooperation between agents was addressed. It was shown that, if agents could enter binding agreements, they would do well to ignore any pricing policy which leaves them both worse off than at the competitive equilibrium. Moreover, it was shown that those cases where the government can "do better" coincide precisely with the cases where agents can "do better."

Chapter four included the results of some numerical experiments into the equilibrium properties of the lobbying system. After selecting specific functional forms for utility functions and the government's pricing rule, implicit best response lobbying functions for agents were derived. These were solved numerically for lobbying equilibria, and their welfare properties were studied. Besides demonstrating that equilibria exist where agents *do* lobby, this chapter displayed evidence that two of the possibilities for welfare outcomes are possible. No examples of the third kind—where the lobbying outcome was not available in the undistorted economy—were discovered. The chapter also related the various properties of agents—their wealth, political influence, and their preference for goods—to the lobbying economy outcomes. There, we showed how changing the level of these properties affected utility and lobbying levels and the government's budget position.

A class of symmetric economies received attention because of the peculiar nature of the lobbying outcomes in these examples. The price didn't change, and markets cleared after lobbying. The utility levels of the two agents were identical, and lobbying always hurt both. This was an example of the prisoners' dilemma nature of some political processes in which strong adversaries are pitted against one another.

Left untouched by this work are several interesting problems in the political process of economic intervention. There is no collective action in the model, as groups and individuals are one and the same. If groups were made up of several members, then collective action problems (free-riding, etc.) could be modeled. Future research can also extend the role of the government from passive to active. Explicit treatment of the make-up and objectives of the government would improve the linkage between the model and both the real world and the political science literature.

The model might also be made dynamic. Many of the political decision problems involve environmental, natural resource, and public goods considerations which are inherently intertemporal. This model, it seems, could add to our understanding of how dynamically-minded agents enter into the political process to alter the course of government policy on these matters.

While the model upon which this study is based awaits these improvements, it is hoped that the results achieved here are useful and instructive. Whenever economic models are asked to account for political behavior, complications arise. At the least, this fact has been demonstrated here. More interesting are the insights into political economic phenomena which obtain from this abstract mathematical model of such phenomena.

REFERENCES

Anderson, K. and Y. Hayami. 1986. *The Political Economy of Agricultural Protection: East Asia in International Perspective.* Sydney: Allen and Unwin.

Applebaum, E. and E. Katz. 1986. Transfer seeking and avoidance: on the full social costs of rent seeking, *Public Choice* 48:175–181.

Arrow, K.J. 1974. *The Limits of Organization.* New York: W.W. Norton and Company.

Arrow, K.J. and G. Debreu. 1954. Existence of an equilibrium for a competitive economy, *Econometrica* 22:265–90. Reprinted in *Mathematical Economics: Twenty Papers of Gerard Debreu.* Cambridge: Cambridge University Press, 1982.

Arrow, K.J. and F. Hahn. 1971. *General Competitive Analysis.* Amsterdam: North-Holland Publishing Co.

Baumol, W.J. 1986. *Superfairness.* Cambridge: The MIT Press.

Becker, G.S. 1983. A theory of competition among pressure groups for political influence, *Quarterly J. of Economics* 93:371–400.

Benassy, J.-P. 1982. Developments in non-walrasian economics and the microeconomic foundations of macroeconomics, in *Advances in Economic Theory.* ed. W. Hildenbrand. Cambridge: Cambridge University Press.

Bhagwati, J. 1980. Lobbying and welfare, *J of Public Economics* 14:355–363.

Bhagwati, J. 1982. Directly unproductive profit-seeking (DUP) activities: a welfare-theoretic synthesis and generalization, *J. of Political Economy* 90:988–1002.

Bhagwati, J. 1983. DUP activities and rent seeking, *Kyklos* 36:634–37.

Bhagwati, J., R.A. Brecher, and T.N. Srinivasan. 1984. DUP activities and economic theory, in *Neoclassical Political Economy: The Analysis of Rent-Seeking and DUP Activities.* ed. D.C. Colander. Cambridge, MA: Ballinger Publishing Co.

Bhagwati, J. and T.N. Srinivasan. 1980. Revenue seeking: A generalization of the theory of tariffs, *J. of Political Economy* 88:1069–87.

Bhagwati, J. and T.N. Srinivasan. 1982. The welfare consequences of directly-unproductive profit-seeking (DUP) lobbying activities, *J. of International Economics* 13:33–44.

Border, K. 1985. *Fixed Point Theorems with Applications to Economics and Game*

Theory. Cambridge: Cambridge University Press.

