



AgEcon SEARCH
RESEARCH IN AGRICULTURAL & APPLIED ECONOMICS

The World's Largest Open Access Agricultural & Applied Economics Digital Library

This document is discoverable and free to researchers across the globe due to the work of AgEcon Search.

Help ensure our sustainability.

Give to AgEcon Search

AgEcon Search
<http://ageconsearch.umn.edu>
aesearch@umn.edu

*Papers downloaded from **AgEcon Search** may be used for non-commercial purposes and personal study only. No other use, including posting to another Internet site, is permitted without permission from the copyright owner (not AgEcon Search), or as allowed under the provisions of Fair Use, U.S. Copyright Act, Title 17 U.S.C.*

Staff Paper Series

Stochastic Energy Demand and the Stabilization Value of Energy Storage

Yacov Tsur and Amos Zemel



DEPARTMENT OF AGRICULTURAL AND APPLIED ECONOMICS

UNIVERSITY OF MINNESOTA

COLLEGE OF AGRICULTURE

ST. PAUL, MINNESOTA 55108

Stochastic Energy Demand and the Stabilization Value of Energy Storage

Yacov Tsur and Amos Zemel

Staff Papers are published by the Department of Agricultural and Applied Economics without formal review.

The University of Minnesota is committed to the policy that all persons shall have equal access to its programs, facilities, and employment without regard to race, color, creed, religion, national origin, sex, age, marital status, disability, public assistance status, veteran status, or sexual orientation.

Stochastic Energy Demand and the Stabilization Value of Energy Storage*

Yacov Tsur¹ and Amos Zemel²

Abstract

The economic value of energy storage to meet peak electricity demand is analyzed with an emphasis on the role of demand uncertainty. The concept of the stabilization value, which measures that part of the benefit of the storage project which is due solely to the stochastic demand components, is defined. The magnitude of the stabilization value, relative to the overall value of energy storage, is evaluated in terms of a simple model that accounts for the relevant characteristics of the electric power utility's production mix. It is found that neglecting the demand uncertainty can seriously bias the benefit assessment of the storage project as well as the determination of the optimal storage capacity.

Keywords: Uncertainty, peak electricity demand, energy storage, stabilization value

*Forthcoming in *Natural Resource Modeling*

¹Department of Agricultural and Applied Economics, University of Minnesota, 1994 Buford Avenue, St. Paul, MN 55108 USA.

²Center for Energy and Environmental Physics, The Jacob Blaustein Institute for Desert Research, Ben Gurion University of the Negev, Sede Boqer Campus, 84990 ISRAEL.

Stochastic Energy Demand and the Stabilization Value of Energy Storage

Yacov Tsur and Amos Zemel

1. Introduction

Electric power demand varies within a day, from day to day and between seasons. Some of these variations follow deterministic trends while other components are random, due, for example, to fluctuations in weather conditions. In the absence of demand uncertainty, planning for a cost efficient energy supply system to meet demand requirements is straightforward. With demand uncertainty the problem is more involved.

In this paper we study how the uncertainty in electric power demand affects the design of a cost efficient energy supply system. In particular, we investigate the role of energy storage as a buffer against demand fluctuations. Energy can be stored in various mechanical forms, e.g., a water reservoir attached to a hydroelectric power station, compressed air or flywheel storage systems. To be concrete, we concentrate here on pumped energy storage, in which water is lifted (pumped) during off-peak periods and used to produce hydro-electricity when demand peaks. With the appropriate investment, this form of energy storage is often feasible. The analysis, however, applies to any form of energy storage.

Due to economies of scale, investment in power stations (conventional or nuclear) that provide the base energy supply (henceforth denoted base units) is indivisible to a large extent. Moreover, operating the base units is inflexible in that it is expensive to change their output rate. It is desirable, therefore, to keep the production rate of the base units constant disregarding whether demand is at a peak or a trough. The supply gap between the base units

production and peak demand is typically filled with smaller back-up units, which are cheaper to build but expensive to run, and by stored energy (if available).

