Testing Market Power with Profit Functions: a Dual Approach with Normalized Quadratic Functions

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Selected Paper prepared for presentation at the Southern Agricultural Economics Association (SAEA) Annual Meeting, Orlando, Florida, 3-5 February 2013

Abstract

The dual relationship between parameters of normalized quadratic cost and profit functions is determined when firms can implement market power on the output market. An approach is developed to test the market power by comparing the profits of a firm with and without market power. Simulations demonstrate the efficiency of this approach.

Key words: Normalized quadratic cost function, Normalized quadratic profit functions, Duality, Market power

Introduction

The price-cost margin (Lerner Index) is usually used to measure the degree of market power that a firm can implement. Because marginal costs are not directly observable, empirical tests of market power normally use the price elasticity of demand to measure market power: a low elasticity implies substantial market power and a high elasticity implies little or no market power. However, these tests do not incorporate the profit information that a firm can obtain by implementing the market power. The profits of a firm are determined by both revenue and costs, which not only depend on the prices and marginal costs, but also rely on the output quantities. In addition, most previous research focus on a single product firm while firms that produce multi-products are popular in many industries. One of the few papers that estimates the market power in a multi-product setting is by Schroeter and Azzam (1990), where a two-output model is presented. Using the dual relationship between profit and cost functions, this article presents an approach to test the market power by comparing the profits under the condition with and without market power. This test incorporates both the input side information (marginal cost) and the output side information (price and output quantity). In addition, this approach naturally incorporates the multi-product characteristics of most modern firms.

Method

Unrestricted Profit Function with Market Power

A cost function with flexible functional form, normalized quadratic cost function with *n* inputs and *m* outputs is defined as

$$C(W,Y) = b_0 + \underset{1*n}{B} * \underset{n*1}{W} + \underset{1*m}{A} * \underset{m*1}{Y} + 0.5 * \underset{1*n}{W'} * \underset{n*n}{BB} * \underset{n*1}{W} + 0.5 * \underset{1*m}{Y'} * \underset{m*m}{CC} * \underset{m*1}{Y} + \underset{1*n}{W'} * \underset{n*m}{AA} * \underset{m*1}{Y} \quad (1),$$

where W is a vector of input prices, Y is a vector of output quantities, and others are parameters in the cost function.

Under the assumption that a firm has market power on the output sides and assuming that the price of a product can only be affected by its own quantity, the price quantity relationship of a product can be defined as $p_i = f(y_i)$. Firms will maximize the profit and the unrestricted profit function can be obtained as a result of following maximization problem:

$$\Pi = \ max\, P(Y) * Y - C(W,Y) \quad (2),$$

where P is a vector of endogenous output prices, $P = [p_1 \quad \cdots \quad p_m]$.

The first order conditions of equation (2) is

$$\frac{\partial \Pi}{\partial Y} = P' + DD * Y - \frac{\partial C(W,Y)}{\partial Y} = 0 \quad (3),$$

$$\text{where} = \begin{bmatrix} \frac{\partial p_1}{\partial y_1} & 0 & \ldots\ldots & 0 \\ 0 & \frac{\partial p_2}{\partial y_2} & \ldots\ldots & 0 \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & \ldots\ldots & \frac{\partial p_m}{\partial y_m} \end{bmatrix}.$$

This is a general condition which states that with market power, a firm's the marginal revenue of producing a product equals its marginal cost. The term $DD * Y$ is the monopoly overcharge, which is the difference between monopoly price and competitive price. With a normalized quadratic cost function, the first order conditions are:

$$P' = A' + CC * Y + AA' * W - DD * Y \text{ (4)}$$

It is not possible to solve the optimal outputs Y^* explicitly because DD is a function of Y. However, without losing generality, we can apply the first-order Taylor series expansion of $p_i = f(y_i)$ around a fixed quantity (i.e. $\overline{y}_0$), which gives:

$$p_i = f(\overline{y}_i) + f'(\overline{y}_i)(y_i - \overline{y}_i) \text{ (5).}$$

As a result, *DD* can by simplified to

$$DD = \begin{bmatrix} f'(\overline{y}_1) & 0 & \ldots\ldots & 0 \\ 0 & f'(\overline{y}_2) & \ldots\ldots & 0 \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & \ldots\ldots & f'(\overline{y}_m) \end{bmatrix} \text{(6).}$$

Plug (6) into the equation (4), and the optimal output qualities can be solved as:

$$Y^* = (CC - DD)^{-1} * (P' - A' - AA' * W) \text{ (7).}$$

The cost at the optimal output levels can be determined by plugging (7) into the equation (1), and the unrestricted profit function can be determined by plugging (7) into the equation (2).

