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A METHODOLOGY FOR VALUING
MULTIPLE-EXERCISE OPTION CONTRACTS FOR WATER

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Abstract

In this paper I use financial derivative pricing theory as a foundation for a computational approach to valuing multiple-exercise option contracts in a natural resources setting. Evidence from the western United States shows that option contracts for water can be even more exotic than many exotic options considered in finance. For instance, one contract negotiated between a municipal water authority and a large agricultural operation allows the municipality to exercise a call option on water up to seven times in a fifteen-year period. This is a highly non-standard option; there is no simple pricing formula to calculate its value. Building on the Black-Scholes option-pricing framework I use dynamic programming techniques to construct a method for valuing such multiple-use option contracts for water.

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1 Water Allocation Dilemmas

Water is, perhaps, the ultimate factor of production. Though rudimentary formulations of production functions rely on two inputs, capital and labor, without water there can be no life and thus no labor and no production. Water plays other more direct roles in an economy too, most obviously as an input for agriculture. By generating electricity, water powers manufacturing and service in all sectors of the economy. In the U.S. in 1998 ten percent (321 billion kilowatt hours) of the electricity generated by utilities was garnered from water sources (U.S Department of Energy, 2000).

From a purely economic standpoint, then, water is a crucial resource and should be allocated with care. From a broader perspective, the decisions individuals, cities, and countries make about how to use or conserve their water resources can have profound effects on regional economic growth, survival of plant and animal species, international relations, and human health and standards of living. Integral to societal activities from food production to recreation, water is distinctive among natural resources due to the tangled web of legal and institutional systems that governs its use at the local, state, national, and international levels. With quality and quantity of water supply emerging as pressing concerns in many areas of the U.S. and the world, efficient allocation mechanisms are becoming more crucial.

Water management or allocation is a delicate balancing act between demand and supply. Growing population and rising concern for instream flows tend to increase demand while water quality problems and global climate change suppress supply or make it less reliable. In order for water to be put to its most valuable use it must be of sufficient quality and adequate quantity, at the correct location and at the proper time. Abundant water in the form of flooding in Minnesota in the spring does not ease the pinch of drought on Colorado agriculture in June. Water allocation mechanisms must be flexible enough to respond to changes in quality, quantity, location, and timing of both water supply and water demand.

1.1 Water Allocation and Transfer Mechanisms

Historically the policy response to excess demand for water was to apply a technical solution. Federal and state governments developed new water supplies, built dams, dug irrigation ditches, and constructed storage facilities. Spurred by decreased federal funding for water projects, the high cost of infrastructure, and swelling environmental concern about dam construction and water diversion, the trend in the U.S. has shifted from infrastructure to institutions. Legislatures and governments are developing and re-designing agencies, regulations, and laws to facilitate water transfers and market mechanisms (Livingston, 1998). For example, the state legislature in Texas has allowed the State Water Development Board to form water banks in an effort to encourage water transfers (Thompson, 1997). Similarly, California has sought to clarify water property rights to facilitate short and long-term transfers (Archibald and Renwick, 1998).

Water transfers differ widely in their characteristics but can be loosely grouped into categories based primarily on the permanence of the trade. Permanent transfers of water right are at one end of the spectrum. In this type of transfer the seller gives up his right, in perpetuity, and no longer has a claim to the yield of that right. Most other transfer mechanisms are temporary and allow water to be moved from one location or user to another but do not change the underlying structure of water rights. California, Colorado, and several other western states permit short-term transfers. These transfers of water are generally limited to a one-year life span and avoid many of the legal hurdles faced by permanent transfers (MacDonnell and Rice, 1994).

The transfer mechanism of greatest interest here is the dry-year option or contingent transfer. This type of transfer is modeled on the more familiar and prevalent option contracts seen in stock markets. The key distinguishing features of options for water are the lifespan of the contract and the possibility of multiple exercise. Unlike standard financial derivatives, water options are often structured to allow the holder to use the option more than one time before the option expires. Option contracts for water are emerging in some states, California for instance, as institutional

and legal modifications allow water users to devise innovative mechanisms to increase reliability of water supply in dry years.

