ASSESSING THE IMPACT OF LAMB PRICE STABILISATION ON PRODUCER WELFARE*

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Abstract

This paper considers the maximising behaviour of small competitive firms engaged in the production of wool and lambs as joint products. Groups of these firms are assumed to have access to the same stochastic technology and are assumed to be expected utility maximisers who must choose input levels *ex ante*. A number of assumptions are made in order that input demands can be expressed as tractable functions of the mean and variance of a random return. The Minimum Reserve Price scheme for wool and a partial lamb price stabilisation scheme are incorporated into the model. Expressions for the producer welfare effects of changes in the degree of partial lamb price stabilisation are derived. The estimation of the parameters of the model is discussed.

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1. Introduction

The lamb industry is comprised of a large number of relatively small and highly competitive firms facing a high degree of uncertainty in the joint production of lambs and wool. Specifically, these outputs are a joint stochastic function of inputs which must be chosen before output prices are known (*ex ante*). Of course wool price uncertainty is currently reduced by the Australian Wool Corporation (AWC) through the administration of a Minimum Reserve Price (MRP) scheme. This paper is motivated by calls for a reduction in lamb price uncertainty through the administration of a lamb price stabilisation scheme (see for example Ogilvie (1983), McSporren (1984) and Scott-McGowan (1984)). The paper is a theoretical one - the results of an empirical application of the model and the estimation techniques outlined below are not yet available.

Price stabilisation proposals are often empirically assessed by simulating the operation of a price stabilisation scheme using a model of agricultural supply response (see for example BAE (1979) and McInnes *et al* (1986)). These empirical models of agricultural supply response tend to account for uncertainty by the inclusion of risk variables which are sometimes crude and often arbitrary (see for example Behrman (1968), Freebairn (1973) and Just (1974)). In this paper, however, we explicitly generate an uncertainty model of supply response from a theoretical base. Specifically, producers are assumed to have access to the same stochastic technology and are assumed to be expected utility maximisers who must choose levels of inputs *ex ante*. As Quiggin (1986) points out, under certain assumptions the behaviour of these firms is formally equivalent to the behaviour of firms which make similar choices with access to a deterministic technology. Models of this genre have been considered by Baron (1970), Sandmo (1971) and others. In these models firms assume some distribution of output prices to prevail in each time period and output is generally derived as some function of the moments of this distribution. We make a seperability assumption on the stochastic technology which is sufficient for these models to apply. In addition we assume that firms hold rational expectations in the sense that the price distribution which they assume prevails in each time period corresponds to the distribution of prices found as the solution to the market equilibrium conditions.

The structure of the paper is as follows. In Section 2 we consider the behaviour of firms in three lamb and wool producing sectors of the Australian economy. Input and planned output choices are specified as functions of the mean and variance of a 'random return to production'. In Section 3 we derive expressions for this 'random return' by assuming that the inverse demand schedules for lambs and wool are stochastic and, in fact, logarithmic. In this section we model the Minimum Reserve Price scheme for wool as a buffer stock scheme with known floor and ceiling prices. Moreover, we model a feasible partial price stabilisation scheme for lambs, and this scheme is simply a tax (subsidy) which is collected (paid) by the stabilisation authority at the end of the production period. In Section 4 we make assumptions regarding the distribution of random variables which give rise to supply and demand uncertainty, and as a result we are more explicit about the mean and variance of the 'random return'. In Section 5 we derive expressions for the cash benefits of certain small or large changes in the degree of partial lamb price stabilisation. In Section 6 we outline techniques by which the parameters of the model can be estimated. The paper is concluded in Section 7.

2. A Model of Firm Behaviour in the Wool and Lamb Producing Sectors

We view the economy as being comprised of a number of sectors including a total of three sectors (indexed k = 1, 2, 3) in which firms produce different kinds of wool and lambs (indexed j = 1, 2 respectively)[1]. In addition we let there be firms in other sectors in the economy which produce, for example, beef cattle and cereals. Indeed, we assume there exist firms which, in the course of one or more production periods, produce adult sheep using lambs as an input. In practice, we find individual farmers vertically and horizontally integrating these firms to the point where a farmer may, for example, own firms which produce different kinds of lambs and wool, firms which produce cereals, and firms which produce adult sheep from lambs. In fact, it is not uncommon for firms in the eastern Australian wheat-sheep zone to be integrated in this way, with the conglomerate of firms being referred to as a "farm", and each firm being referred to as an "enterprise".

We make the simplifying assumption that the relative prices of different types of wool are constant, and that the relative prices of different breeds and crosses of lamb are also constant. Then, by Hick's Aggregation Theorem, we aggregate the differentiable wool outputs of firms in the three sectors into a single output, wool. Likewise, the differentiable lamb outputs of firms in the three sectors are aggregated into a single output, lambs. We write the output of product j from a firm in sector k as q_{kj} ($k = 1, 2, 3; j = 1, 2$).

We make the additional simplifying assumption that in each sector lambs and wool are produced by firms in fixed proportions, so that for any given firm or sector we may consider the two joint outputs of the firm or sector as a single composite output. Indeed, we measure and denote the composite output of a firm in sector k as $q_k = q_{k1} + q_{k2} = q_{k1} + b_k q_{k1}$ where b_k is a factor of proportionality ($b_k \geq 0; k = 1, 2, 3$).

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[1] These sectors are the Merino woolgrowing, non-Merino woolgrowing and prime lamb producing sectors. A fuller description of the activities of firms in each of these sectors is given by O'Donnell (1987).

In sector k each of N_k perfectly competitive firms are assumed to make their production decisions before p_k, the price of the composite output, is known (input levels are chosen *ex ante*). Each firm is assumed to suppose some probability distribution of prices to prevail in each period. We assume that this distribution corresponds to the probability distribution of prices derived from the market equilibrium conditions. In this sense firms are assumed to hold rational expectations.

Each firm is assumed to have access to the same stochastic technology whereby I multiple variable inputs, $x_{k1},\ldots,x_{kI}$, are used to produce the random composite output q_k. The technology may be represented by $q_k = f_k(\theta_k, x_k)$, where $x_k' = (x_{k1},\ldots,x_{kI})$ and where θ_k is a random variable representing a supply-side source of instability in output prices. We make the simplifying assumption that $\theta_k = \beta_k\theta$ where the β_k are constants whose ratios measure the relative standard deviations of the θ_k, and where θ is a random variable with unit mean which gives rise to randomness in the outputs of all sectors. This specification has the property that $\theta_s <> E(\theta_s) \Leftrightarrow \theta_j <> E(\theta_j)$ for all s and j, so that a 'good (bad) season' in one sector implies a 'good (bad) season' in all sectors. We make the additional simplifying assumption that the stochastic technology is multiplicatively separable in $\theta_k = \beta_k\theta$ so that $q_k = \theta_k f_k(x_k)$ and $E(\theta_k)=\beta_k$.

Individual profit is given by $\pi_k = p_k\theta_k f_k(x_k) - w'x_k$ where $w'=(w_1,\ldots,w_I)$ is a vector of input prices. Before considering the maximisation problem it is convenient to reparameterise the profit function using an alternative characterisation of the technology, the cost function. If we let $r_k=p_k\theta_k$ be a measure of the random return to production in sector k then it can be shown that the profit function can be written:

$$\pi_k = r_k\hat{q}_k - C_k(w,\hat{q}_k) \tag{1}$$

where $C_k(w,\hat{q}_k) = \min_{x_k} w'x_k$ subject to $\hat{q}_k = f_k(x_k)$.

