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PAPER 8: ON EVENT UNCERTAINTY AND RENEWABLE RESOURCE MANAGEMENT

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On Event Uncertainty and Renewable Resource Management

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1. Introduction

Hardly a day goes by without one reading or hearing about some environmental disaster creeping at our doorstep or a natural catastrophe of that sort or another soon to occur. We hear about species that are threatened, endangered or already extinguished. We are informed that the accumulation of CO_2 and other greenhouse gases in the atmosphere may lead to global warming with the possibility of severe damages (Nordhaus, 1991, Cline, 1992). We read about aquifers that shrink in size or already are depleted as a result of prolonged pumping above recharge (Apostol, 1993); and about large scale deforestation (Hartwick, 1992), leading to the extinction of incalculable number of species (Colinvaux, 1989) and decreasing the rate of CO_2 removal from the atmosphere.

While some of these alarms may turn out to be premature or even false, others pose real threats on the well-being of the living species on this Earth and of future generations. The mechanisms that drive environmental processes are often poorly understood, and the associated parameters are subject to large uncertainties. Yet, the possibility of irreversible losses requires prudent management because mistakes cannot be fixed.

In this work we offer a framework for analyzing a situation in which the exploitation of a renewable natural resource stock may lead to the occurrence of an undesirable event. Occurrence conditions are not completely known; this is the source of uncertainty in this class of models. The events are classified according to their consequences: The event is irreversible if the resource cannot be used after occurrence; it is partly reversible if occurrence entails a cost (penalty) following which the resource can be reused, possibly with some additional constraints associated with the resolution of the uncertainty regarding the event occurrence.

Irreversible events were studied by Tsur and Zemel (1994a) in the context of groundwater extraction under risk of saline water intrusion. This event is irreversible because its occurrence renders the aquifer obsolete. Partly reversible events were analyzed by Tsur and Zemel (1994b) who discuss fossil fuel combustion and the process of global warming due to the accumulation of greenhouse gases in the atmosphere. This process may trigger a costly event. The event is partly reversible in that its occurrence involves economic penalties, but it does not prevent fossil fuel usage afterwards. The case of a resource stock (e.g., a water stream) that serves both human needs (e.g., irrigation) and as a wild life habitat is studied in Tsur and Zemel (1995). The event in this example corresponds to the extinction of the animal population and it occurs when the resource stock decreases below some (uncertain) critical level. The elimination of the species inflicts the loss of recreational options and other nonuse values (Hanemann, 1994) as well as a decrease in biodiversity (Weitzman, 1992, 1993; Polasky, Solow and Broadus, 1993).

The model here described accommodates these examples as special cases and enables the consideration of two types of event uncertainty: events that occur when the resource stock reaches a threshold level, and events that occur due to external (environmental) conditions. For the first type of events with uncertain threshold levels, uncertainty stems from our ignorance rather than from a truly stochastic process, and gives rise to a common pattern of behavior: a steady-state interval is identified such that optimal state processes initiated within the interval remain in equilibrium. When initiated outside the interval, the optimal state process converges to one of its boundary points. This behavior is contrasted with the non-event (when no event can interrupt) and certainty (when the critical state level is known) situations, and with the external

(environmental) event uncertainty, in all of which the optimal state processes converge to a particular state level. The equilibrium interval gives precise policy prescriptions to environmental claims that the presence of event uncertainties of the first type calls for more preservation. Such claims, however, may not always be valid under event uncertainties of the second type [those due to external conditions, see Clarke and Reed (1994)].

The analysis is built on two strands of literature. The first is the literature on event uncertainty in nonrenewable resources, initiated by Long (1975), Kemp (1976), Loury (1978) and Gilbert (1978), and synthesized by Deshmukh and Pliska (1985). We extend this literature to renewable resources. As it turns out, the introduction of recharge processes brings in the delicate issue of whether to extract at or below the rate of recharge, thus avoiding the event occurrence risk, or to extract above it, reducing the resource stock and taking a risk that the event will occur. These considerations give rise to the steady-state interval mentioned above.

Another precursor of this work is the line of research set forth by Cropper (1976) and Heal (1984) and developed further by Clarke and Reed (1994). Our framework extends this research in several respects. First, we consider ignorance-type event uncertainty, which is somewhat harder to recast in the standard optimal control form. By establishing the monotonicity of the optimal state process we are able to provide a complete formulation of the exploitation model under this type of uncertainty, hence to fully characterize the optimal state process. The equilibrium states are identified in terms of the roots of a simple function of the state variable, and their determination does not require the knowledge of the optimal policy. Moreover, we consider events that are not necessarily terminal: there is life after the event, as we allow for partially

reversible events.