The cost at the optimal output levels is:

$$C(W,Y^{*}) = \beta_0 + \beta_1 * W + \beta_2 * P' + 0.5 * W' * \beta_3 * W + 0.5 * P * \beta_4 * P' + P * \beta_5 * W \quad (8)$$

Where

$$\beta_0 = [b_0 - A * (CC - DD)^{-1} * A' + 0.5 * A * (CC - DD)^{-1} * CC * (CC - DD)^{-1} * A']$$

$$\beta_1 = [B - 2 * A * (CC - DD)^{-1} * AA' + A * (CC - DD)^{-1} * CC * (CC - DD)^{-1} * AA']$$

$$\beta_2 = [A * (CC - DD)^{-1} - A * (CC - DD)^{-1} * CC * (CC - DD)^{-1}]$$

$$\beta_3 = [BB - AA * (CC - DD)^{-1} * CC * (CC - DD)^{-1} * AA']$$

$$\beta_4 = (CC - DD)^{-1} * CC * (CC - DD)^{-1}$$

$$\beta_5 = [(CC - DD)^{-1} * AA' - (CC - DD)^{-1} * CC * (CC - DD)^{-1} * AA' * W]$$

The unrestricted profit function is

$$\Pi = \alpha_0 + \alpha_1 * W + \alpha_2 * P' + 0.5 * W' * \alpha_3 * W + P * \alpha_4 * P' + P * \alpha_5 * W \quad (9)$$

where

$$\alpha_0 = [-b_0 + A * (CC - DD)^{-1} * A' - 0.5 * A * (CC - DD)^{-1} * CC * (CC - DD)^{-1} * A']$$

$$\alpha_1 = [2 * A * (CC - DD)^{-1} * AA' - A * (CC - DD)^{-1} * CC * (CC - DD)^{-1} * AA' - B]$$

$$\alpha_2 = [A * (CC - DD)^{-1} * CC * (CC - DD)^{-1} - 2 * A * (CC - DD)^{-1}]$$

$$\alpha_3 = [AA * (CC - DD)^{-1} * CC * (CC - DD)^{-1} * AA' - BB]$$

$$\alpha_4 = [(CC - DD)^{-1} - 0.5 * (CC - DD)^{-1} * CC * (CC - DD)^{-1}]$$

$$\alpha_5 = [(CC - DD)^{-1} * CC * (CC - DD)^{-1} * AA' - 2 * (CC - DD)^{-1} * AA']$$

Equation (9) indicates that if the market power exists and the price-demand function can be approximated by first-order Taylor series expansion, the unrestricted profit function is also a normalized quadratic function. Equation (9) is a generalization of the results of Gao and Featherstone (2008) who demonstrate the explicit relationship between the parameters in a normalized quadratic cost and a profit function in the case of perfect competition. If there is no market power, the parameters in the unrestricted profit function change to:

$$\alpha_0^* = -b_0 + 0.5 * A * CC^{-1} * A', \alpha_1^* = A * CC^{-1} * AA' - B, \alpha_2^* = -A * CC^{-1},$$

$$\alpha_3^* = [AA * CC^{-1} * AA' - BB], \alpha_4^* = 0.5 * CC^{-1}, \quad \alpha_5^* = CC^{-1} * AA'$$

Test Market Power with Cost and Profit Functions

The cost function defined in equation (1) is the same no matter a firm has market power or not in the output market. If there is no market power, the unrestricted profit function can be recovered by equation (9) where the parameters are defined by α_i^*. The profit (π^p) of a firm without market power can be estimated with the recovered unrestricted profit function.

With the same normalized quadratic cost function, the unrestricted profit function of a firm that may have market power is also a normalized quadratic function. This indicates that a normalized quadratic profit function can be directly estimated with input and output price data without any assumption of the market power that a firm may have. The profit (π^m) of the firm can be estimated using the unrestricted profit function. If the firm does not have market power, the differences in the profits estimated under the conditions with and without market power should be close to zero such that $|\pi^m - \pi^p| < \delta$, where

δ is a small number close to zero. The significance of the market power can be determined by testing whether the difference between $\pi^m and\ \pi^p$ is significantly different from zero or not.

A Simulation Study: A Case of Six Outputs and Three Inputs

Simulations are conducted in three cases: (1) no market power; (2) output price has a linear relationship with output quantity ($p_i = \mathrm{I}_i + D_i * y_i$); and (3) output price has a quadratic relationship with output quantity ($p_i = \mathrm{I}_i + D_i * y_i + E_i y_i^2$). In all three cases, two hundred prices are simulated and 1,000 simulations are conducted. In case one, input and output prices are generated as exogenous variables; in case two and , input prices are generated as exogenous variable, and output quantity and prices are generated using the equation (7) and $p_i = f(y_i)$. The costs and profits are calculated using equation (1) and $P(Y) * Y - C(W, Y)$, respectively. The curvature conditions of the cost function are imposed using Cholesky decomposition methods. The parameters in the cost function and the inverse demand function are reported in table 1.