Despite their differences, all types of water transfers have some common elements. First, the kinds and number of transfers that occur in a given water locality are determined in part by the institutions, laws, and regulations that govern water use. As these institutions and rules have changed in recent years the variety and frequency of transfers has grown. Second, transfers take place only when both the seller and the buyer anticipate gaining from the transaction. Third, since water markets in the United States are not yet thriving in most locations, division of the gains from trade is negotiated separately for each transfer. This can erode the total gains as transaction costs accumulate.

The various transfers mechanisms for water all fall under the general heading of "water markets." Initially water markets referred almost exclusively to permanent rights transfers. Permanent transfers of water rights face numerous obstacles: establishing or adjudicating property rights, assessing and mitigating the effects of the transfer on downstream and third-party users, and evaluating the current value of the future stream of water. Most of these factors increase the transaction costs of permanent rights sales, suggesting that such transfers are not an effective means for addressing short or medium-term water shortages.

Changes in laws and institutions combined with the high transaction costs associated with permanent transfers of water rights have provided the opportunity and incentive to utilize market principles to allocate water among its users. Gollehon (1999) notes that during 1996-1997 in the Western United States, 78% of 282 recorded water transfers were permanent sales of water rights. In contrast, the overwhelming volume of water, 92.8 percent or 2.5 million acre feet[1] , was exchanged in short-term, temporary transactions. Proponents of short-term water transfers argue that market mechanisms can be used even if water property rights are not adequately defined, and

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[1] One acre-foot of water is the amount of water needed to cover one acre of land to a depth of one foot. This is 43,560 cubic feet, enough to fill a 50 meter by 25 meter swimming pool to 9.7 feet deep.

are especially attractive if the transaction costs of trading permanent water rights are prohibitive. (Howitt (1998) is one example). Frederick (1998) argues that markets in many cases increase the value of water by shifting it from low-value agricultural uses to hydropower or urban consumption. For economists, markets for water hold natural appeal: ideally, as supply and demand fluctuate relative to each other, prices will provide signals to market participants and ensure efficient water allocation.

While development of short-term transfers and facilitation of permanent transfers of water rights have eased barriers to efficient water allocation, these mechanisms do not adequately accommodate the medium-term planning needs of many water users. Further, as Howitt (1998) notes, short-term and spot markets appear to disproportionately burden buyers with risk; permanent transfers are riskiest for sellers. Other market arrangements could spread risk more equally between the two parties. Uncertainty about future water supplies and prices, the need to time water availability to meet demand or production constraints, and the irreversibility of investments in water imply that a new market construct may be appropriate: option contracts for water.

The buyers and sellers of call options for water participate because they expect to be better off with the contract than they would be without it. The seller loses a degree of flexibility in his water allocation decisions; he is obligated to provide the water when the buyer strikes. However, the seller receives a measure of immediate compensation, in the form of the option price or premium paid by the buyer when negotiations are complete. The seller also benefits in the event that the call is exercised since the buyer must pay the strike price then. The call buyer's main benefit from the contract is flexibility. Holding the call option entitles the buyer to access a source of water that would have been unavailable otherwise, increasing the buyer's ability to meet water obligations and expanding the range of water allocations choices. The water option price reflects the expected value of this added flexibility in decision-making.

Option contracts for water are an exciting addition to the range of market-based mechanisms for transferring water. Clearly they are related to similar derivative instruments in financial

markets. The point of this essay, though, is that option contracts for water are structurally distinct from financial options and require an innovative pricing approach. Given that option contracts for water are being established between water users in some states, both participants in the agreement must perceive the potential for gains from trade. What is the value of the increased flexibility that is inherent in holding a call option for water?