In such a supply system, energy storage has a dual function: first, it increases the supply of electricity during peak demand, thus substituting for expensive back-up energy; second, it utilizes the surplus in base power during off-peak periods, thus mitigating the "peak load problem" resulting from the cyclical nature of energy demand (see Panzar, 1976). These concepts are well known (see, e.g., Jackson, 1973). What has received less attention is the observation that the benefits from energy storage are greatly enhanced when energy demand has a stochastic component. The purpose of this paper is to investigate the economic value of pumped energy storage which is due to the stochastic components of energy demand: we call this value the *stabilization* value of energy storage.

Why is this concept of interest? Suppose that a pumped energy project can be implemented at some cost and a decision-maker wishes to evaluate the project using benefit-cost approach. If the stabilization value is large relative to the overall value of the project, then assuming that the energy demand is deterministic and ignoring the random components provides a poor approximation of the benefits and can seriously bias assessments of the development project. We demonstrate below that the stabilization value of energy storage can be large.

Our description of the power production system and the associated decision problem is, evidently, oversimplified. The simplification is manifested in the schematic way in which the characteristics of the three components of the production mix are presented. The description of the demand fluctuations is also crude. Electric power demand varies within a day, between seasons and over the years. In general, the daily, seasonal and long-run components of the demand all

contain stochastic elements. In this work, however, we consider only stochastic (short-run) daily variations, and abstract from the issue of demand management—a main subject of peak load pricing models—by taking electricity demand to be determined exogenously and requiring that supply meets demand at all times. In spite of its crudeness, the model contains all the main ingredients required to appreciate the role of the stabilization value in policy decisions regarding energy storage. Extending the analysis to account for more realistic demand patterns would complicate the presentation at no significant payoff in terms of conceptual gains or new insights.

Stabilization and buffer concepts appear in almost any strand of economic literature that involves uncertainty, the underlying idea being that the presence of uncertainty enhances the economic value associated with stocks. Examples include commodity markets (Newbery and Stiglitz, 1981), saving/consumption decisions (Dreze and Modigliani, 1972), energy stockpiling (Devarajan and Weiner, 1989), and water supplies (Tsur, 1989, and Tsur and Graham-Tomasi, 1991). The present effort extends this concept to the case of energy storage under stochastic electricity demand.

A related body of literature appears under the heading of "peak load pricing." Peak load problems occur when the same physical capacity is used to produce a non-storable good during peak and off-peak demand periods (Panzar, 1976). Peak load pricing schemes are widely used to manage electricity demand (see the collection of works edited by Aigner, 1984). Stochastic demand has been studied by Carlton (1977). An extension that allows for storable goods has been proposed, in the energy context, by Jackson (1973). Indeed, the stabilization value concept developed here is a result of the presence of both a storable good (energy) and stochastic demand. The present analysis, however, takes the peak

load pricing scheme and the resulting demand pattern as given and focuses attention on the role of demand uncertainty in energy storage policies.

Energy related studies have often considered the capacity credit to be associated with renewable sources, such as wind or solar energy (Haslett and Diesendorf 1981, Martin and Diesendorf 1982, Carlin 1983, Nozari, Lalli and Kumin, 1986). In this context, the main issue is the role of uncertainty in the production side and the related loss-of-load probability under various scenarios involving energy storage and the connection to the general grid.

The next section lays out the decision problem. Section 3 presents the optimal capacity choice of pumped storage under deterministic and stochastic demand situations. In section 4, the stabilization value of energy storage is defined and its contribution to the total benefit of the project is found to be quite significant. Implications of this finding for energy storage policies are discussed in Section 5.

2. The decision problem

The power supply system consists of base units, back-up units and stored energy. Base units are large conventional (and nuclear) power stations; they involve high investment costs but the production cost is relatively low. Back-up units can come in a smaller scale, involve smaller investment costs but produce expensive energy. These units are operated only when demand exceeds the base capacity. Whenever available, stored energy can replace expensive back-up power production.

Let b denote the capacity of the base units, measured in megawatt (MW). Because it is expensive to change their production rate, base units produce at the constant rate b . The variable cost of producing a unit of base energy is

$\$w_B/\text{MWh}$.

During a day, power demand cycles between peak and low levels. We schematically describe this diurnal variation in terms of two periods: a peak demand period of duration T (hours) and integrated energy demand $Y_H > B = bT$, and a low demand period with a corresponding energy demand Y_L . The uncertainty in energy demand is incorporated by assuming that Y_H contains a random component. Energy demand during low periods, Y_L , can also be random, but is assumed to be sufficiently small so that the surplus base energy is large enough to recharge the storage unit.