We limit consideration to events that occur at once. This is not always the case, as many events evolve gradually over time. Such gradual events, however, are easier to avoid and therefore do not pose the same threat as their abrupt counterparts.

2. Optimal resource management

The state (stock) of the resource S_t evolves over time as a result of human extraction at the rate g_t and of natural recharge at the rate $R(S_t)$, according to

$$dS_t/dt \equiv \dot{S}_t = R(S_t) - g_t. \tag{2.1}$$

The analysis is simplified when $R(S)$ is assumed to be decreasing and concave, vanishing at the resource carrying capacity $\bar{S}$; this typically holds for resources that are exogenously replenished, such as groundwater.

Let $B(g,S)$ represent the instantaneous net benefit generated by g when the stock is at the level S. We assume that B satisfies: $\partial^2 B/\partial g^2 < 0$; $\partial B/\partial S \geq 0$; $\partial^2 B/\partial g \partial S \geq 0$ and $\partial^2 B/\partial S^2 \leq 0$. The benefit functions postulated for the specific examples discussed below are consistent with these conditions.

An extraction plan initiated at a state level S consists of the extraction process g_t and the associated state process S_t, $t \geq 0$. A plan $\{g_t, S_t\}$ is feasible if it satisfies (2.1) and

$$S_0 = S, \ g_t \geq 0 \text{ and } S_t \geq \underline{S}, \tag{2.2}$$

where $\underline{S}$ is the lower bound on the resource level.

2.1. Non-event: When no event can interrupt the plan, the problem is to find the feasible plan that maximizes the discounted stream of benefits:

$$V^n(S) = \underset{\{g_t\}}{\text{Max}} \int_0^\infty B(g_t, S_t) e^{-\rho t} dt \quad \text{s.t. (2.1)-(2.2)}, \qquad (2.3)$$

where ρ is the time rate of discount. We call (2.3) the *non-event* problem and

denote by $\{g_t^n, S_t^n\}$ its optimal plan. It is established (Tsur and Zemel, 1995) that

S_t^n converges monotonically to a unique steady state $\hat{S}$, defined by

$$\begin{cases} \hat{S} = \underline{S} & \text{if } L(\underline{S}) > 0 \\ \hat{S} = \bar{S} & \text{if } L(\bar{S}) < 0 \\ L(\hat{S}) = 0 & \text{otherwise} \end{cases} \qquad (2.4)$$

where the evolution function $L(S)$ is given by

$$L(S) = [\rho - R'(S)]\frac{\partial B(g,S)}{\partial g}\bigg|_{g=R(S)} - \frac{\partial B(g,S)}{\partial S}\bigg|_{g=R(S)}. \qquad (2.5)$$

The properties of R and B ensure that $\hat{S}$ is unique.

Further insight into the meaning of the evolution function $L(S)$ can be gained

by considering, for some arbitrary small constants $h > 0$ and δ, the following

extraction plan, starting at some interior level $S \in (\underline{S}, \bar{S})$,

$$g_t^{\delta h} = \begin{cases} R(S) + \delta, & 0 \le t < h \\ R(S_h), & t \ge h \end{cases}. \qquad (2.6)$$

Let $V^{\delta h}(S)$ denote the benefit associated with $g_t^{\delta h}$ and $W(S) \equiv B(R(S),S)/\rho$ represent

the benefit obtained under the steady state policy $g = R(S)$. It is shown in Tsur

and Zemel (1994a) that

$$V^{\delta h}(S) - W(S) = L(S)\delta h/\rho + o(\delta h). \qquad (2.7)$$

When $L(S) > 0$, there exist $h > 0$ and $\delta > 0$ such that $V^{\delta h}(S) > W(S)$, and the

steady state plan cannot be optimal for this state. Similarly, with $\delta < 0$ it is

seen that interior state levels satisfying $L(S) < 0$ cannot be steady states

either. Thus, the root of $L(S)$ and the boundary points $\underline{S}$ and $\bar{S}$ are the only

possible equilibria.

For a particular specification of R and B, the optimal plan $\{g_t^n, S_t^n\}$ and the

value function $V^n(S)$ are found via standard dynamic programming or optimal control

methods (see Tsur and Zemel, 1995).