2 Structure and Valuation of Financial Options and Water Options

Options are a type of derivative, a financial instrument whose value is based on another, underlying asset. A stock option, for instance, derives its value from the price of a specified stock. Calls and puts are the two basic types of options. A call option gives the holder the right but not the obligation to purchase the underlying asset by a certain date for a specified price. Similarly, a put option gives the holder the right but not the obligation to *sell* the underlying asset by a certain date for a price specified in the option contract. The date agreed upon in the option contract is called the expiration date or exercise date; the listed price is referred to as the strike price or exercise price. Options are further categorized depending on the structure of the exercise date. European style options, for instance, can be used only on the expiration date; by contrast, American options can be exercised any time on *or before* its expiration.

Perhaps the most basic option contract is the European call. On the option's expiration date its holder must decide whether to exercise her right to purchase the underlying asset. If she exercises, she receives a payoff equal to the difference between the current price of the asset and the strike price specified in the option contract. If she does not exercise the option her payoff is zero. More formally, the payoff from the option is

$$\max\left(S_T - K, 0\right), \tag{1}$$

where S_T is the price of the underlying asset at the expiration date, T is the expiration date, and K is the strike price.

An option contract must have both a buyer and a seller. The buyer of a call obtains the right to purchase the underlying asset but the seller of the call is *obligated* to provide the asset if the call is exercised. Thus while the holder of the call retains flexibility, the seller has none. In order to acquire this flexibility about future actions, the call buyer incurs an up-front cost, the option price. Determining the option's value to the buyer is straightforward for a European call but can become quite sticky for other classes of option contracts.

Many options have payoff structures that are more complicated than standard calls and puts; these fall under the general heading of "exotic" options. The payoff from a lookback option, for example, depends on the maximum (or minimum) price achieved by the underlying asset during the lifetime of the option. A few types of exotic options such as European-style compound options can be priced analytically using extensions of known formulas. Most, however, require numerical methods such as binomial trees or numerical quadrature.

Evidence from the western United States shows that option contracts for water can be even more exotic than many exotic options considered in finance. One contract negotiated between a municipal water authority and a large agricultural operation allows the municipality to exercise a call option on water up to eight times in a fifteen-year period. This is a highly non-standard option; there is no simple pricing formula to calculate its value. Standard European call options involve a binary decision of whether or not to exercise on the expiration date. In contrast, there are more than 6,400 ways to exercise the water district's call option seven or fewer times in fifteen years. Clearly this complicated payoff structure increases the difficulty of putting a value on the option contract.

2.1 The Black-Scholes Option Valuation Framework

The challenge of pricing an option is to determine the value of the option, in current dollars, before its maturity date. This is equivalent to putting a dollar value on the flexibility the option holder gains by entering into the contract. The derivative's value, then, is the amount the buyer

pays the seller at the time they make the agreement. On December 9, 1997, Robert C. Merton and Myron S. Scholes received the Alfred Nobel Memorial Prize in Economic Sciences for their roles in developing the conceptual and mathematical frameworks of option pricing theory. The seminal articles by Black and Scholes (1973) and Merton (1973) sparked a torrent of academic and applied research on financial derivatives, which in turn generated modifications, extensions, and innovative applications of the original Black-Scholes model. Merton (1998) provides a brief review of the evolution of option-pricing theory since 1973. Though the original formulation of the Black-Scholes approach is appropriate only for European put and call options on stocks that do not pay dividends, the importance of this early work is unmistakable. As Chriss (1997) states "the basic ideas underlying it are very much the basis for all option pricing models."

