RISK-EFFICIENT IRRIGATION STRATEGIES FOR WHEAT

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Abstract

Agricultural production is risky. When farmers are risk averse, they are likely to put a premium on production methods which reduce perceived risks. Irrigation is generally believed to be a risk-reducing input. By using the concept of stochastic dominance, risk-efficient irrigation policies for wheat grown in central India are identified and quantitative estimates of benefits due to risk reduction are obtained. Such benefits were found to be of a large order of magnitude. The more common methods such as mean variance analysis tended to over-estimate the benefits.

1. Introduction

Agricultural production processes are inherently risky, one of the major sources of risk being the climatic variability. When attitudes to risks are non-neutral, farmers allocate controllable inputs in such a way as to increase or reduce the impact of risk. Thus, it is important to incorporate risk in models of farmer behaviour.

The objective in this paper is twofold. First, risk-efficient irrigation schedules for winter wheat grown in the Raisen district of central India are identified. This is achieved by applying the concept of stochastic dominance with respect to a function (Heyer 1977a, b). Second, the value of investment in irrigation for a group of risk-averse farmers is estimated. For a risk-neutral farmer, this is equal to the expected gain in net returns with irrigation over that without irrigation. As irrigation generally reduces yield risk, additional benefits in the form of reduced risks of low net incomes are obtained by risk-averse farmers. Such additional gains will be quantified.

2. Decision Making Under Uncertainty

One of the most widely applied models for studying decision making under uncertainty is the expected utility model (Schoemaker 1982). The implementation of the model requires that both the probability distribution of outcomes and the risk preferences of decision makers be precisely known. The measurement of risk preferences directly by elicitation of utility function, or indirectly by imputation, is subject to large errors (King and Robison 1981, Schoemaker 1982). Stochastic efficiency criteria are useful when risk preferences cannot be measured accurately. These criteria satisfy the axioms of the expected utility model but do not require precise
measurements of risk preferences. However, as opposed to the complete ordering achieved when risk preferences are precisely known, stochastic efficiency rules provide only a partial ordering.

Stochastic efficiency rules are implemented by pairwise comparisons of cumulative distribution functions (CDF) of outcomes (e.g., net income) resulting from different actions. If the only restriction which can be placed on the nature of the utility function is that more is preferred to less (i.e., the first derivative of the utility function is positive), the first degree stochastic dominance rule (FSD) can be applied. Graphically, the rule requires that, for the distribution $F(Y)$ to be preferred to $G(Y)$, $F(Y)$ should never be to the left of $G(Y)$ but should be to the right of $G(Y)$ for at least one probability point. No assumptions are made about risk preferences of the decision maker. The coefficient of absolute risk aversion (Pratt 1964) may be anywhere between $-\infty$ to $+\infty$. Thus, the rule has a very low discriminatory power.

If it is assumed that the marginal utility is positive but decreases with an increase in income, the second degree stochastic dominance (SSD) rule is applicable. The allowed range on the value of the absolute risk-aversion coefficient is 0 to $+\infty$. This rule is applicable to all risk-averse decision makers. For $G(Y)$ to be dominated by $F(Y)$, the SSD rule requires that:

\[
(1) \quad \int_{-\infty}^{x} [F(Y) - G(Y)] \, dY \leq 0 \quad \text{for all } -\infty < x < +\infty, \text{ and } \quad \int_{-\infty}^{x} [F(Y) - G(Y)] \, dY < \text{ for some } x.
\]

The SSD criteria, although more powerful than the FSD, may still be inadequately discriminatory for many practical applications (Anderson 1974, Anderson, Dillon and Hardaker 1977).

