Use of the Ratio-Delay Method in Processing Plant Operations

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Studies to increase the operating efficiency of packing and processing plants and other establishments that handle farm products have become increasingly important in the marketing research program. These studies have brought to the attention of marketing research people in this Department and the State agricultural experiment stations the techniques of engineering-methods study, a field that had been generally unfamiliar to economists. This paper describes the application of a relatively recent development in this field to a study of operations of pear and apple packing plants.

The ratio-delay method is used in analyzing the use of time in plant operations. Among its advantages are its economy and its susceptibility to statistical evaluation of the reliability of the results obtained.

STUDIES LEADING to greater operating efficiency in plants that pack and process agricultural products have become increasingly important in the marketing research program and are particularly pertinent in the present period of high demand for productive resources. In such studies, the traditional approach has tended to be in terms of average costs computed from the records of a representative sample of plants, but this method frequently yields inadequate or misleading results. Average costs derived from historical records often fail to account for the effects of seasonal variation in volume, excess plant capacity, differences in work methods, and other factors affecting the efficiency of plant operations (*1*).[2] This difficulty has led to a growing interest in the application of industrial engineering techniques to studies of plants.

Examples of the kind of analytical data obtainable with the aid of industrial engineering methods are: Standard unit-time requirements for the performance of essential tasks; estimates of the proportion of working time that is actually used in productive work—the converse of this measure is the proportion of "idle" or "delay" time; and the pattern of flow in handling materials. Such data are basic to an analysis of plant operations aimed at improving efficiency through better work methods and the elimination of bottlenecks in plant operations, or, more generally, through better integration of the plant processes.

The industrial engineering procedure most familiar to agricultural economists probably is the *time study* by which the amount of time required to perform a specific operation is measured by direct observation with a stop watch. This method is most applicable to work performed in repetitive cycles but, if observations are taken for a long enough period, it also will provide estimates of time required for irregular operations and for delays or idle time. If the task to be measured is divided, for purposes of observation, into a series of discrete elements and if performance times for each element are obtained, the time study will provide data for a detailed analysis which may lead to changes in the equipment or work methods that will increase efficiency.

A second industrial engineering method familiar to agricultural economists is the *production study* which consists of a continuous time log of each operation, delay, or other event associated with a particular task. The performance times are obtained by a series of continuous stop-watch readings, each reading being associated on the data sheet with a notation as to the nature of the particular event observed. By sorting these separate observations into appropriate categories, estimates of the average time per operation can be obtained for each category. This type of study is used to obtain unit-time requirements where the pattern of

[1] Credit is due D. G. Malcom for assistance in designing field procedures described in this report; and acknowledgment is made to Herman M. Southworth, Glenn Burrows, and R. G. Bressler, Jr., for suggestions in its preparation.

[2] References in parentheses and italics refer to literature cited at end of paper.

operations is so irregular as to make a time study unsuitable. Or, the production study may be employed to estimate the distribution of the time of a worker who performs more than one task, or to obtain the proportion of his total work time that is spent in a "delay" or "idle" status.

A third method has been introduced more recently by a British statistician (*2*) and has come to be known by industrial engineers as the *ratio-delay method*. This method probably is less familiar to agricultural economists than the time-study or production-study methods; it appears to have excellent possibilities as a tool in marketing research. This paper presents some aspects of its application in a study of costs and efficiency in the operation of pear- and apple-packing houses in California. A question might be raised as to the aptness of the designation, ratio-delay, especially as the applications described in this article are to a much broader category of events than delays. It is suggested that the term, ratio-delay, be associated primarily with a method of procedure rather than a particular kind of event.

An important point is to be stressed with respect to each of these methods. Data obtained by them are based on past performance and each method yields only an estimate of expected or probable performance. Each method is subject to error in that the period observed is only a sample of the total performance.

Choice of Method

In a particular plant the operations often involve many different job classifications and many workers. To analyze the operations it may be necessary to obtain data as to the labor and equipment requirements in each job category. This may require an estimate of the time expended per work unit and of the proportion of time spent in productive work, in a delay or idle status, on work of another category, etc. It also may be essential to obtain the pattern of movement for materials transported by hand truck or fork truck—that is, the transport route for each type of material, the number of units moved per trip, and the number of times the material was moved. The most logical way to obtain this pattern is by observation of the workers involved.

For many jobs, the time requirements per work unit are most easily obtained by time study, but this method is not well adapted to tasks in which the job elements are not well defined—for example, checking packed boxes in a fruit-packing plant to ascertain the size and number of fruits per box. Moreover, the use of the time-study method to obtain data such as the proportion of delay time is unduly expensive if many jobs are to be studied. In fact, if the plant operations are seasonal, as is often the case for marketing facilities, there may be insufficient field time for obtaining these data by time study. Similar handicaps apply to the production-study method.

Thus, under suitable conditions, the ratio-delay method is useful in economizing on field time required for estimating delay proportions and in establishing unit-time requirements for the less well-defined jobs. A modification of the ratio-delay method also is applicable to the problem of defining the pattern of flow in materials handling.