Each firm is assumed to maximise a general von Neumann-Morgenstern utility function, U(.), defined over profits. Using (1) the maximisation problem of firms in sector k can be written:

$$\max_{\hat{q}_k} E\{U[r_k\hat{q}_k - C_k(w,\hat{q}_k)]\} \tag{2}$$

and the first order condition for a maximum is given by:

$$E\{U'(\pi_k)[r_k - C_k^{\hat{q}}(w,\hat{q}_k)]\} = 0 \tag{3}$$

A solution to equation (3) requires, in the first instance, the specification of a utility function. We assume the utility function is exponential with parameter γ: $U(\pi_k) = -\exp(-\gamma\pi_k)$. The first derivative is $U'(\pi_k) = \gamma \exp(-\gamma\pi_k)$ which we approximate using a first order[2] Taylor's series expansion around $\pi_k = 0$. It can be shown that:

$$\hat{q}_k = \frac{\gamma C_k^q(w,\hat{q}_k).[1+\gamma C_k(w,\hat{q}_k)] - [\gamma+\gamma^2 C_k(w,\hat{q}_k)].E\{r_k\}}{\gamma^2 C_k^q(w,\hat{q}_k).E\{r_k\} - \gamma^2.E\{r_k^2\}} \tag{4}$$

Equation (4) is the uncertainty analogue of the supply function. In equation (4) expected supply is written as a non-linear function of the moments of the distribution of the random return to production and of the parameters of the technology and utility function.

Under the simplifying assumption that the production technology exhibits constant returns to scale equation (4) becomes:

$$\hat{q}_k = \frac{E\{r_k\} - C_k(w,1)}{\gamma[E\{r_k^2\} - 2C_k(w,1).E\{r_k\} + C_k(w,1)^2]} \tag{5}$$

Solving (4) or (5) for planned output requires the evaluation of $E\{r_k\}$ and $E\{r_k^2\}$, whereupon input demands are given using Shephard's Lemma. In the following Section we derive expressions for $r_k = p_k\theta_k$ and r_k^2 as functions of θ_k and random variables representing demand side sources of price instability.

3. Specification of Inverse Demand Schedules and Characterisations of Price Stabilisation Schemes

We assume stochastic logarithmic inverse demand schedules so that, in the absence of any stabilisation scheme, the prices of wool and lambs are given by (j=1, 2):

$$P_{ej} = A_j.\xi_j.Q_{ej}^{-1/e_j} \qquad E\{\xi_j\} = 1 \tag{6}$$

where the A_j are nonnegative deterministic functions of any number of exogenous variables, e_j is the (constant) price elasticity of demand for product j, the ξ_j are nonnegative independently distributed random variables representing demand side sources of price instability, and Q_{ej} is the total output of product j variously defined as (j=1,2):

[2] We have followed the suggestion of Newbery and Stiglitz (1981) and delayed the use of Taylor's series approximations until it is no longer analytically tractable to proceed without doing so. Thus, rather than take a second order Taylor's series expansion of $U(\pi_k)$, which yields $U(\pi_k) \cong -1+\gamma\pi_k-\gamma^2\pi_k^2$ and $U'(\pi_k) \cong \gamma-2\gamma^2\pi_k$, we have taken a first order Taylor's series expansion of $U'(\pi_k)$, which yields $U(\pi_k) = -\exp(-\gamma\pi_k)$ and $U'(\pi_k) \cong \gamma-\gamma^2\pi_k$.

$$Q_{ej} = \sum_k Q_{kj} = \sum_k N_k q_{kj} = \sum_k N_k \theta \beta_k \hat{q}_{kj} = \sum_k N_k \theta \bar{q}_{kj} = \theta \sum_k \bar{Q}_{kj} = \theta \bar{Q}_{ej}$$

where the definitions of $\bar{Q}_{ej}$, $\bar{Q}_{kj}$ and $\bar{q}_{kj}$ are obvious. Thus (j=1, 2):

$$P_{ej} = A_j.(\psi_j.\bar{Q}_{ej})^{-1/e_j} \tag{7}$$

where $\psi_j = \theta.\xi_j^{-e_j}$

We begin by modelling the MRP Scheme as a buffer stock scheme with AWC intervention taking the form of sales and purchases of wool according to the following rule:

$$S = \begin{cases} (-\psi_1\bar{Q}_{e1} + A_1^{e_1}.c^{-e_1}) & \text{if } A_1.(\psi_1\bar{Q}_{e1})^{-1/e_1} \geq c \\ 0 & \text{if } f < A_1.(\psi_1\bar{Q}_{e1})^{-1/e_1} < c \\ (-\psi_1\bar{Q}_{e1} + A_1^{e_1}.f^{-e_1}) & \text{if } A_1.(\psi_1\bar{Q}_{e1})^{-1/e_1} \leq f \end{cases} \tag{8}$$

where f and c denote known floor and ceiling prices respectively[3]. The inverse demand schedule for wool becomes:

$$P_{e1} = \begin{cases} c & \text{if } \psi_1 \leq A_1^{e_1}.c^{-e_1}/\bar{Q}_{e1} \\ A_1.(\psi_1\bar{Q}_{e1})^{-1/e_1} & \text{if } A_1^{e_1}.c^{-e_1}/\bar{Q}_{e1} < \psi_1 < A_1^{e_1}.f^{-e_1}/\bar{Q}_{e1} \\ f & \text{if } \psi_1 \geq A_1^{e_1}.f^{-e_1}/\bar{Q}_{e1} \end{cases} \tag{9}$$

In addition we consider a characterisation of a lamb price stabilisation scheme which allows for partial price stabilisation. We assume that the government levies (pays) a variable per unit commodity tax (subsidy) on lambs, and we assume that the size of the per unit tax (subsidy) in a given period depends on the value of A_2 as well as on the (constant) level of average output, the (constant) price elasticity of demand, and realisations of the random variable ψ_2. Indeed, we let the per unit tax (subsidy) on lambs be given by:

$$t = A_2.(\psi_2^{z/e_2} - 1).\psi_2^{-1/e_2}.\bar{Q}_{e2}^{-1/e_2} \tag{10}$$

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[3] In reality the floor price is known but the ceiling price is unknown. Nevertheless, we can maintain the integrity of our theoretical model if we assume that the ceiling price is set using a predetermined and known rule, for example, in order that E(S)=0. The reason we choose not to pursue such a characterisation is that, for the purposes of this paper, it gives rise to an unwarranted degree of econometric complexity.

where z is a government choice variable which determines the degree of partial lamb price stabilisation. When $A_2>0$ and $\psi_2 \leq 1$ (≥ 1) we have that $t \leq 0$ (≥ 0), that is, t is a subsidy (tax). The after-tax (-subsidy) price received for lambs is given by:

$$p_{e2}+t = A_2.\psi_2^{(z-1)/e_2}.\bar{Q}_{e2}^{-1/e_2} \tag{11}$$

Note that $z=0$ implies $t=0$ and no lamb price stabilisation. Moreover, $z=1$ corresponds to perfect lamb price stabilisation.

The total after-tax (-subsidy) return received by a firm in sector k for the sale of both products is given by $p_k(q_{k1}+q_{k2}) = p_{e1}q_{k1} + (p_{e2}+t)q_{k2}$ so that the per unit return on the sale of the joint outputs is simply a weighted average price: $p_k = \{p_{e1}+b_k.(p_{e2}+t_j)\}/(1+b_k)$. Indeed, it follows from (9) and (10) that ($k=1, 2, 3$):

$$p_k = \begin{cases} \left[c+b_k.A_2.\psi_2^{(z-1)/e_2}\bar{Q}_{e2}^{-1/e_2}\right]/(1+b_k) & \text{if } \psi_1 \leq c^* \\ \left[A_1(\psi_1\bar{Q}_{e1})^{-1/e_1}+b_k.A_2.\psi_2^{(z-1)/e_2}\bar{Q}_{e2}^{-1/e_2}\right]/(1+b_k) & \text{if } c^* < \psi_1 < f^* \\ \left[f+b_k.A_2.\psi_2^{(z-1)/e_2}\bar{Q}_{e2}^{-1/e_2}\right]/(1+b_k) & \text{if } \psi_1 \geq f^* \end{cases} \tag{12}$$

where $c^* = A_1^{e_1}.c^{-e_1}/\bar{Q}_{e1}$ and $f^* = A_1^{e_1}.f^{-e_1}/\bar{Q}_{e1}$.