Based on Pratt's (1964) proof that the coefficient of absolute risk aversion represents risk preferences uniquely, Meyer (1977a, b) has proposed a more general stochastic dominance rule, often termed stochastic dominance with respect to a function (SDWRF). If the absolute risk-aversion function of a class of decision makers is bounded by $r_1(Y)$ and $r_2(Y), F(Y)$ is preferred to $G(Y)$ by all decision makers within the preference interval if the utility function $u(Y)$ which minimises

\[
(2) \quad \int_{-\infty}^{+\infty} [G(Y) - F(Y)]u'(Y) \, dY
\]

subject to $r_1(Y) < -u''(Y)/u'(Y) < r_2(Y)$
produces a positive value of the equation. If the minimum is negative
F(Y) does not dominate G(Y). In this case, to check if G(Y) dominates
F(Y), the minimum of the expression

\[ \int_{-\infty}^{+\infty} [F(Y) - G(Y)]u'(Y) \, dY \]

subject to (3) is evaluated. If the minimum is positive, G(Y)
dominates F(Y). If the minimum is again negative, both the
distributions are in the efficient set. The SDWRF criteria cannot
discriminate between the two distributions in such cases.

It is clear from the above discussion that both FSD and SSD are
special cases of SDWRF. The discriminatory power of SDWRF depends on
the width of the preference interval as defined by \( r_1 \) and \( r_2 \). The
desired level of precision can be achieved by selecting an appropriate
range of 'r'. Thus SDWRF allows one to make a tradeoff between the
probability of Type I error (ie, incorrect ranking) which is high in
the explicit utility model and the probability of Type II error (ie,
incomplete ranking) which is high in FSD and SSD criteria. Due to
these flexibilities, SDWRF has become a popular tool used in both
policy research and agricultural extension work (King and Robison
review of SDWRF is provided by Cochran (1986).

When preferences are non-linear, SDWRF can also be used to
calculate the additional benefit resulting from one action over
another. The additional benefit is equal to the amount which a class
of decision makers would be willing to pay, in each state of nature,
and remain indifferent between a dominant distribution and an inferior
alternative (Byerlee and Anderson 1982). If F(Y) dominates G(Y), then
willingness to pay for using the strategy which generates F(Y) over
the alternative generating G(Y) is equal to the horizontal leftward
shift in F(Y) required for both F(Y) and G(Y) to be in the efficient
set. The size of the horizontal shift \( V \) is calculated by satisfying
the following three conditions simultaneously (Bosch and Eidman 1987):

\[ \int_{0}^{1} [G(Y) - F(Y-V)]u'(Y) \, dY > 0 \]

\[ \int_{0}^{1} [G(Y) - F(Y-V-Z)]u'(Y) \, dY \leq 0 \]

\[ r_1(Y) < -u''(Y)/u'(Y) < r_2(Y) \]
Z in equation (6) is a small positive number. When the utility function $u$ is linear, $V$ is equal to the difference between the means of the distributions $G(Y)$ and $F(Y)$. If $r_1(Y) = r_2(Y)$, ie if preferences are precisely known, $V$ is the difference between certainty equivalents of $F(Y)$ and $G(Y)$.

In the context of investment in irrigation, $V$ is an indicator of benefits which farmers derive through the use of water. It is the sum of the expected increase in net income made possible by irrigation and the gain in terms of reduction in production risk compared to a non-irrigated situation. Although it has been generally accepted that irrigation reduces production risks, attempts at quantifying benefits of reduction in risk have been few and generally made under more restrictive assumptions of normally distributed net returns and exponential or quadratic utility functions (Carruthers and Donaldson 1971, Apland, McCarl and Miller 1980, English 1981, Boggess et al 1983). While appraising irrigation projects, no explicit account is generally taken of such additional benefits (Sinha and Bhatia 1982). Hence benefits from irrigation may have been under-estimated in the evaluation of irrigation projects.

3. Empirical Models

Distributions of net returns for several exogenously-specified irrigation schedules were obtained in this study using a simulation model. The simulation model consists of a simplified soil water balance sub-model for wheat grown in the Raisen district of central India, and an equation for predicting yield on the basis of transpiration deficit. Thirty years of daily climatic data were used to drive the model.