Brennan, G. and J.M. Buchanan. 1985. *The Reason of Rules*. Cambridge: Cambridge University Press.

Brock, W.A. and S.P. Magee. 1975. The economics of pork-barrel politics. Report 7511, Center for Mathematical Studies in Business and Economics, University of Chicago.

Brock, W.A. and S.P. Magee. 1978. The economics of special interest politics: The case of the tariff, *American Economic Review* 68:246-50.

Buchanan, J.M. 1980. Rent seeking and profit seeking, in *Toward a Theory of the Rent Seeking Society*. ed. J.M. Buchanan, R.D. Tollison, and G. Tullock. College Station: Texas A&M University Press.

Buchanan, J.M., R.D. Tollison, and G. Tullock, eds. 1980. *Toward a Theory of the Rent Seeking Society*. College Station: Texas A&M University Press.

Buchanan, J.M. and G. Tullock. 1962. *The Calculus of Consent*. Ann Arbor: University of Michigan Press.

Cassing, J.H. and A.L. Hillman. 1985. Political influence motives and the choice between tariffs and quotas, *J. of International Economics* 19:279-290.

Coggins, J., T. Graham-Tomasi, and T.L. Roe. 1988. Existence of equilibria in lobbying economies. Bulletin 88-5, Economic Development Center, University of Minnesota.

Dasgupta, P. and E. Maskin. 1986a. The existence of equilibrium in discontinuous economic games, I: theory, *Review of Economic Studies* 53:1-26.

Dasgupta, P. and E. Maskin. 1986b. The existence of equilibrium in discontinuous economic games, II: applications, *Review of Economic Studies* 53:27-41.

Debreu, G. 1952. A social equilibrium existence theorem, *Proceedings of the National Academy of Sciences*, 38:886-93. Reprinted in *Mathematical Economics: Twenty Papers of Gerard Debreu*. Cambridge: Cambridge University Press, 1982.

Debreu, G. 1959. *The Theory of Value*. New Haven: Yale University Press.

Debreu, G. 1982. Existence of competitive equilibrium, in *Handbook of Mathematical Economics*. ed. K. Arrow and M. Intriligator. Amsterdam: North-Holland Publishing Company.

Diewert, W.E., M. Avriel, and I. Zang. 1981. Nine kinds of quasiconcavity and concavity, *J. of Economic Theory* 25:397-420.

Dinopoulos, E. 1983. Import competition, international factor mobility and lobbying responses: The Schumpeterian industry case, *J. of International Economics* 14:395-410.

Downs, A. 1957. *An Economic Theory of Democracy*. New York: Harper and Brothers.

Feenstra, R.C. and J.N. Bhagwati. 1982. Tariff seeking and the efficient tariff, in *Import Competition and Response*. ed. J.N. Bhagwati. Chicago: The University of Chicago Press.

Findlay, R. and S. Wellisz. 1982. Endogenous tariffs, the political economy of trade restrictions, and welfare, in *Import Competition and Response*. ed. J.N. Bhagwati and T.N. Srinivasan. Chicago: University of Chicago Press.

Findlay, R. and S. Wellisz. 1983. Some aspects of the political economy of trade restrictions, *Kyklos* 36:469-481.

Frey, B., W. Pommerehne, F. Schneider, and G. Gilbert. 1984. Concensus and dissensus among economists: An empirical inquiry, *American Economic Review* 74:986-994.

Geanakoplos, J. and H. Polemarchakis. 1986. Existence, regularity and constrained suboptimality of competitive allocations when the asset market is incomplete, in *Essays in Honor of Kenneth J. Arrow*, Vol. III. ed. W. Hildenbrand, R. Starr and D. Starrett. London: Cambridge University Press.

Geller, W. 1986. An improved bound for approximate equilibria, *Review of Economic Studies* 53:307-308.

Hahn, F. 1980. General equilibrium theory, *The Public Interest* (Special Issue):123-38.

Hansen, J.M. 1985. The political economy of group membership, *American Political Science Review* 79:79-96.

Harberger, A.C. 1954. Monopoly and resource allocation, *American Economic Review* 44:77-87.

Hardin, R. 1982. *Collective Action*. Baltimore: The Johns Hopkins University Press.

Harsanyi, J.C. 1977. *Rational Behavior and Bargaining Equilibrium in Games and Social Situations*. Cambridge: Cambridge University Press.

Hildenbrand, W. 1982. Core of an economy, in *Handbook of Mathematical Economics*. ed. K. Arrow and M. Intriligator. Amsterdam: North-Holland.