From equation (12) the definitions of $r_k = p_k\bar{q}_k\theta$ and r_k^2 are obvious and in the following Section we make distributional assumptions on θ and ξ_j ($j=1, 2$) in order to derive expressions for the expected values of these variables.

4. The Mean and Variance of the Random Return to Production

In our notation, if $Y=\log X$ is normally distributed with mean μ and variance σ^2 then X is lognormally distributed and we write $X \sim \Lambda(\mu,\sigma^2)$. In addition, we use $\Lambda(x|\mu,\sigma^2)$ to denote the cumulative distribution function of X so that $\Lambda(x|\mu,\sigma^2) = \Pr(X \leq x)$ when $X \sim \Lambda(\mu,\sigma^2)$.

We assume that θ and ξ_j ($j=1, 2$) are independent lognormally distributed random variables with unit means and variances. Then $\theta \sim \Lambda(-0.5\sigma_\theta^2,\sigma_\theta^2)$ and $\xi_j \sim \Lambda(-0.5\sigma_{\xi j}^2,\sigma_{\xi j}^2)$ ($j=1, 2$). Moreover, from the distribution of the sum of independent normally distributed random variables ($j=1, 2$):

$$\psi_j = a\xi_j^{-\theta_j} \sim \Lambda(-0.5[\sigma_\theta^2 - \theta_j\sigma_{\xi j}^2],\ \sigma_\theta^2 + \theta_j^2\sigma_{\xi j}^2) \qquad (13)$$

which we abbreviate as $\psi_j \sim \Lambda(\mu_j, \sigma_j^2)$ where the definitions of μ_j and σ_j^2 are obvious.

We make use of two theorems, the first of which is a generalisation of two results presented by Aitchison and Brown (1957):

<u>Theorem 1</u>: Let $x \sim \Lambda(\mu, \sigma^2)$ and let those parts of the distribution for which $x \le l$ and $x \ge h$ be removed, so that the distribution of x is *truncated* at these points. Then

$$E\{x^k | l \le x \le h\} = \exp(k\mu + 0.5k^2\sigma^2) . [\Lambda(h|\mu + k\sigma^2, \sigma^2) - \Lambda(l|\mu + k\sigma^2, \sigma^2)] / [\Lambda(h|\mu, \sigma^2) - \Lambda(l|\mu, \sigma^2)]$$

<u>Proof</u>: O'Donnell (1987)

<u>Theorem 2</u>: Let $(x, y) \sim \Lambda(\mu_x, \mu_y, \sigma_x^2, \sigma_y^2, \rho)$ where $\rho = \mathrm{Corr}(\ln x, \ln y) = \sigma_{xy}/\sigma_x\sigma_y$. Let those parts of the distribution for which $y \le l$ and $y \ge h$ be removed, so that the joint distribution of x and y is *truncated* at these points. Then

$$E\{x^k | l \le y \le h\} = \exp(k\mu_x + 0.5k^2\sigma_x^2) . [\Lambda(h|\mu_y + k\sigma_{xy}, \sigma_y^2) - \Lambda(l|\mu_y + k\sigma_{xy}, \sigma_y^2)] / [\Lambda(h|\mu_y, \sigma_y^2) - \Lambda(l|\mu_y, \sigma_y^2)]$$

<u>Proof</u>: O'Donnell (1987)

A straightforward application of Theorem 1 yields:

$$E\{p_k\} = \Big[c.\Lambda(c^*|\mu_1, \sigma_1^2) + f.[1 - \Lambda(f^*|\mu_1, \sigma_1^2)]$$

$$+ A_1.\bar{Q}_{e1}^{-1/e_1}.\exp(0.5\sigma_1^2/e_1 - \mu_1/e_1^2).[\Lambda(f^*|\mu_1 - \sigma_1^2/e_1, \sigma_1^2) - \Lambda(c^*|\mu_1 - \sigma_1^2/e_1, \sigma_1^2)]$$

$$+ b_k.A_2.\bar{Q}_{e2}^{-1/e_2}.\exp(\mu_2(z-1)/e_2 + 0.5\sigma_2^2(z-1)^2/e_2^2)\Big]/(1+b_k) \qquad (14)$$

The evaluation of $E\{r_k\}$ and $E\{r_k^2\}$ is more problematic. It can be shown that the expectations $E\{r_k\}$ and $E\{r_k^2\}$ include expectations which we can write in the general form $E\{\theta^\alpha.\eta^\beta.\xi_2^\kappa | a \le \theta_\eta \le b\}$

where $\eta = \xi_1^{-e_1} \sim \Lambda(0.5\, e_1 \sigma^2_{\xi 1},\ e_1^2 \sigma^2_{\xi 1})$.

Since θ, η and ξ_2 are independent log-normally distributed random variables we have that $E\{\theta^{\alpha} \cdot \eta^{\beta} \xi_2^{\kappa} | a \le \theta_\eta \le b\} = E\{\theta^{\alpha} | a \le \theta_\eta \le b\} . E\{\eta^{\beta} | a \le \theta_\eta \le b\} . E\{\xi_2^{\kappa}\}$ and these expectations can each be evaluated using Theorem 2. Specifically, let $x=\theta$ and let $y=\theta_\eta$ so that:

$$E\{\theta^{\alpha} | a \le \theta_\eta \le b\} = \exp(0.5\sigma_\theta^2 \alpha(\alpha - 1)) . [\Lambda(b|\mu_1 + \alpha\sigma_\theta^2, \sigma_1^2) - \Lambda(a|\mu_1 + \alpha\sigma_\theta^2, \sigma_1^2)] / [\Lambda(b|\mu_1, \sigma_1^2) - \Lambda(a|\mu_1, \sigma_1^2)]$$

Similarly, if we let $x=\eta$ and $y=\theta_\eta$ then a straightforward application of Theorem 2 yields:

$$E\{\eta^{\beta} | a \le \theta_\eta \le b\} = \exp(0.5\, \beta\, e_1 \sigma_{\xi 1}^2 (1 + \beta e_1)) . [\Lambda(b|\mu_1 + \beta e_1^2 \sigma_{\xi 1}^2, \sigma_1^2) - \Lambda(a|\mu_1 + \beta e_1^2 \sigma_{\xi 1}^2, \sigma_1^2)] / [\Lambda(b|\mu_1, \sigma_1^2) - \Lambda(a|\mu_1, \sigma_1^2)]$$

so that we may write:

$$E\{r_k\} = \phi_1(a_k, b_k, A_1, A_2, \bar{Q}_{\theta 1}, \bar{Q}_{\theta 2}, e_1, e_2, \sigma^2_{\xi 1}, \sigma^2_{\xi 2}, \sigma^2_{\theta}, z, c, f) \tag{15}$$

$$E\{r_k^2\} = \phi_2(a_k, b_k, A_1, A_2, \bar{Q}_{\theta 1}, \bar{Q}_{\theta 2}, e_1, e_2, \sigma^2_{\xi 1}, \sigma^2_{\xi 2}, \sigma^2_{\theta}, z, c, f) \tag{16}$$

where the functions $\phi_1(.)$ and $\phi_2(.)$ are provided in Appendix 1. Upon substituting equations (15) and (16) into equations (4) or (5) our model of firm behaviour is complete.