Procedures in Ratio-Delay Studies[3]

The ratio-delay method is essentially a sampling process which involves: (1) a machine or worker whose activity is divided into several categories, (2) a large number of instantaneous and, for practical purposes, random and independent observations of the work, and (3) the theory that the ratio of the *number of observations* in any one category to the total number of observations will yield a reliable estimate of the ratio of *time expended* in that category to the total time. The process can be visualized more easily, perhaps, by first considering how observations are made in the field.

As a preliminary step, the work performed at each work station is studied and a written summary or job description of the operations is prepared. The observer thus familiarizes himself with the details of each job and is prepared to classify properly the events to be noted in the ratio-delay study. A schematic plant lay-out may be drawn to record the locations of the work stations (fig. 1) to be observed and for use in planning the route to be followed by the observer. Tours of the plant may be made over this route and on each tour the activity of the worker at each station may be classified.

To avoid bias in classifying the observations, they should be made on an instantaneous basis, with care to eliminate any tendency to anticipate what the work status should be or unconsciously to ex-

[3] For general discussions of ratio-delay procedures, see references (*5*), (*6*), and (*7*).

ercise a preference for recording the work status in one way or another. For example, a kindhearted observer might unconsciously prefer to record a worker as "working" rather than "idle." The kind of observation desired may be described as that resulting if the observer were to wear special goggles equipped with a camera shutter. If the shutter were operated the instant the work station was visible, an instantaneous observation of the work status would be obtained.

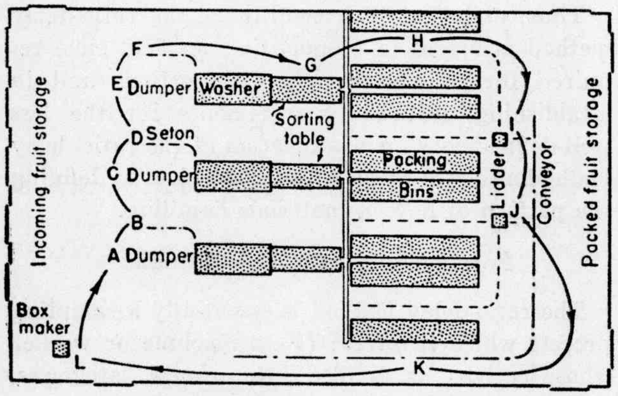

FIGURE 1.—Plant lay-out and ratio-delay observation route.

To assure consistent classification of the observations (both from the standpoint of consistency of one observation to another and from one observer to another) careful definitions must be drawn as to the character of each category of work status. Each observation should be recorded with a tally mark in the appropriate column of a data sheet. For example, the record for one job in a fruit-packing plant—the dumping of fruit from field lugs to a conveyor at three dumping stations—is illustrated in figure 2. A separate record should be made of the observations at each work station, although it may be desirable in later analysis of the data to aggregate the observations for several stations. For example, if identical operations are performed at several work stations, a ratio-delay proportion relating to the total work performed at the several stations may be wanted. In this event, observations on the individual stations may be aggregated to obtain an over-all proportion.

The ratio-delay method is statistically acceptable if the observations are random, independent, and unbiased, and if the number of observations is sufficiently large. These conditions require that all possible events have an equal chance of being included in the sample. As the ratio of delay observations to total observations is the basis for an estimate of the proportions in which the total time is divided, it is consistent with a random sampling procedure to record the same delay, if it is a long one, on successive trips.[4] A necessary precaution is that visits to a particular station not coincide with a cyclical delay, otherwise considerable bias may be introduced.

As the events observed in the ratio-delay study occur over a time period—and past events have no chance of being included in the sample—a truly random sample is not possible. But the desired conditions probably are approximated if a continuous round of observations are taken and if the starting time and the duration of each trip are arranged so as to avoid visiting the various work stations on a regular time cycle. The procedure of continuous sampling probably will approximate a random sample if applied to a situation in which the delays are random in distribution.[5]

RATIO-DELAY STUDY						
Plant: A Date: 7-14-50		No. of workers	WORKING	NOT WORKING		
Job	Sta.			Break for lots	Equip. failure	Other
Dumper	A	1	HH HH HH HH	HH I		
Dumper	C	1	HH HH HH II	HH I	II	I
Dumper	E	1	HH HH HH HH	HH I		

FIGURE 2.—Sample ratio-delay data sheet.

Applications in a Packing-House Study

To indicate how the ratio-delay method may be used in plant studies, several illustrations are given of its application in a current study of deciduous fruit-packing houses in California. This work has included an intensive study of operations in 22 plants in which the number of job classifications varied from about 12 to 45 and the total number of

[4] In the literature concerning the ratio-delay study, the usual instruction regarding long delays is either to schedule the observations so that each tour is slightly longer than the longest delay; or if the same delay extends into more than one observation trip, to count the delay only once. The procedure suggested in this paper will result in a more reliable estimate of the delay time.

[5] This sampling procedure becomes less appropriate the more the delays under observation deviate from random occurrence. If delays occur with regularity or are cyclical, the delay probably can best be measured directly, rather than by sampling. For example, sampling should be discontinued during lunch and rest periods.

workers per plant ranged from 25 to 180. The ratio-delay method was employed to obtain three types of data: (1) The proportion of "delay" (nonproductive time) in relation to total working time. (2) Time requirements per work unit for specific jobs. (3) The flow pattern in materials handling.