As a supply system is required to meet demand at all times, there must be some idle capacity during periods of low demand. This is primarily composed of back-up capacity which is cheaper than base capacity and can be turned off and on at negligible costs. The unit production cost of back-up electricity is denoted by $\$w_K/\text{MWh}$; as base energy is cheaper than back-up energy, $w_B < w_K$.

The high cost of producing back-up energy often makes it profitable to substitute back-up energy with stored energy. Due to inherent inefficiencies in the storage and discharge processes, the production cost associated with stored energy, $\$w_P/\text{MWh}$, is somewhat larger than w_B , but is still lower than w_K . Of course, replacing some back-up capacity by storage capacity entails investment costs. Let $C(X)$ represent the imputed (per day) investment cost associated with a storage project of capacity X . The (short-run) decision problem consists of finding the optimal storage capacity X , given B and the production and investment costs. We study this decision problem, concentrating on the role of uncertainty in energy demand.

3. Optimal storage

In this section we compare the optimal choices of the storage capacity under two scenarios. In subsection 3.1 the case of a stable (non stochastic) peak demand, $Y_H = M > B$ is considered. In subsection 3.2 we analyze the case where Y_H is a random variable distributed according to a cumulative distribution function F and the corresponding density $f \equiv F'$, with $E\{Y_H\} = M$ and $\text{Var}\{Y_H\} = \sigma^2$. In both cases we assume that the production costs $w_B < w_p < w_K$ and the base energy capacity B are given. The imputed investment cost function for a storage project of capacity X is assumed to take the form $C(X) = cX + c_0$, where c_0 and c are non-negative constants, representing, respectively, the fixed investment cost and the additional cost per unit of storage capacity.

3.1. Stable peak demand

Suppose that Y_H is stable at the mean, i.e., $Y_H = M$, and $M > B$. During periods of low power demand, the base units can supply the entire demand. At peak demand, pumped and back-up energy are needed to meet demand requirements, and the corresponding cost consists of the sum of the costs associated with the three components of the production mix. Base energy always contributes $w_B B$ to the supply cost. The cost of pumped energy consists of the investment cost $c_0 + cX$ and the supply cost, which depends on whether $M-B \leq X$ or $M-B > X$. In the former case, $M-B$ units of pumped energy are supplied and contribute $w_p(M-B)$ to energy cost. In the latter case $M-B > X$ and the entire storage capacity X is supplied at a cost of $w_p X$. Finally, back-up units supply the residual demand $M-(B+X)$ only when $M-B > X$, in which case they incur the cost $w_K[M-(B+X)]$.

Using the indicator function $I(\cdot)$ that takes the value one when its argument is true and zero otherwise, the energy supply cost for positive storage capacity

X can be compactly expressed as

$$G_M(X) = w_B B + w_p(M-B)I(M-B \leq X) + w_p X I(M-B > X) + w_K[M-(B+X)]I(M-B > X) + cX + c_0 \quad (3.1)$$

When $X=0$, the investment cost term should be dropped, and G_M undergoes a discontinuity, the jump being equal to c_0 . After some algebraic manipulations, Eq. (3.1) reduces to

$$G_M(X) = \text{constant} + (w_K - w_p)[M-(B+X)]I(M-B > X) + cX \quad X > 0$$

describing a piecewise linear function with a slope discontinuity at $X=M-B$, (the "constant" is independent of X). The optimal storage capacity for stable peak demand is easily found:

$$X_M^* = \begin{cases} M - B & \text{if } (w_K - w_p - c)(M-B) \geq c_0 \\ 0 & \text{otherwise} \end{cases} \quad (3.2)$$

The cost saving due to the storage project, which we call the storage value for stable demand and denote by V_M , is given by $G_M(0) - G_M(X_M^*)$. To calculate V_M , use Eqs. (3.1)-(3.2): $G_M(M-B) = w_B B + w_p(M-B) + c(M-B) + c_0$ and $G_M(0) = w_B B + w_K(M-B)$. Hence:

$$V_M = \begin{cases} (w_K - w_p - c)(M-B) - c_0 & \text{if } (w_K - w_p - c)(M-B) \geq c_0 \\ 0 & \text{otherwise} \end{cases} \quad (3.3)$$