5. <u>The Effect of a Large or Small Change in the Degree of Partial Lamb Price Stabilisation on the Welfare of Producers in Sector k</u>

Consider a change in z, c or f after the start of the production period, that is, after input decisions have been made. We refer to the effect on welfare precipitated by a such a change as the impact effect. In this section we begin by considering the impact effect of a <u>large</u> change in z on the welfare of producers in sector k.

If we let π_k and π^o_k respectively denote the random profits of individual firms in sector k before and after a large change in the degree of lamb price stabilisation, the impact benefit of the change in the stabilisation program, B_k, is found as the solution to $E\{U(y_k - w'x_k)\} = E\{U(y^o_k - w'x_k - B_k)\}$ where y_k and y^o_k respectively represent incomes before and after the change. If the utility function is additively or multiplicatively seperable then it can be shown that B_k is also the solution to $E\{U(y_k)\} = E\{U(y^o_k - B_k)\}$. Following Newbery and Stiglitz (1981) we take second-order Taylor series expansions of each side around the mean of y_k and equate these expansions. We let R denote the coefficient of relative risk aversion evaluated at the mean of y_k and we find, eventually, that:

$$\frac{B_k}{\bar{y}_k} = \frac{\Delta\bar{y}_k}{\bar{y}_k} + \frac{1}{2}.R.\left[Var(y_k)/\bar{y}_k^2 - Var(y^o_k)/\bar{y}_k^2 - (\Delta\bar{y}_k - B_k)^2/\bar{y}_k^2\right] \tag{17}$$

where $\bar{y}_k = E\{y_k\}$, $\bar{y}^o_k = E\{y^o_k\}$ and $\Delta\bar{y}_k = \bar{y}^o_k - \bar{y}_k$.

Equation (17) can be approximated by:

$$\frac{B_k}{\bar{y}_k} \doteq \frac{\Delta\bar{y}_k}{\bar{y}_k} + \frac{1}{2}.R.\left[Var(y_k)/\bar{y}_k^2 - Var(y^o_k)/\bar{y}_k^2\right] \tag{18}$$

The first term on the right hand side is referred to by Newbery and Stiglitz (1981) as the 'transfer benefit'. The second term is referred to as the 'risk benefit'. Initially we assume there is no lamb price stabilisation (z=0) so that the mean and variance of initial income are given by:

$$E\{y_k\} = \bar{y}_k = \hat{q}_k.E\{r_k\} \qquad \text{and} \qquad Var(y_k) = \hat{q}_k^2.[E\{r_k^2\} - E\{r_k\}^2] \tag{19}$$

Let the stabilisation authority choose a degree of stabilisation $0<z\leq 1$ so that the mean and variance of income are then given by:

$$E\{y^o_k\} = \bar{y}^o_k = \hat{q}_k.E\{r^o_k\} \qquad \text{and} \qquad Var(y^o_k) = \hat{q}_k^2.[E\{r_k^{o2}\} - E\{r^o_k\}^2] \tag{20}$$

where the definition of r_k^o is obvious. Thus, for an exponential utility function with coefficient of absolute risk aversion γ, equation (18) becomes:

$$\frac{B_k}{\bar{y}_k} \doteq \left[E\{r^o_k\} - E\{r_k\} - 0.5\,\gamma.\hat{q}_k.[E\{r_k^2\} - E\{r_k\}^2 - E\{r_k^{o2}\} + E\{r^o_k\}^2]\right]/E\{r_k\} \tag{21}$$

where the expectations can be evaluated using equations (15) and (16).

Now consider a change in z, c or f which gives rise to changes in planned outputs. We refer to the effect on welfare precipitated by such a change as the long-run effect. If we let π_k and π'_k respectively denote the random profits of individual firms in sector k before and after a <u>large</u> change in the degree of lamb price stabilisation, the associated long-run benefit of the stabilisation program, B_k, is found as the solution to $E\{U(\pi_k)\} = E\{U(\pi'_k - B_k)\}$. It can be shown that:

$$\frac{B_k}{\bar{\pi}_k} = \frac{\Delta\bar{\pi}_k}{\bar{\pi}_k} + \frac{1}{2}.R.\left[Var(\pi_k)/\bar{\pi}_k^2 - Var(\pi_k')/\bar{\pi}_k^2 - (\Delta\bar{\pi}_k - B_k)^2/\bar{\pi}_k^2\right] \tag{22}$$

where $\bar{\pi}_k = E\{\pi_k\}$, $\bar{\pi}_k' = E\{\pi_k'\}$, $\Delta\bar{\pi}_k = \bar{\pi}_k' - \bar{\pi}_k$ and R is now evaluated at $\bar{\pi}_k$.

Moreover, equation (22) can be approximated by:

$$\frac{B_k}{\bar{\pi}_k} \doteq \frac{\Delta\bar{\pi}_k}{\bar{\pi}_k} + \frac{1}{2}.R.\left[Var(\pi_k)/\bar{\pi}_k^2 - Var(\pi_k')/\bar{\pi}_k^2\right] \tag{23}$$

In the absence of a lamb price stabilisation scheme (z=0) profit may be represented by (1) with a mean and variance given by:

$$E\{\pi_k\} = \bar{\pi}_k = \dot{q}_k.E\{r_k\} - C_k(w, \dot{q}_k) \quad \text{and} \quad Var(\pi_k) = \dot{q}_k^2.[E\{r_k^2\} - E\{r_k\}^2] \tag{24}$$

Again, let the stabilisation authority choose a degree of stabilisation $0<z\leq 1$ so that profit is given by:

$$\pi_k' = r_k'\dot{q}_k' - C_k(w, \dot{q}_k') \tag{25}$$

where the definitions are obvious. The mean and variance of profit are given by:

$$E\{\pi_k'\} = \bar{\pi}_k' = \dot{q}_k'.E\{r_k'\} - C_k(w, \dot{q}_k') \quad \text{and} \quad Var(\pi_k') = \dot{q}_k'^2.[E\{r_k'^2\} - E\{r_k'\}^2] \tag{26}$$

Thus, having assumed an exponential utility function, and under the assumption of constant returns to scale, we may write (23) as:

$$\frac{B_k}{\bar{\pi}_k} \doteq \Big[\dot{q}_k'.E\{r_k'\} - \dot{q}_k.E\{r_k\} - C_k(w,1).(\dot{q}_k' - \dot{q}_k)$$

$$-0.5\gamma.\left[\dot{q}_k'^2[E\{r_k'^2\} - E\{r_k'\}^2] - \dot{q}_k^2[E\{r_k^2\} - E\{r_k\}^2]\right]\Big] / \dot{q}_k.[E\{r_k\} - C_k(w,1)] \tag{27}$$

where $E\{r_k\}$ and $E\{r_k^2\}$ are given by equations (15) and (16) and the remaining expectations are given by:

$$E\{r_k'\} = \varnothing_1(\beta_k, b_k, A_1, A_2, \bar{Q}_{e1}', \bar{Q}_{e2}', e_1, e_2, \sigma_{\xi1}^2, \sigma_{\xi2}^2, \sigma_{\theta}^2, z, c, f) \tag{28}$$

$$E\{r_k'^2\} = \varnothing_2(\beta_k, b_k, A_1, A_2, \bar{Q}_{e1}', \bar{Q}_{e2}', e_1, e_2, \sigma_{\xi1}^2, \sigma_{\xi2}^2, \sigma_{\theta}^2, z, c, f) \tag{29}$$

where $\bar{Q}^{*}_{ej} = \sum_k N_k \bar{q}^{*}_{kj} = \sum_k N_k \beta_k \dot{q}^{*}_{kj}$ for $j=1,2$.