In the context of financial derivatives that are traded on organized exchanges, the Black-Scholes formula supplies the value of an option as function of five variables: strike price of the option, time until expiration, price of the underlying stock, risk-free interest rate, and volatility of the stock. For a thorough yet accessible explication of the Black-Scholes differential equation and pricing formulas see Hull (1997). Under a set of assumptions listed below, Black and Scholes revealed that the value of a financial option does not depend on the expected rate of return on the underlying stock. To derive their option pricing formula, Black and Scholes posited several assumptions (Hull, 1997). These are:

1. Price of underlying asset follows a geometric Brownian motion process.

2. Market participants are permitted to short-sell the asset, with full use of proceeds.

3. No transaction costs or taxes exist.

4. The underlying asset is perfectly divisible.

5. No dividends accrue during the life of the option.

6. Arbitrage opportunities are not available.

7. Trading of the asset is continuous.

8. The risk-free interest rate is constant, strictly positive, and compounded continuously.

While these assumptions may be justifiable to some extent in the presence of well-established, active markets, the validity of some them is doubtful in nascent or thinly-traded water markets. As the list of assumptions inherent in the model suggests, the Black-Scholes method may not be suitable for wholesale adoption in this sphere of resource economics. Many of the conditions upon which the pricing formulas are predicated are absent from water markets at their current stage of development. In the succeeding paragraphs of this section I argue that the applicability of the Black-Scholes framework to water markets is tenuous, suggesting the need for an improved option pricing model for water.

The beauty of the Black-Scholes framework is that, given five inputs, the formula calculates the exact value of the option. The mathematics underlying the pricing formula quickly become treacherous, however, as the complexity of the option structure increases. On the whole, American options prove more difficult than European ones; even Black and Scholes failed to find a closed-form solution for an American put. Computational methods such as hill-climbing algorithms and other techniques apply in cases where the differential equation is relatively easy to discern but difficult to solve analytically.

2.2 Suitability of Black-Scholes Assumptions in the Context of Water Markets

Transaction costs are conspicuously absent from the Black-Scholes pricing formulas. Instead, the approach computes option values as though market prices reflect the total cost of buying or selling the underlying asset. Even in efficient and active arenas like the large stock exchanges, however, transaction costs are present in the form of brokerage fees and commissions. Leland (1985) noted that the presence of transaction costs makes continuous portfolio adjustment prohibitively expensive. In 1992 Boyle and Vorst extended Leland's analysis to a discrete-time framework

using a variation of Cox, Ross, and Rubinstein's (1979) numerical technique for valuing options. Both papers concluded that relaxing the transaction cost assumption of the Black-Scholes model has a distinct impact on the valuation results: rather than yielding a unique price for the option, the modified approach that includes transaction costs determines bounds on the option price.

All water exchanges are regulated or constrained in some way, usually by state or federal government. This governmental intervention generates uncertainties and imposes costs on both buyers and sellers in water markets (Colby, 1990). Such policy-induced transaction costs are only a subset of costs incurred in water transfers, beyond the purchase price of the commodity. Conveyance, storage, and treatment costs also contribute to the transaction cost load borne by water buyers (Lund and Israel, 1995). For further study of transaction costs in water markets see Archibald and Renwick (1998) and Colby *et al.* (1989). From a market perspective, transaction costs raise the price of water and reduce the amount of water exchanged (Easter *et al.*, 1998).

The no-arbitrage assumption fails to hold when two assets that are virtually identical are being sold at different prices. An arbitrage opportunity is defined as "an investment strategy that guarantees a positive payoff in some contingency with no possibility of a negative payoff and with no net investment (Dybvig and Ross, 1989)." A direct implication of the absence of arbitrage is the law of one price: two perfect substitutes must trade at the same price. When identical assets are available at different prices, the savvy investor will buy the asset at the low price and sell immediately at the high price, gaining certain positive payoff with no net investment. In thinly-traded exchanges such as evolving water markets, arbitrage opportunities do arise, violating the Black-Scholes model's no-arbitrage condition.