2.2. Certainty: Suppose now that an event occurs as soon as S_t reaches a certain threshold level X, and assume that X is known. The event is undesirable in two respects. First, occurrence reduces the value of the resource. Let $\varphi(S)$ be the post-event value function, representing the value of the resource upon occurrence (when $S = X$). The undesirability of the event implies that $V^n(S) > \varphi(S)$. Furthermore, we require that a delayed occurrence is preferred to an immediate occurrence. That is, $V^n(S)-\varphi(S) \geq e^{-\rho t}[V^n(S_t^n)-\varphi(S_t^n)]$ for all $S_t^n < S$ along the optimal non-event plan. The right-hand side of this inequality represents the present value of a future loss caused by an occurrence at a later time t, which cannot exceed the loss associated with an immediate occurrence. The post-event value functions associated with the various types of events discussed below are consistent with these requirements.

With $T > 0$ denoting the event occurrence time ($T = \infty$ if the event never occurs) and X substituting $\underline{S}$ in (2.2), the allocation problem for $S > X$ is formulated as

$$V^c(S,X) = \max_{\{g_t, T\}} \int_0^T B(g_t, S_t)e^{-\rho t}dt + e^{-\rho T}\varphi(X)$$

$$\text{s.t. (2.1)-(2.2) and } S_T = X. \tag{2.8}$$

We call (2.8) the *certainty* problem and denote by $\{g_t^c, S_t^c\}$ the associated optimal plan. As in the non-event case it is verified (Tsur and Zemel, 1995) that the optimal solution S_t^c (at least one in the case of multiple solutions) evolves monotonically in time.

Since the event is unwanted, it is not optimal to decrease S_t^c to the occurrence level X when $\hat{S} > X$. This is so because the non-event process S_t^n stays above X and $V^n(S)$ exceeds the value of any plan that goes through X.[1] Thus, when

$\hat{S} > X$ the optimal certainty process coincides with the non-event process.

When $\hat{S} \leq X$, following the non-event path would eventually trigger the event, as S_t^n must pass through X on its way to $\hat{S}$. This may not be desirable if the event is too costly. How then to decide whether or not it pays to trigger the event? Just above X the planner must decide whether to decrease S_t^c further (and collect the post-event benefit $\varphi(X)$), or remain above X and enjoy a benefit (arbitrarily close to) $W(X)$. It is intuitively clear that the former policy is advantageous when $\varphi(X) > W(X)$ while the latter policy is preferable otherwise. Indeed, Tsur and Zemel (1995) established the following rule:

(a) If $\hat{S} > X$, then $S_t^c = S_t^n$.

(b) If $\hat{S} \leq X$ and $W(X) \geq \varphi(X)$, then S_t^c decreases asymptotically towards X but never reaches it at a finite time. In this case, S_t^c is found by solving

$$V^c(S,X) = \underset{\{g_t\}}{\text{Max}} \int_0^\infty B(g_t, S_t) e^{-\rho t} dt \quad \text{s.t. (2.1)}, \ g_t \geq 0, \ S_0 = S \text{ and } S_t > X.$$

(c) If $\hat{S} \leq X$ and $W(X) < \varphi(X)$, then S_t^c will reach X following which the post-event policy will be enacted: $\{S_t^c, \ t \in [0,T]\}$ and T are found by solving

$$V^c(S,X) = \underset{\{g_t, T\}}{\text{Max}} \int_0^T B(g_t, S_t) e^{-\rho t} dt + e^{-\rho T} \varphi(X) \quad \text{s.t. (2.1)-(2.2) and } S_T = X.$$

2.3. Uncertainty: Often X is incompletely known and can be specified in terms of the distribution and density functions $F(S) = \Pr\{X < S\}$ and $f(S) = dF/dS$. The distribution F is assumed to be smooth (twice continuously differentiable) over $[\underline{S}, \overline{S}]$) where $\underline{S}$ is the highest state level at which the event is bound to occur.

For any extraction plan $\{g_t, S_t\}$, the distribution on X induces a distribution on the occurrence time T. Starting at a pre-event state S, we search for the exploitation policy corresponding to

$$V(S) = \underset{\{g_t\}}{\text{Max}} \ E_T\left\{\int_0^T B(g_t,S_t)e^{-\rho t}dt + e^{-\rho T}\varphi(S_T)\,\big|\,T>0\right\} \quad \text{s.t. (2.1)-(2.2),} \qquad (2.9)$$

where E_T represents expectation with respect to the distribution of T. We call (2.9) the *uncertainty* problem and denote by $\{g_t^*,S_t^*\}$ the corresponding optimal plan.