Wheat was assumed to be sown on the 298th Julian day and harvested on the 54th Julian day of the following year. Although these dates vary slightly from year to year, they represent a typical pattern in the district under study. The growing season was divided into four stages, namely: sowing to ear initiation (40 days), ear initiation to flowering (25 days), flowering to soft dough (25 days) and soft dough to harvest (31 days). In using this classification,

1. However, Meyer (1987) has shown that these assumptions are not always necessary for the mean-variance analysis to be theoretically consistent with the expected utility model. A sufficient condition for the mean-variance analysis to be valid is that net returns be a positive linear function of the stochastic variable.
response to irrigation is presumed to differ among these stages but to remain constant within a stage.

Yield of wheat is predicted using a transpiration-based model estimated by Pandey (1985). The specification used is:

\[
Y_c = a \prod_{i=1}^{4} (T/TP)^{\lambda_i} \exp[e_i],
\]

where,  
\( Y \) = actual yield,  
\( T \) = actual transpiration,  
\( TP \) = potential transpiration,  
\( a, \lambda \) = model parameters,  
\( i \) = growth stage index,  
\( t \) = time index, and  
\( e \) = normal random variate \([E(e)=0, E(e^2)=\sigma^2]\).

For estimating the model, data on \( T \) and \( TP \) for various growth stages are required. Since these data were not directly available, they were estimated using the soil water balance model. The details of the soil water balance and its validation are discussed by Pandey (1986).

Irrigation experiments conducted by Tomar, Gupta and Tomar (1981) for three years (1974/75-1976/77) each consisting of four treatments were used as the basic data source. \( T/TP \) for the first stage was close to unity in all 12 observations as a pre-sowing irrigation was provided to all treatments. Hence \( \lambda \) could be estimated for the last three stages only.

A note is in order for the specification of the error structure used in the model. Just and Pope (1978) have shown that a multiplicative error structure such as in equation (8) implies that marginal risk increases with an increase in \( T/TP \). In the present case, as \( T/TP \) approaches unity marginal risk can be expected to decrease. Despite the appropriateness of the Just and Pope specification for the present study, the limited number of data points (only 12) precluded any reliable estimation of marginal risk coefficients.

The estimate of the yield response equation is:

\[
\log Y = 1.57 + 0.52 \log(T/TP)^2 + 0.09 \log(T/TP)^3 + 0.16 \log(T/TP)^4
\]

\[ (0.05) \quad (0.24) \quad (0.11) \quad (0.05) \]

\( R^2 = 0.90 \)  
\( n = 12 \)
The estimated standard errors are in parentheses.

The antilog of the intercept provides an estimate of the potential yield of wheat in the absence of moisture stress. Its value for equation (9) is 4.8 t/ha. Such high yields are unlikely to be realized on farmers' fields due to poorer environmental conditions and inadequacy of complementary inputs in comparison to those in experimental plots. Accordingly, the estimated value of the intercept was shifted down to represent a more realistic yield of 1 t/ha on farmers' fields.

The model was used to predict wheat yield for various irrigation schedules. The basic structure of the schedules is presented in Table 1. The range of water application was varied from 10 to 60 mm in steps of 10 mm. Thus, there are six sets of eight schedules each. The first seven schedules of each set are derived using all possible combinations of skipping irrigation in the last three stages. The last schedule (i.e., the eighth of each set) corresponds to the existing practice of irrigating in all four stages.

### TABLE 1
The Basic Structure of Irrigation Schedules

<table>
<thead>
<tr>
<th>Schedule</th>
<th>Growth Stage 1</th>
<th>Growth Stage 2</th>
<th>Growth Stage 3</th>
<th>Growth Stage 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>x†</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>x</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>x</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>x</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>x</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>x</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>x</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

† x if water not applied, / if water applied.

In the first set of schedules, the quantity of water applied is 10 mm. For example, in the third schedule, 10 mm of water is applied on the first day of each of the stages 2, 3 and 4. In the second set, 20 mm of water is assumed to be applied when irrigated. Altogether 49 schedules were thus generated by adding a final non-irrigated
treatment. The possibility of varying the level of water application between stages is not considered in these schedules.