Note the all-or-nothing character of the solution (3.2) in this case. The optimal storage capacity obtains the constant level $M - B$ even when V_M is very small. In this region, even small errors in the value of M can cause X_M^* to change abruptly from 0 to $M - B$. This unstable behavior disappears when the uncertainty in the peak demand is taken into account. The significance of the factors in the criterion of Eq. (3.2) is easily recognized: First, one must have $c < w_K - w_p$ so that the gain from replacing a unit of back-up energy by pumped energy more than compensates for the unit investment cost c . Then, the potential replacement capacity $M - B$ must be large enough to regain the fixed investment

cost c_0 .

3.2. Uncertain peak demand

When Y_H is a random variable distributed according to F , the cost of energy supply depends on the realization of Y_H . The storage choice is determined so as to minimize expected cost. Following Eq. (3.1), the energy supply cost given that demand equals Y_H is

$$G_{Y_H}(X) = w_B B + w_p(Y_H - B)I(Y_H \leq X+B) + w_p X I(Y_H > X+B) + w_K[Y_H - (B+X)]I(Y_H > X+B) + cX + c_0.$$

Taking expectation with respect to Y_H yields

$$\begin{aligned} G(X) \equiv E\{G_{Y_H}(X)\} &= w_B B + w_p \int_B^{X+B} sf(s)ds - w_p B[F(X+B) - F(B)] + w_p X[1 - F(X+B)] + \\ &\quad + w_K \int_{X+B}^{\infty} sf(s)ds - w_K(B+X)[1 - F(X+B)] + cX + c_0, \end{aligned} \quad (3.4)$$

The first order condition for a minimum requires that optimal storage X^* satisfies

$$\partial G(X^*)/\partial X = w_p[1 - F(X^* + B)] - w_K[1 - F(X^* + B)] + c = 0,$$

from which we obtain

$$F(X^* + B) = 1 - \frac{c}{w_K - w_p}, \quad (3.5)$$

provided $F(B) < 1 - \frac{c}{w_K - w_p}$ or, put differently, $c < (w_K - w_p)[1 - F(B)]$; otherwise,

$X^* = 0$. It is easy to verify that $\partial^2 G(X^*)/\partial X^2 = (w_K - w_p)f(X^* + B)$, thus X^* minimizes $G(X)$ whenever $f(X^* + B) > 0$ (as $w_K - w_p > 0$). Yet, the condition $c < (w_K - w_p)[1 - F(B)]$ is not sufficient to ensure a non vanishing solution. A necessary condition that takes the c_0 jump of $G(X)$ at $X=0$ into account (in analogy to the condition of Eq. 3.2), is derived below.

The value of a storage project of capacity X , denoted by $V(X)$, equals the cost saving it generates, i.e., $V(X) = G(0) - G(X)$. Using (3.4)-(3.5) and some

algebraic manipulations, we obtain

$$G(X^*) = w_B B + w_K \int_B^{\infty} s f(s) ds - w_P [1-F(B)]B - (w_K - w_P) \int_B^{X^*+B} s f(s) ds - cB + c_0.$$

and

$$G(0) = w_B B + w_K \int_B^{\infty} s f(s) ds - w_K B [1-F(B)].$$

Therefore,

$$V(X^*) \equiv G(0) - G(X^*) = (w_K - w_P) \int_B^{X^*+B} s f(s) ds - (w_K - w_P) [1-F(B)]B + cB - c_0. \quad (3.6)$$

Using $(w_K - w_P) \int_B^{X^*+B} B f(s) ds = (w_K - w_P) [1-F(B)]B - cB$, $V(X^*)$ reduces to

$$V(X^*) = (w_K - w_P) \int_B^{X^*+B} (s-B) f(s) ds - c_0. \quad (3.7)$$

Since the decision not to undertake the project is always feasible, $V(X^*)$ must be positive for the project to be profitable. Thus, a necessary condition for $X^* > 0$ is

$$c_0 < (w_K - w_P) \int_B^{X^*+B} (s-B) f(s) ds. \quad (3.8)$$

When $\sigma \rightarrow 0$, $f(s)$ is very small except for $s \cong M$. Thus, $s-B$ can be approximated by $M-B$ and taken out of the integral which reduces to $F(X^*+B) - F(B)$. Since $M-B \gg \sigma$, $F(B)$ can be neglected, while $F(X^*+B)$ is given by Eq. (3.5). We find that (3.8) reduces to the condition $c_0 < (w_K - w_P - c)(M-B)$, derived for stable peak demand with $Y_H = M$.