As the process evolves in time, the distributions of X and T are modified to account for the information that X must lie below $\underset{\tau\in[0,t]}{\text{Min}} \{S_\tau\}$; for otherwise the event would have occurred at some time τ before t. The expected benefit (2.9), thus, depends on all history to time t. When S_t evolves monotonically in time, $\underset{\tau\in[0,t]}{\text{Min}} \{S_\tau\} = S_t$ or S_0 if S_t is non-increasing or non-decreasing, respectively. It turns out that at least one of the *optimal* S-trajectories corresponding to (2.9) evolves monotonically in time, allowing us to restrict attention to monotonic trajectories.

A detailed proof of the monotonicity property is given in Tsur and Zemel (1995). The property is due to the autonomous nature of the problem, as the functions B(g,S), R(S) and F(S) do not depend explicitly on time. It is shown that knowing, at any state level along the optimal trajectory, that the event has not yet occurred cannot affect earlier decisions. Prior to occurrence, no need ever arises to update the optimal plan (implying that the open- and closed-loop solutions are the same).

If a non-decreasing state trajectories is chosen, it is known at the outset that the event will never occur and the allocation problem is the same as the non-event situation.

For non-increasing state processes, the distribution of T, induced by that of X, is given by

$$1-F_T(t) \equiv \Pr\{T>t\,|\,T>0\} = \Pr\{X<S_t\,|\,X<S_0\} = F(S_t)/F(S_0)$$

and the allocation problem is that of[2]

$$V^a(S) = \max_{\{g_t\}} \int_0^\infty \{B(g_t, S_t) + \lambda(S_t)[g_t - R(S_t)]\varphi(S_t)\} \frac{F(S_t)}{F(S_0)} e^{-\rho t} dt$$

$$\text{s.t. } (2.1)-(2.2), \tag{2.10}$$

where $\lambda(S) \equiv f(S)/F(S)$ is a measure of the risk that the event will occur

following a small decline in the resource stock from a pre-event level S. $\lambda(S)$ is

assumed to decrease with S. We call (2.10) the *auxiliary* problem and denote by

$\{g_t^a, S_t^a\}$ its optimal plan. In similarity with the previous problems, S_t^a evolves

monotonically in time.

Although (2.10) is motivated by (2.9), we stress that the auxiliary and

uncertainty problems are distinct, because they share the same objective only when

the trajectory corresponding to the latter problem decreases. In fact, we

expect that when S_t^* increases it will coincide with S_t^n, and when it deceases it

will follow S_t^a. The behavior of S_t^n has been characterized above. Now, it is easy

to verify that while passing through state levels below the non-event equilibrium

$\hat{S}$, the optimal trajectory S_t^* must increase. Thus, the auxiliary problem is

relevant only for $S \geq \hat{S}$, and $\hat{S}$ replaces $\underline{S}$ in the application of condition (2.2) to

(2.10). Since $\lambda(S)$ does not increase, the auxiliary and non-event problems are

distinct only when $\lambda(\hat{S}) > 0$.

The dynamic behavior of S_t^a is determined by the auxiliary evolution function

$$L^a(S) = L(S) - \rho\lambda(S)[W(S) - \varphi(S)]. \tag{2.11}$$

In similarity with the non-event problem, the roots of $L^a(S)$ and the boundary

points $\hat{S}$ and $\bar{S}$ are the only possible equilibria. With $\lambda(\hat{S}) > 0$ and

$W(\hat{S}) = V^n(\hat{S}) > \varphi(\hat{S})$, we find that $L^a(\hat{S}) < L(\hat{S}) = 0$.

When $L^a(\hat{S})$ has at most one root in $[\hat{S}, \bar{S}]$, a state $\hat{S}^a$ is uniquely defined by:

$$\begin{cases} L^a(\hat{S}^a) = 0 & \text{if } L^a(\bar{S}) \geq 0 \\ \hat{S}^a = \bar{S} & \text{if } L^a(\bar{S}) < 0 \end{cases}$$ (2.12)

In this case, the analysis of the non-event problem can be repeated (with obvious modifications) to show that S_t^a converges to $\hat{S}^a$ from any initial state. Having characterized the non-event and auxiliary plans, the uncertainty trajectory can also be characterized by:

(i) S_t^* increases at S levels below $\hat{S}$ (for which $L(S) < 0$).

(ii) S_t^* decreases at S levels above $\hat{S}^a$ (for which $L^a(S) > 0$).

(iii) All state levels in $[\hat{S}, \hat{S}^a]$ (for which $L(S) \geq 0$ and $L^a(S) \leq 0$) are equilibrium states of S_t^*.