THE PROPORTION OF DELAY TIME.—In the simplest case, this involves a classification of the observations into only two categories. The data given in figure 2, for example, would be grouped into two classes, "working" — 57 observations — and "not working"—21 observations—and the delay proportion computed as the ratio of delay observations to total observations. If the estimated delay proportion is represented by p, this ratio in the example is:

$$p = 21/78 = 0.269$$

As it may prove desirable to have information regarding the causes underlying the total delays observed, subgroups of delay observations might be obtained. Thus, in the data in figure 2, 18 observations were recorded under "break for lots"[6] and the proportion of observations in this category is:

$$p = 18/78 = 0.231$$

The foregoing ratios of instantaneous observations are estimates of the proportions in which the total time was divided. Thus, we estimate that of the total time about 73 percent was actual working or productive time and 27 percent was total-delay time. Delay due to break for lots is estimated as 23 percent of the total time.

As the delay proportions are based on sampling data, the question of reliability of the estimates obtained is important. This problem is considered in detail later in this paper, but it is well at this point to note some empirical evidence as to the accuracy of the observed proportions. In several instances it was feasible to make concurrent ratio-delay and production studies of a particular job. These parallel studies make possible a comparison between delay proportions obtained by direct measurement in a production study and sample proportions obtained in a ratio-delay study (table 1). Although the two sets of proportions do not correspond exactly, they are reasonably consistent, particularly in view of the small number of ratio-delay observations.

[6] "Break for lots" refers to the suspension of the dumping operation while changing from the fruit of one grower to the fruit of another.

Time Requirements Per Work Unit for Specific Jobs

When time requirements per work unit are obtained by time study, the operations comprising the task usually are divided into work "elements" and the performance time determined separately for each element. The element times then are summed to obtain a cycle time. The element times are useful in a detailed analysis of work methods, but for some purposes it is sufficient to obtain only the average total cycle time. A simple way to ascertain such times is to divide the actual *productive time* in a given period by the number of work units completed during that period.

Obtaining an estimate of the productive time, as distinguished from the total time, is a simple matter if the delay proportion is available. Consider, for example, the task of dumping fruit from field lugs at the sorting table conveyor recorded in figure 2. For the 1-day period illustrated, the total delay proportion for the three dumpers was 0.269; for the complete study—over a 4-day period—their delay proportion was 0.321. Conversely, their proportion of *productive* work time was (1-0.321) = 0.679.

Their aggregate total working time per hour was 180 man-minutes. This total, when adjusted to allow for two 10-minute rest periods per 8 hours (not counted as a delay in the ratio-delay studies), reduces to 172.5 minutes of total available work time per plant-hour. A further adjustment by the proportion of productive work, p = 0.679, gives an estimate of 117.2 minutes of productive dumper time per hour.

During the 4-day period, the average rate at which field lugs were dumped (as reported in the plant records) was 837 lugs per hour of plant operation. The productive time per work unit (in this instance, dumping one field lug) then is: productive time = 117.2/837 = 0.140 minutes per lug.

Within the limits of accuracy of the ratio-delay proportion, this represents the average time required per work unit, excluding time lost through delays, equipment breakdown, rest period, and so on.

The foregoing computation leads to another comparison of results derived from ratio-delay proportions with data obtained by direct measurement. In nine of the packing houses the time required for

TABLE 1.—*Comparison of delay percentages observed by ratio-delay and production-study methods*

Plant	Type of delay	Job	Production study: delay proportion	Ratio-delay study		
				Delay proportion	Number of observations	Estimated standard error S_p
A	BFL[1]	Set-on	0.174	0.167	67	0.046
A	BFL	Dumper	.174	.167	67	.046
A	BFL	Sorter	.174	.197	67	.049
B	Conveyor stopped	Lidder	.157	.184	49	.055
B	BFL	Dumper	.031	.038	52	.026
C	BFL	Set-on	.269	.218	78	.047
C	BFL	Dumper	.269	.239	78	.051
C	BFL	Sorter	.269	.251	78	.049

[1] BFL = Break for lots.

dumping field lugs was obtained directly by time study. The average times thus estimated are compared (table 2) with average times derived from the ratio-delay proportion in the manner described above. The two sets of estimates of average dumping times are reasonably consistent but some qualifications must be noted. Neither estimate is, in any sense, an absolutely true value. The ratio-delay proportion is subject to sampling errors, as is the time-study average. If it is granted that reasonably consistent estimates were obtained by each method, it still must be recognized that the time-study observations were confined to an operating period of only 10 to 20 minutes and to a single dumper per plant. The ratio-delay observations, on the other hand, are the aggregate of observations made during a period of 3 or 4 days in each plant and, for the plants having more than one dumper, the summaries represent the aggregate of observations on all the dumpers.

