COMPUTER MODELLING OF RANGELAND REGENERATION IN THE ARID ZONE OF WESTERN AUSTRALIA

By

K M WANG*, R B HACKER+, R K LINDNER* AND G S RICHMOND*

*Agricultural Economics, School of Agriculture
The University of Western Australia
+Rangeland Management, W A Department of Agriculture

Contributed Paper presented to the 32nd Annual Conference of the
Australian Agricultural Economics Society
University of Melbourne
9-11 February

The financial support of the Australian Wool corporation is gratefully acknowledged.

Abstract

Historically, sheep overstocking has occurred in the pastoral areas of Australia. This has caused a decline in range productivity. This trend is likely to continue unless long term management strategies which facilitate an improvement in range condition are devised. This study used stochastic dynamic programming to derive near-optimal decision rule for the stocking rate. Stochastic dynamic programming requires the derivation of transition probabilities between different states of range condition under various management strategies. A pastoral sheep-grazing simulation model was constructed to derive these transition probabilities. This model is driven mainly by rainfall and stocking rate, which simulates vegetation dynamics and sheep production under different management strategies. The asymptotic optimal decision rule shows that stocking rate is correlated positively with rainfall season and with forage availability. The marginal value of perennial forage is relatively more important in a dry season than in other seasons. The marginal value of ephemeral forage can be either positive or negative depending on season and available perennial and ephemeral forage.

Introduction

Historically, a combination of a lack of knowledge and a myopic approach to range management has tended to encourage woolgrowers to overstock dryland grazing properties. As a result, dominant shrub species which are an important feed source for sheep were reduced from dense to scattered communities within the rangelands and the valuable perennial species were replaced by less desirable pasture plants. In many instances, where stock tended to be concentrated around specific areas, ground cover was completely destroyed and soil erosion occurred. This resulted in a significant degradation of rangeland condition in many pastoral woolgrowing areas of Australia, which in turn has reduced the future financial viability of woolgrowers. This decline in rangeland productivity and woolgrower's long-run financial viability is likely to continue in the future unless optimal management strategies are discovered which allow woolgrowers to rehabilitate their financial capability, as well as the condition of their range.

The purpose of this study is to identify optimum rangeland management strategies to guide pastoral wool growers in the choice of stocking rate patterns over time, and of whether or not to apply different types of treatments to rehabilitate rangeland carrying capacity. These treatments include reseeding and water ponding, in conjunction with grazing management.

Since the rangeland environment is both dynamic and stochastic in nature and involves very significant intertemporal effects, a stochastic optimal control approach was adopted in this study to derive near-optimal rangeland management policies. This approach requires the derivation of the transition probabilities between different states of range condition under various management strategies. A simulation model of rangeland regeneration was constructed to derive these probabilities for various stocking rates and treatments. This model simulates vegetation dynamics, animal production, and the interaction among vegetation, grazing and cultural treatments. The derivation of optimal stocking rates was approximately formulated as a finite state Markov decision process and solved by combining linear programming and Bellman's successive approximation method. Optimal decisions on whether to apply a treatment or not for a given range condition will not be analysed until the results derived for optimal stocking rates have been cross checked using a validated simulation model.

A Conceptual Framework of Rangeland Management

The characteristics of the decision-making process under uncertainty in the rangeland environment are presented by Figure 1. As indicated by the figure, the range manager must periodically make management decisions after monitoring the state of the rangeland ecosystem to achieve the management objectives. The evolution of the rangeland ecosystem is affected by management decisions, and by climatic sequences (mainly rainfall events). As a consequence, the initial state of the rangeland ecosystem will be transformed into the next period states. In addition to the dynamic transition of the range condition, there is an economic return which comes from animal products. Again, at the beginning of each period, the manager observes the state of the rangeland ecosystem and thereby makes his decisions about stocking rate, etc. which together with the climate will affect the evolution of the system. The evolution of the ecosystem will generate economic returns as well as the following period state. This management decision cycle is repeated.

A Markovian Decision Model

The above management framework can be closely represented by a Markovian decision model. The formulation of a Markovian decision model involves the following components: an objective function; sets of state, exogenous and decision (control) variables; and a set of state transition equations. In the application of this decision model to rangeland management, the objective function is assumed to represent the motives of the decision-maker. In this study, it is assumed that the range manager is risk neutral. Therefore, the objective function can be specified as maximization of expected present value of the stream of net profits received. The state variables must be observable, and capture as closely as possible those aspects of past history which influence the changes of the rangeland condition and profits, and upon which the manager bases his decision. In this study we assume that range condition is jointly described by forage biomasses and desirable perennial plant density. The most

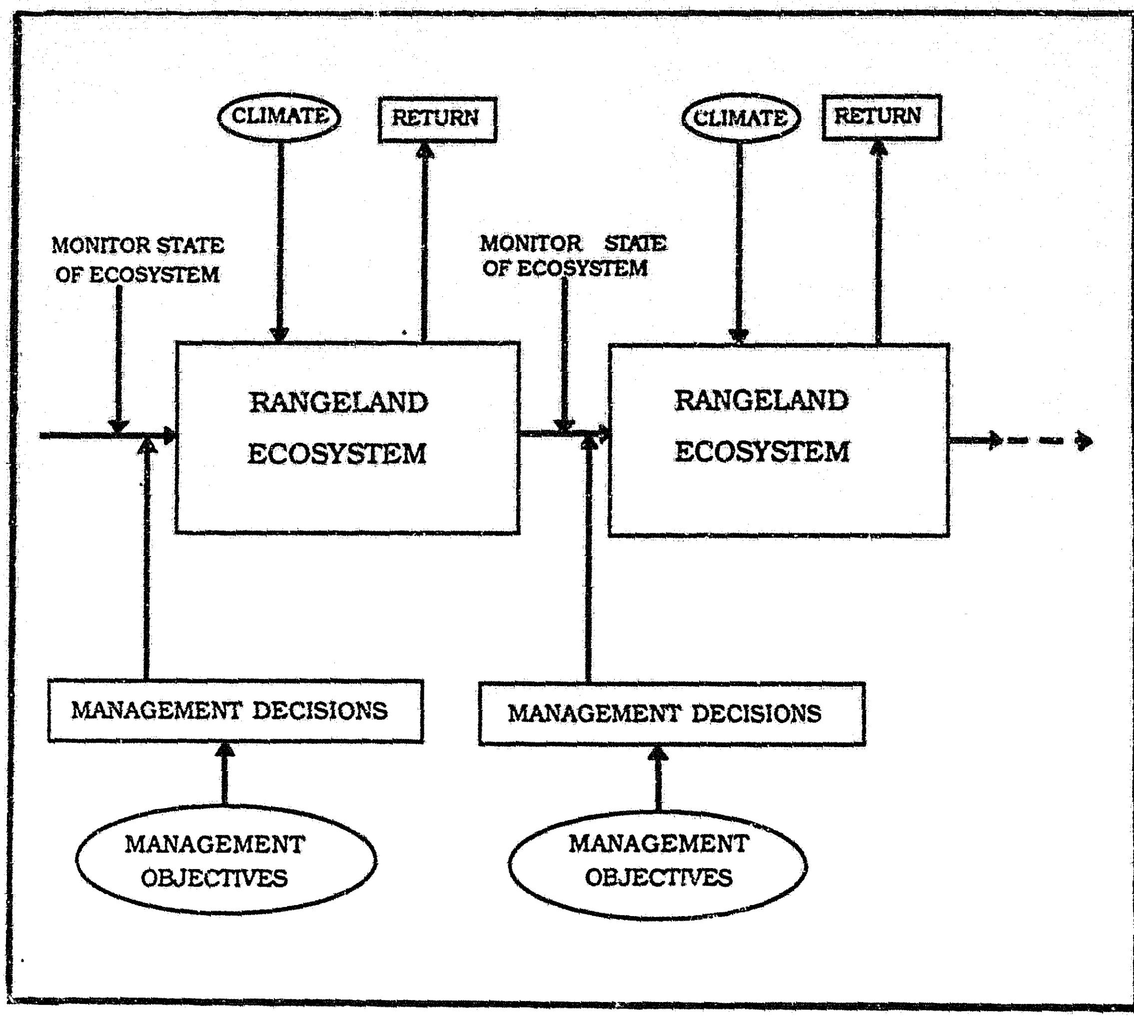

FIGURE 1. A CONCEPTIONAL FRAMEWORK OF RANGELAND MANAGEMENT

important exogenous variable in the pastoral area is rainfall which is both stochastic and beyond the control of the range manager. The control variables include those variables such as stocking rate and range regeneration investments (e.g. reseeding, bush clearing, fencing, cultivation, ponding etc.) which can be manipulated by the manager to influence range condition and future profits. The set of state transition equations make up the model of range dynamics which describes the evolution of rangeland condition over time. In other words, the model relates future range condition to current range condition for a given sequences of decisions and climatic patterns.

The formulation of the decision problem is to discover the optimal time paths of decision (control) variables to maximize the expectation of the objective function subject to the transition probabilities which are derived from state transition equations and a set of initial values for the states and exogenous variables. Mathematically, the formulation of the decision model can be specified as follows:

$$\text{Maximize } E_0 \sum_{t=0}^{\infty} \alpha^t \Pi(x_t, u_t, w_t) \qquad \text{(A.1)}$$

$$\text{Subject to } x_{t+1} = f(x_t, u_t, w_t) \qquad \text{(A.2)}$$

$$x_0 = \bar{x} \qquad \text{(A.3)}$$

where E_0 is the expectation held at initial period;

α is the discount factor;

$\Pi(x_t, u_t, w_t)$ is the periodic net return function;

x, u and w are the vector of state variables, the vector of control variables and the vector of random (or exogenous) variables, respectively;

$f(x_t, u_t, w_t)$ is a set of transition equations;

$\bar{x}$ is a vector of the initial values for the state variables.