It is seen that the optimal process under uncertainty converges to the boundaries of $[\hat{S}, \hat{S}^a]$ from any initial state outside this interval, and remains constant when initiated at any state within the interval. At $\hat{S}^a$ the expected loss due to event occurrence is so high that entering the interval cannot be optimal, though without the event risk doing so would yield a higher benefit. Indeed, the steady-state interval is due to the difference between the evolution functions L(S) and $L^a(S)$, namely $\lambda(S)\rho[W(S)-\varphi(S)]$, which measures the expected benefit loss from an event following immediately a policy that extracts above recharge. Within $(\hat{S}, \hat{S}^a)$, the expected loss more than outweighs the benefit, and extraction above recharge is not desirable.

The case in which $L^a(\hat{S})$ admits multiple roots in $[\hat{S}, \bar{S}]$ is more involved and is discussed in Tsur and Zemel (1994a).

2.4. Environmental uncertainty: As shown above, the equilibrium interval emerges because the uncertainty problem reduces to two distinct problems, namely the non-event problem and the auxiliary problem, according to whether the state variable increases or decreases with time. This is obviously due to the type of

uncertainty considered, which reflects our ignorance with regard to the exact location of the critical level. Increasing trajectories cannot trigger the event, and uncertainty does not affect the expected benefit. In order to further illustrate the relation between the equilibrium interval and this type of uncertainty, we consider now uncertainty of a different origin, which does not give rise to equilibrium intervals.

Assume, following the catastrophic pollution model of Cropper (1976), that occurrence is not entirely due to the extraction plan driving the resource below the critical level, but that it can be influenced by random (exogenous) environmental conditions. Specifically, we take λ to be the hazard rate of the event occurrence, so that $Pr(T \leq h | T > 0) = \lambda h + o(h)$ for all S. With constant λ, T is distributed exponentially with

$$Pr(T > t | T > 0) \equiv 1 - F_T(t) = \exp\{-\int_0^t \lambda d\tau\} = e^{-\lambda t} \quad \text{and} \quad f_T(t) = \lambda e^{-\lambda t}.$$

The objective function is

$$E_T\left\{\int_0^T B(g_t,S_t)e^{-\rho t}dt + e^{-\rho T}\varphi(S_T) \mid T > 0\right\}$$

$$= E_T\left\{\int_0^\infty B(g_t,S_t)e^{-\rho t}I(T > t)dt + e^{-\rho T}\varphi(S_T) \mid T > 0\right\}$$

$$= E_T\left\{\int_0^\infty B(g_t,S_t)e^{-\rho t}I(T > t)dt + e^{-\rho T}\varphi(S_T) \mid T > 0\right\}$$

$$= \int_0^\infty B(g_t,S_t)e^{-\rho t}[1-F_T(t)]dt + \int_0^\infty e^{-\rho t}\varphi(S_t)f_T(t)dt$$

$$= \int_0^\infty \{B(g_t,S_t) + \lambda\varphi(S_t)\}e^{-(\rho+\lambda)t}dt,$$

and the allocation problem is

$$V^e(S) = \underset{\{g_t\}}{\text{Max}} \int_0^\infty \{B(g_t, S_t) + \lambda\varphi(S_t)\}e^{-(\rho+\lambda)t}dt, \quad \text{s.t. } (2.1)-(2.2), \qquad (2.13)$$

giving rise to

$$L^e(S) = L(S) + \lambda[\frac{\partial B(g,S)}{\partial g}|_{g=R(S)} - \varphi'(S)] \qquad (2.14)$$

for the environmental uncertainty evolution function, where L(S) is the non-event

evolution function (2.5). In similarity with the previous problems, the optimal

plan corresponding to (2.13) is monotonic, approaching an equilibrium state which

must be either a root of $L^e(S)$ or an endpoint. However, (2.13) describes the

expected benefit regardless of whether the trajectory increases or decreases,

hence it corresponds to the full uncertainty problem. Thus, no equilibrium

interval can emerge. A concrete example of this type of uncertainty is treated in

the following section.

3. Examples

We present here several examples of event uncertainty. The first example,

(Tsur and Zemel 1994a), corresponds to groundwater extraction under uncertainty

with regard to saline water intrusion. Since occurrence renders the aquifer

useless, the event is irreversible. The second example, (Tsur and Zemel 1994b),

concerns policy implications of possible consequences of global warming due to

the accumulation of greenhouse gases in the atmosphere. In the third example,

(Tsur and Zemel 1995), we consider the exploitation of resources that serve both

human needs and a wildlife population: exploiting the resource beyond an

incompletely known threshold level entails the extinction of the wildlife

population. Finally, we return to the aquifer example, but consider events the

occurrence of which is determined by exogenous factors; the uncertainty is of the

environmental type.