Several of the assumptions built into the general Black-Scholes framework serve to ensure that option writers and buyers can indeed devise a riskless hedge, or replicating portfolio. Derivation of the pricing formulas relies heavily on the existence of the riskless portfolio and also requires the ability to rebalance the portfolio over time so that it continues to mimic exactly the payoff of the option being valued. Assumptions allowing short selling, continuous trading, and divisibility of

assets, in addition to the assumptions regarding arbitrage and transaction costs, ensure availability of this dynamic riskless hedge. These assumptions do not translate well when applied to water markets. Thinness in water markets undermines the validity of these assumptions, making the possibility of devising a replicating portfolio dubious at best. Water markets are not active enough to facilitate creating and maintaining the riskless hedge or to guarantee that the value of the hedge is known with certainty.

The assumption that the underlying asset's price follows a geometric Brownian motion process may also be problematic in the context of water markets since it contains a series of additional, implicit assumptions: changes in the price of the asset follow a continuous-time, non-stationary, Markov process, and are normally distributed. The price of water is a stochastic process, defined by a probability law describing how price will evolve over time. One implication of this is that we can predict the probability that water price will be in a given range at a specified future time. Is the stochastic process non-stationary and Markov? Non-stationarity requires the possibility that the expected value of water price grow without bound and that the variance of price increase over time. To satisfy the conditions of a Markov property, only the current price of water can be relevant to predicting tomorrow's price. Given the seasonality of water use and supply, the Markov assumption is questionable. This suggests the possibility of easing the Itô process assumption by exploring different types of stochastic forces driving water price. Data from Texas suggest that a mean-reverting process, for instance, may bear further investigation.

Recent formulations of option valuation in water markets naturally mimic the conceptual and mathematical framework developed by Black, Scholes, and Merton (see Howitt (1998) and Watters (1995)). As I argue above, the assumptions of Black-Scholes option pricing formulas may not be fully justifiable in the context of water markets. Furthermore, the Black-Scholes approach loses elegance and usefulness when applied to non-standard options.

2.3 Highly Exotic Option Contracts for Water

Option contracts for water are far more complicated than the comfortingly simple vanilla options described earlier. In some respects these water options are also more complex than the exotic options I mentioned earlier in this essay. Several option contracts emerged in California in the early and middle 1990s, when drought descended or threatened almost annually. These contracts serve as evidence of the variety that exists in water options, their differences from financial options, and their complicated payoff structures. The Metropolitan Water District in Los Angeles (MWD) designed four of the five earliest contracts; the California Department of Water Resources arranged the fifth. The water providers were typically irrigators from nearby water districts.

These contracts exhibit features distinct from financial derivatives. For example, each agreement is negotiated individually, there is no standard format. In addition, strike deadlines tend to cluster in April and May to accommodate the planning needs of agricultural users. Most importantly, a large percentage of the contracts are long-term and allow for multiple exercises of the option. This is consistent with institutional, theoretical, and simulation-based research on water options. Hamilton *et al.* (1989) and Clark and Abt (1993) attribute both of these characteristics to the option-like transfer agreements they analyze. In a simulation of water supply options for the Fort Collins, Colorado area, Michelsen and Young (1993) construct a contract over 20 years with multiple exercise both permitted and expected.

Water options also differ from financial derivatives because quantity of the underlying asset plays a prominent role in option contracts for water. While financial options usually specify the number of units of stock, for instance, that are entailed in the derivative, water options tend to allow some leeway in the volume of the commodity to be exchanged. The Dudley Ridge agreement, an example from California, included an April 1 deadline for determining the maximum volume of water for purchase. Flexible quantity provisions are reflections of the uncertainty surrounding both future demand for water and the future yield from ownership of a certain water right.

Quantity can also play the role of a trigger in options contracts for water. Dudley Ridge is a good illustration of this: exercise of the option was contingent on the quantity of water MWD received from the State Water Project.

The distinctive attributes of option contracts for water and especially their frequent inclusion of multiple-exercise clauses provide a financial engineering challenge. Exotic options are priced using a range of numerical techniques; vanilla options rely on the Black-Scholes formula. Neither of these approaches is designed to handle multiple exercise of a single option. This calls for computational innovation to value multiple-exercise options. Though the concern here is focused on water, the methodology is applicable or adaptable to any underlying asset.

