# The Effect of Fire Risk on the Critical Harvesting Times for Pacific Northwest Douglas-Fir When Carbon Price Is Stochastic

**Selmin F. Creamer, Alan Genz, and Keith A. Blatner**

The forest owner's decision regarding when to harvest, based on forest's current worth, is analyzed using the real options approach for a representative Pacific Northwest Douglas-fir stand when the carbon price is stochastic and there is a fire risk. The problem is framed as a linear complementarity problem and solved using the fully implicit finite difference method combined with a penalty method. The fire risk results in lower option values and earlier critical harvesting times, whereas a wider carbon price range (\$0–\$100 versus \$0–\$10) produces contrary results and more responsiveness to the parameter changes.

***Key Words:*** Chicago Climate Exchange Sustainably Managed Forest Project, carbon financial instrument, geometric mean-reverting process

From 1849 to 1976, optimal forest management schemes were primarily based on the timber harvest returns and the concept of sustainable harvest levels. The introduction of the joint production of timber revenues and amenities by Hartman (1976) led to consideration of forests' potential to sequester carbon starting in the 1990s. This was well known scientifically and further emphasized during the Intergovernmental Panel on Climate Change as a mitigation strategy by the forestry experts.

In an environment where timber benefits (both market and non-market) have been the major motivation for forest ownership, carbon sequestration incentives for the forest owners add an important motivating factor to forest management decisions. The effect of the incentives to sequester carbon has been investigated within the deterministic framework. Nevertheless, most of the studies examined the effect of these incentives when timber and carbon prices were constant and there was no risk of a catastrophic fire. It should be noted that as the optimal rotation period lengthens, the exposure to uncertainties in the context of forest management increases via changes in timber and carbon prices, interest rate fluctuations, and natural disasters.

Fire is considered one of the primary sources of uncertainty in the forest economics literature. Most studies have examined fire risk under a deterministic framework. In the event of a fire, the forest owner is expected to experience a reduction in returns; however, it is difficult to make the same assumption when there are changing parameters in the model (Amacher, Ollikainen, and Koskela 2009). Stochastic processes are considered the best method for modeling the continuously changing parameters because the stochastic process models are built to incorporate drift and volatility into these parameters (Amacher, Ollikainen, and Koskela 2009).

In this paper, the forest owner's decision regarding when to harvest her or his forest and how much it is currently worth is analyzed for a representative Douglas-fir stand when carbon price follows a geometric mean-reverting process and there is a risk of fire. The analysis is formulated

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under the real options framework that incorporates the value of the forest owner's flexibility in making an irreversible harvesting decision. The model is structured using the linear complementarity problem, in order to compare the forest owner's strategy for postponing the harvest versus harvesting the stand. The problem is solved using the fully implicit finite difference method combined with a penalty method as in Insley and Rollins (2005).

This paper extends the work of Insley and Rollins (2005) by incorporating the carbon benefits and a fire risk into the forest management decisions. The optimal harvesting decisions within the real options framework have been investigated when timber and carbon benefits are jointly produced (Chladná 2007, Guthrie and Kumareswaran 2009) or when timber benefits are produced with a fire risk (Insley and Lei 2007). To our knowledge, no prior studies have investigated the optimal harvesting decisions within the real options framework when timber and carbon benefits are considered together with a fire risk.[1] Finally, this paper also presents the first real options model application to the Pacific Northwest Douglas-fir.

## Brief Literature Review

Samuelson's seminal work (1976) marked a departure from the previously examined basic questions and led to a different set of questions in the forest economics literature (Newman 2002), where an economic approach to the harvesting question became the standard (Amacher, Ollikainen, and Koskela 2009). Hartman (1976) in the same year extended the single rotation model by including the value in situ that forests provided as public goods. After the 1980s, the impact of different policy instruments on optimal rotation age, uncertainty in prices, and stand growth including uncertainty brought on by the catastrophic events, such as fire, were examined closely within the Faustmann framework (Newman 2002).

During the last thirty years of the forest economics literature, various types of uncertainties (timber price, stand growth, amenity benefits, interest rate, and fire risk) were incorporated into the optimal harvesting problem via extending the Faustmann model (Clarke and Reed 1989, Willassen 1998, Sødal 2002, Saphores 2003, Duku-Kaakyire and Nanang 2004, Daigneault, Sohngen, and Miranda 2010) and Wicksellian tree-cutting problem (Alvarez and Koskela 2003), or using the real options framework (Morck, Schwartz, and Stangeland 1989, Clarke and Reed 1989, Reed and Clarke 1990, Thomson 1992, Conrad 1997, Thorsen 1999, Saphores, Khalaf, and Pelletier 2002, Duku-Kaakyire and Nanang 2004, Insley and Rollins 2005, Khajuria, Kant, and Laaksonen-Craig 2009).

Prior to and in conjunction with the progress of real option literature, Norstrøm (1975), Brazee and Mendelsohn (1988), Haight and Holmes (1991), Plantinga (1998), and Gong (1999) provided early examples of reservation price policies. In particular, Plantinga (1998) analytically demonstrated the potential use of reservation price policies as a way to incorporate the option value[2] into the calculation of optimal rotation length. He set the reservation price equal to the price where the expected returns from harvesting earlier were equal to the expected returns from harvesting later.

Morck, Schwartz, and Stangeland (1989), Clarke and Reed (1989), Reed and Clarke (1990), and Thomson (1992) presented the earliest examples in which the optimal harvesting rules were examined within the real options framework.