3.1. Irreversible event: Groundwater extraction under salinity risk

Consider a coastal aquifer exploited for domestic, irrigation or industrial purposes. At time t, S_t is the aquifer stock, g_t is the extraction rate and $R(G_t)$ denotes the natural recharge rate. When the stock level decreases to a threshold level X, saline water penetrates and ruins the aquifer. The critical level X is known only up to a probability distribution $F(S) = Pr(X \leq S)$.

Let Y(g) be the profit generated by extracting at the rate g and C(S) be the unit extraction cost when the stock is at the level S. The net instantaneous benefit is $B(g,S) = Y(g)-C(S)g$. We assume that $Y(0) = 0$, Y(g) is increasing and strictly concave, and C(S) is non-increasing and convex. Since the aquifer is useless after the event, the post-event value function $\varphi(S)$ vanishes at all S.

This benefit function satisfies all the conditions on B, specified above, and gives rise to the following the non-event evolution function (Eq. 2.5):

$$L(S) = [\rho-R'((S)][Y'(R(S))-C(S)] + C'(S)R(S)$$

yielding a unique non-event equilibrium state $\hat{S}$ as defined in (2.4). The steady state value is $V^n(\hat{S}) = W(\hat{S}) = [Y(R(\hat{S}))-C(\hat{S})R(\hat{S})]/\rho$.

Since $\varphi(S) = 0$ for all S, the auxiliary evolution function (2.11) becomes

$$L^a(S) = L(S) - \lambda(S)\rho W(S).$$

Suppose that $\underline{S} < \hat{S} < \overline{S}$ (if $\hat{S} = \overline{S}$ the aquifer does not admit profitable exploitation, and the analysis easily extends to the case $\hat{S} = \underline{S}$) so that $L(\hat{S}) = 0$. Since $W(\hat{S}) > 0$ at $\hat{S}$ (otherwise the steady state extraction rate vanishes, which is permitted only at $\overline{S}$), $L^a(\hat{S}) = -\lambda(\hat{S})\rho W(\hat{S}) < 0$, provided $\lambda(\hat{S}) > 0$. Since $W(\overline{S}) = 0$, and $L(\overline{S}) > 0$, we see that $L^a(\overline{S}) > 0$. Thus, $L^a(S) = 0$ must have solutions in $[\hat{S},\overline{S}]$.

When $L^a(S) = 0$ has a unique solution, equation (2.12) can be invoked to define $\hat{S}^a$ as the root of $L^a(S)$. Applying the uncertainty results of Section 2,

we conclude that the optimal extraction rate exceeds, is equal to or falls short of recharge at stock levels above $\hat{S}^a$, in $[\hat{S}, \hat{S}^a]$, or below $\hat{S}$, respectively.

If $L^a(S)$ has multiple roots in $[\hat{S}, \overline{S}]$, let $\hat{S}^a_l$ and $\hat{S}^a_u$ denote the smallest and largest of these roots. Tsur and Zemel (1994a) established that any state in $[\hat{S}, \hat{S}^a_l]$ must be a steady state. If the initial state lies below $\hat{S}$, the optimal policy is to recover the stock to the level $\hat{S}$ and to extract $R(\hat{S})$ thereafter. Above $\hat{S}^a_u$ it is always optimal to extract above recharge and to decrease the stock. The steady state to which the stock converges in this case must be one of the roots of $L^a(S)$ for which $L^{a\prime}(S) > 0$. Which of these roots is the optimal steady state must be determined in each case, depending on the particular specification of Y, C, R and F.

3.2. Partial reversibility: Accounting for global warming risks.

The combustion of fossil fuel produces CO_2 and other gases that accumulate in the atmosphere and, via the greenhouse effect, may lead to global warming. This process and its possible consequences are subject to a lively debate: some claim that a catastrophe is inevitable if CO_2 emission is not controlled; others maintain that the overall effect of a likely rise in the Earth's average temperature is beneficial; other opinions are scattered between these two extreme views (Cline, 1992; Nordhaus, 1991, 1994). The view taken here is closer to the pessimistic end of this spectrum, assuming that as soon as the atmospheric concentration of greenhouse gases reaches a certain (unknown) threshold level it triggers an undesirable event, inflicting an economic loss ψ.

To be consistent with the general framework of Section 2, the state variable should be defined as the purity level of the atmosphere, with $\overline{S}$ denoting the maximum purity (say natural CO_2 concentration before the industrial revolution) and $\underline{S}$ is the minimum purity, i.e., the carrying capacity of

atmospheric pollution. The resource, in this case, corresponds to atmospheric purity. It is more natural, however, to define the state variable in terms of the pollution level. To do that, let $G = \bar{S} - S$, so that $G = 0$ corresponds to zero pollution and $\bar{G} = \bar{S} - \underline{S}$ corresponds to maximum pollution.