In the above situation, considerable variation might be expected between the rates of operation—and, hence, the observed average times per work unit—existing during a 10- to 20-minute time study and the average for a 3- or 4-day period. One might also expect to find variation in the rate of operation by different dumpers in the same plant. The likelihood of such variation is greatly reduced by the circumstances applying to the dumping job. An essential requirement is to maintain an even flow of fruit to each sorting and packing line. This means that the dumper is "line paced" and that his rate of operation must be approximately uniform so long as the rate of line operation does not vary appreciably. A fairly uniform rate of operation per line over several days is not uncommon. In fact, in some packing plants a mechanical timer is employed to regulate the rate of work of the dumper.

MATERIALS-HANDLING PATTERN.—The objective in this application of the ratio-delay study was to obtain a quantitative description of how the fruit and packing materials were transported within the plant. The data desired were the proportion of total trips in a given category that occurred over each transport route, the number of units transported per trip, and the number of times each unit of material was moved before final disposition.

The application of ratio-delay procedures is illustrated by reference to the observations on the receiving of incoming fruit at a pear-packing plant. The work consists of unloading palletized lots of

TABLE 2.—*Comparison of unit-time requirements for dumping field lugs as ascertained from plant-output data and ratio-delay data*

Plant	Number of observations	Ratio-delay proportion	Estimated standard error of proportion S_p	Estimated average productive time per work unit	
				From ratio-delay	From time study
A	252	0.329	0.030	0.138	0.140
B	87	.103	.033	.082	.078
C	104	.077	.026	.162	.153
D	300	.010	.006	.155	.156
E	98	.041	.020	.163	.156
F	164	.143	.028	.245	.237
G	41	.122	.051	.094	.095
H	138	.109	.026	.251	.251
I	276	.119	.029	.165	.127
J	72	.111	.037	.143	.137

full field lugs, using fork-truck equipment, and transporting them to a sampling station, or to a temporary storage, or directly to the dumping station (fig. 3). The work area was divided into numbered zones for convenience in recording. Each fork truck was treated as a separate observation unit and a data sheet was prepared for each unit. Similar arrangements were made with respect to other trucking jobs in the plant and an observation route was laid out in the way described in discussing the general procedures in ratio-delay studies.

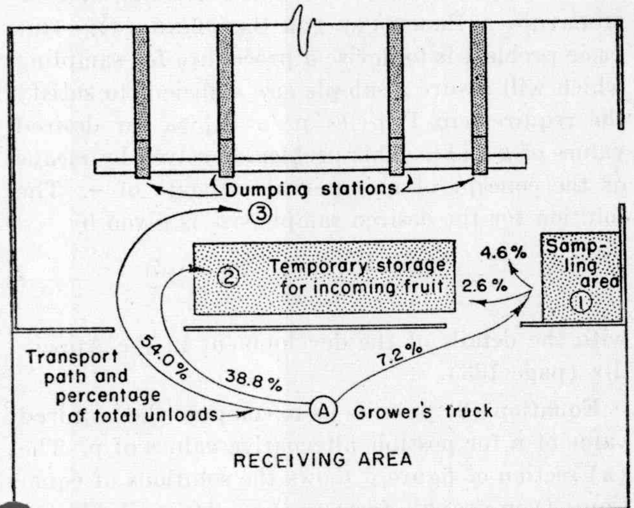

FIGURE 3.—Lay-out of packing-house receiving area, with transport paths for unloading growers' trucks and percentage of total unloads in each path.

On each tour of the observation route the observer made "instantaneous" ratio-delay observations by which each fork truck was classified as "working" or "not working." If a fork truck was working, the observer also noted the load being carried and the transport path followed. These data were recorded systematically by truck and by transport path (fig. 4). In this study 350 observations were made on the entire receiving operation. There were 324 of these in a "working" category; 126 of them applied to removing pallets from growers' trucks and transporting them to a sampling station, or to a temporary storage, or directly to a dumping station. The ratio of observations in each transport path to the total observations on removing pallets from the growers' trucks provides a description of the transport pattern. Thus, it is estimated that 7.2 percent of the total pallets unloaded were transported to the sampling station, 38.8 percent to the temporary storage, and 54.0 percent directly to the dumping station. All the pallets placed in temporary storage (38.8 percent of the total received) required two separate movements before reaching the dumping station.

Plant: A | Observer: E T | Date: 8·1·50

Transport path From	Transport path To	Material trans-ported	No of units per trip	OBSERVATIONS: WORKING	NOT WORKING: Waiting	NOT WORKING: Service truck	NOT WORKING	W'k'r. not obs'v'd	W'k'r, other work
A	1	Field lugs	36	IIII					
A	2	Field lugs	36	卌 卌 卌 III					
A	3	" "	36	卌 卌 卌 卌 卌					
—	—				卌 I	II			卌 II

FIGURE 4.—Data sheet for estimating materials-handling pattern from ratio-delay observations.

Sampling Procedures and Reliability of Estimates

The preceding illustrations from actual plant studies indicate that estimates can be based on ratio-delay proportions that are reasonably consistent with estimates based on direct measurement. But more rigorous tests as to reliability are needed. The objective of the ratio-delay procedure is to estimate the proportion of the elements in a population that possesses a given characteristic. This resolves into a statistical problem of sampling for attributes possessed by elements of the population, rather than for value measurements of these elements, and estimating the true proportion from the proportion observed in the sample.