### *Carbon Benefits*

The introduction of the joint production of timber revenues and amenities by Hartman (1976) led to consideration of forests' capacity to sequester carbon starting in the 1990s. Englin and Callaway (1993), Hoen and Solberg (1994), and Van Kooten, Binkley, and Delcourt (1995) focused on the optimal rotation age of a stand when timber and

[1] Daigneault, Sohngen, and Miranda (2010) investigated the effects of forest carbon sequestration credits on optimal forest management for a Douglas-fir stand in the Pacific Northwest when there was a fire risk, posing the problem as an infinite horizon, discrete-time, stochastic dynamic optimization model. A Faustmann type approach was used to incorporate the endogenous fire risk where timber and carbon prices were assumed constant. The results indicated that a probability of fire always resulted in a lower rotation age relative to the deterministic case with no fire risk. Higher carbon prices resulted in longer rotations even when there was a high probability of a fire.

[2] Plantinga (1998) defines the option value as a premium over the expected net present value of a timber stand, reflecting the opportunity cost of harvesting now and foregoing the option to delay harvest until information on future stand values is revealed.

carbon benefits were jointly produced. Only recently, the joint production of timber and carbon was investigated under the real options approach. Chladná (2007) and Guthrie and Kumareswaran (2009) were among the first studies in which timber and carbon benefits were analyzed jointly within the real options framework.

Chladná (2007) formulated the single rotation optimal harvesting problem using a real options framework when forest carbon prices follow a geometric Brownian motion and timber prices follow a mean-reverting process. The problem was solved moving backwards with respect to decision time points using a dynamic programming algorithm. It was determined that as the responsibility to pay back at time of harvest increased, the optimal rotation period decreased, except when the carbon price was constant. Later on, Guthrie and Kumareswaran (2009) analyzed the impact of two carbon credit payment schemes (land- and tree-based) on the timing of the harvest and replanting-abandonment decision for multiple rotations within the real options framework when timber prices followed a mean-reverting process. It was shown that both schemes discouraged deforestation. While the land-based payment scheme shortened the rotation period, the tree-based scheme lengthened it.

### *Fire Risk*

Martell (1980) and Routledge (1980) were among the first studies to examine the impact of a fire on the optimal rotation problem. However, Reed (1984) was the first to study the problem of how forest owners behaved facing a probability of a forest fire on a theoretical level using the Faustmann framework. Furthermore, Reed (1993) was among the first studies in which the impact of a fire risk on the optimal harvesting decision was analyzed when there was a stochastic process in the model.

Reed (1993) examined the decision to harvest or conserve old-growth forests when future amenity values for the standing forest and timber revenues follow a geometric Brownian motion. He extended the model incorporating a catastrophic fire risk. Recently, Insley and Lei (2007) studied the impact of a fire risk on the optimal harvesting at the stand level within the real options framework when timber prices follow a mean-reverting process. It was determined that as the probability of fire increased, the critical harvesting price and value of the option to delay harvest were reduced, and harvesting occurred sooner.

## Model Background

The forest owner's opportunity to harvest can be thought of as a real option analogous to holding a call option, giving the forest owner the right but not the obligation to harvest the forest anytime. Furthermore, the value of the option to delay harvest can be considered the market value of standing trees at different merchantable stock volumes and ages. When the forest owner harvests, she or he exercises the option to delay harvest and gives up the possibility of postponing harvest until the new information arrives.

The price of carbon $P^c$ evolves based on the geometric mean-reverting process over time, in which $\bar{P}^c$ is the normal level of $P^c$, $\sigma_{p^c}$ is the volatility of the process, $\alpha$ is the speed of reversion, $z_{p^c}$ is the Wiener process, $dz_{p^c}$ is the increment to the standard Wiener process, $t$ is time, and $dt$ is the time interval:

$$(1) \qquad dP^c = \alpha(\bar{P}^c - P^c)dt + \sigma_{p^c} P^c dz_{p^c} .$$

Equation (1) illustrates that the carbon price reverts to $\bar{P}^c$ in the long run and the variance rate $(\sigma_{p^c} P^c)$ is expected to grow with $P^c$. When $P^c$ is zero, the variance rate goes to zero.

The merchantable volume of timber in cubic feet per acre is represented as

$$X = g(t) = e^{\Psi_0 - \frac{\Psi_1}{t}}$$

(Englin and Callaway 1995), in which $\Psi_0$ and $\Psi_1$ are the stand growth parameters and $t$ is time in years. The timber growth in cubic feet per acre is

$$g'(t) = \frac{\Psi_1}{t^2}\left(e^{\Psi_0 - \frac{\Psi_1}{t}}\right)$$

and can be written as a function of the volume of timber $X$ illustrated in equation (2):

$$(2) \qquad g'(X) = \frac{\left(\Psi_0 - \ln[g(t)]\right)^2 g(t)}{\Psi_1} = \frac{[\Psi_0 - \ln(X)]^2 X}{\Psi_1} .$$

The volume of timber per acre grows based on the deterministic process:

(3) $$dX = g'(X)dt\,.$$

In the analysis, it is assumed that the forest owner is a small project participant in the Pacific Northwest and registered voluntarily in a Chicago Climate Exchange (CCX) Sustainably Managed Forest Project. The forest owner gives up or is issued the CFIs (tradable instruments in CCX representing the exchange allowance or offsets) for the decreases (via harvesting) or increases (via stand growth) in the forest carbon stock (Chicago Climate Exchange, Inc., 2009). The project starts when the trees are planted (baseline of the project) and ends when the trees are harvested (end of the CCX market period). The payments or surrendering of the CFIs occur at the end of the CCX market period when harvesting is completed (when the project ends).

The forest owner's problem is to choose the optimal harvesting time for a representative even aged stand of trees per acre. The problem is formulated based on a single rotation. The goal is to characterize the solution conditions for the stopping problem by obtaining a partial differential equation in terms of the value function (value of the option).