The constraint of transition equations (A.2) can be replaced by the transition probability function $P_{ij}(u_k)$. A transition probability is defined as the probability that the next state will be j given that the current state is i and control $u=u_k$ is applied. It can be specified as $P_{ij}(u_k)=P(x_{t+1}=j \mid x_t=i,\ u_t=u_k)$. Therefore, it can be calculated from the transition equations because the state in period t+1 is a random variable, its conditional distribution depends on the current state and control.

A Simulation Model of Rangeland Regeneration in the Arid Zone of Western Australia

In order to derive the transition probilities a simulation model of rangeland regeneration in the Western Australian arid zone was constructed to simulate responses of range forage species and sheep intake of feeds under different stocking rates, treatments anu a variety of seasonal patterns. The essence of the model which simulates forage evolution and sheep production at paddock level, is illustrated in Figure 2. A four monthly interval, viz. three seasons: January-April, May-August and September-December, was used for the simulation. This division of the year corresponds to the periods of unreliable summer rainfall, reliable winter ranifall and reliable summer drought respectively.

The driving variables for the simulation model are rainfall, potential evaporation and temperature. These variables affect the soil store water. A soil water balance submodel (WATBAL) developed by Fitzpatrick et at. (1967) was used to derive the soil moisture index in terms of growth periods (number of wet pentads). A wet pentad is defined as a wet upper soil store during a five day period.

The soil moisture index together with the management decisions drive the vegetation dynamics. Vegetation is assumed to consist of three related components: ephemeral forage biomass, perennial forage biomass and desirable perennial plant density. The dynamics of each component are represented by a set of difference equation which add recruitment to the initial stock and subtract disappearance due to sheep consumption and other reasons. Recruitment basically is determined by the growth rate and initial stock level. Growth rate is usually affected by the stock level and the environmental factors such as rainfall and temperature. The total voluntary or actual feed intake of sheep within the paddock is mainly determined by liveweight, physiological state of the sheep, stocking rate, forage availability and quality. The total available perennial forage biomass together with the total sheep intake of perennial forage determines the degree of grazing pressure on the paddock. This grazing pressure together with

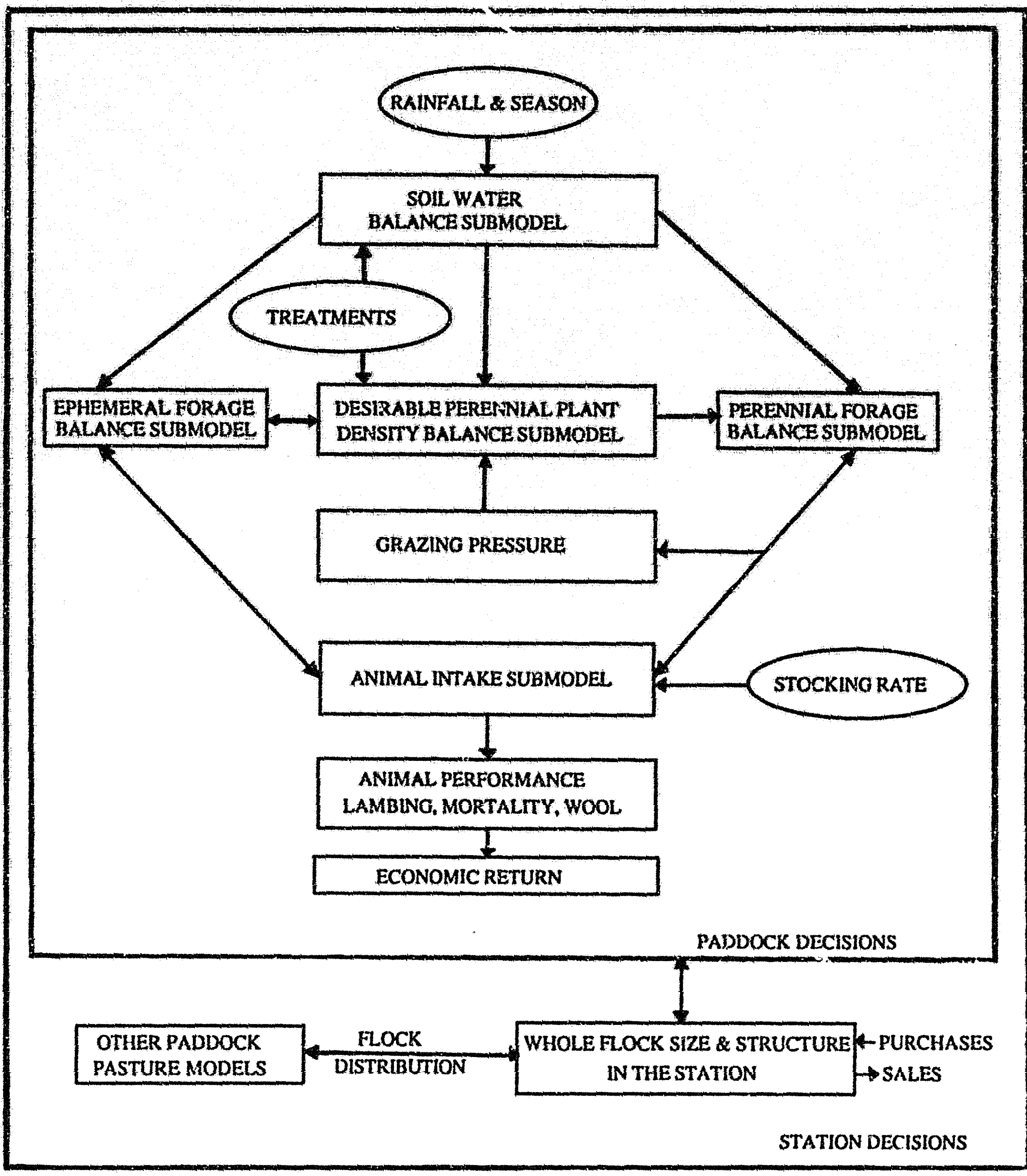

FIGURE 2. SIMULATION MODEL OF RANGELAND ECOSYSTEM

climatic factors and the application of treatments will influence the transition of the population of the desirable perennial species, which in turn will influence the forage biomass in the subsequent seasons. Sheep forage intake and stocking rate affect sheep mortality, lambing and wool production which determine the income to the woolgrower.

This single paddock simulation model was constructed for making paddock-level decisions such as stocking rate and whether to apply a treatment. However, in the future it will be incorporated into a station-level model to derive optimal strategies for the station-level decisions such as flock distribution, flock composition, timing of shearing, sheep purchases and sales, etc..

The main components of the simulation model are presented by the following submodels:

Soil water balance submodel

The soil water balance submodel describes changes over time in the percentage of soil water holding capacity in the soil and uses a set of soil moisture balance equations to estimate soil moisture in two stores of soil over a five-day (pentad) period. Store A corresponds to the soil moisture available to plants for evapotranspiration. Store B is the soil moisture held by the soil at less than the wilting point. Soil moisture is modelled separately in these two zones. A growth period (or wet pentad) begins whenever store A is recharged after being zero. If store B is also zero, then ranifall less runoff has to exceed 0.5 times the potential evaporation for resuming growth. In the study, the WATBAL model was used to estimate the frequency of wet pentads for a station in the arid zone of W.A.. Climatic records for over 69 years were used. The vegetation type of the station was typical of a chenopod dominant plant community.

Vegetation dynamic submodel

The vegetation dynamic submodel is composed of three components: desirable perennial plants density S_t^p, perennial forage biomass S_t^f and ephemeral forage biomass S_t^h. Each component is described by a difference equation (V.1), which has a four-month time step to reflect the rainfall season. Each difference equation indicates that the sum of initial stock level

S_t^i plus recruitment R_t^i minus the disappearance D_t^i from the stock makes up the season-ending stock S_{t+1}^i. Paddock-level recruitment is assumed to be a weighted average of the recruitment in the areas under different treatments (a fraction β is used to calculate the weight). Similarily, stock disappearance is a weighted average of the stock which disappeares in the areas under different treatments.

$$S_{t+1}^i=S_t^i+R_t^i-D_t^i \qquad i=p,h,f \qquad (V.1)$$

$$R_t^i=(1-\beta_1-\beta_2)R_{t,0}^i+\beta_1 R_{t,1}^i+\beta_2 R_{t,2}^i \qquad 0\le\beta_1\le1,\ 0\le\beta_2\le1 \qquad (V.2)$$

$$D_t^i=(1-\beta_1-\beta_2)D_{t,0}^i+\beta_1 D_{t,1}^i+\beta_2 D_{t,2}^i \qquad (V.3)$$

where

S_t^i is initial size of stock i at season t;

R_t^i is recruitment to stock i during season t;

D_t^i is disappearance from stock i during season t; where i=p for desirable perennial plants; i=f for perennial forage biomass; i =h for ephemeral forage biomass;

β_1 is the proportion of paddock under treatment 1;

β_2 is the proportion of paddock under treatment 2.

Recruitment of desirable perennial plants R_t^p is defined as the number of newly established young desirable perennial plants aged two years. Equation (R.1) assumes that the actual recruitment is proportional to the potential recruitment R_t^{p*}. The potential recruitment is defined as the gap between the environmental saturation level S_{max}^p and the current density S_t^p. This gap is adjusted by the proportional rate i.e. adjustment coefficient γ to give the actual recruitment. Since the establishment of a desirable perennial plant comes from the successful germination of the seeds and the survival of the following seedlings, the adjustment coefficient is calculated by multiplication of three indices: replacement capacity index RCI_{t-6}, germination index GMI_{t-6} and survival index SI_t. Each index ranges between 0 and 1.

$$R_{t,j}^p=\gamma R_t^{p*} \qquad j=0,1,2;\ 0\le\gamma\le1 \qquad (R.1)$$

$$=RCI_{t-6}GMI_{t-6}SI_t(S^p_{max}-S^p_t) \qquad (R.1.1)$$

where

$R^p_{t,j}$ is the recruitment of perennial plant under treatment j, where j=0 for no treatment; j=1 for treatment 1, i.e. cultivation; j=2 for treatment 2, i.e. ponding and reseeding;

γ is adustment coefficient which represents the proportion of potential recruitment that can be realized in season t under treatment j;

R^{p*}_t is the potential recruitment of desirable perennial plants in season t;

S^p_{max} is the desirable perennial plant density at climax condition;

RCI_{t-6} is the replacement capacity index which is used as a proxy for seed stocks;

GMI_{t-6} is the germination index during season t-6;

SI_t is the seedling survival index for recruitment.