Statistical Properties of a Sample Proportion

In samples of n random and independent observations drawn from a given population, the proportion, p, which possesses a certain attribute is a random variable which follows the binomial probability law. It can be shown that the average (expected) value of p for an indefinitely large number of random samples of size n is equal to π, the probability that a given observation will possess the attribute, and the variance of p is $\frac{\pi(1-\pi)}{n}$.

For finite populations sampled with replacement, π would represent the proportion in the population possessing the attribute. Replacement restrictions can be eliminated without serious error if the population is absolutely large and the sample is relatively small.

Because the evaluation of the discrete probabilities of the binomial distribution is extremely tedious for large values of n, the distribution of p is frequently approximated. The normal distribution is appropriate when it can be assumed that $n\pi$ is greater than 5, (*3*). Or, in cases when $n\pi \leqq 5$ can be assumed and n is large, the Poisson distribution is a satisfactory approximation. The normal approximation has theoretical validity, since it is the limiting form of the binomial distribution as n becomes infinitely large. The limit is approached most rapidly when $\pi = 0.50$, and least rapidly when π is near zero or one. The use of the normal approximation for problems of the type considered in this paper generally results in oversampling. Much of what follows is based on the assumption that p is normally distributed with mean equal to π and variance equal to $\frac{\pi(1-\pi)}{n}$, that is, $E(p) = \pi$ and $\sigma_p^2 = \frac{\pi(1-\pi)}{n}$

Certain probability statements may be made about p for a given value of π and a given sample size n. For example, we may say that the probability that p differs from π by not more than $t_\alpha \sigma_p$ is equal to α. Symbolically, this is written as

$$P[-t_\alpha \sigma_p \leqq p - \pi \leqq t_\alpha \sigma_p] = \alpha \quad (1)$$

in which t_α is found in a table of central areas under the normal curve for a given value of α. The use of $S_p = \sqrt{\frac{p(1-p)}{n}}$, as in tables 1 and 2, for σ_p gives fairly reasonable approximations to probability statements for testing the reliability of a proportion in a sample of a given size, provided n is large. A few of the commonly used values of α and the corresponding values of t_α are:

α	t_α
.68	1.000
.90	1.645
.95	1.960
.99	2.576

Suppose we are interested in knowing how large n should be, *for a given value of* π, such that $P[-\theta \leqq p-\pi \leqq \theta] = \alpha$ where θ is a preassigned admissible error and α is the desired probability level. From equation (1) it is evident that the value of n must be such that

$$\theta = t_\alpha \sigma_p = t_\alpha \sqrt{\frac{\pi(1-\pi)}{n}}$$

The solution for n is

$$n = \frac{t_\alpha^2 \, \pi(1-\pi)}{\theta^2} \quad (2)$$

Note that n increases directly with the desired probability level (since t_α increases with α) and reaches a maximum when $\pi = 0.50$ for any given values of θ and α. The above expression for n assumes a knowledge of the value of π, but in any practical problem, π is unknown; hence a satisfactory basis for estimating the required sample size is not provided. A more valid approach follows.

Interval Estimation of π and Its Use in Formulating a Procedure for Sampling (*4*).—Our basic problem is to devise a procedure for sampling which will assure a sample size sufficient to satisfy the requirement $P[-\theta \leqq p-\pi \leqq \theta] \geqq \alpha$ for desired values of θ and α. This problem is solved by means of the concept of an interval estimate of π. The solution for the desired sample size is given by

$$n = \frac{t_\alpha^2 (\theta + p')(1 - \theta - p')}{\theta^2} \quad (3)$$[7]

with the details of the development in the Appendix (page 133).

Equation (3) permits us to compute the required value of n for possible alternative values of p. The (A) section of figure 5 shows the solutions of equation (3) in graphic form for $\theta = 0.05$ and selected values of α. The (B) section of figure 5 shows graphic results for $\alpha = .95$ and selected values of θ. These charts give the paired values of n and p necessary to satisfy conditions postulated by assigned values of θ and α.

Several procedures, varying as to details, may be followed in the use of figure 5 to draw samples which have the required paired values of n and p. In one method, the observer takes an initial sample of n_o observations, where n_o is the value of n when $p' = 0$, i.e., when $p = 0$ or 1. For example, if $\theta = 0.05$ and $\alpha = .95$, then n_o, from figure 5, is 73.[8] If this initial sample actually results in $p = 0$ or 1, no larger sample is required. If $p \neq 0$ or 1, the observer notes from figure 5 the sample size corresponding to the preliminary sample proportion and takes additional observations to obtain the total sample size indicated. This does not assure that the total sample size and the value of p obtained thereby will exactly satisfy the conditions set by θ and α, unless the final value of p is the same as

[7] $p' = p$ when $p \leqq \frac{1}{2}$, and $p' = 1 - p$ when $p \geqq \frac{1}{2}$.