At the current period $t$, the current values of carbon and timber prices and stand volume are known, but the future values of carbon price are unknown. The decision making process is expressed as an optimal stopping problem where the forest owner faces a decision at each time increment: to harvest immediately or to postpone the harvest.

The forest owner has the option to receive immediate/termination payoff from the harvest at the current time or receive the expected discounted value of additional rents from the change in carbon prices and stand volume (additional growth) between the current and future decision time. The decision process is split into two parts: the *immediate period* and *continuation period*.

*Model Without a Fire Risk*

The decision of the forest owner at each point in time is to choose the greater of the immediate/terminal payoff or the value of continuing the rotation. The *Bellman equation* for this optimal stopping problem in finite time is illustrated in equation (4):

(4)
$$V(P^c, X, t) = \max\begin{bmatrix} f(P^c, P^t, X), P^c\gamma g'(X) \\ +\left(\dfrac{1}{1+r}\right)E\left(V(P^{c'}, X', t') \mid P^c, X\right)\end{bmatrix}.$$

$V(P^c, X, t)$ is the value of the option to harvest the forest at time $t$, $f(P^c, P^t, X)$ is the immediate/termination payoff [equation (5)], and

$$\left[P^c\gamma g'(X) + \left(\frac{1}{1+r}\right)E\left(V(P^{c'}, X', t') \mid P^c, X\right)\right]$$

is the value of continuing the rotation.

$P^c\gamma g'(X)$ is the flow of rents from letting the stand grow for an additional period, $r$ is the discount rate, $X'$ is the next period's volume of timber given the current volume of timber $X$, $P^{c'}$ is the next period's carbon price given the current carbon price $P^c$, and $t'$ is the next period.

(5) $$f(P^c, P^t, X) = P^t X - P^c(1-\eta)\gamma X + P^c\gamma g'(X)\,.$$

In equation (5), $P^t X$ are the timber benefits, $P^c(1-\eta)\gamma X$ is the payback of the carbon benefits after timber is harvested, and $P^c\gamma g'(X)$ are the carbon benefits received from the growth in the carbon stock. $\gamma$ is the estimate for converting the growing stock volume in 1,000 metric board feet (MBF) per acre to the average carbon content in metric tons per acre, $g'(X)\gamma$ is the quantity of carbon sequestered, $P^t$ is the net price of timber per MBF, $\eta$ is the fraction of the harvested timber going into the long-term storage, and $P^c$ is the price of carbon in metric tons. Equation (5) is formulated based on Chladná (2007) and the CCX Forestry Carbon Sequestration Project Protocol illustrating the payoff increase via the stand growth or payoff decrease via harvesting (to reflect the change in the carbon stock).

The continuous time version of the *Bellman equation in the continuation region* after rearranging equation (4) is represented below:

(6)
$$rV(P^c, X, t) = \max\left[P^c\gamma g'(X) + E\frac{\left[dV(P^c, X, t)\right]}{dt}\right].$$

Equation (6) states that the total return on the forest $rV(P^c, X, t)$ has two components. $P^c\gamma g'(X)$ is the flow of rents from letting the forest stand grow an additional period.

$$E\frac{[dV(P^c, X, t)]}{dt}$$

is the expected capital gain from potentially harvesting the forest one period later and contains the uncertainty in the value of the next period's carbon price through the drift and volatility parameters.

Equation (6) can be thought of in terms of an equilibrium condition (no arbitrage condition) in which the opportunity cost of holding the option is equal to the benefits received from holding the option. It can be written openly as in equation (7).

Equation (7) characterizes the solution conditions for the optimal stopping problem in terms of the value function where $V(P^c, X, t)$ is twice differentiable in $P^c$ and once differentiable in $t$:

$$(7)\quad rV(P^c, X, t) = P^c\gamma g'(X) + \frac{1}{2}V_{P^cP^c}P^{c2}\sigma_{P^c}{}^2 + \alpha(\bar{P}^c - P^c)V_{P^c} + V_X g'(X) + V_t.$$

*Model With a Fire Risk*

The *Bellman equation* for the optimal stopping problem when there is a risk of fire is illustrated in equation (8):

$$(8)\quad V(P^c, X, t) = \max\begin{bmatrix} f(P^c, P^t, X), P^c\gamma\left(g'(X) - \lambda X\right) \\ +\left(\frac{1}{1+r}\right)E\left(V(P^{c'}, X', t') \mid P^c, X\right)\end{bmatrix}.$$

The volume of timber per acre grows based on the process below in the case of a fire:

$$(9)\qquad dX = g'(X)dt - Xdq^3$$

$$dq = 0 \text{ with probability } 1 - \lambda dt$$

$$= 1 \text{ with probability } \lambda dt,$$

where $dq$ is the Poisson differential equation, $q$ denotes the Poisson process, $\lambda$ is the mean arrival rate of a fire, $\lambda dt$ is the probability that a fire will occur during a time interval of length $dt$, and $1 - \lambda dt$ is the probability that a fire will not occur during a time interval of length $dt$.

The Poisson differential equation (9) is analogous to the Ito process. The volume of timber $X$ changes continuously and deterministically where $g'(X)$ represents the deterministic change. There is also a possibility that a Poisson event may occur; when it does, $X$ changes by $-X$, so does the value of the option $V(P^c, X, t)$. It is assumed that the fire risk exists only during the continuation period.