Finally, although our interest centres on large changes in z we might consider a stabilisation authority which makes small adjustments to the degree of partial lamb price stabilisation in the short term. In the remainder of this section we consider the impact effect of this fine tuning on producer welfare.

We write the maximised value of the expected utility of profits for firms in sector k as:

$$W_k = E\{U[p_k \theta \bar{q}_k - C_k(w, \dot{q}_k)]\} \tag{30}$$

Following Newbery and Stiglitz (1981) the impact effect of a small change in the degree of partial lamb price stabilisation on producer welfare can now be found by partially differentiating both sides of this expression with respect to z. Since $\partial x_{ki}/\partial z = 0$ we have that:

$$\frac{\partial W_k}{\partial z} = E\left[U'(\pi_k)\frac{\partial p_k}{\partial z}\theta \bar{q}_k\right] \tag{31}$$

Partially differentiating (12) with respect to z yields:

$$\frac{\partial p_k}{\partial z} = \left[b_k . A_2 . \bar{Q}_{e2}^{-1/e_2} . \psi_2^{z-1/e_2} . \ln \psi_2\right] / e_2(1+b_k) \tag{32}$$

so that (31) becomes an expression which involves the expected values of the products of normally and lognormally distributed random variables. We choose to approximate these expectations using a sequence of Taylor series expansions. The expression we derive is given in Appendix 2. For convenience we write:

$$\frac{\partial W_k}{\partial z} \doteq \phi_3(r, w, \beta_k, \dot{q}_k, b_k, A_1, A_2, \bar{Q}_{e1}, \bar{Q}_{e2}, e_1, e_2, \sigma^2_{\xi 1}, \sigma^2_{\xi 2}, \sigma^2_{\theta}, z, c, f) \tag{33}$$

The cash equivalent benefit of a differential change in z is found after dividing this result by the marginal utility of expected profits.

6. Estimating the Parameters of the Model

In this section we generalise the model to $t=1,\ldots,T$ time periods so that $n=1,\ldots, N=N_1+N_2+N_3$. We discuss techniques by which the parameters of the production technology, the parameters of the inverse demand schedules, and a group of remaining parameters can be estimated seperately.

(i) The Parameters of the Production Technology.

For illustrative purposes we choose to consider the estimation of a CRTS Cobb-Douglas technology which we recognise as being theoretically restrictive[4] but which can be written in a form which is linear in the parameters. We have chosen to exemplify the estimation of the parameters of the technology using the production function rather than the cost or input demand functions because the parameter restrictions implied by the CRTS assumption are easier to impose[5]. We assume[6] that observations on the w_{nt}, x_{nt} and q_{nt} are available for all n and t.

The CRTS Cobb-Douglas production function for firm n in time period t can be written[7]:

$$\hat{q}_{nt} = \prod_{j=1}^{J} x_{njt}^{\alpha_{kj}} \tag{34}$$

where $\sum_{j=1}^{J} \alpha_{kj} = 1$. We substitute for $\hat{q}_{nt}$ and rearrange to obtain:

$$q_{nt} = \theta_t \beta_k . \prod_{j=1}^{J} x_{njt}^{\alpha_{kj}} \tag{35}$$

After taking logs we have that:

$$\ln q_{nt} = \ln \beta_k + \sum_{j=1}^{J} \alpha_{kj} \ln x_{njt} + \ln \theta_t \tag{36}$$

where $\ln \theta_t$ is normally distributed. The (Tx1) vector $\tau = [\ln \theta_1 \; .... \; \ln \theta_T]'$ has the following properties:

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4 It can be shown that the 'elasticity of substitution' between each pair of inputs must be identically one, and it is difficult to find an *a priori* justification for the imposition of such a restriction.

5 Even though estimating the production or cost functions is more likely to give rise to omitted variable bias.

6 In fact this data has been constructed from Australian Sheep Industry Survey (ASIS) data generously supplied by the Bureau of Agricultural Economics. In our empirical work we have constructed data for six input groups: land, labour, capital, livestock, equipment and supplies and services and overheads.

7 Notice that this particular Cobb-Douglas specification does not include a constant term. Recall from Section 2 that $f_k(\theta_k, x_k)$ is multiplicatively seperable in θ_k and that $\theta_k = \theta\beta_k$. It follows that the parameter β_k is indistinguishable from the constant of the more usual Cobb-Douglas specification. This is not the case for all technologies and this is the reason we have chosen to write the model of Section 2 in its more general form.

$$E\{\varepsilon\} = -0.5\sigma_\theta^2 j_T \quad \text{and} \quad E\{\varepsilon\varepsilon'\} = \sigma_\theta^2 I_T + 0.25\sigma_\theta^4 j_T j_T' \tag{37}$$

where I_T is a (TxT) identity matrix and j_T is a (Tx1) vector of ones. At this point we choose to add η_{nt}, an independently and identically distributed normal random variable which has the properties:

$$E\{\eta_t\} = 0 j_N \qquad E\{\eta_t \eta_t'\} = \sigma_\eta^2 I_N \quad \text{and} \quad E\{\eta_t \eta_s'\} = 0 I_N \quad (t \neq s) \tag{38}$$

where $\eta_t = [\eta_{1t} \cdots \eta_{Nt}]'$ is an (Nx1) vector.

The η_{nt} have been introduced to reflect measurement error in the dependent variable and equation (36) can be now be written:

$$\ln q_{nt} = [\ln \beta_k - 0.5\sigma_\theta^2] + \sum_{j=1}^{I} \alpha_{kj} \ln x_{njt} + u_{nt} \tag{39}$$

where $u_{nt} = (\eta_{nt} + \ln(\theta_t) + 0.5\sigma_\theta^2)$. We now define the vectors and matrices:

$y_t = [\ln q_{1t} \ \ldots\ldots \ \ln q_{Nt}]'$ as (Nx1)

$\alpha = [\ln(2\beta_1/\sigma_\theta^2) \quad \ln(\beta_2/\beta_1) \ldots \ln(\beta_K/\beta_1) \quad \alpha_{11} \ldots \alpha_{1I} \ \alpha_{21} \ldots \alpha_{KI}]'$ as (K(I+1)x1)

$x_{nt} = [\ln x_{n1t} \ldots \ln x_{nIt}]$ as (1xI)

X_{jt} as the (NxI) matrix $[x_{1t} \ldots \ x_{Nt}]'$ where rows which correspond to $k \neq j$ have been replaced by a row of zero's.

d_j as an (Nx1) dummy variable vector with elements taking on the value one if k=j and the value 0 otherwise.