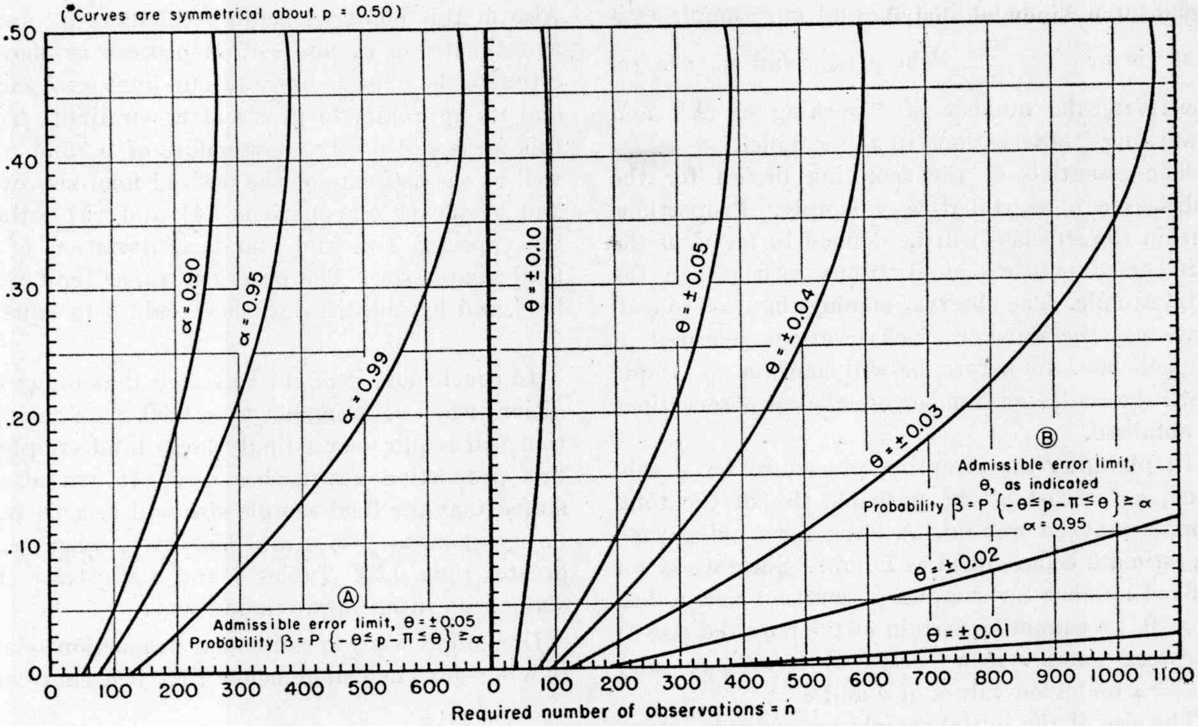

FIGURE 5.—Number of observations required to obtain ratio-delay proportions with given limits of error and stated probability.

the preliminary value obtained from the n_o observations. However, with n_o fairly large, as it usually will be, the final sample size which results will ordinarily be very near to the actual requirement, being over or under by a small number.

A second method may be followed which begins, as above, with the observer taking the required n_o observations and stopping at this point, if $p = 0$ or 1. If $p = 0$ or 1, he continues sampling and computing p until he obtains for the first time a sample proportion for which the required n, as given by figure 5, is less than the n actually supplied. In reaching this last observation a "one-by-one" sequence need not be followed throughout, but only in the neighborhood of the indicated final n. For earlier proportions of the sampling sequence, the computation of the sample proportion and a check of the required n need be made only at intervals. For example, if the initial sample indicated a need for 300 observations, a second check might be made after 50 or 100 additional observations—or a check might be made at the end of each day's study. The size of these increments of observations could be progressively decreased as the final n was approached.

ESTIMATING SUBCLASS PROPORTIONS.—The theoretical discussion thus far has been limited to the simple case of a proportion of one attribute relative to all the elements in a given population. This was the situation in our earlier example of estimating the delay proportion, π_d, at a dumping station, with respect to total time. But in some instances we are interested in a proportion relative to some subclass within the total population. This was true for our illustration of activity at the receiving station of a pear-packing plant where we were interested in proportions relative to the "working" subclass, or to the secondary subclass "unloading grower's trucks."

In a generalized form this situation is one in which a worker (or set of similar workers) has mutually exclusive jobs A, B, . . . , N which he does while actively working. We are interested in estimating π_{W_A}, the proportion of the "working" time

8 Values of n_o for certain other levels of θ and α may be obtained from figure 5, or n_o may be computed from the equation,

$$n_o = \frac{t_\alpha^2 (1 - \theta)}{\theta}$$

spent in working on job A, and our sample estimate is $p_{W_A} = \frac{n_{W_A}}{n_W}$, where n_{W_A} and n_w are respectively the number of "working on A" and "working" observations in the sample.

The essentials of the sampling design for the subclass can be stated very simply. Proportions within the subclass will be defined in terms of the number of subclass observations rather than the total sample. The observer employs figure 5 to estimate n_W, the number of observations required in the subclass. As before, he will continue to sample until the indicated number of subclass observations is obtained.

In planning the sampling procedure for a subclass, a forecast of the probable size of the total sample may be wanted. A basis for making such an estimate is developed as follows. Since n_W is not a fixed number for samples of constant size, unless $\pi_d = 0$, we cannot be certain of the required size of our total sample if we desire $P[-\theta \leq p_{W_A} - \pi_{W_A} \leq \theta] \geq \alpha$ for given values of θ and α.