The continuous time version of the *Bellman equation in the continuation region* is represented in equation (10):

$$(10)\quad rV(P^c, X, t) = \max\begin{bmatrix} P^c\gamma\left(g'(X) - \lambda X\right) \\ +E\frac{[dV(P^c, X, t)]}{dt}\end{bmatrix}.$$

Equation (10) characterizes the solution conditions for the optimal stopping problem in terms of the value function and can be written openly as in equation (11), in which $V(P^c, 0, t)$ represents the value of the bare timberland:

$$(11)\quad (r+\lambda)V(P^c, X, t) = P^c\lambda\left(g'(X) - \lambda X\right) + \frac{1}{2}V_{P^cP^c}P^{c2}\sigma_{P^c}{}^2 + \alpha(\bar{P}^c - P^c)V_{P^c} + V_X g'(X) + V_t + \lambda V(P^c, 0, t).$$

**Linear Complementarity Problem and Boundary Conditions**

The forest owner's decision whether to harvest or postpone harvesting is constructed using a linear complementarity problem. Equations (7) and (11) are rearranged as in equations (12) and (13) to reflect the time in terms of time to maturity ($\tau$) rather than the calendar time ($t$), after letting $\tau = T - t$ and substituting $V(P^c, X, t)$ with $-V(P^c, X, \tau)$:

[3] In Insley and Lei (2007), the volume of timber is illustrated as a deterministic function of the stand age $t$ in which the stand age was a stochastic variable based on the time of the last harvest and the occurrence of a fire.

(12)

$$rV(P^c, X, \tau) - \begin{bmatrix} P^c \gamma g'(X) + \frac{1}{2} V_{P^c P^c} P^{c2} \sigma_{P^c}{}^2 \\ + \alpha(\bar{P}^c - P^c) V_{P^c} + V_X g'(X) - V_\tau \end{bmatrix} \geq 0$$

$$(13) \quad (r+\lambda)V(P^c, X, \tau) - \begin{bmatrix} P^c \gamma \left( g'(X) - \lambda X \right) \\ + \frac{1}{2} V_{P^c P^c} P^{c2} \sigma_{P^c}{}^2 \\ + \alpha(\bar{P}^c - P^c) V_{P^c} \\ + V_X g'(X) - V_\tau \\ + \lambda V(P^c, 0, \tau) \end{bmatrix} \geq 0 .$$

The forest owner will continue to postpone harvesting as long as the *return required* over the investment for holding the option brings the *actual return* (value in brackets). Once the required return is greater than the actual return, then it is optimal for the forest owner to harvest. If equations (12) and (13) hold with strict equality, the optimal policy for the forest owner is to postpone harvesting.

$$(14) \qquad V(P^c, X, \tau) - f(P^c, P^t, X) \geq 0 .$$

Equation (14) illustrates the condition where the option value can never go below the early exercise payoff. If $V(P^c, X, \tau) < f(P^c, P^t, X)$, then $-V(P^c, X, \tau) + f(P^c, P^t, X) > 0$ would represent the riskless profit and the option would be immediately exercised. If we assume that there is no arbitrage opportunity, then equation (14) should hold. The option value must be at least as great as the return from harvesting immediately. If equation (14) holds with strict equality, the optimal policy for the forest owner is to harvest immediately.

In order to be able to choose between two actions—to harvest or to postpone harvesting—either equations (12) and (13) or equation (14) should hold with equality, referring to either waiting or harvesting immediately. If equations (12) and (13) and equation (14) are both identically zero, then the forest owner would be indifferent between holding the option and harvesting.

The linear complementarity problem is solved using the fully implicit finite difference method combined with a penalty method (Zvan, Forsyth, and Vetzal 1998). The penalty method assures that the value of the option can never go below the value of harvesting immediately (immediate payoff) (Insley 2002). Two optimal stopping conditions—value matching and smooth pasting[4]—must hold at the time of stopping as we derive equations (7) and (11) under the assumption of continuation. However, these conditions do not need to be specified openly because they are natural results of the linear complementarity formulation (Friedman 1988, Insley 2002). The linear complementary formulation eliminates the dependence on free boundary (Wilmott, Howison, and Dewynne 1993, Insley 2002).

The boundary conditions are specified to solve the linear complementarity problem numerically. The conditions for the timber price and time in Insley and Rollins (2005) are assumed for the carbon price and time in this paper. As the carbon price $P^c$ goes to zero, $dP^c$ goes to $\alpha\bar{P}^c$ (no boundary condition is required), and as $P^c$ goes to infinity, $V_{P^c P^c}$ is set to zero. As the stand growth $g'(X)$ goes to zero, the stand volume $X$ approaches the maximum volume and the value function $V(P^c, X, t)$ stops growing. The maximum stand volume is calculated based on the particular timber growth function used in this paper. As the terminal time $T$ becomes large, it is assumed that value of the option $V(P^c, X, t)$ is equal to zero, representing the terminal condition for the option.

## Parameter Estimates

### *Carbon Price*

The geometric mean reversion process $dP^c = \alpha(\bar{P}^c - P^c)dt + \sigma_{P^c} P^c dz_{P^c}$ is illustrated as a discrete time approximation (Insley 2002):

$$(15) \quad P_t^c - P_{t-1}^c = \alpha\bar{P}^c \Delta t - \alpha \Delta t P_{t-1}^c + \sigma_{P^c} P_{t-1}^c \sqrt{\Delta t \varepsilon_t} ,$$

where $\varepsilon_t$ is $N(0,1)$. Equation (15) is rewritten as

$$(16) \qquad \frac{P_t^c - P_{t-1}^c}{P_{t-1}^c} = z(1) + z(2)\frac{1}{P_{t-1}^c} + e_t ,$$

[4] The value-matching condition ensures that the value function is equal to the immediate payoff, and the smooth-pasting condition ensures that the change in the value of function (with respect to the carbon price) is equal to the change in the immediate payoff (with respect to the carbon price) at the optimal stopping time.

where

$$z(1) = -\alpha\Delta t,\ z(2) = \alpha\overline{P}^c\Delta t$$

and

$$e_t = \sigma_{P^c}\varepsilon_t\sqrt{\Delta t}\ .$$

The parameters are estimated based on the average monthly time series data (Figure 1) calculated from the daily CFI contract prices. The parameter estimates are $\overline{P}^c = 1.6921$, $\alpha$ = 0.07212, and $\sigma_{P^c}$ = 0.0837. The standard error estimate for the regression model is 0.24166, with $R^2$ = 0.4 percent and where $\Delta t$ is 1/12 (one month).