Lower and upper bounds \((r_1\) and \(r_2\)) on the values of absolute risk-aversion coefficient defined over net income for a representative farm are required for implementing SDWF. Evidence indicates that poor farmers in India are mostly risk averse (Binswanger 1980, Antle 1987). Hardaker and Ghodake (1984) have calculated 'r' from the estimates of partial risk-aversion coefficients reported by Binswanger (1980). Their estimates range from \(1.03\times10^{-4}\) to \(2.67\times10^{-3}\). Based on Anderson, Dillon and Hardaker (1985), Pandey (1986) calculated 'r' as the ratio of relative risk-aversion coefficient to the total wealth. The estimate is equal to \(4\times10^{-5}\). In the light of Raskin and Cochrane's (1986) comments about the pitfalls in transferring the estimates of coefficient of risk aversion estimated in a particular situation to another, four ranges for risk-aversion coefficients are used. The ranges specified are 0 to 0.00004, 0.00004 to 0.0004, 0.0004 to 0.004, and 0.004 to 0.04.

Thirty years of daily rainfall and evaporation data were used for predicting T and TP for each irrigation schedule. Yield was predicted by substituting the estimated value of T and TP in equation (5) and allowing for random variations in the error term. Randomness was explicitly incorporated because parameters of the regression equation are themselves random (Anderson 1976). Following Mihram (1972), different seeds for generating random numbers were used for each of the schedules. Thus, estimated yields incorporate stochasticity in the climatic variables in the estimated parameters of the model.

Net returns were calculated by subtracting all variable costs from gross returns. Variable costs included were of three types: the input costs which were fixed for all treatments (fertilizers, labour, etc); the cost of irrigation which varied according to the quantity of water applied and the number of applications; and the harvesting costs which varied according to the yield of the crop. In gross returns was also included the market value of wheat straw. Summary measures of distributions of yield, net returns and quantity of water applied for 15 schedules which are efficient in the sense of FSD are presented in Table 2.

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2. These are the only sources of stochasticity considered in this paper. Price risk was not included because its effect is invariant to irrigation strategies.
### Table 2

**Summary Statistics of Distributions of Yield, Net Returns and Water Applied for FSD Schedules**

<table>
<thead>
<tr>
<th>Schedule number</th>
<th>Yield</th>
<th>Net Income</th>
<th>Average quantity of water applied (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>Standard Coeff of deviation</td>
<td>Coefficient of skewness (t/ha) (t/ha)</td>
</tr>
<tr>
<td>S22</td>
<td>0.76</td>
<td>0.12</td>
<td>0.18</td>
</tr>
<tr>
<td>S23</td>
<td>0.73</td>
<td>0.13</td>
<td>0.39</td>
</tr>
<tr>
<td>S27</td>
<td>0.85</td>
<td>0.08</td>
<td>0.76</td>
</tr>
<tr>
<td>S28</td>
<td>0.69</td>
<td>0.19</td>
<td>0.73</td>
</tr>
<tr>
<td>S29</td>
<td>0.72</td>
<td>0.16</td>
<td>0.56</td>
</tr>
<tr>
<td>S31</td>
<td>0.79</td>
<td>0.12</td>
<td>-0.53</td>
</tr>
<tr>
<td>S35</td>
<td>0.91</td>
<td>0.08</td>
<td>-0.25</td>
</tr>
<tr>
<td>S37</td>
<td>0.75</td>
<td>0.14</td>
<td>-0.02</td>
</tr>
<tr>
<td>S38</td>
<td>0.83</td>
<td>0.08</td>
<td>0.02</td>
</tr>
<tr>
<td>S39</td>
<td>0.81</td>
<td>0.13</td>
<td>-0.07</td>
</tr>
<tr>
<td>S42</td>
<td>0.87</td>
<td>0.09</td>
<td>0.63</td>
</tr>
<tr>
<td>S43</td>
<td>0.96</td>
<td>0.06</td>
<td>-0.55</td>
</tr>
<tr>
<td>S45</td>
<td>0.80</td>
<td>0.10</td>
<td>0.36</td>
</tr>
<tr>
<td>S'6</td>
<td>0.87</td>
<td>0.07</td>
<td>0.79</td>
</tr>
<tr>
<td>S49</td>
<td>0.56</td>
<td>0.16</td>
<td>0.36</td>
</tr>
</tbody>
</table>

Results are presented for one unit area. However, all stochastic efficiency analyses were conducted by scaling net returns up to the representative farm size (10 ha).