3   An Algorithm for Valuing Multiple-Exercise Options

Here I describe my dynamic programming method for valuing multiple-use options. I characterize the multiple-exercise option scenario as a finite-horizon, stochastic dynamic programming problem with discrete and finite state and action spaces and discrete time periods. I then give an overview of the numerical techniques I use to approximate the dynamic programming problem's solution. The model relies on assumptions about specific parameter values and the functional forms of the objective function and constraint equations, which I also detail here.

Option contracts for water present interesting possibilities for managing water allocation. The contracts negotiated in California exhibit characteristics that distinguish them from both standard and exotic financial derivatives. The divergence of financial and water options prompts the central question of this research: what valuation methodology is appropriate for long-term, multiple-exercise option contracts?

3.1   Multiple-Exercise Options: General Framework

Dixit and Pindyck (1994) present dynamic programming as an analytical approach for valuing "real options," or investment opportunities. Their method is simply standard theory cast in a framework of investment decision-making with uncertainty and irreversibility. The model allows

for finite or infinite planning horizons, facilitates both discrete-time and continuous-time analysis, and accommodates a range of stochastic processes that may drive uncertainty. There are two primary drawbacks to the dynamic programming approach, one practical and one less palatable. On the practical side, computational requirements for solving dynamic programming systems can become onerous; this concern is mitigated by access to sufficiently powerful computers and efficient solution algorithms. More troubling is dynamic programming's treatment of the discount rate as an exogenous variable. The selected discount rate should reflect the risk inherent in the investment opportunity under scrutiny, but quantifying the risk in terms of a discount rate is an imprecise endeavor. The capital asset pricing model is one common tool for determining this discount rate.

The decision of whether and when to use a multiple-exercise option is a variation on the common problem of dynamic programming: optimizing the sum of current and expected future rewards. The option holder knows that he may exercise the call option once per period, up to $\bar{R}$ times in m periods. The option contract also specifies the strike price, K. The holder knows the current price of the asset, S_t, but must estimate next period's price based on assumptions about the future price path. Exercising the option today yields a payoff equal to the difference between S_t and K but the payoff from exercising in later periods is uncertain. In contrast, foregoing use of the option today earns the option holder zero monetary reward in the current period while preserving the exercise right for possible use in the future. After the contract expires at the end of period m, however, any exercises of the option that remain unused are worth nothing.

In each period during the life of the contract, then, the option holder must decide whether or not to strike. Though he knows the immediate reward from striking in the current period, $(S_t - K)$, his future payoffs depend on the realized price of the asset in each future period and on his option use history. Since he cannot exceed the maximum number of exercises for his call, striking today has an irreversible and limiting effect on his ability to secure higher payoffs later. Uncertainty, irreversibility, and the importance of timing characterize this derivative holder's decision, making it an excellent candidate for a dynamic programming solution algorithm.

3.2 Standard Dynamic Programming Formulation

Translating this multiple-exercise option pricing problem into a standard dynamic programming formulation requires one control variable and two state variables. The control variable, x_t, represents the decision of whether to strike ($x_t = 1$) or hold ($x_t = 0$) in the current time period. The two states are p_t, the natural log of the price of water in the current period, and R_t, the number of option exercises left in the contract. More formally,

$$
\begin{aligned}
p_t &\in \quad \mathcal{S}_1 = [\tilde{p}, \bar{p}], \\
R_t &\in \quad \mathcal{S}_2 = \{0, 1, 2, \ldots, \bar{R}\}, \\
x_t &\in \quad \mathcal{X} = \{0, 1\}, \text{ and} \\
t &\in \quad \{0, 1, 2, \ldots m\}.
\end{aligned}
$$

Here, $\tilde{p}$ and $\bar{p}$ are lower and upper limits respectively on the log price, parameters of the problem. Note that since the option cannot be used after the maximum number of exercises is attained, $x_t = 0$ necessarily when $R_t = 0$.