*Stand Growth*

The stand growth parameters $\Psi_0$ = 10.28 and $\Psi_1$ = 67.78 are for the Pacific Northwest Douglas-fir (Englin and Callaway 1995).

*Carbon Storage*

The estimate for converting the growing stock volume in cubic feet to average carbon content in metric tons ($\gamma$) is 0.01152 for the Pacific Northwest Douglas-fir (Birdsey 1992). It is further assumed that 20 percent of the harvested timber goes into the long-term storage ($\eta$ = 0.2).

*Timber Stumpage Price*

An average annual value of Douglas-fir softwood stumpage in dollars between 1965 and 2005 for western Washington and western Oregon is used. The average annual stumpage price for Douglas-fir ($P_t$) is \$340 per 1,000 board feet (MBF) in constant (2000) dollars. The stumpage price is based on the sales of saw timber from national forests during this period.

*Value of Bare Timberland, Harvesting Cost, and Discount Rate*

The value of the bare timberland is assumed to be \$500 per acre for the Pacific Coast region, and a harvesting cost of \$200 per MBF is assumed based on the information received from a timberland investment advisory firm. Harvesting cost is converted to cubic meters per acre for the numerical solution. A discount rate of 5 percent is chosen for this paper (Daigneault, Sohngen, and Miranda 2010).

*Average Fire Arrival Rate*

The fire return interval for Pacific Northwest Douglas-fir is assumed to be 50 years (Morrison and Swanson 1990). The probability of a forest fire arriving in any given year (fire arrival rate) is 1/50 ($\lambda$).

## Results

The linear complementarity problem is solved using the penalty method and the method of characteristics as in Insley and Rollins (2005).[5] The earliest critical time for harvesting is reached once the value of the option line touches the immediate payoff line at the point where the two optimal stopping conditions—value matching and smooth pasting—are satisfied. Figure 2 illustrates the graphs representing the earliest possible harvesting times in terms of stand volume and age. Without a fire risk, optimal stopping conditions are satisfied when the stand is approximately 37.75 years old and the merchantable stock volume is 137.05 cubic meters. With the incorporation of a fire risk, these conditions are met when the stand is approximately 32.07 years old and the merchantable stock volume is 99.68 cubic meters. The primary difference between these two models results from an increase in the discount rate by 2 percent, due to the incorporation of a fire risk (from 5 percent to 7 percent) in the base model.

Incorporation of the carbon benefits (specifically, the amount that the forest owner has to pay back after harvesting) results in the value of the option decreasing for a fixed stock volume. The forest owner may harvest any time after the optimal stopping conditions are met, even though it may be more profitable not to harvest since the value of the option continues to increase as the stand volume increases. The value of the option increases faster during the early years of the stand age, when the stock volume is low, in comparison with later years after the growth starts to decline gradually (when the stand is approximately 35

[5] Insley and Rollins (2005) estimated the critical timber price above which it is optimal to harvest when the timber price is stochastic and the forest owner's revenues are just based on the timber benefits.

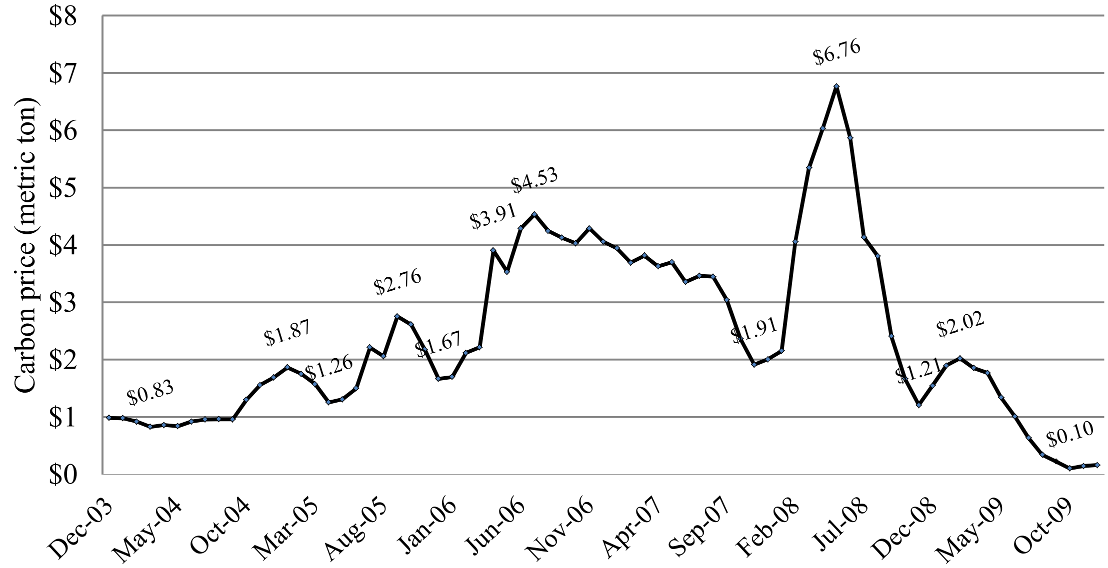


**Figure 1. Monthly Average Carbon Price Between December 2003 and December 2009 Based on Daily CFI (Carbon Financial Instrument) of CCX (Chicago Climate Exchange)**