The distributions of yields and net returns are positively skewed in most cases. This agrees with the observations made by Day (1965) and Walker and Subba Rao (1982). The usual assumption of normality hence seems inappropriate.

4. **Analysis**

A microcomputer software developed by Coh, Raskin and Cochran (1987) was used for stochastic efficiency analyses. The program also allows for the identification of quasi-first- and second-degree stochastic dominance. The distributions presented in Table 2 are quasi-FSD. For quasi-FSD, the bounds on r(Y) are set wide enough to
include essentially all observed risk-preference behaviour. For quasi-SSD, the lower bound is set equal to zero. The bounds are set automatically by the program such that the absolute size of the relative risk-aversion coefficient never exceeds 100.

5. Results

Of the 49 distributions considered, 15 were quasi-FSD (Table 2). The schedule of irrigating in all four stages, which is recommended by extension workers in the region, was dominated in the sense of FSD. Only four schedules (S35, S43, S45, S46) were quasi-SSD. It is not possible to discriminate among these schedules on the basis of the usual SSD criteria. All four schedules have a comparable average net income but average water use for schedules S45 and S46 is much lower compared to that for schedules S35 and S43. Thus schedules S45 or S46 may be preferable if reduced water usage is also one of the objectives.

Results of SDWF are presented in Table 3. For a low level of risk aversion (0 ≤ r ≤ 0.00004), the risk-efficient schedule is S43. The schedule is also the one which maximises the average net return. At a very high level of risk aversion (0.004 ≤ r ≤ 0.04), S43 ceases to be risk-efficient. S45 and S46 are preferable due to their risk-reducing effects, even though mean net incomes for these schedules are lower. S43 and S46 are risk-efficient schedules if the maximum size of 'r' is 0.004. These schedules correspond to the application of 60 mm of water in stages 2, 3 and 4 and the application of 60 mm of water in stages 2 and 4, respectively.

<table>
<thead>
<tr>
<th>Risk aversion interval</th>
<th>Dominant schedules</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 to 0.00004</td>
<td>S43</td>
</tr>
<tr>
<td>0.00004 to 0.0004</td>
<td>S43</td>
</tr>
<tr>
<td>0.0004 to 0.004</td>
<td>S43, S46</td>
</tr>
<tr>
<td>0.004 to 0.04</td>
<td>S45, S46</td>
</tr>
</tbody>
</table>

TABLE 3

Results of SDWF
More risk-averse farmers might be expected to apply a higher quantity of risk-reducing inputs, such as water. However, within the risk-preference interval considered in this study, the average level of water application associated with risk-efficient schedules seems to decrease with an increase in risk aversion. This behaviour can be explained on the basis of an increased positive skewness of net returns associated with water-conserving schedules (S45 and S46) and higher net returns at lower tails compared to S43 (Figure 1). Both these factors increase the utilities of water-conserving schedules (Tsiang 1972, Hammond 1974).