With the parameters in the in the cost function, the parameters in the unrestricted profit function under perfect competition are calculated and reported in table 2. With parameters in table 2, the profits under perfect competition (π^p) are calculated with equation (9) where the parameters are defined by α_i^*. Then the simulation is conducted with the following steps: (1) The profit (π^m) under perfect competition, linear price-demand function, and quadratic price-demand function are generated by $P(Y) * Y - C(W, Y)$ and random errors following normal distribution are added to π^m such that $\pi^{m*} = \pi^m + \varepsilon$, to simulate the observed profits; (2) π^{m*} are used to estimate the unrestricted profit function with the normalized quadratic form; (3) The null hypothesis that the predicted π^{m*} are not statistically significantly different from π^{p} is tested. The steps 1 to 3 are repeated 1,000 times and the statistics of the p-values for the hypothesis tests are reported in table 3. The results in table 3 indicate that when market power exists (case 2 and case 3), the null hypothesis that there is no market power is rejected for all the 1,000 simulations. However, the test fails to reject the null hypothesis in some cases when there is no market power. Further investigation shows that in about 12 cases (1.2%), the null hypothesis of no market power are failed to be rejected at the 10% significance level.

Conclusion

This article presents an approach to test the market power using the dual relationship between the normalized cost and profit function. We first extend the results of Gao and Featherstone (2008) by demonstrating that the unrestricted profit function corresponding to the normalized quadratic cost function is also a quadratic function. We then present the a simple approach to test the market power by comparing the profit under the condition of market power and perfect competition. Our simulation results show that the approach successfully rejects the null hypothesis that there is no market power when market power exist. In addition, although the approach rejects the null hypothesis of no market power in the case of perfect competition, the number of rejections only accounts for 1.2% of the total simulation cases.

References:

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Table 1. Parameters of the Cost and Price-demand Function used in Simulation

Parameter	Value	Parameter	Value	Parameter	Value	Parameter	Value
b0	35.00	CC		AA			
B		C11	59.25	A11	-43.59	I1	2978.90
B1	0.20	C12	23.74	A12	30.00	I2	4379.09
B2	0.30	C13	27.72	A13	551.61	I3	7066.66
B3	0.40	C14	49.96	A14	100.00	I4	12631.43
A		C15	35.69	A15	30.00	I5	11287.63
A1	30.00	C16	12.09	A16	104.87	I6	6466.87
A2	298.71	C22	27.87	A21	-66.54	D1	-3.48
A3	60.00	C23	31.34	A22	188.18	D2	-3.96
A4	3.00	C24	65.48	A23	103.26	D3	-6.93
A5	70.00	C25	51.91	A24	20.00	D4	-0.60
A6	10.00	C26	14.88	A25	299.37	D5	-5.09
BB		C33	84.24	A26	31.74	D6	-4.48
B11	-13.42	C34	102.82	A31	-1.72	E1	-3.48
B12	-21.20	C35	83.30	A32	120.48	E2	-3.96
B13	-6.50	C36	61.38	A33	-21.53	E3	-6.93
B22	-53.00	C44	231.03	A34	134.56	E4	-0.60
B23	-17.50	C45	185.52	A35	30.00	E5	-5.09
B33	-6.25	C46	97.65	A36	49.98	E6	-4.48
		C55	171.53				
		C56	81.84				
		C66	86.49				

Table 2. Parameters of the Unrestricted Profit Function Corresponding to the Cost Function under Perfect Competition

Parameter	Value	Parameter	Value	Parameter	Value
α_0^*	11833.489	α_4^*		α_5^*	
α_1^*		PC11	0.027	PA11	-2.283
PB1	-5064.677	PC12	-0.034	PA12	-20.017
PB2	14959.625	PC13	0.000	PA13	20.629
PB3	8103.616	PC14	0.002	PA14	4.420
α_2^*		PC15	0.005	PA15	-2.003
PA1	8.914	PC16	-0.005	PA16	-12.759
PA2	-82.686	PC22	0.292	PA21	-6.723
PA3	14.828	PC23	-0.058	PA22	49.326
PA4	26.572	PC24	-0.083	PA23	-6.564
PA5	0.031	PC25	-0.011	PA24	-24.374
PA6	-27.690	PC26	0.100	PA25	10.646
α_3^*		PC33	0.045	PA26	14.925
PB11	9935.567	PC34	0.009	PA31	-3.960
PB12	-2379.759	PC35	0.002	PA32	30.005
PB13	-2948.385	PC36	-0.035	PA33	-8.412
PB22	12277.938	PC44	0.064	PA34	-4.232
PB23	3898.707	PC45	-0.033	PA35	-4.784
PB33	3658.165	PC46	-0.034	PA36	11.244
		PC55	0.046		
		PC56	-0.006		
		PC66	0.064		

Table 3. Statistic of P-Values of the Hypothesis Test there There is No Market Power

Case	N	Mean	Std	Min	Max
No Market Power	1000	0.62	0.24	0.02	1.00
Market Power (Linear Price-Demand Function)	1000	0.00	0.00	0.00	0.00
Market Power (Quadratic Price-Demand Function)	1000	0.00	0.00	0.00	0.00