Replacement capacity index is used as a proxy for the seed stock of desirable perennial plants in the soil and represents the capacity of the community to replenish itself. It is a function of the relative density of desirable perennial plants DP_{t-6} and type of treatments T_{t-6}. The time is lagged six seasons because it takes about two years for the seedlings to reach maturity. All the funtional forms in the study are shown in the Appendix A.1. The functional form assumed for replacement capacity index is a truncated hyperbolic in type (1.1). For the treatment involving reseeding, the function is specified by (1.3) and (1.4) which assume that the action of reseeding can immediately bring the replacement capacity to its maximum level after which this capacity decline due to loss of seed reserves through germination and other factors.

$$RCI_{t-6}=f_1(DP_{t-6},T_{t-6}) \qquad (R.1.2)$$

where

DP_{t-6} is the relative density of desirable perennial plants;

T_{t-6} is the types of treatment during season t-6.

Germination index is a function of wet pentads NWP_{t-6}, treatments and temperature b_t. A negative exponetial function (2.1) was assumed. The effects of temperature on germination is accounted by a seasonal factor b_t in (2.1).

$$GMI_{t-6}=f_2(NWP_{t-6},T_{t-6},b_t) \qquad (R.1.3)$$

where

NWP_t is climatic driving variable in season t, # growth periods, i.e. wet pentads;

b_t is a seasonal factor which is used to account for temperature effects on germination;

Survival index is a function of wet pentads, treatments, grazing pressure G_t and density of ephemeral forage DH_t. Time is lagged through zero to five periods because a successful survival requires two years to complete. The functional form assumed is a multiplication of three indices: soil moisture index SMI_t, grazing pressure index GI_t and ephemeral competition index CI_t. Soil moisture index is assumed to be a truncated linearly increasing function of wet pentads NWP_t (3.2)-(3.4). Different treatments and different seedling ages have different intercepts and slopes for this function. Grazing pressure index for the seedling survival is assumed to be a hyperbolic function of grazing pressure in the paddock (3.5). Ephemeral competition index is assumed to be a truncated linear decreasing function of the density of ephemeral forage biomass HD_t (3.6). Again, there are different slopes and intercepts for different seedling ages.

$$SI_t=f_3(NWP_{t-i},T_{t-i},G_{t-i},DH_{t-i}) \qquad i=0,1,\ldots 5 \qquad (R.1.4)$$

where

G_{t-i} is grazing pressure for the survival of perennial seedlings at stage t-i;

DH_{t-i} is the existing ephemeral forage density during season t-i.

Recruitment of perennial forage biomass R_t^f is calculated by discounting the potential growth rate PGR_t^f by two indices: soil moisture index SMI_t^f and growth capacity index GCI_t. Potential growth rate is defined by (R.2.1) as a gap between the environmental saturation level K_t^f and the current level of perennial forage biomass. Soil moisture index is a function of wet pentads and treatments. The functional form assumed is truncated linear in type (4.1). Growth capacity index is a function of perennial forage density DF_t. The functional form assumed is hyperbolic in type (5.1). The environmental carrying capacity is a function of relative density of desirable perennial plants. A negative exponential functional form (6.1) was assumed.

$$R^f_{t,j}=PGR^f_t SMI^f_t GCI_t \qquad j=0,1,2 \qquad (R.2)$$

where

PGR^f_t is potential growth rate of perennial forage biomass;

SMI^f_t is soil moisture index for the perennial forage growth;

GCI_t is growth capacity index for the perennial forage.

$$PGR^f_t=K^f_t-S^f_t \qquad (R.2.1)$$

where

K^f_t is the carrying capacity of perennial forage during season t.

$$SMI^f_t=f_4(NWP_t,T_t) \qquad (R.2.2)$$

$$GCI_t=f_5(DF_t) \qquad (R.2.3)$$

$$K^f_t=f_6(DP_t) \qquad (R.2.4)$$

where

DF_t is relative density of perennial forage biomass at the beginning of season t.

Recruitment of ephemeral forage biomass R^h_t is calculated by discounting the potential growth rate PGR^h_t by the soil moisture index SMI^h_t. Potential growth rate is defined by (R.3.1) where K^h_t is the environmental carrying capacity of ephemeral forage biomass. Soil moisture index is a function of wet pentads, initial ephemeral forage biomass, environmental ephemeral carrying capacity and treatments. The functional form assumed is truncated linear in type (7.1)-(7.4). Environmental ephemeral carrying capacity is a function of relative density of desirable perennial plants. The functional form assumed is negative exponential in type (8.1).

$$R^h_{t,j}=PGR^h_t SMI^h_t \qquad j=0,1,2 \qquad (R.3)$$

where

PGR^h_t is potennial growth rate of ephemeral forage biomass;

SMI^h_t is soil moisture index for the ephemeral forage growth;

$$PGR^h_t=K^h_t-S^h_t \qquad (R.3.1)$$

where

K^h_t is the carrying capacity of ephemeral forage biomass during season t;

$$SMI^h_t=f_7(NWP_t,S^h_t,K^h_t,T_t) \qquad (R.3.2)$$

$$K^h_t=f_8(DP_t) \qquad (R.3.3)$$

Disappearance of desirable perennial plants D^p_t is computed by multiplying the initial level of plant population by its disappearance rate d_t. The disappearance rate is disaggregated into two parts: natural mortality rate m_0 and the environmental factors induced mortality rate m_1. The environmental factors considered are grazing pressure, wet pentads, desirable perennial plant density and treatments. m_1 is calculated by discounting potential mortality rate by a weighted average index which include the above environmental factors (see (9.1)-(9.7)).

$$D^p_{t,j}=d_t S^p_t \qquad j=0,1,2 \qquad (D.1)$$

where

d_t is seasonal disappearance rate of desirable perennial plants;

$$d_t=m_0+m_1 \qquad (D.1.1)$$

$$m_1=f_9(G_{t-i},NWP_{t-i},DP_{t-i},T_{t-i}) \qquad i=0,1,\ldots 5 \qquad (D.1.2)$$

where

m_0 is natural mortality rate of desirable perennial plants;

m_1 environmental factors induced mortality rate of desirabel perennial plants.

Disappearance of perennial forage biomass D_t^f consists of two pools: intake by sheep and other losses due to non-consumptive reasons $DNCF_t$. Sheep intake is calculated by multiplying stocking rate U_t by the perennial forage intake of sheep FIP_t. The non-consumptive losses mainly are drying of the plant material. Therefore, it is calculated by multiplying the existing forage biomass by the drying rate δ_1. Since the drying losses and sheep intake occur at the same time a simultaneous equation system was used to explain these two pools. The drying rate of perennial forage biomass is calculated by discunting the maximal drying rate by two indices, density index DIF_t and soil moisture index $SMIF_t$, respectively (10.1). Density index is assumed truncated linear (10.2)and soil moisture index is hyperbolic in type (10.4).

$$D_{t,j}^f = U_t FIP_t + DNCF_t \qquad j=0,1,2 \qquad (D.2)$$

where

U_t is stocking rate of sheep per hectare in season t, sheep/ha;

FIP_t is the perennial forage intake of sheep, kg d.m./sheep;

$DNCF_t$ is the losses of perennial forage due to non-consumptive reasons, kg dm/ha;

$$DNCF_t = \delta_1 (S_t^f + R_t^f - U_t FIP_t) \qquad (D.2.1)$$

where

δ_1 is the drying rate of perennial forage during season t.

$$\delta_1 = f_{10}((S_t^f + R_t^f)/K_t^f, NWP_t, T_t) \qquad (D.2.2)$$

Disappearance of ephemeral forage biomass D_t^h is divided into three pools: intake by sheep, drying losses SL_t and trampling losses TL_t. Sheep intake is computed by multiplying stocking rate by the ephemeral forage intake of sheep FIE_t. Drying losses are calculated by (D.3.1) where δ_2 is the ephemeral forage drying rate. The drying rate is a function of ephemeral forage density DH_t, wet pentads and treatments. The calculation and functional form for the ephemeral drying rate are the same as those for perennials (11.1)-(11.3). Trampling losses are calculated by equation (D.3.2) where δ_3 is the trampling loss rate of ephemeral forage biomass. Trampling loss rate is a function of stocking rate. The functional form assumed is negative exponential in type (12.1). Trampling losses and drying losses are simultaneously determined with sheep intake.

$$D^h_{t,j}=U_t FIE_t+SL_t+TL_t \qquad j=0,1,2 \qquad (D.3)$$

where

FIE_t is the ephemeral forage intake of sheep, kg d.m./sheep;

SL_t is the non-consumptive losses of ephemeral forage due to is senescence;

TL_t is the non-consumptive losses of ephemeral forage due to trampling;

$$SL_t=\delta_2(S^h_t+R^h_t-U_t FIE_t-TL_t) \qquad (D.3.2)$$

where

δ_2 is the drying rate of ephemeral forage biomass during season t under treatment j;

$$TL_t=\delta_3(S^h_t+R^h_t-U_t FIE_t-SL_t) \qquad (D.3.3)$$

δ_3 is trampling loss rate of ephemeral forage biomass during season t;

$$\delta_2=f_{11}(DH_t,NWP_t,T_t) \qquad (D.3.4)$$

$$\delta_3=f_{12}(U_t) \qquad (D.3.5)$$

Sheep intake submodel

Sheep intake is composed of two parts: ephemeral forage intake and perennial forage intake. Ephemeral forage intake is calculated by discounting potential sheep intake by ephemeral forage availability index AIE_t and quality index QIE_t. AIE_t is a function of relative availability of ephemeral forage