The dominance of schedules S43 and S46 implies that farmers using these schedules are better off than the ones without an access to irrigation (schedule S49). The value of irrigation (V) is the maximum amount the existing users of S45 (or S46) will be willing to pay to continue using S43 (or S46). As mentioned before, V measures the value to farmers at only one of the end points in the preference interval ($r_1$, $r_2$). Farmers at the other end of the preference interval will always be willing to pay more than V. Thus, upper and lower limits on the value of V can be identified. The lower limit $V_L$ is as defined

![Figure 1](image-url)

**FIGURE 1**
CDF of Net Returns
above. The upper limit ($V_U$) is the horizontal shift in the CDF of the dominant distribution required for all users of the dominant strategy to switch over to the undominated strategy. The two limits converge with the convergence of the preference interval. If the risk-aversion coefficient is precisely defined, $V = V_U - V_L$ is the difference between certainty equivalents of the dominant and the dominated distribution. It is common to calculate certainty equivalents as a linear combination of mean and variance (Freund 1956) under the dual assumptions of an exponential utility function and normally distributed net returns. The estimates of $V$ obtained using the exponential utility, moment-generating function approach (EUMGF) as implemented by Yassour, Zilberman, and Rausser (1981) are presented in Table 4 for normal and gamma distributions along with those obtained using SDWRF.

### Table 4

<table>
<thead>
<tr>
<th>Risk-aversion interval</th>
<th>Benefits of Irrigation†</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>SDWRF</td>
</tr>
<tr>
<td>$r_1$</td>
<td>$r_2$</td>
</tr>
<tr>
<td>0</td>
<td>0.00004</td>
</tr>
<tr>
<td>0.00004</td>
<td>0.0004</td>
</tr>
<tr>
<td>0.0004</td>
<td>0.004</td>
</tr>
</tbody>
</table>

† Dominant and dominated schedules used in these calculations are S43 and S49 respectively.

For the first preference interval, $V_L$ is Rs 216/ha. This is obtained using $r_1 = 0$. It is simply the difference between the means of net returns associated with S43 and S49. Both $V_L$ and $V_U$ increase with an increase in the coefficient of risk aversion. At low levels of risk aversion, estimates of $V$ under SDWRF are similar to those under normal and gamma distribution assumptions. However, with an increase in the risk aversion coefficient, the assumptions of normal and gamma distribution resulted in over-estimate and under-estimate,
respectively. For example, \( V_U \) for the third interval using SDWRF is three times lower and 1.5 times higher than the corresponding values under normal and gamma distribution assumptions, respectively. SDWRF may be considered to provide a better estimate of the true \( V \) because no assumptions about the nature of distribution is made. The assumption of normal distribution, which is commonly used in risk analysis, tended to over-estimate \( V \), the severity of over-estimation increasing with the size of \( \tau \).

Taking the third preference interval as the relevant one, the estimate of \( V_L = Rs 318 \) is the sum of the benefits from increase in mean net return and benefits from risk reduction. The latter is the difference between \( V_L \) for the third and the first preference intervals (i.e., \( Rs 318 - Rs 216 = Rs 102 \)). In the present case, benefits due to risk reductions seem to be as high as 47 per cent (i.e., \( Rs 102/Rs 216 \)) of the benefits in terms of increase in mean net returns. Thus risk-reducing inputs such as irrigation can improve farmers’ welfare substantially by reducing income risks. Benefits from investments for supplying such inputs can be seriously under-estimated if, as is the general practice, only the difference between mean net returns is considered.

6. Summary and Conclusions

Risk-efficient irrigation schedules for wheat were identified using a generalised stochastic dominance. The policy of applying 60 mm of water in growth stages 2, 3 and 4 was found to be risk efficient at low levels of risk aversion. Efficient schedule for a higher level of risk aversion was to skip irrigation in the second stage. This contrasts with the extension advice of applying about 60 mm in each of the four stages. The usual rationale for an intensive irrigation as a risk-reducing strategy is not supported by this study. In fact, increased risk aversion within the preference interval examined resulted in reduced water usage.

The benefits of irrigation to risk-averse farmers were also calculated using the stochastic dominance rule. The results indicate that benefits in terms of reduction in risk may be a significant proportion of the difference in mean net returns with and without irrigation. Such benefits, of course, increase with an increase in risk aversion. The more common mean-variance analysis tended to over-estimate benefits from risk reduction. Benefits of irrigation may be significantly under-estimated in the appraisal of irrigation projects if non-linearity in risk preferences is not allowed for.
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