The state variables both change from period to period but their dynamics are quite distinct. The number of exercises remaining decreases whenever the option buyer strikes, evolving according to:

$$
R_{t+1} = R_t - x_t. \tag{2}
$$

The price of water follows a more complicated path, a generalized Weiner process:

$$
p_{t+1} = p_t + \mu + \sigma \epsilon_{t+1} \tag{3}
$$

with μ and σ denoting mean and volatility as usual, and $\epsilon \sim N(0,1)$.

The objective is to maximize, over the life of the contract, the present value of the sum of expected returns from exercising the option,

$$
\max_{x \in \{0,1\}} \sum_{t=0}^{T} \left(\frac{1}{1+r} \right)^t x_t \left(e^{p_t} - K \right)
$$

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subject to the state equations above, non-negativity conditions, and the following constraints:

$$\sum_{t=0}^{m} x_t \leq \bar{R}$$

$$K = 1.$$

For convenience, the strike price is normalized to 1.

In a given period the reward function is a function of the two state variables, log price and number of uses remaining:

$$f(x_t, p_t) = x_t \left(e^{p_t} - K \right) \tag{4}$$

Recall that p_t is the natural log of the asset price, thus $e^{p_t} = P_t$, the actual asset price. For this problem the value function $V_t(p_t, R_t)$ is the value of the option at time t if the price of water is p_t and there are R_t uses of the call remaining. The value function must satisfy Bellman's equation:

$$V_t(p_t, R_t) = \max_{x \in \{0,1\}} \left\{ x_t \left(e^{p_t} - K \right) + \frac{1}{1+r} E \left[V_{t+1} \left(p_{t+1}, R_{t+1} \right) \right] \right\} \tag{5}$$

From the transition equations for p and R, this becomes

$$V_t(p_t, R_t) = \max_{x \in \{0,1\}} \left\{ x_t \left(e^{p_t} - K \right) + \frac{1}{1+r} E \left[V_{t+1} \left(p_t + \mu + \sigma \epsilon_{t+1}, R_t - x_t \right) \right] \right\}.$$

The boundary condition needed to solve the dynamic programming problem arises from the terms of the option contract; the value function equals zero with certainty for $t > m$:

$$V_t(p_t, R_t) = 0 \text{ for } t > m.$$

3.3 Solution Algorithm Summary

My Matlab program begins with ten parameters and yields four outputs. Four of the parameters are identical to those used in the Black-Scholes pricing formula: length of option life in years, option strike price, annual interest rate, and annual volatility in log price. Since I treat log price as a discrete variable, rather than a continuous one, and because my approach computes option prices for a range of possible initial prices, I must prespecify the number of discretization points

for price and the upper and lower limits on log price. Mean increase in annual log price and number of quadrature points are used in conjunction with annual volatility to approximate the probability distribution of log prices for water. The final parameter is dictated by the nature of the options the program values: maximum number of uses (exercises) during the life of the option.

The four outputs are a vector of price nodes, the function definition structure, the option prices, and optimal exercise decisions. The price nodes output vector is simply the discretized log prices converted into actual prices. The function definition structure indicates the approximating functions used for R and p. The option valuation coefficients and optimal policies are structured as multidimensional arrays consisting of one page for each time period, each page with a column for each exercise permitted and a row for each log price state. Thus the researcher can determine the optimal policy and option price for each possible state of log price and number of uses remaining, in any time period.

The solution methodology entails five main steps: defining the problem size and state spaces, initializing and defining the state transition matrices, initializing and defining the reward matrix, solving the model using backward recursion, and reshaping output arrays. The solution program computes all the necessary numbers to value a multiple-exercise option for any initial starting price for the asset, within the specified range. It also determines, for each possible state and time period, whether to use the option.