$X_t = [\, j_N \ d_2 \ldots d_K \ X_{1t} \ X_{2t} \ \ldots X_{Kt}\,]$ as (N x K(I+1))

We can now write the period t model as:

$$y_t = X_t \alpha + \eta_t + \ln\theta_t j_N + 0.5\sigma_\theta^2 j_N \tag{40}$$

The complete set of NT observations may be written as:

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_T \end{bmatrix} = \begin{bmatrix} X_1 \\ X_2 \\ \vdots \\ X_T \end{bmatrix} \alpha + \begin{bmatrix} \eta_1 \\ \eta_2 \\ \vdots \\ \eta_T \end{bmatrix} + \begin{bmatrix} \ln \theta_1 \otimes j_N \\ \ln \theta_2 \otimes j_N \\ \vdots \\ \ln \theta_T \otimes j_N \end{bmatrix} + \begin{bmatrix} 0.5\sigma_\theta^2 \otimes j_N \\ 0.5\sigma_\theta^2 \otimes j_N \\ \vdots \\ 0.5\sigma_\theta^2 \otimes j_N \end{bmatrix}$$

Using obvious definitions the complete set of NT observations may be written more compactly as:

$$y = X\alpha + \eta + \tau \otimes j_N + 0.5\sigma_\theta^2 \otimes j_{NT} \qquad (41)$$

which is the form of an error components model. Specifically we have, by our earlier assumptions, that:

$$E\{\eta\} = 0 j_{NT} \qquad E\{\eta\eta'\} = \sigma_\eta^2 I_{NT} \qquad \text{and} \qquad E\{\eta\tau' \otimes j_N'\} = 0 I_{NT} \qquad (42)$$

so that the properties of the composite error term $u = \eta + \tau \otimes j_N + 0.5\sigma_\theta^2 \otimes j_{NT}$ are:

$$E\{u\} = 0 j_{NT}$$

and

$$\begin{aligned} E\{uu'\} &= \sigma_\eta^2 I_{NT} + \sigma_\theta^2 I_T \otimes j_N j_N' \\ &= I_T \otimes [\sigma_\theta^2 j_N j_N' + \sigma_\eta^2 I_N] \\ &= I_T \otimes V \\ &= \Omega \end{aligned} \qquad (43)$$

The covariance matrix Ω is block diagonal because the disturbance vectors associated with different time periods are uncorrelated. The parameters of the technology can be estimated by maximum likelihood or by using the estimated generalised least squares procedure outlined in Judge *et al* (1982, p.490-494).

(ii) The Parameters of the Inverse Demand Schedule for Wool.

For time periods prior to the introduction of a wool price stabilisation scheme, the inverse demand schedule for wool in the t^th^ period can, after taking logarithms, be written as:

$$\ln p_{e1t} = \ln A_1 - 1/e_1 . \ln Q_{e1t} + \ln \xi_{1t} \qquad (44)$$

where we assume that A_1 is a constant[8] and where $\ln \xi_{1t}$ is, from Section 4, an independent and identically distributed normal random variable with a mean equal to the negative of one half of its variance: $\ln \xi_{1t} \sim N(-0.5\sigma_{\xi 1}^2, \sigma_{\xi 1}^2)$. In any of the T periods the t^{th} observation may be written in the following compact form:

$$y_t = \begin{cases} C_t & \text{if } x_t\beta + u_t \geq C_t \\ x_t\beta + u_t & \text{if } F_t < x_t\beta + u_t < C_t \\ F_t & \text{if } x_t\beta + u_t \leq F_t \end{cases} \tag{45}$$

where $C_t = \ln c_t$
$F_t = \ln f_t$
$y_t = \ln p_{o1t}$
$x_t = [1; \ln Q_{o1t}]$
$\beta = [(\ln A_1 - 0.5\sigma_{\xi 1}^2); (-1/a_1)]'$
and $u_t = \ln \xi_{1t} + 0.5\sigma_{\xi 1}^2$

The properties of the disturbance term $u=(u_1,....u_T)$ are, given our earlier assumptions: $E\{u\} = 0j_T$ and $E\{uu'\} = \sigma_{\xi 1}^2 I_T$. Equation (45) is the form of a censored regression model. For notational convenience we let $\sigma^2 = \sigma_{\xi 1}^2$ and write the likelihood function for the T independent observations corresponding to equation (45) as:

$$L = \prod_0 \Phi((F_t - x_t\beta)/\sigma) \prod_1 [1 - \Phi((C_t - x_t\beta)/\sigma)] \prod_2 (1/\sigma)\phi((y_t - x_t\beta)/\sigma) \tag{46}$$

where Π_0 denotes the product over those t for which $x_t\beta + u_t < F_t$
Π_1 denotes the product over those t for which $x_t\beta + u_t > C_t$
Π_2 denotes the product over those t for which $F_t < x_t\beta + u_t < C_t$

and Φ and ϕ are the distribution and density function respectively of a standard normal variable. The logarithm of the likelihood function is:

$$\ln L = \sum_0 \ln[\Phi((F_t - x_t\beta)/\sigma)] + \sum_1 \ln[1 - \Phi((C_t - x_t\beta)/\sigma)]$$

$$- (n_2/2)\ln\sigma^2 - (1/2\sigma^2)\sum_2 (y_t - x_t\beta)^2 \tag{47}$$

where n_2 is the number of time periods for which $F_t < x_t\beta + u_t < C_t$. The first derivatives are:

8 Extending the specification to the case where A_1 is a function of any number of exogenous variables is trivial, provided the functional form of A_1 is a convenient one (multiplicative).

$$\frac{\partial \ln L}{\partial \beta} = (-1/\sigma)\sum_0 [\phi((F_t - x_t\beta)/\sigma)/\Phi((F_t - x_t\beta)/\sigma)]\,x_t'$$

$$+ (1/\sigma)\sum_1 [\phi((C_t - x_t\beta)/\sigma)/(1-\Phi((C_t - x_t\beta)/\sigma))]\,x_t'$$

$$+ (1/\sigma^2)\sum_2 (y_t - x_t\beta)\,x_t' \qquad (48)$$

and

$$\frac{\partial \ln L}{\partial \sigma^2} = (1/2\sigma^2)\sum_0 [\phi((F_t - x_t\beta)/\sigma)/\Phi((F_t - x_t\beta)/\sigma)](F_t - x_t\beta)$$

$$- (1/2\sigma^2)\sum_1 [\phi((C_t - x_t\beta)/\sigma)/(1-\Phi((C_t - x_t\beta)/\sigma))](C_t - x_t\beta)$$

$$- (n_2/2\sigma^2) - (1/2\sigma^4)\sum_2 (y_t - x_t\beta)^2 \qquad (49)$$

and the maximum likelihood estimators are defined as the solutions to the equations obtained by setting equations (48) and (49) to zero. Because these equations are non-linear in the parameters they must be solved iteratively. Maddala (1983) claims that the likelihood function is globally concave and this implies that a standard iterative method such as Newton-Raphson or method of scoring is guaranteed to converge.

(iii) The Parameters of the Inverse Demand Schedule for Lambs.

Set z=0 so that the inverse demand schedule for lambs in the t^{th} period can, after taking logarithms, be written as:

$$\ln P_{e2t} = \ln A_2 - 1/e_2 . \ln Q_{e2t} + \ln \xi_{2t} \qquad (50)$$

where, again, we assume that A_2 is a constant and where $\ln \xi_{2t}$ is independently normally distributed: $\ln \xi_{1t} \sim N(-0.5\sigma_{\xi 1}^2, \sigma_{\xi 1}^2)$. Again, equation (50) may be re-written in a more compact form:

$$y = X\beta + u \qquad (51)$$

where the definitions are analogous to the definitions associated with equation (45). Moreover, the properties of the disturbance term u are: $E(u) = 0j_T$ and $E(uu') = \sigma_{\xi 2}^2 I_T$. It follows that ordinary least squares applied to equation (51) will yield best linear unbiased estimates of β and $\sigma_{\xi 2}^2$.

(iv) The Remaining Parameters in the Model.