$(S_t^h+R_t^h-DNCE_t)/(a*U_t)$. QIE_t is a function of the digestibility of ephemeral forage DG_t^h. Perennial forage intake is calculated in a same way as ephemeral except that it includes an additional index, sheep diet preference index $(1-AIE_tQIE_t)$. In this preference index, it is assumed that sheep select ephemeral forage preferentially. The consumption of perennial forage is limited by the extent to which the potential sheep intake is satisfied by the ephemeral forage consumption. Digestibilities of ephemeral and perennial forage are a function of their drying rate, respectively. The functional form assumed is hyperbolic in type (17.1) & (18.1).

$$FIE_t=aAIE_tQIE_t \qquad (I.1)$$

where

a is the conversion factor of potential sheep intake, a=180kg dm/sheep;

AIE_t is ephemeral forage availability index for sheep intake;

QIE_t is ephemeral forage quality index for sheep intake.

$$FIP_t=a(1-AIE_tQIE_t)AIP_tQIP_t \qquad (I.2)$$

where

AIP_t is perennial forage availability index for sheep intake;

QIP_t is perennial forage quality index for sheep intake.

$$AIE_t=f_{13}(S_t^h,R_t^h,DNCE_t,U_t) \qquad (I.3)$$

$$QIE_t=f_{14}(DG_t^h) \qquad (I.4)$$

where

DG_t^h is digestibility of ephemeral forage.

$$AIP_t=f_{15}(S_t^f,R_t^f,DNCF_t,U_t) \qquad (I.5)$$

$$QIP\ =f_{16}(DG_t^f) \qquad (I.6)$$

where

DG_t^f is digestibility of perennial forage.

$$DG_t^h = f_{17}(\delta_2) \tag{I.7}$$

$$DG_t^f = f_{18}(\delta_1) \tag{I.8}$$

Grazing prossure

Grazing pressure in the paddock is defined as the ratio of the perennial intake of sheep to the perennial forage availability. This is an index of the degree to which the pasture is grazed.

$$G_t = \frac{U_t * FIP_t}{S_t^f + R_t^f - DNCF_t} \tag{G.1}$$

Sheep performance submodel

Sheep population is increased by birth and decreased by mortality. Since this is a paddock model there is only one class of animal, e.g. breeding ewes or wethers, contributed for any simulation. In the environment of rangeland in W.A., lambing occurs at the end of season 2 and is a function of the plane of nutrition NI in current period and previous three periods. The lagged effects of nutrition are included to reflect the influence on lambing of factors such as ewe body condition at mating, early pregnancy nutrition and late pregnancy nutrition. The plane of nutrition is a function of sheep intake. The functional form assumed is linear in type (20.1). Mortality rate consists of two parts: natural mortality rate M_0 and the plane of nutrition induced mortality rate M_1. The functional form assumed for M_1 is exponential in type (21.1).

Wool clip per sheep is a function of digestible dry matter intake $DDMI_t$. The functional form assumed is linear in type (22.1). Digestible dry matter intake is calculated by equation (S.7).

$$TLS_{t+1} = A_t * U_t * (1 + L_t - M_t) \tag{S.1}$$

where

A_t is the area25Xsuitable for grazing in the paddock;

FLS_{t+1} is sheep flock size at the end of season t, # sheep/paddock;
L_t is lambing rate as a percentage of whole ewe population;
M_t is seasonal sheep mortality rate.

$$L_t = f_{19}(NI_{t-i}) \qquad i=0,1,\ldots 3 \qquad (S.2)$$

where
NI_t is the nutritional index for sheep during season t.

$$NI_{t-i} = f_{20}(FIE_t, FIP_t) \qquad (S.3)$$

$$M_t = M_0 + M_1 \qquad (S.4)$$

where
M_0 is natural seasonal death rate of sheep;
M_1 is the sheep mortality rate induced by the plane of nutrition.

$$M_1 = f_{21}(NI_t) \qquad (S.5)$$

$$WCS_t = f_{22}(DDMI_t) \qquad (S.6)$$

$$DDMI_t = FIE_t DG_t^h + FIP_t DG_t^f \qquad (S.7)$$

where
WCS_t is the average wool production per sheep per season, kg/sheep;
$DDMI_t$ is the digestible dry matter intake per sheep.

Seasonal net profit function

The net profit is calculated by selling the sheep and wool minus the marketing costs, shearing costs, other variable costs, initial expenditure for purchasing the sheep and treatment costs.

$$\Pi_t = ((PS_t - MS_t + (PW_t - MW_t)WCS_t - SC_t - VC_t)(1+L_t-M_t)/(1+r) - PB_t)A_t * U_t$$
$$- A_t \sum_{j=0}^{2} C_t^j \qquad (P.1)$$

where
PS_t is sale price of a sheep, dollars/sheep;
MS_t is marketing costs for selling a sheep, dollars/sheep;
PW_t is price of wool, dollars/kg;

MW_t is marketing costs for selling wool, dollars/kg wool;

SC_t is shearing costs per sheep, dollars/sheep;

VC_t is other variable costs per sheep eg. mulesing, mustering, maintenance, fuel and labour etc, dollars/sheep;

r is seasonal interest rate, r=2.67%;

PB_t is purchase price of sheep, dollars/sheep;

C^j is total costs of applying a treatment j, dollars/ha.

Method

Define V(i) as the maximum expected value of the discounted stream of returns in (A.1) for a given initial state x_0=i. The formulation of the range management problem can be solved by following Howard's dynamic programming (DP) method:

$$V(i)=\max_u \left(\bar{\pi}(i,u)+\alpha\sum_{j=1}^{N} P_{ij}(u)V(j)\right) \qquad (M.1)$$

where $\bar{\pi}$ is expected net profits: P_{ij} is transition probability under a given decision u, and i, j are the beginning state and ending state, respectively. The time subscripts were dropped because the solution is stationary. There are three methods for solving the above functional equation: successive approximation, policy iterative process and linear programming (LP) (see Derman 1970, Bertsekas 1976, Taha 1982). Successive approximation is essentially the DP algorithm, which starts by assigning an arbitrary value to V(j) and solving (M.1)) for V(i), V(j) is then replaced by V(i) and the process is repeated until the value of V(i) converges.

The two other methods, policy iteration and LP, determine the optimal policy in a finite number of iterations. They require solution of linear systems of equations or of a linear program of dimension as large as the number of points in the state space. The policy iterative process starts with an arbitrary policy, and consists of two basic components, called the value determination step and the policy improvement step. An improved policy is generated at every iteration. The iterative process ends when two successive policies are identical and the associated policy and value function are the optimal solutions. LP method transforms (M.1) into a LP format which is a minimization problem. The optimal solution can be determined by the degenerate constraints (i.e. slack equals zero).

The optimal solutions for stocking rate and treatments in this study can be determined by a two step procedure. First, by formulating the problem with only one control variable, e.g.stocking rate, and applying (M.1) to solve it. Second, after the optimal stocking rate is derived a set of simulation runs can be processed to obtain the optimal treatment decisions. The reduced forms of the rangeland ecosystem without applying treatments are as follows:

$$S^f_{t+1}=f(S^p_t,S^f_t,S^h_t,U_t,NWP_t,B_t,s_t) \qquad (M.2)$$

$$S^h_{t+1}=f(S^p_t,S^f_t,S^h_t,U_t,NWP_t,B_t,s_t) \qquad (M.3)$$

$$S^p_{t+1}=f(S^p_{t-i},S^f_{t-j},S^h_{t-j},U_{t-j},NWP_{t-i},B_{t-j},s_t) \qquad (M.4)$$
$$i=0,1,\ldots,6; \quad j=0,1,\ldots,5$$

in which

B_t is the effects of other exogeneous variables (mainly the parameters);

s_t is seasonal dependant index, s=1 for season 1, s=2 for season 2 and s=3 for season 3;

other variables have the same notation as before.

Note that there are a total of 31 state variables in the system. Equation (M.2) & (M.3) require that current perennial plant density (S^p_t), perennial forage biomass (S^f_t), ephemeral forage biomass (S^h_t) be entered as state variables. Equation (M.4) requires that first- through sixth-order lagged perennial plant densities, first- through fifth-order lagged perennial forage biomass, first- through fifth-order lagged ephemeral forage biomass, first- through fifth-order lagged sheep stocking rates and first- through sixth-order lagged number of wet pentads be entered as extra state variables. Moreover, since the model has three decision stages within the year we need to include seasonal index s as another state variable. Thus, the total number of state variables is 31. Stocking rate U_t is the only control variable and NWP_t is the random variable. Hence, in order to apply DP to the above rangeland simulation model, there is a total of 31 state variables. Since it is impractical to solve a model containing 31 state variables, some sort of approximation has to be developed.

The procedure of approximation adopted in the study is based on Burt, et al (1980). The idea of approximation is to use an approximation to V(j) in (M.1) and solve the one-stage maximization problem since the functional equation would be a static optimization problem if the function V(j) were known. There are various ways to arrive at an approximation to V(j). The better approximation to the original problem would give the solution closer to the opimum. In this study the proposed approximation method is to use a simplified model containing only three state variables to estimate V(j). All other state variables are suppressed. Three state variables are, perennial forage biomass, ephemeral forage biomass and seasonal index. This approximation can greatly simplify the calculations, although ideally desirable perennial plant density should also be included. The above approximation is based on the proposition that the three state variables carry the most information in the range management decision process and the amount of independent variation between perennial forage biomass and desirable perennial plant density are relatively small. In the simulation run of the reduced model, the suppressed state variables are kept at their long-run expected values or their average values. The approximation is to replace V(j) in (M.1) by an estimate of the solution V^* of the following three-state variable DP problem,

$$V^*(S^h,S^f,s)=\max_u \{\Pi(S^h,S^f,s,U)+\alpha\sum_{j=1}^{N} P_{ij}(U)V^*(S^h+R^h-D^h,S^f+R^f-D^f,s)\} \quad (M.5)$$

where seasonal index s will follow the following transition equations;

$$s_{t+1}=s_t+1 \quad \text{for season 1 and season 2} \quad (M.6)$$
$$=1 \quad \text{for season 3}$$

Equation (M.5) is solved using the LP method, which is a minimization problem with the number of activity equal thedimension in the state space and the number of constraints equal the dimensions of the state space multiplying the control space. The optimal solution occurs where the slack variable equals zero (i.e. the constraint is degenerate).

