Book Reviews

Statistical Inference in Dynamic Economic Models. By COWLES COMMISSION RESEARCH STAFF MEMBERS AND GUESTS. Edited by TJALLING C. KOOPMANS, with introduction by JACOB MARSCHAK. John Wiley & Sons, Inc., New York. 438 pages. 1950. (Cowles Commission Monograph No. 10)

AS EARLY AS 1945, writers in this new field systematically omitted important statements of fundamentals, theorems, derivations, and proofs, with a passing reference to Monograph No. 10, as "soon to appear." Probably not since the 20-year gestation of Schumpeter's treatise on business cycles has a work been so conspicuous by its absence. As this field is comparatively unfamiliar to many economists and statisticians, it is desirable to review some of the background leading to the extensive research which has produced the contributions to the volume.

Economists have recognized and accepted the representation of all economic variables of an economy or major segment in terms of a mutually interdependent set defined by a system of simultaneous equations—following the concepts of Walras and Pareto. Most economists and statisticians would be surprised to learn that their procedures in separately fitting individual economic equations to observational data are frequently inconsistent with the postulate of mutual economic interdependence.

Trygve Haavelmo first called attention to the inconsistency of this procedure in, *The Statistical Implications of a System of Simultaneous Equations* (Econometrica, January 1943). He pointed out that the simultaneous character of a system of economic equations and the mutual and simultaneous determination of a set of interdependent economic variables imposed logical restrictions upon the estimating procedure used to calculate statistical constants for these equations. To Ragnar Frisch is credited the first suggestions leading to development of this new line of analysis.

Intensive study in the field has been sponsored by the Cowles Commission for Research in Economics and the University of Chicago, culminating in the present volume—a product of pooled research and exploration by an outstanding group of econometricians. Contributing members here represented include T. C. Koopmans, R. L. Anderson, T. W. Anderson, T. Haavelmo, H. Hotelling, L. Hurwicz, R. B. Leipnik, H. E. Mann, J. Marschak, H. Rubin, and A. Wald. Meyer Girshick was a major contributor to development of the basic theory and its application.

Monograph No. 10 is a basic reference work, source book, and treatise, on the theory relating to the new methodology. Its language is essentially statistical and mathematical. Much statistical theory, particularly that part dealing with multivariate analysis, is presumed to constitute part of the preliminary equipment of the reader. As the discussion and development proceeds in terms of vectors and matrices and matrix operations, the prospective reader will want to brush up on at least the elementary operations of matrix algebra. Numerical illustrations and examples are exceedingly rare.

There is need for a volume on the order of Ezekiel's, *Methods of Correlation Analysis,* to bridge the gap between the elaborate and abstract theory of Monograph 10 and the numerical applications. Until a work of that character is published the applied research worker, who lacks sufficient mathematical training to proceed directly from the basic theory, will be obliged to imitate procedures demonstrated in the few articles in which these methods have been applied. Outstanding among them is the article by M. A. Girshick and Trygve Haavelmo, *Statistical Analysis of the Demand for Food—Examples of Simultaneous Estimation of Structural Equations* (Econometrica, April 1947).

This reviewer would advise the student to read the 1943 article of Haavelmo, then the article by Koopmans, *Statistical Estimation of Simultaneous Economic Relations* (Journal of the American Statistical Association, December 1945), and then the above article by Girshick and Haavelmo by way of introduction to the Monograph.

The analysis by Girshick and Haavelmo utilizes the "limited-information maximum-likelihood" method of estimation to determine values of the

structural coefficients—also known as the "reduced form" method—involving sacrifice of certain *a priori* information. This method is touched on at points in the Monograph and is discussed in the chapter by T. W. Anderson. The reduced-form method has been used in most applied analyses where a single equation is to be determined or a few out of a larger set of equations. It yields consistent estimates of parameters, which approach population values in the probability sense as sample size increases. But these estimates are less efficient, with larger sampling variance of the estimated constants, than those obtained by the more detailed "information-preserving maximum-likelihood" method. Principal emphasis of Monograph No. 10 is directed toward development of the latter method—particularly Section II, Measuring the Equation Systems of Dynamic Economics, by Koopmans, Rubin, and Leipnik. The working out of these theories is not complete, and unsolved problems and approximate solutions are indicated at certain points.

This method is often called structural analysis. An economic model of the total economy or a particular self-contained segment is set up as a system of simultaneous equations of prescribed form. Decisions as to form of equations and choice of variables appearing in each, utilize part of the *a priori* information of the model. Variables are divided into an "endogenous" group whose current values constitute the jointly dependent variables and an "exogenous" group whose current values are not influenced by any other variable in the system. The exogenous group together with lagged values of all variables constitute the "predetermined" variables which resolve the simultaneous determination of the jointly dependent variables. The statistical stochastic device which is adopted is the so-called shock model in which random disturbances are assumed to affect entire equations of the system, but variables are assumed to be measured free of error.

A major contribution of the new theory is the concept and technique of "identification" of particular equations and of the entire model—a logical problem that precedes estimation. A collection of observed data simply specifies a probability distribution and the statistical method is required to estimate the most probable consistent values of the structural coefficients for those equations which are identifiable. If certain equations are not identified, a number of alternative structures would yield the identical set of observed values of the variables and hence would be indistinguishable on the basis of the observed data. This situation is met frequently in observed scatters of price-quantity data which could be generated by any pair of a double infinity of alternative pairs of demand and supply functions. In such a situation additional information must be injected into the system in order to stabilize or identify the supply function, or the demand function, or both.

The estimation problem is treated systematically by applying the criterion of maximum likelihood, but this application becomes involved because of the presence of *a priori* information in the form of various linear and bilinear side conditions or restrictions upon the structural coefficients and the distribution function of the disturbances.

Dynamic character is achieved by use of lagged variables and by possibility of arbitrarily altering certain structural coefficients to fit changed present or future situations.

This reviewer believes that it is only a question of time before these concepts and methods for statistical analysis of economic data will become generally accepted and utilized. Therefore scientific economists would seem to have an obligation to study this newest analytical tool.

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