The natural rate of removal of greenhouse gases from the atmosphere is $R(S) = R(\bar{S} - G) \equiv Q(G)$, where $Q(0) = 0$ and $Q(G)$ is increasing and concave. The net benefit function is specified as

$$B(g,G) = Y(g) - C(G),$$

where $C(G)$ is a non-decreasing and convex function describing the flow of costs (unrelated to the event) associated with pollution. X is the pollution level at which the event occurs, known up to the probability distribution $F(G) = Pr(X \leq G)$. Following the event, CO_2 emission is restricted not to exceed the natural removal rate in order to avoid further occurrences. Tsur and Zemel (1994b) have shown that if the event occurs under the optimal policy, it never pays to reduce the pollution level, hence the post-event emission equals $Q(G)$ and the post-event value function is given by $\varphi(G) = W(G) - \psi$.

With these specifications, the non-event evolution function (2.5) specializes to

$$L(G) = [\rho + Q'(G)]Y'(Q(G)) - C'(G)$$

and $L(G)/[\rho + Q'(G)]$ decreases in G. The non-event steady state to which the non-event optimal state converges is defined by (2.4):

$$\begin{cases} \hat{G} = \bar{G} & \text{if } L(\bar{G}) > 0 \\ \hat{G} = 0 & \text{if } L(0) < 0 \\ L(\hat{G}) = 0 & \text{otherwise} \end{cases}$$

The evolution function of the auxiliary problem specializes to

$$L^a(G) = L(G) - \lambda(G)\rho\psi.$$

Since $\lambda(G)$ is non-decreasing, $L^a(G)/[\rho + Q'(G)]$ decreases with G, and $\hat{G}^a$ of (2.12)

is unique. Our analysis then implies that the optimal uncertainty state process converges to the boundaries of $[\hat{G}^a, \hat{G}]$ from any initial state outside this interval and remains in equilibrium inside the interval.

3.3. Coexistence and Competition between human and wildlife populations

Here the framework of Section 2 is applied to a situation in which off-stream diversion of water for irrigation (or other human needs) may lead to the extinction of some wildlife, say fish, population. The resource state S measures the water level at some crucial point along the river, R(S) represents net natural replenishment rate, and the off-stream diversion rate is denoted by g. As in the groundwater case, the instantaneous net benefit is of the form

$B(g,S) = Y(g)-C(S)g$, with $Y(g)$ increasing and strictly concave and $C(S)$ non-increasing and convex.

Excessive off-stream diversion will lead to the elimination of the wildlife population if the water level decreases below a critical value, inflicting a penalty ψ. The penalty represents biodiversity, recreational and nonuse benefits forgone due to extinction, and is taken as an exogenous parameter. Unlike the previous example, the post-event diversion process proceeds with no further restrictions. With events of this nature, the post-event value function takes the form $\varphi(S) = V^n(S) - \psi$.

The non-event evolution function and the corresponding equilibrium state $\hat{S}$ are the same as in the groundwater example. The auxiliary evolution function reflects the different form of φ:

$$L^a(S) = L(S) + \rho\lambda(S)[V^n(S)-W(S)-\psi].$$

Since $V^n(\hat{S}) = W(\hat{S})$ and $L(\hat{S}) = 0$, $L^a(\hat{S}) = -\rho\lambda(\hat{S})\psi < 0$. Tsur and Zemel (1995) have established that $L^a(S)$ can have at most one root in $[\hat{S},\bar{S}]$: either $L^a(S)$ is negative for all S in $[\hat{S},\bar{S}]$, in which case $\hat{S}^a = \bar{S}$, or $L^a(\hat{S}^a) = 0$ for some $\hat{S}^a$ in

$(\hat{S}, \bar{S}]$ (cf. 2.12). Again, the optimal state process S_t^* must converge to the nearby boundary of $[\hat{S}, \hat{S}^a]$ when initiated outside this interval and remain at equilibrium within the interval. The upper boundary $\hat{S}^a$ depends on the penalty ψ: $\hat{S}^a$ increases with ψ. When the penalty is so high that $L^a(\bar{S})$ is negative, $\hat{S}^a = \bar{S}$, and it is never optimal to extract above recharge. In this case, preservation is guaranteed.