The other state variables in the simulation model are gradually phased into the expected present value function over six additional stages because the longest time-lag is six periods. Each stage the new lagged state variables will be introduced into the recursive equation (M.1) and additional iteration contained an increment to the number of state variables will be done to provide an improved approximation to V(j) in (M.1). The final iteration implicitly contained 31 state variables and generated a solution for a given initial state of the model.

The decision about whether to apply a treatment under a given range condition can be determined after the derivation of optimal stocking rates and their corresponding net present values. If the effects of a treatment on improving the range condition can last for 20 years, a simulation run of the regeneration model for 20 years can be done to calculate the potential increment to the profits resulted from improved range condition. Since after 20 years the transition of range condition will follow the same way as no treatment the treatment costs can be used to compare with the the expectation of net present value of potential augumented profits for a twenty-year period due to the treatments.

Data and implementation

The implementation of the simulation model requires biological and economic data to estimate the parameters and a compute package to solve the simultaneous nonlinear equations. Due to lack of data, some of the parameter values in the functions are informed guesses based on results reported in the literature or in unpublished material from communication with research workers, so the model presented herein is preliminary. Validation of the model using rangeland monitoring data is still in progress.

The state space for ephemeral forage and perennial forage was partitioned in 200 kg dry matter (kg dm)/ha intervals from 0-1200 kg dm/ha and 0-1400 kg dm/ha, respectively. The control space for stocking rates was partitioned into three rates, 0.1, 0.3 and 0.5 sheep/ha, respectively. These partitions are made to include possible ranges for the variables in the rangeland. It is also necessary to define the initial states and climatic patterns of the rangeland ecosystem The initial values for the state variables used in the study are given by Table 1, which correspond

TABLE 1. INITIAL STATE OF THE RANGELAND ECOSYSTEM, 1982

SEASON		S^p plants/ha	S^f kg dm/ha	S^h kg dm/ha	U sheep/ha	NWP # pentads
1980	1	5300	850	40	0.14	11
	2	5500	880	30	0.14	22
	3	5700	950	390	0.14	11
31	1	5850	830	30	0.12	8
	2	5150	680	45	0.12	16
	3	4940	630	15	0.12	2

to the 1980-1981 data for the station. The distribution of wet pentads derived from climatic records are presented by Table 2.

A list of the variables, functional forms and parameter values used in the simulation are presented in Appendices. The solutions of the nonlinear equations in the model were solved by the software package, TK!Solver on an NEC APCIV personal computer. The optimal solutions were calculated by using LP method which is to solve a minimization problem with 126 activities and 378 contraints.

Empirical results

The optimal decision rules for stocking rates derived from the simplified model containing only three state variables are useful in showing the general structure of the decision rule for grazing management. Since the stochastic process of rangeland dynamics is formulated as a periodically stationary system,i.e. three seasonal rainfall patterns per year, the optimal decision rules are also periodic over time and can be shown in the following three two-dimensional tables (Tables 3 to 5). These tables illustrate the optimal stocking rules and their corresponding net present values under three seasons.

As indicated by Table 3, the optimal stocking rates for season one are increasing with both ephemeral and perennial forage biomasses. For the lowest forage availability i.e. less than 200 kg dry matter (dm)/ha for both ephemeral and perennial forage at the beginning of season 1, the optimal stocking rate is 0.1 sheep/ha. The medium stocking rate 0.3 sheep/ha is optimal for most of the forage availability ranging from 0 to 400 kg dm/ha for perennials combined with 0-1200 kg dm/ha for ephemerals, although there are some exceptions to this pattern. The higher stocking rate of 0.5 sheep/ha appears optimal for higher levels of perennial availability (approximately 400-1400 kg dm/ha). Generally, the optimal stocking rate seems to vary with increasing perennial vegetation more so than with increasing available ephemeral forage, which is consistent with the long term nature of the range regeneration problem. The net present value also increases with a rise in the availability of perennials. However, this trend does not apply to the ephemerals. The lowest net present value of $80.92/ha occurs when both ephemeral and

TABLE 2. PROBABILITY DISTRIBUTION OF WET PENTADS DERIVED FROM WATBAL MODEL FOR THE STATION

NWP	FREQUENCY			CUMULATIVE DISTRIBUTION FUNCTION		
# PENTADS	SEASON 1	SEASON 2	SEASON 3	CDFI	CDF2	CDF3
0	2	0	9	0.03	0	0.13
1	3	0	10	0.07	0	0.28
2	5	0	11	0.14	0	0.43
3	4	0	9	0.2	0	0.56
4	9	0	9	0.33	0	0.69
5	3	1	1	0.38	0.01	0.71
6	11	0	0	0.54	0.01	0.71
7	4	2	4	0.59	0.04	0.76
8	4	0	5	0.65	0.04	0.84
9	5	2	6	0.72	0.07	0.93
10	3	0	1	0.77	0.07	0.94
11	4	0	2	0.83	0.07	0.97
12	1	2	1	0.84	0.1	0.99
13	2	3	0	0.87	0.15	0.99
14	2	3	0	0.9	0.19	0.99
15	2	2	1	0.93	0.22	1
16	1	3	0	0.94	0.26	1
17	3	4	0	0.99	0.32	1
18	0	6	0	0.99	0.41	1
19	0	7	0	0.99	0.51	1
20	1	6	0	1	0.6	1
21	0	5	0	1	0.68	1
22	0	8	0	1	0.79	1
23	0	4	0	1	0.85	1
24	0	11	0	1	1	1
TOTAL	69	69	69			

TABLE 3 OPTIMAL STOCKING RATES & THE CORRESPONDING NET PRESENT VALUE FOR SEASON 1

EPHEMERAL FORAGE '00 KG DM/HA	PERENNIAL FORAGE '00 KG DM/HA 0-2	2-4	4-6	6-8	8-10	10-12	12-14
			----sheep/ha---- -------$/HA------				
0-2	.1	.3	.3	.5	.5	.5	.5
	80.92	81.16	81.24	81.36	81.42	81.46	81.48
2-4	.3	.3	.5	.5	.5	.5	.5
	80.93	81.20	81.28	81.37	81.42	81.47	81.48
4-6	.3	.3	.5	.5	.5	.5	.5
	80.96	81.21	81.29	81.36	81.43	81.47	81.48
6-8	.3	.3	.5	.5	.5	.5	.5
	80.97	81.20	81.30	81.39	81.41	81.45	81.47
8-10	.3	.3	.5	.5	.5	.5	.5
	80.97	81.19	81.30	81.38	81.40	81.44	81.46
10-12	.3	.3	.5	.5	.5	.5	.5
	80.96	81.18	81.29	81.36	81.39	81.43	81.45

TABLE 4 OPTIMAL STOCKING RATES & THE CORRESPONDING NET PRESENT VALUE FOR SEASON 2

EPHEMERAL FORAGE '00 KG DM/HA	PERENNIAL FORAGE '00 KG DM/HA 0-2	2-4	4-6	6-8	8-10	10-12	12-14
0-2	.5	.5	.5	.5	.5	.5	.5
	82.92	83.10	83.16	83.20	83.21	83.21	83.24
2-4	.5	.5	.5	.5	.5	.5	.5
	82.92	83.07	83.16	83.17	83.21	83.21	83.24
4-6	.5	.5	.5	.5	.5	.5	.5
	82.93	83.10	83.18	83.18	83.21	83.21	83.24
6-8	.5	.5	.5	.5	.5	.5	.5
	82.93	83.09	83.15	83.19	83.20	83.20	83.23
8-10	.5	.5	.5	.5	.5	.5	.5
	82.92	83.09	83.15	83.18	83.20	83.20	83.23
10-12	.5	.5	.5	.5	.5	.5	.5
	82.92	83.09	83.14	83.18	83.20	83.20	83.22

TABLE 5 OPTIMAL STOCKING RATES & THE CORRESPONDING NET PRESENT VALUE FOR SEASON 3

EPHEMERAL FORAGE '00 KG DM/HA	PERENNIAL FORAGE '00 KG DM/HA 0-2	2-4	4-6	6-8	8-10	10-12	12-14
0-2	.1	.1	.3	.3	.5	.5	.5
	78.88	79.08	79.25	79.34	79.42	79.51	79.59
2-4	.1	.3	.3	.3	.5	.5	.5
	78.89	79.12	79.28	79.35	79.45	79.53	79.60
4-6	.1	.3	.3	.3	.5	.5	.5
	78.94	79.14	79.28	79.35	79.45	79.52	79.60
6-8	.1	.3	.3	.5	.5	.5	.5
	78.93	79.14	79.28	79.36	79.45	79.53	79.59
8-10	.1	.3	.3	.5	.5	.5	.5
	78.94	79.13	79.27	79.35	79.44	79.50	79.58
10-12	.1	.3	.3	.5	.5	.5	.5
	78.94	79.11	79.26	79.33	79.42	79.50	79.57

perennial forage are below 200 kg dm/ha and with stocking rate 0.1 sheep/ha. The highest net present value of $81.48/ha occurs when ephemeral availability is within the range of 200-400 kg dm/ha combined with perennials at 1200-1400 kg dm/ha.

Table 4 indicates that the optimal stocking rate for season 2 is 0.5 sheep/ha over the whole range of the available vegetation. This is in agreement with the expectation that a heavier stocking rate should be employed during a season when rainfall is more likely to occur. The net present values again increase with a rise in perennial forage levels and show no systematic pattern of change with increasing ephemeral forage biomass. The lowest net present value of $82.92 occurs with the lowest forage availabities, 0-200 kg dm/ha for both ephemeral and perennials. The highest net present value of $83.24/ha occurs when available ephemeral forage is in the 0-200 kg dm/ha range, and perennial forage is in the 1200-1400 kg dm/ha range.