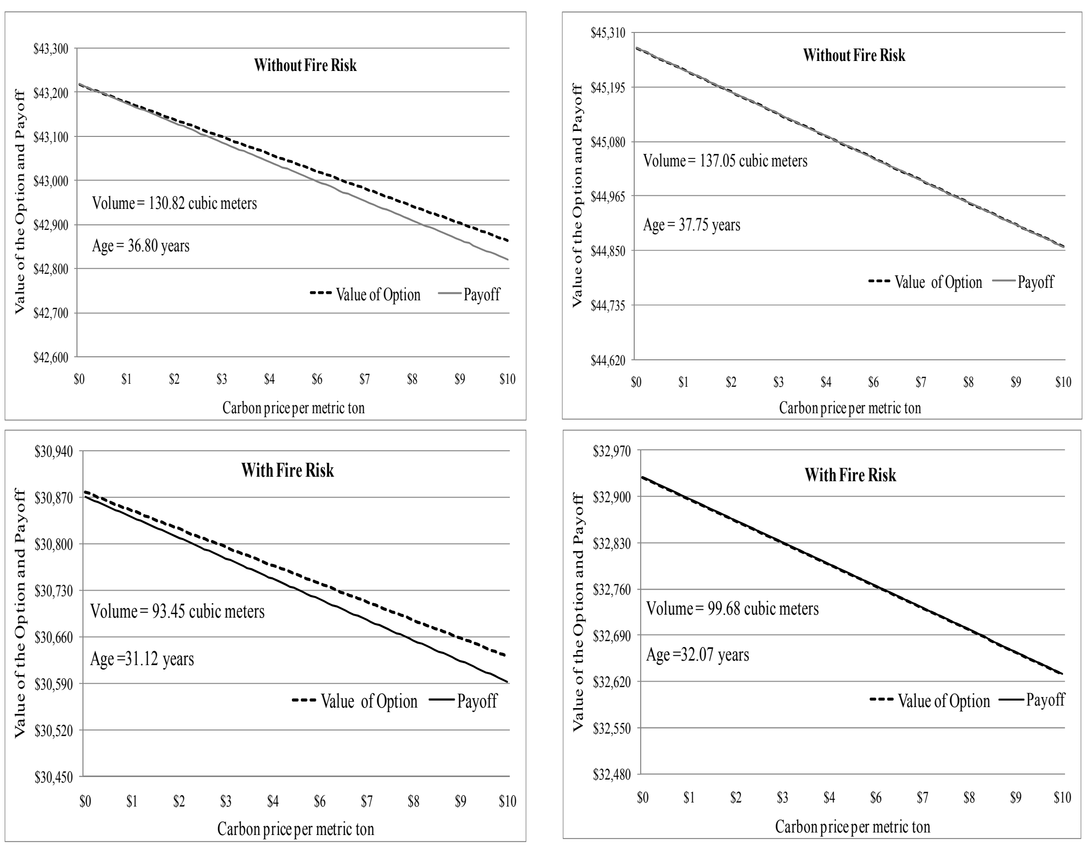


**Figure 2. Graphs Illustrating the Earliest Critical Times for Harvesting**

years old and the stock volume is 118.36 cubic meters).

As part of the sensitivity analysis, three additional fire return intervals are simulated: 25 years (9 percent), 75 years (6.3 percent), and 100 years (6 percent). Figure 3 illustrates the graphs representing the discount rate versus the stand age and stand volume at the earliest critical harvesting times. It is determined that as the fire return interval declines, the discount rate increases; consequently, the earliest critical harvesting time occurs sooner.

Various volatility, mean reversion rate, and long term carbon price figures are simulated. The changes do not affect the earliest critical harvesting times for the given price range ($0–$10). Nevertheless, the values of the options for both models are responsive to these parameter changes when the maximum price is set at $100.

When the maximum carbon price is set at $100, optimal stopping conditions are met once the stand is approximately 42.61 years old and the merchantable stock volume is 168.20 cubic meters for the model without a fire risk (as opposed to 37.75 years and 137.05 cubic meters in the original model). For the model with a fire risk, these conditions are satisfied when the stand is approximately 35.85 years old and the merchantable stock volume is 124.59 cubic meters (as opposed to 32.07 years and 99.68 cubic meters in the original model). Table 1 illustrates the stand volumes and ages at the earliest critical harvesting times for both models when the carbon price ranges are set for $0–$10 and $0–$100. The effect of a fire risk on the stand volume and age is greater when the carbon price range is set for $0–$10 (37.49 percent versus 35.00 percent), and the effect of setting maximum carbon price at $100 is greater when there is a fire risk (24.99 percent versus 22.73 percent).

Figure 4 shows the graphs representing the earliest possible harvesting times in terms of the stand volume and age when 100 percent of the harvested timber goes into the long-term storage. The positive slope is an outcome of the elimination of the responsibility to pay back the CFIs at time of harvest from the immediate payoff [$f(P^c, P^t, X) = P^tX - P^c(1-\eta)\gamma X + P^c\gamma g'(X)$ becomes $f(P^c, P^t, X) = P^tX + P^c\gamma g'(X)$]. The earliest critical harvesting times for both models are the same as in the original models (when the long-term storage is assumed to be 20 percent).

The values of the options are different for both models up to the point where the optimal stopping conditions are satisfied for the model without a fire risk (stand is 37.75 years old and stock volume is 137.05 cubic meters) (Table 2). After this point, both models illustrate the same values of the options (Table 3).