3.4. Irreversible event: Groundwater extraction under environmental uncertainty.

Consider again the aquifer of example 3.1, managed under the risk that an event will render it useless. This time, however, occurrence is not restricted to the state process arriving at the unknown critical level. Rather, an event (such as the discovery of a cheaper resource by a competitor, or the destruction of the resource by an accident in a nearby pollution source), can occur at any state level according to the hazard rate λ. With $\varphi \equiv 0$, the evolution function (2.14) reduces to $L^e(S) = L(S) + \lambda[Y'(R(S)) - C(S)]$. The arguments for the uniqueness of $\hat{S}$ apply for $L^e(S)$ as well, implying that this evolution function can have at most one root in $[\underline{S}, \bar{S}]$. For $S \geq \hat{S}$, when $L(S) \geq 0$, the second term of $L^e(S)$ is positive and $L^e(S)$ cannot vanish. Thus, the root of $L^e(S)$, corresponding to the equilibrium state of this uncertainty problem, must lie **below** $\hat{S}$. This behavior is in sharp contrast with the results of the previous examples, in which uncertainty implies a more prudent management. The reason for this is clear [Clarke and Reed, (1994)]: since the extraction policy does not affect the occurrence probability, the manager is encouraged to extract as much as he can before the aquifer is ruined. Indeed, with $\varphi \equiv 0$, the allocation problem (2.13) reduces to the non-event problem, with $(\rho + \lambda)$ replacing ρ as the effective discount rate. Obviously, this kind of event uncertainty cannot give

rise to equilibrium intervals.

4. Summary

In this chapter we offer a unified framework for a systematic investigation of the effects of uncertainty on natural resource exploitation policies. The uncertainty concerns the conditions leading to the occurrence of an undesirable event. The events can differ in their consequences: Irreversible events render the resource useless, while partly reversible events imply an economic penalty, but they do not prohibit further exploitation. Another classification is according to occurrence conditions: Some events occur if, and only if, the resource stock declines to a critical level which is not a-priori known. Other events are subject to environmental uncertainty and do not depend entirely on the resource stock.

An important common feature of all these models is the monotonic evolution of the state process. In other respects, the processes vary significantly. Events that occur at the critical level call for prudent exploitation and more preservation. This is manifested by an equilibrium interval, inside which extraction equals the rate of recharge. The interval is clearly associated with the uncertainty - without occurrence risk it pays to extract above the recharge rate when the resource level is within the interval. The equilibrium interval is easily determined in terms of the roots of certain functions of the state variable, which we call *the evolution functions*. Explicit knowledge of the optimal trajectories is not required for this task. In this manner the evolution functions conveniently provide policy prescriptions aimed at determining when extraction should not exceed recharge.

The equilibrium interval is unique to events whose occurrence conditions

depend entirely on the resource state. When the event occurs as a result of conditions external to the resource state, no such intervals emerge and the optimal state converges to a particular level. For such events, it is even possible that the event uncertainty encourages more exploitation rather than preservation because the prudent policy does not imply reduced occurrence risks.

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Endnotes

1 If S_t^c decreases to X, then $V^c(S,X) = \int_0^T B(g_t^c, S_t^c)e^{-\rho t}dt + e^{-\rho T}\varphi(X) <$

$\int_0^T B(g_t^c, S_t^c)e^{-\rho t}dt + e^{-\rho T}V^n(X) \le V^n(S)$. The first inequality follows from $V^n(X) > \varphi(X)$

and the second holds because the state trajectory that coincides with S_t^c for

$t \in [0,T]$ and follows S_t^n that departs from X thereafter is feasible but may not be

optimal for the non-event problem.

2 The density of T is $f_T(t) \equiv dF_T(t)/dt = f(S_t)[g_t-R(S_t)]/F(S_0)$ with the hazard

rate $f_T(t)/[1-F_T(t)] = \lambda(S_t)[g_t-R(S_t)]$. By expressing the expectation in (2.9) as

$E_T\left\{\int_0^\infty B(g_t,S_t)I(T>t)e^{-\rho t}dt + e^{-\rho T}\varphi(S_T) \mid T>0\right\}$, with $I(\cdot) = 1$ or $I(\cdot) = 0$ when its

argument is true or false, respectively, and noting that $E_T\{I(T>t) \mid T>0\} = 1-F_T(t)$

$= F(S_t)/F(S_0)$, the objective of (2.9) for non-increasing S trajectories becomes

$\int_0^\infty \{B(g_t,S_t)+\lambda(S_t)[g_t-R(S_t)]\varphi(S_t)\}\frac{F(S_t)}{F(S_0)}e^{-\rho t}dt$, which leads to (2.10).