As a concrete example, suppose that Y_H is distributed uniformly over $[\alpha, \beta]$,

with α and β two positive constants such that $\alpha < \beta$, $B < (\alpha+\beta)/2 = M$, and $(\beta-\alpha)^2/12 = \sigma^2$. Using (3.5) and $F(x) = (x-\alpha)/(\beta-\alpha)$, $\alpha \leq x \leq \beta$, we find

$$X^* = \left(1 - \frac{c}{w_K - w_P}\right)(\beta-\alpha) + \alpha - B = \sqrt{12}\sigma \left(\frac{1}{2} - \frac{c}{w_K - w_P}\right) + M - B \quad (3.9)$$

provided the right-hand side is positive; otherwise, $X^* = 0$. When $\sigma = 0$, this result reduces to $X_M^* = M - B$, derived above.

Using Eqs. (3.7) and (3.9), we also find

$$V(X^*) = (w_K - w_P)X^{*2}/\sqrt{48}\sigma - c_0 \quad \text{if } \alpha \leq B \quad (3.10)$$

$$\begin{aligned} V(X^*) &= (w_K - w_P)[X^{*2} - (\alpha-B)^2]/2(\beta-\alpha) - c_0 = (w_K - w_P - c)(X^* + \alpha - B)/2 - c_0 \\ &= (w_K - w_P - c)[M - B - \sqrt{3}\sigma c/(w_K - w_P)] - c_0 \quad \text{if } \alpha > B \end{aligned} \quad (3.11)$$

Again, for $\sigma = 0$, Eq. (3.11) reduces to $V_M = (w_K - w_P - c)[M - B] - c_0$, in agreement with Eq. (3.3).

4. The stabilization value of energy storage

When Y_H is assumed to be stable at the mean, the storage capacity is chosen at X_M^* . If Y_H is truly random, this choice is sub-optimal: the optimal choice is X^* , which yields the storage benefit $V(X^*)$. The cost associated with the sub-optimal decision is the difference in cost saving between a project of capacity X_M^* and the optimal project. This difference, which we call the *stabilization value* of energy storage and denote by SV , is given by $SV = V(X^*) - V(X_M^*)$. The stabilization value measures the economic benefit from energy storage which is due to the random component of peak power demand. In terms of the production costs, $SV = G(X_M^*) - G(X^*)$, which is non-negative (since X^* minimizes G).

The formalism of Section 3 provides all the necessary relations to derive the stabilization value for arbitrary demand distributions and system parameters. We investigate first the magnitude of SV in terms of a specific example, in which the distribution of Y_H is assumed to be uniform over its domain. Following

Section 3, closed form expressions as well as some numerical examples are given.

It is found that the contribution of the stabilization value to the total benefit of the project can be quite significant. Finally, the generalization of these results to arbitrary demand distributions is presented.

Suppose that Y_H is distributed uniformly over $[\alpha, \beta]$, with α and β two positive constants such that $\alpha < \beta$, $B < (\alpha + \beta)/2 = M$, and $(\beta - \alpha)^2/12 = \sigma^2$. From Eq. (3.9), $X^* = \sqrt{12}\sigma \left(\frac{1}{2} - \frac{c}{w_K - w_P} \right) + M - B$. When $\alpha \leq B$, Eq. (3.4) specializes to $V(X) = (w_K - w_P)X(2X^* - X)/\sqrt{48}\sigma - c_0$, which reduces to $V(X^*) = (w_K - w_P)X^{*2}/\sqrt{48}\sigma - c_0$, in agreement with Eq. (3.10). Applying this result to $X_M^* = M - B = X^* - \sqrt{12}\sigma \left(\frac{1}{2} - \frac{c}{w_K - w_P} \right)$, we find

$$SV \equiv V(X^*) - V(X_M^*) = (w_K - w_P)(X^* - X_M^*)^2/\sqrt{48}\sigma = \sqrt{3}\sigma(w_K - w_P) \left(\frac{1}{2} - \frac{c}{w_K - w_P} \right)^2 \quad (4.1)$$