The size of the initial sample necessary to secure n_{w_o} observations in the "working" category could be estimated from the sample when it grew to perhaps 100 observations in total size. At this point we could estimate π_w and its complement $\pi_d = 1 - \pi_w$, from the sample values of p_w and p_d. Without presenting the basis for the following, we could say that the expected size of the initial sample, n_o, required to reach the necessary value of n_{w_o} would be given approximately by[9]

$$n'_o = \frac{n_{W_o}}{p_w} \qquad (4)$$

with an approximate estimated standard deviation, s_{n_o}, of

$$s_{n_o} = \sqrt{\frac{n_{w_o} p_d}{p_w}} \qquad (5)$$

Assuming a normal distribution for the size of the initial sample, we could say that there was only about a 1-percent chance that the size of the initial sample would exceed

$$n''_o = \frac{n_{w_o}}{p_w} + 2.236 \sqrt{\frac{n_{w_o} p_d}{p_w}} \qquad (6)$$

When n_{w_o} was reached, we would proceed to determine the final size of n_w, as in the simple case.

[9] The negative binomial probability law is the basis for this development. Equations (4) and (5) are only sample estimates of $E(n_o)$ and σ_{n_o} formed by substituting p's for π's in the population expressions.

Also at this point we could recalculate p_w and p_d as estimates of π_w and π_d and proceed as above to estimate the expected size of our final total sample and its approximate practical upper limit. To do this we would use the new values of p_w and p_d, as well as the estimate of the desired final size of n_w, and by means of equations (4) and (5) estimate the expected size and standard deviation of the final sample size. The practical upper limit would be found by substituting these values in equation (6).

In conclusion, it might be stated that under conditions when π_w is smaller than 0.50, the final situation will require exceedingly large total samples as this proportion approaches zero. It can also be stated that the final sample size will become fairly large when π_{W_A} is near 0.50 even when π_w is greater than 0.50. Tables 3 and 4 illustrate these statements in an approximate way.

One might want to estimate a proportion relative to a category one stage below that just considered.

TABLE 3. — *Estimated average size and practical upper limit of the initial sample in a subclass proportion problem* ($\theta = \pm 0.05$, $\alpha = .95$)

p_{w_s}*	n_{w_o} required	Estimated average size	Estimated size with about 1% probability of being exceeded
.90	73	81	88
.50	73	146	174
.10	73	730	919

* p_{w_s} is proportion of "working" observations calculated when initial sample has become fairly large, prior to reaching $n_w = n_{w_o}$. Values given in table are only assumed for illustrative purposes.

TABLE 4. — *Estimated average size and practical upper limit of the total sample in a subclass proportion problem* ($\theta = \pm 0.05$, $\alpha = .95$)

p_{W_A}*	p_w**	Estimated final n_w required	Estimated average size	Estimated size with about 1% probability of being exceeded
.10 or .90	.822	196	238	255
.20 or .80	.822	289	352	373
.45 or .55	.822	385	469	495

* p_{W_A} is the proportion, *in the initial sample*, that the "working on A" observations are of the "working" observations.

** p_W is the proportion, *in the intitial sample*, that the "working" observations are of the total.

(Values given for both these proportions are assumed only for illustrative purposes.)

The procedures already given could be generalized to cover such cases. A total sample of exorbitant size would be required unless the subclass could be isolated and reduced to a simpler case. If this is not feasible, procedures other than the "ratio-delay" method could be used—that is, time or production studies.

Some Practical Considerations

In the application of the ratio-delay method to plant studies, there are numerous practical considerations. Some of these are noted below.

The effectiveness of the sampling design will depend on how fully it is possible to anticipate all categories in which estimates are wanted, including estimates relating to subclasses. The critical effect of these "anticipations" on the sampling design emphasizes the importance of the initial step in the ratio-delay study—the analysis of activity in each job category.

A sequential sampling procedure, or some adaptation of it, may be advantageous in conducting the observation tours of the plant, suggested earlier as a suitable field procedure. In regular tours of a plant, different delay proportions probably will be observed at each station and the total number of tours required will be determined by the station having the proportion requiring the largest n; also by the aggregate number of observations per tour that are obtained in a particular job category. It is probable that the necessary n will be attained at some stations sooner than at others. If such a station—or a group of stations—can be deleted from the observation route with appreciable saving in the observation time, the field time will be reduced, and it may be possible to obtain more complete data by focusing more quickly on operations that require greater accuracy or more detailed data.

In ascertaining delay proportions, the ratio-delay method usually will be less costly to apply than either the production-study or time-study method. In the plant studies here cited, for example, the field time required in the ratio-delay study is estimated to have required 80 percent less time than would have been necessary to obtain a one-day production study of each job. This estimate is greater than has been reported in other studies; estimated savings of 33 to 70 percent have been noted in other reports (*5*), (*6*), (*8*).