Table 5 indicates that optimal stocking rate generally increases with increasing available forage. The optimal stocking rates are 0.1 sheep/ha for low availablility of perennials (i.e. roughly below 200 kg dm/ha), 0.3 sheep/ha for medium availability (i.e. about 200-800 kg dm/ha), and 0.5 sheep/ha for high perennial availability (i.e. more than 800 kg dm/ha), although there are some exceptions. The net present values for season 3 rise with perennial forage availability, but show no systematic relation to available ephemeral forage biomass. The minimal value of $78.88/ha occurs when the available forage is 0-200 kg dm/ha for both ephemerals and perennials. The highest net present value of $79.60/ha occurs at the combination of the ephemeral availability of 200-400 kg dm/ha and the perennial availability of 1200-1400 kg dm/ha.

Comparing optimal stocking rates among the three seasons, it can be seen that the stocking rates in season 2 are greater than those in season 1 and the stocking rates in season 1 are greater than those in season 3. The net present values among the three seasons also follow the same sequence as the optimal stocking rates. These patterns are consistent with the expectation that higher stocking rate should be optimal for a higher probability of increasing forage availability which could result from a higher probability of rainfall events.

Marginal value (or shadow price) of the state variables are given by the partial derivatives of the optimal value function V(x) with respect to the state variables. The discrete approximations of marginal values of perennial and ephemeral forage biomasses are calculated from Table 3 to 5 and given by Table 6a to 6c and Table 7a to 7c, respectively. As illustrated by Table 6a to 6c marginal value of perennial forage is positive every where and generally decreases with the increasing availabilities. Table 7a to 7c indicate that the marginal value of ephemeral forage is negative for the high availabities but positive for the low availabilities although there are some exceptions in season 2. Comparing the magnitude of marginal values of ephemeral and perennial forage, we can claim that the perennial forage is more important than ephemeral forage in making range management decisions.Comparing marginal values of perennial forage among three seasons, we know that the perennial forage is most important during dry season (season 3) but less important when the season is wet (season 3).

The optimal decision rule of stocking rates derived from using all information available in the beginning period (season 1 in 1982) is 0.5 sheep/ha and the net present value is $79.74/ha. The decision rule of optimal stocking rate after 1982 cannot be determined until the value of the state variables are observed and the entire approximation computational process with the observed data is repeated. However, the asymptotic optimal decision rule derived from the simplified model can be used as a general guide for the choice of stocking rate.

Discussion

Optimal stocking rates increase with increasing available perennial vegetation more so than with increasing available ephemeral forage. The associated net present values also increase with increasing available perennial forage but show no systematic pattern of changes with increasing ephemerals. These patterns are caused by the strong Markov chain in the perennial forage biomass. Generally speaking, the higher autocorrelation exists in a state variable, the better the state variable can capture the essence of the system and vice versa. Thus, optimal stocking rates and net present values would be affected mainly by the variation of perennial availability.

TABLE 6a MARGINAL VALUES OF PERENNIAL FORAGE BIOMASS FOR SEASON 1

EPHEMERAL FORAGE	PERENNIAL FORAGE KG DM/HA					
KG DM/HA	100-300	300-500	500-700	700-900	900-1100	1100-1300
			------cents/kg dm------			
100	603	197	296	150	96	43
300	693	183	248	125	105	43
500	620	194	183	167	98	39
700	571	260	204	67	94	58
900	549	264	197	64	92	57
1100	559	260	189	79	81	58

TABLE 6b MARGINAL VALUES OF PERENNIAL FORAGE BIOMASS FOR SEASON 2

EPHEMERAL FORAGE	PERENNIAL FORAGE KG DM/HA					
KG DM/HA	100-300	300-500	500-700	700-900	900-1100	1100-1300
			------cents/kg dm------			
100	453	153	88	36	2	75
300	373	219	25	98	2	75
500	429	208	10	58	1	71
700	419	148	91	35	2	68
900	414	141	92	39	1	68
1100	412	141	90	41	18	51

TABLE 6c MARGINAL VALUES OF PERENNIAL FORAGE BIOMASS FOR SEASON 3

EPHEMERAL FORAGE	PERENNIAL FORAGE KG DM/HA					
KG DM/HA	100-300	300-500	500-700	700-900	900-1100	1100-1300
			------cents/kg dm------			
100	505	428	225	182	237	194
300	567	391	190	234	209	182
500	487	357	180	249	176	198
700	507	346	201	234	193	171
900	479	348	197	234	152	212
1100	430	366	182	238	198	173

TABLE 7a MARGINAL VALUES OF EPHEMERAL FORAGE BIOMASS FOR SEASON 1

EPHEMERAL FORAGE KG DM/HA	PERENNIAL FORAGE KG DM/HA 100	300	500	700	900	1100	1300
	------cents/kg dm------						
100-300	6	96	82	34	9	18	18
300-500	87	14	25	-40	3	-5	-9
500-700	27	-22	44	65	-36	-40	-21
700-900	8	-15	-10	-17	-20	-22	-23
900-1100	-37	-27	-31	-40	-25	-36	-36

TABLE 7b MARGINAL VALUES OF EPHEMERAL FORAGE BIOMASS FOR SEASON 2

EPHEMERAL FORAGE KG DM/HA	PERENNIAL FORAGE KG DM/HA 100	300	500	700	900	1100	1300
	------cents/kg dm------						
100-300	7	-73	-7	-71	-9	-9	-9
300-500	5	61	49	34	-6	-7	-10
500-700	4	-7	-67	15	-9	-8	-11
700-900	-6	-11	-18	-17	-12	-13	-13
900-1100	-7	-9	-9	-11	-9	-8	-9

TABLE 7c MARGINAL VALUES OF EPHEMERAL FORAGE BIOMASS FOR SEASON 3

EPHEMERAL FORAGE KG DM/HA	PERENNIAL FORAGE KG DM/HA 100	300	500	700	900	1100	1300
	------cents/kg dm------						
100-300	33	95	59	24	76	48	36
300-500	126	46	-12	2	17	-17	-1
500-700	-22	-1	-13	8	-7	10	-17
700-900	4	-25	-22	-21	-26	-67	-26
900-1100	5	-45	-27	-41	-37	9	-30

The marginal values of ephemeral forage are negative for some higher ranges. This phenomenon may be caused by the fact that the ephemeral forage dry out very quickly and their digestibilities also drop very fast when the availability of ephemeral forage is high. The lower digestibility of ephemeral forage would reduce the total digestible dry matter intake of sheep to an extent that the total wool production starts to decline. Thus, the marginal value of ephemeral forage will be negative at some level of the ephemeral forage biomass. The marginal values of perennial forage in season 3 are the highest in the year. This indicates that the perennial forage is very important for sheep survival during the dry season because there are only scarce ephemeral forage available to sheep.

The optimal stocking rates for season 2 are 0.5 sheep/ha over the whole rage of forage availability. This may indicate that the upper limit of the range for stocking rate should be increased. However, optimal stocking rates for all three seasons in this study are derived on the assumption that the costs of stocking rate adjustment among three seasons are zero and negligible. In reality, if the adjustment costs are expensive the model should take them into account and the optimal stocking rate so derived would be much lower.

There are problems in simplifying the model. As mentioned before it was hoped that the model of only three state variables might catch the essence of the original model due to the linear correlation between perennial forage biomass and desirable perennial plants. The results presented above suggest another way to simplify the model, which only containing the following three state variables: total forage availability $S_t^f+S_t^h$, desirable perennial plant density S_t^p and desirable perennial seedling density s_t^p. Since the ephemeral forage biomass is relative unimportant it can be added into perennial forage to form a total forage availability $S_t^f+S_t^h$. State variable, season, in the original model is dropped because we will aggregate the seasonal model into an annual model to reduce the time lags. However, in the aggregated annual model we have to pre-specify the stocking rates among three seasons. Although this is a major shortcoming in aggregating the model it can take stocking rate adjustment costs into account because each pre-specified stocking rate will incur a different costs. Desirable perennial plant density should be explicitly taken into account due to the strong Markov chain. Since there are two years for a young seedling to reach maturity a distinguish between populations (i.e. cohort) of young plant and adult plant may be important. There is an evident need to test the optimal decisions derived by these tw different types of simplificaitons.

The method required to derive optimal decisions on whether to apply a cultural treatment will vary with the type of treatment. If the application of a treatment is a periodic decision such as reseeding, clearing bush, chemical spraying, fencing, etc, so that its effect only lasts for a limited period, then the optimal treatment decision needs to be derived by combining the treatment decision with choice of stocking rate, thus increasing the dimension of decision space. The optimal frequency of treatment application will be an important decision to be made, and could be determined by adding another state variable F, which is number of periods since treatment was last applied. Note that such a variable F is deterministic and subject to the following transition equations:

$$F_{t+1} = \begin{cases} F_t+1 & \text{for not applying treatment and } F_t<F_m \\ 1 & \text{for applying treatment;} \\ F_m & \text{for not applying treatment and } F_t=F_m \end{cases}$$

where F_m is the periods over which treatment effects persist. The variable F affects the transition probability of range dynamics. The optimal value of F so derived is the optimal frequency of applying a treatment for each range condition.

If a treatment is not a periodic decision such as ponding and/or cultivation which are both "once in a life-time" decisions, the treatment effects can remain indefinitely. Thus, the optimal frequency of applying the treatment is not important, and whether to apply the treatment or not at current range condition turns out to be the only decision required about the treatment. And the optimal decision can be derived by the above analysis.

Futher revision of the simulation model also is needed. Three important factors currently omitted from the model which need to be included are: firstly, the effects of other animals in the paddock such as cattle, goat, kangaroo and rabbit; secondly, the variable of dry forage biomass; and thirdly, the costs of stocking rate adjustment among three seasons.