Since $c_0 \geq 0$, the share of the stabilization value in the total value of energy storage is bounded by the relation

$$\frac{SV}{V(X^*)} \geq \frac{(X^* - X_M^*)^2}{X^{*2}}, \quad \text{equality holding when } c_0 = 0. \quad (4.2)$$

When $\alpha > B$, the derivation is quite similar, although some care must be exercised in the application of Eq. (3.4), since the lower limit of the integrals is α rather than B and $F(B) = 0$ in this case. For $X > \alpha - B$, one finds $V(X) = V(X^*) - (w_K - w_P)(X - X^*)^2/\sqrt{48}\sigma$, so Eq. (4.1) holds for $\alpha > B$ as well. The corresponding bound on $SV/V(X^*)$ is, however, stronger. Using the first form of Eq. (3.11), $V(X^*) = (w_K - w_P)[X^{*2} - (\alpha - B)^2]/2(\beta - \alpha) - c_0$, we find

$$\frac{SV}{V(X^*)} \geq \frac{(X^* - X_M^*)^2}{X^{*2} - (\alpha - B)^2} = \frac{(X^* - X_M^*)^2}{X^{*2} - (X_M^* - \sigma\sqrt{3})^2}, \quad \text{equality holding when } c_0 = 0. \quad (4.3)$$

It is seen that the relative importance of SV depends on σ and on the relative

unit gain $(w_K - w_P)/c$. For $\sigma = M - B$ and $(w_K - w_P)/c = 10$, we find from Eq. (4.2) that $SV/V(X^*) \geq 34\%$, whereas for $(w_K - w_P)/c = 1.5$, the same relation gives $SV/V(X^*) \geq 186\%$. Neglecting the random components of peak power demand entails a benefit loss of more than 34% in the first example and more than 186% in the second, due to the sub-optimal capacity choice. The latter result, (which means that $V(X_M^*) < 0$), manifests the basic difference between the two methods of assessing the project benefit: when demand uncertainty is taken into account, the low relative unit gain entails low optimal capacity, $X^* \cong 0.42(M - B)$. On the other hand, assuming a stable peak demand would lead the planner to the choice of $X_M^* = M - B$, which is insensitive to such details. In the particular example at hand, such a choice inflicts a negative expected value.

The discussion above is based on the assumption that both assessment schemes suggest that the project is profitable (although they differ in the proposed optimal capacity value). Indeed, under certain circumstances they may produce conflicting conclusions on whether the project is worth while at all. If, for example, the relative unit gain assumes the value $(w_K - w_P)/c = 1.25$, Eq. (3.9) gives a negative value for X^* , implying that the project should not be undertaken. Assuming that the demand is stable, one might erroneously decide that X_M^* is the optimal choice if c_0 is small enough.

Similar results can be obtained for arbitrary error distributions. Following the derivation of Eq. (3.7), one finds

$$SV = (w_K - w_P) \int_M^{X^* + B} (s - M)f(s)ds \quad \text{if } M < X^* + B \quad (4.4)$$

(with an obvious modification when $M > X^* + B$). Combining Eqs. (4.4) and (3.7), we obtain the following bound

$$\frac{SV}{V(X^*)} \geq \frac{\int_{X^*+B}^M (s-M)f(s)ds}{\int_B^{X^*+B} (s-B)f(s)ds}, \quad \text{equality holding when } c_0 = 0. \quad (4.5)$$

It is seen that when $M \approx B$, $X_M^* \approx 0$, the stabilization value constitutes nearly the entire benefit of the storage project. Indeed, for $B = M$ the storage project will always be rendered unprofitable under the assumption of a stable peak demand. Accounting for uncertainty, the solution of Eq. (3.5) can produce positive values of X^* if $(w_K - w_P)/c$ is large enough. Again we see that neglect of the demand uncertainty can lead to a wrong policy decision; in this case a storage project of positive expected value will not be undertaken.