The ratio-delay sample may be more representative than a time study or a production study, for it may easily be composed of an aggregation of observations taken over a period of days or weeks (assuming no essential changes in the plant organization or working conditions during the period of observation) and thus may reflect typical conditions more accurately than would isolated time studies or a production study confined to one day.

If made on a department or plant-wide basis, the ratio-delay study can provide, in a sense, a simultaneous measure of delay at all points and so is an excellent device for indicating how effectively plant operations are integrated, and at what points improvements in work methods to eliminate delays would be most beneficial. These relationships would not be so clearly revealed by a succession of isolated production or time studies.

The ratio-delay data may be less biased than the production- or time-study data from the standpoint of the worker's reaction to observation, since the worker is under observation in the ratio-delay study for very short periods. Even so, in the particular study referred to in this paper, some worker reaction was noted in a few instances. The reaction usually was in the nature of a make-work tendency. An experienced observer, however, can offset abnormal worker reaction: For example, he can obtain a "flash" observation on entering the work place; he can make his observation after having passed the work place; or he can observe from across the plant.

The ratio-delay method shares a common handicap with the production- and time-study techniques—that is, the bias introduced by the rate at which a particular individual works. It is conceivable, and not unlikely, that delay time is observed for some individuals whose output is governed by a production line only because they work at an abnormally rapid rate and thus work themselves out of a job. Conversely, the bias for a slow worker would be in the other direction. Owing to the nature of the ratio-delay study, any such bias appears difficult to eliminate. But if observations on several workers are aggregated to obtain the ratio-delay proportion, the effect of rate-of-working by an individual would tend to average out.

Appendix

Sample Size Based on the Interval Estimate of π

An interval estimate of π can be made whenever we can find two functions of our sample observations which are independent of the specific value of π, and for which there is a pre-assigned probability α, that the interval formed by these two functions will include the unknown but fixed value of π. The length and position of such intervals will vary from sample to sample; but for an indefinitely large number of samples, α proportion of them will cover π.

Let $L_{1\alpha}$ and $L_{2\alpha}$ be the desired functions; then $(L_{1\alpha}, L_{2\alpha})$ will be the confidence interval for π. We shall now determine the general formulas for $L_{2\alpha}$ and $L_{1\alpha}$, under the normality assumption. Starting with the probability statement,

$$P\left[-t_\alpha \sqrt{\frac{\pi(1-\pi)}{n}} \leqq p-\pi \leqq t_\alpha \sqrt{\frac{\pi(1-\pi)}{n}}\right] = \alpha,$$

we can transform it to the equivalent statement,

$$P\left[n(p-\pi)^2 - t_\alpha^2\,\pi(1-\pi) \leqq 0\right] = \alpha. \quad (2)$$

Setting the expression inside the brackets in equation (2) equal to zero, we may solve for π in terms of n, p, and t_α to obtain,

$$\pi = \frac{(2np + t_\alpha^2) \pm t_\alpha\sqrt{t_\alpha^2 + 4np(1-p)}}{2(n + t_\alpha^2)}$$

In the above equation, let the value given with the positive and negative roots of the radical be respectively $\pi_{2\alpha}$ and $\pi_{1\alpha}$. For all values of π between $\pi_{1\alpha}$ and $\pi_{2\alpha}$, $n(p-\pi)^2 - t_\alpha\,\pi(1-\pi)$ is less than zero. Therefore, our original probability statement is equivalent to

$$P\left[\pi_{1\alpha} \leqq \pi \leqq \pi_{2\alpha}\right] = \alpha$$

and the interval $(\pi_{1\alpha}, \pi_{2\alpha})$ is our confidence interval for π with level of probability α. This proves that the functions $L_{2\alpha}$ and $L_{1\alpha}$ are given by $L_{2\alpha} = \pi_{2\alpha}$ and $L_{1\alpha} = \pi_{1\alpha}$.

In connection with our basic problem, if we set $L_{2\alpha} - p = \theta$ or $p - L_{1\alpha} = \theta$ depending on whether $p < \frac{1}{2}$ or $p > \frac{1}{2}$, and solve the resulting expression for n, we will secure the sample size which will satisfy the requirement of

$$P\left[-\theta \leqq p-\pi \leqq \theta\right] \geqq \alpha.$$

It can be shown that p will always be contained in the interval $(L_{1\alpha}, L_{2\alpha})$, or in cases when $p = 0$ or 1 it will be one of the end points. For $p<\frac{1}{2}$ $L_{2\alpha}$ is farther from p than $L_{1\alpha}$ for a given value of n and α, and vice versa for $p>\frac{1}{2}$. Because of symmetry, $L_{2\alpha}-p'$ is equal to $(1-p')-L_{1\alpha}$ for $p'\leqq\frac{1}{2}$ where $L_{2\alpha}$ is computed for $p = p'$ and $L_{1\alpha}$ is computed for $p = 1 - p'$, for fixed values of n and α. The expression for $L_{2\alpha}-p'$ is given by

$$\frac{t_\alpha^2(1-2p') + t_\alpha\sqrt{t_\alpha^2 + 4np'(1-p')}}{2(n + t_\alpha^2)}$$

The solution for n is,

$$n = \frac{t_\alpha^2(\theta + p')(1 - \theta - p')}{\theta^2}$$

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