The remaining parameters in the model are γ and b_k $(k=1, 2, 3)$. One equation in our model which expresses a relationship between these parameters is equation (5). An equivalent form of equation (5), for individual n in time period t, is the following[9]:

$$q_{nt} = (\theta_t/\gamma).[E\{r_{kt}\}-C_k(w_{nt},1)]/[E\{r_{kt}^2\}-2C_k(w_{nt},1)E\{r_{kt}\}+C_k(w_{nt},1)^2] \qquad (52)$$

Using equations (15) and (16) and after appropriately defining new variables x_{jkt} $(j=1,\dots,5)$ the logarithm of equation (52) can be written as:

$$\begin{aligned} y_{nt} = \alpha_0 &+ \ln[(1/(1+b_k)).(x_{1kt}+b_k x_{2kt})-C_{nt}] \\ &- \ln[(1/(1+b_k)^2).(x_{3kt}+b_k x_{4kt}+b_k^2 x_{5kt}) \\ &-2C_{nt}(1/(1+b_k)).(x_{1kt}+b_k x_{2kt})+C_{nt}^2] + \ln\theta_t \end{aligned} \qquad (53)$$

where $y_{nt} = \ln q_{nt}$
$\alpha_0 = \ln(1/\gamma)$
$C_{nt} = C_k(w_{nt}, 1)$

and a typical x_{jkt} is given by:

$$x_{2kt} = \exp[(0.5\sigma_\theta^2(1-1/e_2)+\ln\theta_t).(-1/e_2)].Q_{e2t}^{-1/e_2}.A_2.\beta_k \qquad (54)$$

With the exception of C_{nt} the explanatory variables in the non-linear equation (53) are unobserved. An intuitively reasonable procedure is to replace $\ln\theta_t$ and the x_{jkt} $(j=1,\dots,5)$ with variables constructed using estimates from parts (i) to (iii)[10]. Then equation (53) becomes:

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[9] Our notation implies that input prices are free to vary between individuals, and this variation is intended to reflect variation in the quality of inputs available to different individuals. We assume that all individuals face the same prices for labour, equipment, supplies, and services. However, individuals are assumed to face different prices for land, capital and livestock unless they are engaged in the same productive activity (belong to the same sector) and are located in the same geographical area (state and zone). In this case, the same input prices are faced by all individuals.

[10] An estimate of $\ln\theta_t$ can be obtained by a method suggested by Judge *et al* (1982, p.494-495). Our decision to replace $\ln\theta_t$ with this estimate forces a certain consistency between the apparent realisations of the random variable θ_t in equations (41) and (53). Moreover, this replacement causes the error structure to be considerably simplified (it does not give rise to an error components model).

$$y_{nt} \doteq \alpha_0 + \ln[(1/(1+b_k)).(\hat{x}_{1kt} + b_k\hat{x}_{2kt}) - C_{nt}]$$

$$- \ln[(1/(1+b_k)^2)(\hat{x}_{3kt} + b_k\hat{x}_{4kt} + b_k^2\hat{x}_{5kt})$$

$$-2C_{nt}(1/(1+b_k)).(\hat{x}_{1kt} + b_k\hat{x}_{2kt}) + C_{nt}^2] + \ln\hat{e}_t \qquad (55)$$

At this point we choose to introduce an independent normally distributed random variable v_{nt} with the following properties:

$$E(v_t) = 0j_N$$

and

$$E[v_t v_t'] = \sigma_v^2 I_N$$

where

$$v_t = (v_{1t} \ldots v_{Nt})'$$

We introduce the v_{nt} in order to reflect measurement error in the dependent variable as well as the fact that (55) is only an approximate relationship. We choose to write the set of N observations corresponding to time period t as:

$$y_t = g(\hat{X}_t, Z_t, b) + v_t \qquad (56)$$

where

$$y_t = (y_{1t} \ldots y_{Nt})'$$

$$\hat{X}_t = [\ln\hat{e}_t j_N, (\hat{x}_{11t} j_{N_1} \ldots \hat{x}_{1Kt} j_{N_K})', \ldots, (\hat{x}_{51t} j_{N_1} \ldots \hat{x}_{5Kt} j_{N_K})']$$

$$Z_t = [j_N, (C_{1t} \ldots C_{Nt})']$$

and

$$b = (\alpha_0, b_1, \ldots, b_K)$$

so that the complete set of NT observations may be written:

$$\begin{bmatrix} y_1 \\ \vdots \\ y_T \end{bmatrix} = \begin{bmatrix} g(\hat{X}_1, Z_1, b) \\ \vdots \\ g(\hat{X}_T, Z_T, b) \end{bmatrix} + \begin{bmatrix} v_1 \\ \vdots \\ v_T \end{bmatrix} \qquad (57)$$

or

$$y = h(\hat{X}, Z, b) + v \qquad (58)$$

where the definitions are obvious and where $E[v]=0_{NT}$ and $E[vv']=\sigma_v^2 I_{NT}$. Equation (58) is of the form of a non-linear latent variables model. Estimators for these types of models are available (see Aigner *et al* (1984)) but in most cases a number of important statistical properties remain unknown. Aigner *et al* (1984) suggest that h(.) be approximated by a polynomial of required accuracy before applying the algorithm of O'Neill *et al* (1969).

7. Conclusion

Whereas many price stabilisation proposals are empirically assessed using a more or less *ad hoc* model of agricultural supply response, we have chosen to develop a model of supply response which is founded in the economic theory of firm behaviour under uncertainty. The model is then tailored to an assessment of a partial lamb price stabilisation scheme along lines proposed by Newbery and Stiglitz (1981). Finally, we outline methods by which the parameters of the model can be estimated.

The paper extends the theoretical price stabilisation studies of Tisdell (1963), Subotnik and Houck (1976) and Turnovsky (1978) by modelling producers as expected utility maximisers and by allowing for partial price stabilisation. The analysis generalises the theoretical study of Newbery and Stiglitz (1981, p.304ff) to the multiple input case, with output price uncertainty due to randomness in both d[illegible]nd and supply.

From an econometric viewpoint it is interesting to note that the theoretical model of Sections 2 to 5 implies the estimation of an error components model, a censored regression model and a non-linear latent variables model.

Appendix 1

$$E(r_k) = \Big[\exp[0.5\sigma_\theta^2(1+(z-1)/e_2)((z-1)/e_2)-0.5\sigma_{\xi 2}^2 z(1-z)].\bar{Q}_{e2}^{-1/e_2}.A_2.b_k$$

$$+\ \Delta(c^*|\mu_1+\sigma_\theta^2,\sigma_1^2).c + [1-\Delta(f^*|\mu_1+\sigma_\theta^2,\sigma_1^2)].f$$

$$+\ \exp[0.5\sigma_\theta^2(1-1/e_1).1/e_1].[\Delta(f^*|\mu_1+(1-1/e_1)\sigma_\theta^2,\sigma_1^2)-\Delta(c^*|\mu_1+(1-1/e_1)\sigma_\theta^2,\sigma_1^2)]$$

$$.[\Delta(f^*|\mu_1-e_1\sigma_{\xi 1}^2,\sigma_1^2)-\Delta(c^*|\mu_1-e_1\sigma_{\xi 1}^2,\sigma_1^2)]$$

$$.A_1.\bar{Q}_{e1}^{-1/e_1}/[\Delta(f^*|\mu_1,\sigma_1^2)-\Delta(c^*|\mu_1,\sigma_1^2)]\Big].\beta_k/(1+b_k)$$

$$E(r_k^2) = \Big[\exp[\sigma_\theta^2(1+(z-1)/e_2)(1+2(z-1)/e_2)+\sigma_{\xi 2}^2(1-z)(1-2z)].\bar{Q}_{e2}^{-2/e_2}.b_k^2.A_2^2$$

$$+\ \exp[\sigma_\theta^2].[\Delta(c^*|\mu_1+2\sigma_\theta^2,\sigma_1^2).c^2+[1-\Delta(f^*|\mu_1+2\sigma_\theta^2,\sigma_1^2)].f^2]$$