When $X^* - X_M^*$ is not too large, $G(X)$ can be approximated by a second-order Taylor expansion, $G(X) \cong G(X^*) + (w_K - w_P)f(X^*+B)(X-X^*)^2/2$, (see the discussion following Eq. 3.5), in which case Eq. (4.4) simplifies to

$$SV \cong (w_K - w_P)f(X^*+B)(X_M^* - X^*)^2/2. \quad (4.6)$$

It is easy to check that Eq. (4.6) specializes to Eq. (4.1) when f is the uniform density.

Another result which appears peculiar to the uniform distribution holds, in fact, for every symmetric distribution: observe in Eq. (3.9) that $X^* = X_M^*$ whenever $(w_K - w_P)/c = 2$. However, Eq. (3.5) implies that X^*+B equals the median in this case. Therefore, the two results for the optimal storage capacity agree for every distribution for which the mean and the median coincide. For such distributions the stabilization value is expected to be small if the relative unit gain is roughly equal to two, but can be considerable otherwise.

5. Concluding Comments

Daily cycles of electricity demand contain stochastic components due, for instance, to the variability of weather conditions. A significant part of the benefit associated with energy storage is due solely to these stochastic components. We denote this benefit the stabilization value of energy storage. The term "stabilization" signifies the role of energy storage in stabilizing the stochastic demand fluctuations and reducing the dependence on expensive back-up electricity.

Explicit expressions and simple lower bounds on the relative size of the stabilization value (compared to the overall benefit of energy storage) have been derived. Under some circumstances, the contribution of the stabilization value is considerable. Thus, failing to account for the uncertainty in peak power demand (e.g., by erroneously assuming that demand is stable at the mean), leads to sub-optimal investment choices, and in some cases to wrong decisions on whether or not a storage project should be undertaken.

The significance of the stabilization value of energy storage depends on a few parameters which vary from place to place. The approach presented here, however, is general and applicable for a wide class of situations characterized by the structure of electricity demand, details of the various elements in the utility's production mix, and the costs associated with feasible energy storage projects. The data required to determine the relevant parameters is usually available to the utilities, so the application of this analysis to realistic situations should be straightforward.

References

- Aigner, D.J. (ed.), Welfare econometrics of peak-load pricing for electricity, *Journal of Econometrics*, 26 Supplement (1984).
- Carlin J., A theoretical model for estimating the capacity credit of wind power in the presence of correlation between wind speed and electricity demand, *Solar Energy*, 31, 511-515 (1983).
- Carlton D.W., Peak load pricing with stochastic demand, *The American Economic Review*, 67, 1006-1010 (1977).
- Devarajan, S. and R.J. Weiner, Dynamic policy coordination: Stockpiling for energy security, *Journal of Environmental Economics and Management*, 16, 9-22 (1989).
- Dreze, J.H. and F. Modigliani, Consumption decisions under uncertainty, *Journal of Economic Theory*, 5, 308-355 (1972).
- Haslett, J. and M. Diesendorf, The capacity credit of wind power: a theoretical analysis, *Solar Energy*, 26, 391-401 (1981).
- Jackson, R., Peak load pricing model of an electric utility using pumped storage, *Water Resources Research*, 9, 556-562 (1973).
- Martin, B. and M. Diesendorf, Optimal thermal mix in electricity grids containing wind power, *Electrical power & Energy systems*, 4, 155-161 (1982).
- Newbery, D.G.M. and J.E. Stiglitz, *The theory of commodity price stabilization*, Clarendon press, Oxford (1981).
- Nozari, A., J.C. Lalli and H. Kumin, A random walk approximation for a solar energy storage system, *Solar Energy*, 37, 127-134 (1986).
- Panzar, J.C., A neoclassical approach to peak load pricing, *The Bell Journal of Economics*, 7, 521-530, (Autumn 1976).
- Tsur, Y., The Stabilization role of groundwater when surface water supplies are uncertain: The implications for groundwater development, *Water Resources Research*, 26, 811-818 (1990).
- Tsur, Y. and T. Graham-Tomasi, The buffer value of groundwater with stochastic surface water supplies, *Journal of Environmental Economics and Management* 21, 201-224 (1991).