As the stand growth declines, the increase in the value of the options gradually declines. For example, when carbon price is 0.10, the value of the option increases from $61,740 to $88,490 (43 percent) in approximately 15 years (when the stand age is 60.24 years); similarly, the value of the option increases from $88,490 to $111,140 (25 percent) and from $111,140 to $129,660 (17 percent) in approximately 15 years (Table 3).

## Conclusion

This paper presents an addition to the existing real options literature investigating the optimal harvesting decisions, as it extends the work of Insley and Rollins (2005) by incorporating the carbon benefits and a fire risk into the forest management decisions, and possibly presents the first real options model application to the Pacific Northwest Douglas-fir. Pacific Northwest Douglas-fir's high capacity for average carbon storage, wildfire risk, and sustainably managed forest projects introduced by the Chicago Climate Exchange have been the main motivating factors for this paper.

The forest owner's decision regarding when to harvest her or his forest at the earliest possible time and its current worth is analyzed using the real options framework. In order to compare the forest owner's strategy for postponing the harvest versus harvesting the stand, the analysis is framed as a linear complementarity problem, eliminating the need to explicitly specify the value-matching and smooth-pasting conditions. The linear complementarity problem is discretized using the fully implicit finite difference method and solved with a penalty method, which is considered an improvement over the projected successive over-relaxation method (Insley 2002). In the current models, the forest owner can harvest as soon as the optimal stopping conditions (value matching and smooth pasting) are met, even though it may be more profitable not to harvest since the value of the option continues to increase as the stock

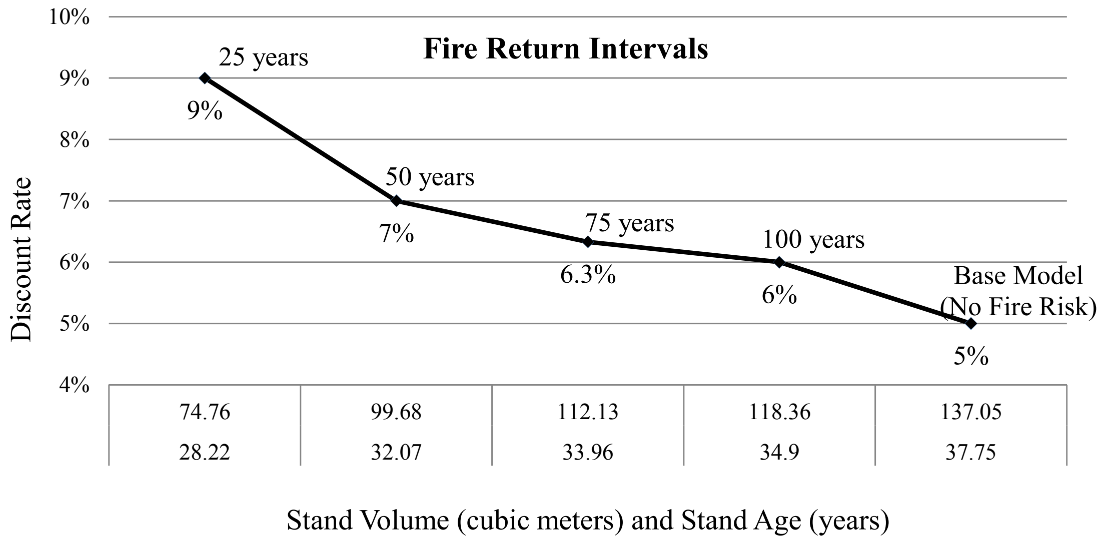


**Figure 3. Stand Volume (first row) and Ages (second row) at the Earliest Critical Harvesting Times for Various Fire Return Intervals: 25, 50, 75, and 100 Years**

**Table 1. Sensitivity Analysis: Stand Volumes and Ages at the Earliest Critical Harvesting Times**

| | | Carbon Price Range | | |
|---|---|---|---|---|
| | | $0–$10 | $0–$100 | Carbon Price Effect |
| Stand volume (cubic meters) | Model with a fire risk | 99.68 (32.07) | 124.59 (35.85) | 24.99% |
| | Model without a fire risk | 137.05 (37.75) | 168.20 (42.61) | 22.73% |
| Fire risk effect | | 37.49% | 35.00% | |

Note: Values in parentheses represent the stand ages (years). Fire arrival rate is 0.02.

continues to grow. Nevertheless, as the stand ages and growth starts to slow down and declines, the increase in the value of the options starts to decline gradually as well.

This paper not only confirms the results of Insley and Lei (2007), but also demonstrates that a catastrophic fire risk reduces both the value of the option to delay harvest and the earliest critical harvesting times for the carbon price ranges investigated here. The reductions due to a fire risk are greater for the carbon price range of $0–$10, suggesting that a catastrophic fire risk would have a more prominent effect in a market where the demand for the carbon offset credits is moderately fixed, while the supply is boundless, as in the case of CCX. Given that, the forest owners who are already participating in these markets may exhibit more willingness to mitigate the risk of catastrophic fires. It is also possible that the forest owners whose forests are facing infrequent or no catastrophic fires may be more likely to participate in these voluntary markets, especially when the carbon prices are high.