Summary and conclusion

This paper uses stochastic dynamic programming to solve the range regeneration management problem with respect to decisions about stocking rate. Decisions about whether to apply a treatment or not can be determined after the derivation of the optimal stocking rate. A simulation model was developed to investigate the vegetation response to cultural treatments and different stocking rates. Transition probabilities were derived by simplifying this model.

The optimal decision rule was derived by an approximation method to reduce the state space. The asymptotic optimal decision rule is periodic stationary, corresponding to the three rainfall seasons. As would be expected, high stocking rates are optimal in the high rainfall season, and low stocking rates are best in the dry season. The marginal values of perennial forage biomass are positive but those of ephemeral forage can be either positive or negative. The perennial forage biomass seems to be more important during the dry season when available forage is relatively scarce. Although the solution presented is preliminary it is consistent with conventional wisdom about choice of stocking rate for range regeneration under uncertainty. Further research is needed to validate the simulation model and to test the near-optimal decision rule derived in this study.

REFERENCES

Bertsekas, D. P. (1976), "Dynamic Programming and Stochastic Control", New York: Academic Press.

Burt, O. R. (1972), "A dynamic economic model of pasture and range investments", American J. of Agr. Econs., 53:197-205.

Burt, O. R., Won W. Koo, and N.J. Dudley (1980), "Optimal stochastic control of U.S. wheat stocks and exports", American J. of Agr. Economics, 62: 173-187.

Derman, C. (1970), "Finite state Markovian decision processes", New York: Academic Press.

Fitzpatrick, E. A., R. O. Slatyer and A. I. Krishnan (1967), "Incidence and duration of periods of plant growth in central Australia as estimated from climatic data", Agr. Meteorol., 4:389-404.

Harrington, G. N., A. D. Wilson and M. D. Young (1984), "Management of Australia's Rangelands", Division of Wildlife and Rangelands Research. CSIRO, Australia.

Larry Karp and Arden Pope, III (1984), "Range management under uncertainty", American J. of Agr. Econos., 66:437-446.

Stoddart, L. A., Smith, A. D. and T. W. Box (1975), "Range management", 3rd edition, McGraw-Hill Book Company.

Taha, H. A. (1982), "Operations research: an introduction", 3rd ed Macmillan Publishing Co. Inc. New York.

Appendix

A.1 A List of Functional Forms in the Simulation Model

Replacement capacity index

$$RCI_{t-6,j}=\min(b_{10}+\frac{b_{11}DP_{t-6}}{1+b_{12}DP_{t-6}},1) \qquad \text{for } j=0,1 \tag{1.1}$$

$$DP_{t-6}=\frac{S^{p}_{t-6}}{S^{p}_{max}} \tag{1.2}$$

$$RCI_{t-6,j}=\min\{[RCI_{t-6,j}+DRCI\prod_{i=1}^{t}(1-\lambda_{t-i})(1-GMI_{t-i,2})],1\} \tag{1.3}$$

for periods after reseeding

$$=1 \quad \text{for period of reseeding occurring.} \tag{1.4}$$

$$DRCI=1-RCI_{t}^{*} \tag{1.5}$$

Germination index

$$GMI_{t-6,j}=b_{t}(1-\exp(-b_{20,j}(NWP_{t-6})^{2}) \tag{2.1}$$

Survival index

$$SI_{t,j}=\prod_{i=0}^{5}(SMI_{t-i,j}GI_{t-i}CI_{t-i}) \tag{3.1}$$

Soil moisture index for seedling survival

$$SMI_{t-i,0}=\min(b_{300}-b_{301}i+\frac{b_{300}+b_{301}i}{b_{302}+i}NWP_{t-i},1) \tag{3.2}$$

$$SMI_{t-i,1}=\min(b_{310}-b_{311}i+\frac{b_{312}+b_{311}i}{b_{313}+i}NWP_{t-i},1) \tag{3.3}$$

$$SMI_{t-i,2}=\min(b_{320}-b_{321}i+\frac{b_{322}+b_{321}i}{b_{323}+i}NWP_{t-i},1) \tag{3.4}$$

Grazing pressure index for seedling survival

$$GI_{t-i}=\min(\frac{b_{330}(G_{t-i}-1)}{1-b_{331}(G_{t-i}-1)},1) \tag{3.5}$$

Ephemeral competition index for seedling survival

$$CI_{t-i}=\min[b_{340}-(b_{341}+b_{342}i)(HD_{t-i}-b_{343}),1] \tag{3.6}$$

$$HD_{t-i}=\frac{S^{h}_{t-i}+R^{h}_{t-i,j}}{k^{h}_{t-i}} \tag{3.7}$$

Soil moisture index for perennial forage recruitment

$$SMI^{f}_{t,j}=\min(b_{40,j}NWP_{t},1) \tag{4.1}$$

Growth capacity index

$$GCI_{t}=b_{50}+\frac{b_{51}S^{f}_{t}/K^{f}_{t}}{1+b_{52}S^{f}_{t}/K^{f}_{t}} \tag{5.1}$$

Perennial forage carrying capacity

$$K^{f}_{t}=b_{60}(1-\exp(-b_{61}\frac{S^{p}_{t}}{S^{p}_{max}})) \tag{6.1}$$

Soil moisture index for ephemeral forage recruitment

$$W=b_{70,j}NWP_{t}+b_{71,j}\frac{S^{h}_{t}-K^{h}_{t}}{K^{h}_{t}}+b_{72,j}\frac{S^{h}_{t}-K^{h}_{t}}{K^{h}_{t}}NWP_{t} \tag{7.1}$$

$$SMI^{h}_{t,0}=\max(W,0) \tag{7.2}$$

$$SMI^{h}_{t,1}=\begin{cases} W & \text{if } 1\geq W\geq 0 \\ 1 & \text{if } W>1 \\ 0 & \text{otherwise} \end{cases} \tag{7.3}$$

$$SMI^{h}_{t,2}=\begin{cases} W & \text{if } 1\geq W\geq 0 \\ 1 & \text{if } W>1 \\ 0 & \text{otherwise} \end{cases} \tag{7.4}$$

Ephemeral forage carrying capacity

$$K^{h}_{t}=b_{80}\exp(-b_{81}(\frac{S^{p}_{t}}{S^{p}_{max}}-b_{82})^{2}) \tag{8.1}$$

Environmental factors induced mortality of desirable plants

$$m_{1}=(1-m_{0})(b_{900}GPI^{d}_{t}+b_{901}SMI^{d}_{t,j}DI^{d}_{t}+b_{902}GPI^{d}_{t}SMI^{d}_{t,j}DI^{d}_{t}) \tag{9.1}$$

$$GPI^{d}_{t}=\prod_{i=0}^{5}GPI_{t-i} \tag{9.2}$$

$$GPI_{t}=\max(\frac{b_{903}(G_{t}-1)}{1-b_{904}(G_{t}-1)}+1,0) \tag{9.3}$$

$$SMI^{d}_{t,j}=\prod_{i=0}^{5}SMIP_{t-i,j} \tag{9.4}$$

$$SMIF_{t-i,j}=\max\left(\frac{1-NWP_{t-i}}{MNWP_{t-i}-j},0\right) \qquad j=0,1,2 \tag{9.5}$$

$$DID_t=\prod_{i=0}^{5} DI_{t-i} \tag{9.6}$$

$$DI_{t-i}=b_{905}D_{t-i} \tag{9.7}$$

Perennial forage drying rate

$$\delta_1=a_{10}DIF_t SMIF_{t,j} \tag{10.1}$$

$$DIF_t=a_{11}+a_{12}FD \qquad \text{for } NWP_t\neq 0 \tag{10.2}$$
$$=1 \qquad \text{for } NWP_t=0$$

$$FD_t=\frac{S_t^f+R_t^f}{K_t^f} \tag{10.3}$$

$$SMIF_{t,j}=a_{13}-\frac{a_{14,j}NWP_t}{1-a_{15,j}NWP_t} \tag{10.4}$$

Ephemeral forage drying rate

$$\delta_2=a_{20}DIE_t SMIE_{t,j} \tag{11.1}$$

$$DIE_t=a_{21}+a_{22}ED_t \qquad \text{for } NWP_t\neq 0 \tag{11.2}$$
$$=1 \qquad \text{for } NWP_t=0$$

$$SMIE_{t,j}=a_{23}-\frac{a_{24,j}NWP_t}{1-a_{25,j}NWP_t} \qquad j=0,1,2 \tag{11.3}$$

Trampling loss rate

$$\delta_3=1-\exp(-a_{26}U_t) \tag{12.1}$$

Ephemeral forage availability index

$$AIE_t=(1-\exp(-a_{30}(\frac{S_t^h+R_t^h-DNCE_t}{a*U_t}))) \tag{13.1}$$

Ephemeral forage quality index

$$QIE_t=\exp(-a_{40}(DG_t^h-a_{41})^2) \tag{14.1}$$

Perennial forage availability index

$$AIP_t=(1-\exp(-a_{50}(\frac{S_t^f+R_t^f-DNCF_t}{a*U_t}))) \tag{15.1}$$

Perennial forage quality index

$$QIP_t = \exp(-a_{80}(DG^f_t - b_{81})^2) \tag{16.1}$$

Digestibility functions

$$DG^h_t = a_{70} - \frac{a_{71}\delta_2}{1-a_{72}\delta_2} \tag{17.1}$$

$$DG^f_t = a_{80} - \frac{a_{81}\delta_3}{1-a_{82}\delta_3} \tag{18.1}$$

Lambing rate function

$$L_t = a_{90}(1-\exp(-a_{91}WNI^2_t)) \quad \text{if } \mathrm{mod}(t,3) \neq 2 \tag{19.1}$$
$$= 0 \quad \text{if } \mathrm{mod}(t,3) = 2$$

$$WNI_t = \sum_{i=0}^{3} w_i NI_{t-i} \tag{19.2}$$

$$NI_{t-i} = \frac{FIH_{t-i} + FIP_{t-i}}{s} \tag{20.1}$$

Sheep mortality

$$M_1 = (1-M_0)(\exp(-a_{92}(NI^2_t))) \tag{21.1}$$

Wool clip

$$WCS_t = a_{93}\ DDMI_t \tag{22.1}$$

A.2 A List of Variable Definitions in the Simulation Model

S_t^i initial size of stock i at season t;