Hillman, A.L. and H.W. Ursprung. 1988. Domestic politics, foreign interests, and international trade policy, *American Economic Review* 78:729–745.

Hirschleifer, J. 1985. The expanding domain of economics, *American Economic Review* 75:53–68.

Johnson, H.G. 1960. The cost of protection and the scientific tariff, *J of Political Economy* 68:327–45.

Just, R., E. Lutz, A. Schmitz, and S. Turnovsky. 1978. The distribution of welfare gains from price stabilization, *J. of International Economics* 8:551–563.

Krueger, A.O. 1974. The political economy of the rent-seeking society, *American Economic Review* 64:291–303.

Krueger, A.O. 1988. The political economy of controls: American sugar, *N.B.E.R.* Working Paper No. 2504.

Lindbeck, A. 1976. Stabilization policies in open economies with endogenous politicians, *American Economic Review* 66:1–19.

Mas-Colell, A. 1985. *The Theory of General Economic Equilibrium: A Differentiable Approach.* Cambridge: Cambridge University Press.

Mayer, W. 1984. Endogenous tariff formation, *American Economic Review* 74:970–985.

Mayer, W. and R. Reizman. 1987. Endogenous choice of trade policy instruments, *J. of International Economics* 23:377–381.

Moe, T.M. 1980. *The Organization of Interests: Incentives and the Internal Dynamics of Political Interest Groups.* Chicago: The University of Chicago Press.

Mundell, R. 1962. Review of janssen, free trade, protection and customs unions, *American Economic Review* 52:622.

Nash, J.F. 1950. Equilibrium points in n-person games, *Proceedings of the National Academy of Sciences*, U.S.A. 36:48–49.

Nelson, D. 1988. Endogenous tariff theory: a critical survey, *American J. of Political Science* 32:796–837.

North, D. 1984. Three approaches to the study of institutions, in *Neoclassical Political Economy: The Analysis of Rent-Seeking and DUP Activities.* ed. D.C. Colander. Cambridge, MA: Ballinger Publishing Co.

Olson, M. 1965. *The Logic of Collective Action.* Cambridge: Harvard University Press.

Peltzman, S. 1976. Toward a more general theory of regulation, *J. of Law and Economics* 19:211-240.

Pittman, R. 1988. Rent-seeking and market structure: Comment, *Public Choice* 58:173-185.

Rausser, G.C., E. Lichtenberg, and R. Lattimore. 1982. Developments in theory and empirical applications of endogenous governmental behavior, in *New Directions in Econometric Modeling and Forecasting in U.S. Agriculture*. ed. G.C. Rausser. New York: North-Holland.

Roe, T. and E. Yeldan. 1988. An open economy model of political influence and competition among rent seeking groups. Bulletin 88-1, Economic Development Center, University of Minnesota.

Samuels, W.J. and N. Mercuro. 1984. A critique of rent-seeking theory, in *Neoclassical Political Economy: The Analysis of Rent-Seeking and DUP Activities*. ed. D.C. Colander. Cambridge, MA: Ballinger Publishing Co.

Shepsle, K.A. and B.R. Weingast. 1984. Political solutions to market problems, *American Political Science Review* 78:417:434.

Starrett, D.A. 1988. *Foundations of Public Economics*. Cambridge: Cambridge University Press.

Stigler, G.J. 1975. *The Citizen and the State: Essays on Regulation*. Chicago: The University of Chicago Press.

Timmer, C.P. 1986. *Getting Prices Right: The Scope and Limits of Agricultural Price Policy*. Ithaca: Cornell University Press.

Tollison, R.D. 1982. Rent seeking: a survey, *Kyklos* 35:575-602.

Tullock, G. 1967. The welfare costs of tariffs, monopolies, and theft, *Western Econ. J.* 5:224-232.

Tullock, G. 1980a. Rent seeking as a negative-sum game, in *Toward a Theory of the Rent-Seeking Society*. ed. J.M. Buchanan, R.D. Tollison, and G. Tullock. College Station: Texas A&M University Press.

Tullock, G. 1980b. Efficient rent seeking, in *Toward a Theory of the Rent-Seeking Society*. ed. J.M. Buchanan, R.D. Tollison, and G. Tullock. College Station: Texas A&M University Press.

Wellisz, S. and J.D. Wilson. 1986. Lobbying and tariff formation: A deadweight loss consideration, *J. of International Economics* 20:367-375.

Young, L. and S.P. Magee. 1986. Endogenous protection, factor returns and resource allocation, *Review of Economic Studies* 53:407-419.