$$+\ \exp[0.5\sigma_\theta^2(2+(z-1)/e_2)(1+(z-1)/e_2)-0.5\sigma_{\xi 2}^2 z(1-z)].\bar{Q}_{e2}^{-1/e_2}.2.b_k.A_2$$

$$.[\Delta(c^*|\mu_1+(2+(z-1)/e_2)\sigma_\theta^2,\sigma_1^2).c+[1-\Delta(f^*|\mu_1+(2+(z-1)/e_2)\sigma_\theta^2,\sigma_1^2)].f]$$

$$+\ \exp[\sigma_\theta^2(1-1/e_1)(1-2/e_1)+\sigma_{\xi 1}^2].\bar{Q}_{e1}^{-2/e_1}.A_1^2$$

$$.[\Delta(f^*|\mu_1+(2-2/e_1)\sigma_\theta^2,\sigma_1^2)-\Delta(c^*|\mu_1+(2-2/e_1)\sigma_\theta^2,\sigma_1^2)]$$

$$.[\Delta(f^*|\mu_1-2e_1\sigma_{\xi 1}^2,\sigma_1^2)-\Delta(c^*|\mu_1-2e_1\sigma_{\xi 1}^2,\sigma_1^2)]/[\Delta(f^*|\mu_1,\sigma_1^2)-\Delta(c^*|\mu_1,\sigma_1^2)]$$

$$+\ \exp[0.5\sigma_\theta^2(2-(1/e_1)+(z-1)/e_2).(1-(1/e_1)+(z-1)/e_2)-0.5\sigma_{\xi 2}^2 z(1-z)]$$

$$.[\Delta(f^*|\mu_1+(2-(1/e_1)+(z-1)/e_2)\sigma_\theta^2,\sigma_1^2)-\Delta(c^*|\mu_1+(2-(1/e_1)+(z-1)/e_2)\sigma_\theta^2,\sigma_1^2)]$$

$$.[\Delta(f^*|\mu_1-e_1\sigma_{\xi 1}^2,\sigma_1^2)-\Delta(c^*|\mu_1-e_1\sigma_{\xi 1}^2,\sigma_1^2)]$$

$$.\bar{Q}_{e1}^{-1/e_1}.\bar{Q}_{e2}^{-1/e_2}.2.b_k.A_1.A_2/[\Delta(f^*|\mu_1,\sigma_1^2)-\Delta(c^*|\mu_1,\sigma_1^2)]\Big].\beta_k^2/(1+b_k)^2$$

Appendix 2

Substituting equation (32) into equation (31) yields:

$$\frac{\partial W_k}{\partial z} = E\left[U'(\pi_k).\theta^{1+(z-1)/e_2}.\xi_2^{1-z}.\ln\theta_2\right].A_2.\bar{q}_k.\bar{Q}_{e2}^{-1/e_2}.\frac{b_k}{e_2(1+b_k)}$$

$$= \left[E\left[U'(\pi_k).\theta^{1+(z-1)/e_2}.\xi_2^{1-z}.\ln\theta\right] - e_2.E\left[U'(\pi_k).\theta^{1+(z-1)/e_2}.\xi_2^{1-z}\ln\xi_2\right]\right]$$

$$.A_2.\bar{q}_k.\bar{Q}_{e2}^{-1/e_2}.\frac{b_k}{(1+b_k)} \qquad \text{(A1)}$$

The utility function is exponential with parameter γ. We expand the subjects of the expectations in a first order Taylor series expansion around $\pi_k = 0$ so that:

$$U'(\pi_k).\theta^{1+(z-1)/e_2}.\xi_2^{1-z}.\ln\theta \doteq (\gamma-\gamma^2\pi_k).\theta^{1+(z-1)/e_2}.\xi_2^{1-z}.\ln\theta \qquad \text{(A2)}$$

and

$$U'(\pi_k).\theta^{1+(z-1)/e_2}.\xi_2^{1-z}.\ln\xi_2 \doteq (\gamma-\gamma^2\pi_k).\theta^{1+(z-1)/e_2}.\xi_2^{1-z}.\ln\xi_2 \qquad \text{(A3)}$$

We expand (A2) in a second order Taylor series expansion around $\theta=1$. Under our assumptions on the distribution of the random variables in the model we eventually find that:

$$E\{U'(\pi_k).\theta^{1+(z-1)/e_2}.\xi_2^{1-z}.\ln\theta\} \doteq [\exp(\sigma_\theta^2)-1]$$

$$.\Big[\exp(0.5z(z-1)\sigma_{\xi 2}^2).\Big[(\gamma+\gamma^2 C_k(w,\bar{q}_k)).(0.5+(z-1)/e_2)$$

$$-\gamma^2.\frac{\bar{q}_k}{(1+b_k)}.[(1.5+(z-1)/e_2).\{c-c.\Delta(c^*|-0.5\sigma_{\xi 1}^2,\sigma_{\xi 1}^2) + f.\Delta(f^*|-0.5\sigma_{\xi 1}^2,\sigma_{\xi 1}^2)\}$$

$$+ (1.5-1/e_1+(z-1)/e_2).A_1.\bar{Q}_{e1}^{-1/e_1}.\{\Delta(c^*|0.5\sigma_{\xi 1}^2,\sigma_{\xi 1}^2)-\Delta(f^*|0.5\sigma_{\xi 1}^2,\sigma_{\xi 1}^2)\}\Big]\Big]$$

$$-\gamma^2.\frac{b_k}{(1+b_k)}.A_2.\bar{q}_k.\bar{Q}_{e2}^{-1/e_2}.(1.5+2(z-1)/e_2).\exp((1-z)(1-2z)\sigma_{\xi 2}^2)\Big] \qquad \text{(A4)}$$

where $c^* = A_1.\bar{Q}_{e1}^{-1/e_1}/c$ and $f^* = A_1.\bar{Q}_{e1}^{-1/e_1}/f$.

We expand (A3) in a second order Taylor's series around $\xi_2=1$. Eventually we find that:

$$E[U'(\pi_k).e^{1+(z-1)/e_2}.\xi_2^{1-z}.\ln\xi_2\} \doteq [\exp(\sigma_{\xi 2}^2)-1]$$

$$.\Big[(0.5-z).\exp(0.5\sigma_\theta^2(1+(z-1)/e_2)(z-1)/e_2).\Big[\gamma+\gamma^2 C_k(w,\bar{q}_k)$$

$$-\gamma^2.\frac{\bar{q}_k}{(1+b_k)}.\{c.\Delta(c^*|\mu_1+(1+(z-1)/e_2)\sigma_\theta^2,\sigma_1^2)+f-f.\Delta(f^*|\mu_1+(1+(z-1)/e_2)\sigma_\theta^2,\sigma_1^2)\}\Big]$$

$$-\gamma^2.\frac{\bar{q}_k}{(1+b_k)}.\Big[b_k.A_2.\bar{Q}_{e2}^{-1/e_2}.(1.5-2z).\exp(0.5\sigma_\theta^2(2+(z-1)/e_2)(1+(z-1)/e_2))$$

$$+(0.5-z).A_1.\bar{Q}_{e1}^{-1/e_1}.\{\Delta(f^*|\mu_1+(1+(z-1)/e_2)\sigma_\theta^2,\sigma_1^2)^2$$

$$-\Delta(c^*|\mu_1+(1+(z-1)/e_2)\sigma_\theta^2,\sigma_1^2)\}.\exp(0.5\sigma_\theta^2(1-1/e_1+(z-1)/e_2)((z-1)/e_2-1/e_1))\Big]\Big] \quad \text{(A5)}$$

Equations (A4) and (A5) are substituted into equation (A1) and for convenience the result is written as equation (33).

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