3.4   Assumptions of the Solution Approximation

All of economics rests on assumptions designed to simplify a complex reality so that it can be simulated and modeled. Of course, this multiple-exercise option pricing model and the computer code that approximates it also rely on a host of suppositions. Some of these assumptions are clearly technical, some are more fundamental, and they vary in the degree of difficulty entailed in relaxing them. I group the assumptions into four categories based on their primary emphasis: 1) structures and terms of the option contract, 2) technical assumptions, 3) transition process from

state to state, and 4) assumptions that mimic Black-Scholes.

The assumptions about the structure of the option contract are generally not restrictive and are easy to revise. The option can be exercised at most once in a given time period. The total number of uses of the option must be strictly greater than zero. The number of periods in the life of the option contract is strictly greater than the total number of uses allowed. First strike can occur immediately, in period 0. Finally, the strike price is constant across time periods. The strike price assumption would be inconvenient but not difficult to change and would alter the reward matrix definition.

The technical assumptions facilitate the dynamic programming solution algorithm. Possibly the strongest assumption in this area is treating price as a discrete variable. The program divides the price state into equal intervals; the upper and lower limits on log price and the number of discretization point are parameters of the program. The actual number of discretization points is two fewer than the specified parameter (if the parameter is an odd number) due to rounding and the need to avoid listing the same price twice. Discretizing the price space rather than construing it as continuous more accurately represents actual price movements in water markets.

My formulation of the multiple-exercise option pricing problem as a dynamic programming exercise presupposes that the log price of water evolves according to a Markov process. Specifically, the probability distribution of future price depends only on the currently observed price and the action the call holder chooses:

$$\Pr(p_{t+1} = \hat{p} \mid x_t = x, p_t = p, R_t = R) = \Pr(\hat{p} \mid x, p, R).$$

The future price of water does not depend on the price history beyond the current price. The Markov assumption is unnecessary for the state describing remaining uses. R is deterministic; its value is known with certainty once the current price and action are revealed.

The Markov assumption is a crucial one for my model; it enables me to use this particular formulation of the dynamic programming technique. The Bellman equation (5), summarizes

the agent's dilemma: balancing immediate payoff from exercising the option against the present value of expected payoffs from using the option in future periods. If the future price of water depends on more than one period of price history, the dynamic programming representation of the multiple-exercise option valuation problem becomes more complex.

The model rests on an additional assumption about the stochastic process driving the price of water: log price evolves according to

$$p_{t+1} = p_t + \mu + \sigma\epsilon_{t+1}$$

where $\epsilon \sim N(0,1)$. The change in log prices is thus stochastic with mean μ and standard deviation (volatility) σ. Since parameters μ and σ do not depend on log price this is a Wiener process. The stochastic process for water price, however, is the same process Black and Scholes assumed: geometric Brownian motion. The trend and volatility terms for this process are functions of time and the underlying variable, price.[2] The key link is that both price and log price are affected by the same source of uncertainty, ϵ.

The remaining notable assumptions of the model mimic those of Black and Scholes: no transaction costs or taxes apply and no arbitrage opportunities exist. Because water that is traded must be transferred using available infrastructure, and due to the difficulty of coordinating potential buyers and sellers, arbitrage positions may arise. Transaction costs are likely present, too, as asserted earlier.

4 Beyond the Model: The Need for Reliable Parameters

The usefulness of this model's results depends not only on a carefully constructed theoretical and computational framework, but on the accuracy of its parameters. The specified values for the mean and volatility of the price process must represent the true path of log price of water as closely as possible for the model to serve as a useful tool in water management and policy

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[2] See Hull (1997; p. 221) for explanation of a parallel scenario based on stock prices.

decision-making. Augmenting the preceding theoretical and algorithmic analysis with empirical evaluation will ensure high quality inputs to increase the probability of high quality output, test the assumptions about the stochastic process for log price, suggest a more appropriate price process, and simulate comparative statics to discern which parameters have the greatest effect on multiple-exercise option prices in this model. The next step in this research is estimation of model parameters using data from short-term water trades in the Texas Rio Grande.

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