R_t^i recruitment to stock i during season t;

D_t^i disappearance from stock i during season t; where i=p for desirable perennial plants; i=f for perennial forage biomass; i =h for ephemeral forage biomass;

β_1 the proportion of paddock under treatment 1;

β_2 the proportion of paddock under treatment 2;

$R_{t,j}^p$ the recruitment of perennial plant under treatment j, where j=0 for no treatment;j=1 for treatment 1, i.e. cultivation; j=2 for treatment 2, i.e. ponding and reseeding;

γ adjustment coefficient which represents the proportion of potential recruitment that can be realized in season t under treatment j;

R_t^{p*} the potential recruitment of desirable perennial plants in season t;

S_{max}^p the desirable perennial plant density at climax condition;

$RCI_{t-6,j}$ the replacement capacity index as a proxy for seed stocks under application of treatment j;

$GMI_{t-6,j}$ the germination index during season t-6 under treatment j;

$SI_{t,j}$ the seedling survival index for recruitment under treatment j;

$SMI_{t-i,j}$ soil moisture index for the survival of perennial seedlings at stage t-i under treatment j;

GI_{t-i} grazing pressure index for the survival of perennial seedlings at stage t-i;

CI_{t-i} ephemeral competition index for the survival of perennial seedlings at stage t-i; where t-i also represents the age of the seedlings, i=0 for seedling aged 20-24 months; i=1 for 16-20 months; i=2 for 12-16 months; i=3 for 8-12 months; i=4 for 4-8 months; i=5 for 0-4 months;

G_{t-i} the grazing pressure index in the paddock during season t-i;

NWP_t climatic driving variable in season t, # growth periods i.e. wet pentads;

K_{t-i}^h the carrying capacity of ephemeral forage during season t-i;

HD_{t-i} the existing ephemeral forage density during season t-i;

D_{t-6} the desirable perennial plant density relative to its saturation level;

DRCI the increment of replacement capacity by reseeding;

λ_{t-i}	loss rate of seed reservo duo to non-germinative reason during season t-i;
RCI_t^*	replacement capacity index at the time when treatment 2 applied;
$GMI_{t-6,j}$	the germination index of desirable perennial seeds during season t-6 under treatment j;
b_t	seasonal factor which is used to account for temperature effects on the germination, it changes as season varies;
mod(t,3)	remainder of division of t by 3;
PGR_t^f	potential growth rate of perennial forage biomass;
$SMI_{t,j}^f$	soil moisture index for the perennial forage growth under c treatment j;
GCI_t	growth capacity index for the perennial forage.
K_t^f	the carrying capacity of perennial forage biomass;
PGR_t^h	potential growth rate of ephemeral forage biomass,
$SMI_{t,j}^h$	soil moisture index for the ephemeral forage growth under treatment j;
b_{80}	the maximal .arrying capacity of ephemeral forage biomass,
$d_{t,j}$	seasonal disappearance rate of desirable perennial plants under treatment j;
m_0	natural mortality rate of desirable perennial plants;
m_1	environmental factors induced mortality rate of desirabel perennial plants;
GPI_t	grazing pressure index for perennial plant mortality during season t;
GPI_t^d	the cumulative grazing pressures for perennial plants disappearance;
$SMI_{t,j}^d$	the cumulative soil moisture index for perennial plant mortality under treatment j;
$SMIP_{t,j}$	the soil moisture index for perennial plant mortality during season t under treatment j;
$MNWP_{t-i}$	the mode of the distribution of NWP_{t-i} during season t-i.
DID_t	the cumulative density index for perennial plant mortality;
DI_t	the density index for perennial plant mortality;
FIE_t	the ephemeral forage intake of sheep, kg d.m./sheep;
U_t	stocking rate of sheep per hectare in season t, sheep/ha;
$DNCE_{t,j}$	the losses of ephemeral forage due to non-consumptive reasons under treatment j, kg/ha d.m.;

SL_t the non-consumptive losses of ephemeral forage due to senescence;

TL_t the non-consumptive losses of ephemeral forage due to trampling;

δ_2 the drying rate of ephemeral forage biomass during season t under treatment j;

δ_3 trampling loss rate of ephemeral forage biomass during season t;

DIE_t density index for ephemeral senescence;

$SMIE_{t,j}$ the soil moisture index for ephemeral senescence under treatment j.

$DNCP_{t,j}$ the losses of perennial forage due to non-consumptive reasons under no treatment;

δ_1 the drying rate of perennial forage during season t under treatment j;

DIP_t the density index for perennial senescence;

PD_t the density of perennial forage biomass during season t;

$SMIP_t$ the soil moisture index for perennial senescence;

PIP_t the perennial forage intake of sheep, kg d.m./sheep;AIE_t ephemeral forage availability index for sheep intake;

AIP_t perennial forage availability index for sheep intake;

QIE_t ephemeral forage quality index for sheep intake;

QIP_t perennial forage quality index for sheep intake;

DG_t^h digestibility of ephemeral forage;

DG_t^f digestibility of perennial forage;

a the conversion factor of potential sheep intake;

FLS_{t+1} sheep flock size at the end of season t, # sheep/paddock;

L_t lambing rate as % of whole ewe population;

M_t seasonal sheep mortality rate as a % of sheep population;

NI_t nutritional index for sheep;

WNI_t cumulative nutritional index for sheep;

A_t the area suitable for grazing in the paddock;

a_{50} the maximal lambing rate;

w_i weight assigned to each season nutritional index;

M_0 natural seasonal death rate of sheep;

M_1 the plane of nutrition induced sheep mortality;

WCS_t the average wool production per sheep per season,kg/sheep;

$DDMI_t$ the digestible dry matter intake per sheep;

PS_t sale price of a sheep, dollars/sheep;

MS_t	marketing costs for selling a sheep, dollars/sheep;
PW_t	price of wool, dollars/kg;
MW_t	marketing costs for selling wool, dollars/kg wool;
SC_t	shearing costs per sheep, dollars/sheep;
VC_t	other variable costs per sheep eg. mulesing, mustering, maintenance, fuel and labour etc, dollars/sheep;
r	seasonal interest rate;
PS_t	purchase price of sheep, dollars/sheep;
C^j	total costs of applying a treatment j, dollars/ha.

A.3 A List of Values for the Parameters in the Simulation Model

Equation	Parameter	Value	Parameter	Value
(1.1)	b_{10}	0.01	b_{11}	1.98
	b_{12}	0.0		
(1.2)	S^{p}_{max}	8000.0		
(1.3)	λ_{t-1}	0.11		
(2.1)	b_{1}	1.0	b_{2}	0.8
	b_{3}	0.9	$b_{20,0}$	0.1872
	$b_{20,1}$	0.3	$b_{20,2}$	0.4
(3.2)	b_{300}	0.5	b_{301}	0.1
	b_{302}	7.0		
(3.3)	b_{310}	0.85	b_{311}	0.13
	b_{312}	0.35	b_{313}	6.0
(3.4)	b_{320}	0.8	b_{321}	0.16
	b_{322}	0.2	b_{323}	5.0
(3.5)	b_{330}	-2.0	b_{331}	0.0
(3.6)	b_{340}	1.0	b_{341}	2.0
	b_{342}	0.16	b_{343}	0.5
(4.1)	$b_{40,0}$	1/24	$b_{40,1}$	1/23
	$b_{40,2}$	1/22		
(5.1)	b_{50}	0.05	b_{51}	11.76
	b_{52}	11.38		
(6.1)	b_{60}	1400.0	b_{61}	2.33
(7.1)	$b_{70,0}$	1/24	$b_{70,1}$	1/23
	$b_{70,2}$	1/22	$b_{71,0}$	1/11
	$b_{71,1}$	2/21	$b_{71,2}$	1/10
	$b_{72,0}$	-1/264	$b_{72,1}$	-2/483
	$b_{72,2}$	-1/220		
(8.1)	b_{80}	1200.0	b_{81}	6.4
	b_{82}	0.4		
(9.1)	m_{0}	1/150	b_{900}	0.2
	b_{901}	0.4	b_{902}	0.4
(9.3)	b_{903}	2.0	b_{904}	0.0
(9.7)	b_{905}	1.0		
(10.1)	a_{10}	0.5		
(10.2)	a_{11}	0.9	a_{12}	0.1

(10.4)	a_{13}	1.0	$a_{14,0}$	0.04125
	$a_{15,0}$	0.0	$a_{14,1}$	0.0505
	$a_{15,1}$	-0.0094	$a_{14,2}$	0.02[illegible]1
	$a_{35,2}$	-0.0362		
(11.1)	a_{20}	1.0		
(11.2)	a_{21}	0.8	a_{22}	0.2
(11.3)	a_{23}	1.0	$a_{24,0}$	0.0375
	$a_{25,0}$	0.0	$a_{24,1}$	0.0589
	$a_{25,1}$	-0.0230	$a_{24,2}$	0.0975
	$a_{25,2}$	-0.0667		
(12.1)	a_{26}	0.11		
(13.1)	a_{30}	0.48	a	180.0
(14.1)	a_{40}	4.51	a_{41}	0.7
(15.1)	a_{50}	4.51		
(16.1)	a_{60}	0.48	a_{61}	0.65
(17.1)	a_{70}	0.7	a_{71}	0.2
	a_{72}	0.0		
(18.1)	a_{80}	0.65	a_{81}	0.2
	a_{82}	0.0		
(19.1)	a_{90}	0.95	a_{91}	3.8
(19.2)	w_0	0.5	w_1	0.2
	w_2	0.2	w_3	0.1
(21.1)	M_0	0.0167	a_{92}	2.77
(22.1)	a_{93}	0.015		