The wider carbon price range ($0–$100 versus $0–$10) increases the value of the option to delay harvest and earliest critical times for both models (with and without a fire risk) considered here, suggesting that the carbon supply can be expected

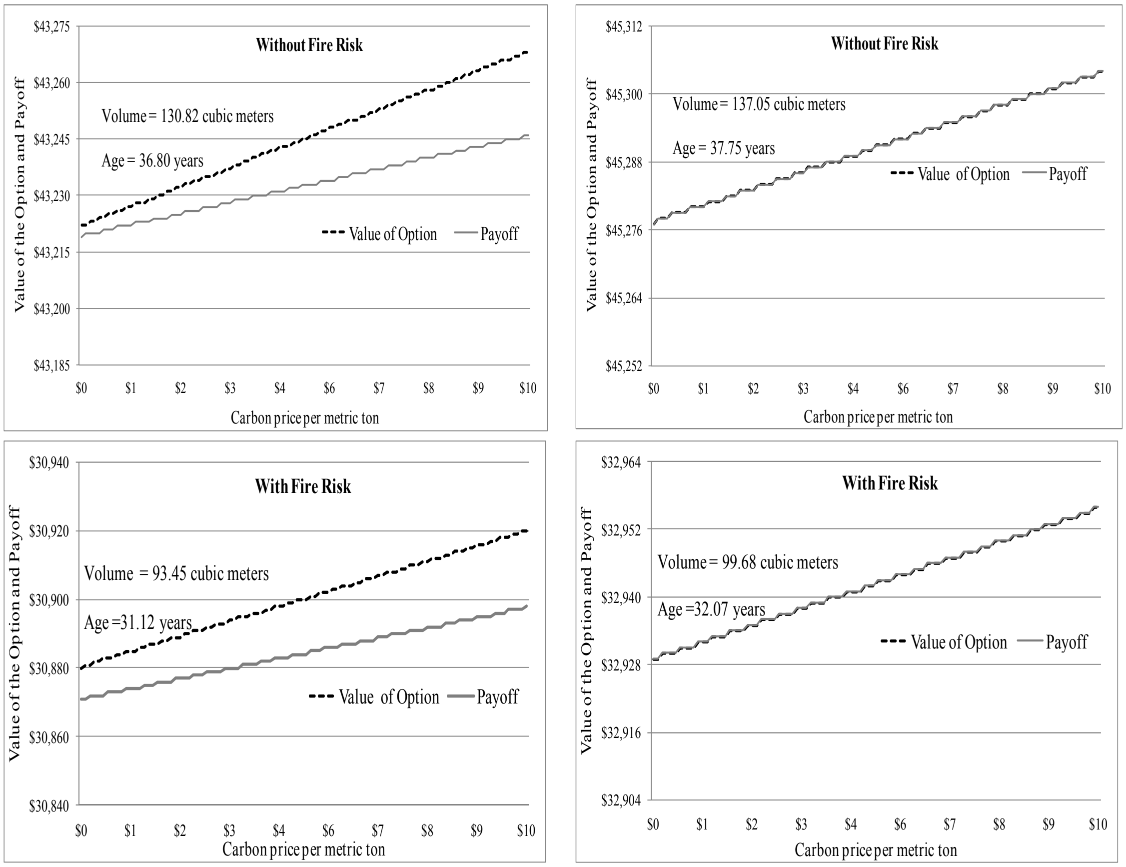


**Figure 4. Graphs Illustrating the Earliest Critical Times for Harvesting When the Long-Term Storage Is 100 Percent**

to increase when the carbon prices are high regardless of the frequency of catastrophic fires. Particularly, the increase is greater for the model considering a fire risk when the carbon price range is set for $0–$100. However, it is also expected that the carbon supply goes down with a catastrophic fire risk regardless of the carbon price range, as the earliest critical harvesting times get shorter. Hence, it may be reasonable to conclude that carbon sequestration policies should account for the differences in the forests' susceptibility to catastrophic fires.

The practical value of the models illustrated in this paper may depend on the ownership motivations of the forest owner. For example, a timber-oriented forest owner may be more interested to know the earliest possible harvesting times for both models, while it may not be that critical for a non–timber-oriented forest owner, especially if the forest owner does not mind waiting since the option value and immediate payoff produce the same dollar values for both models, once the stand is 37.75 years old.

The current model can be further extended by including stochastic timber price and/or making jump/fire size random (due to a fire risk). Furthermore, it may be more realistic to build the model by letting the fire risk change over time, at the expense of complicating the model, as the frequency of forest fires is expected to increase in the future as a result of global warming. Finally, considering an endogenous fire risk (in which risk depends on the stand management or the stage of stand development) rather than treating fire damage and occurrence as exogenous may also enhance the applied contribution of the models discussed here.

**Table 2. Value of the Options and Payoffs without a Fire Risk and with a Fire Risk**

| Carbon Price (metric ton) | 93.45 Cubic Meters, 31.12 Years | | 105.91 Cubic Meters, 33.01 Years | | 118.36 Cubic Meters, 34.90 Years | | 130.82 Cubic Meters, 36.80 Years | |
|---|---|---|---|---|---|---|---|---|
| | Option Value | Payoff | Option Value | Payoff | Option Value | Payoff | Option Value | Payoff |
| 0.10 | 32,728 (30,876) | 30,868 | 35,913 (34,984) | 34,984 | 39,392 (39,100) | 39,100 | 43,215 (43,215) | 43,215 |
| 1.01 | 32,720 (30,853) | 30,843 | 35,898 (34,955) | 34,955 | 39,368 (39,067) | 39,067 | 43,182 (43,179) | 43,179 |
| 2.02 | 32,712 (30,828) | 30,815 | 35,882 (34,923) | 34,923 | 39,344 (39,031) | 39,031 | 43,146 (43,139) | 43,139 |
| 3.03 | 32,704 (30,803) | 30,787 | 35,867 (34,891) | 34,891 | 39,319 (38,995) | 38,995 | 43,110 (43,098) | 43,098 |
| 4.04 | 32,696 (30,778) | 30,759 | 35,852 (34,859) | 34,859 | 39,295 (38,958) | 38,958 | 43,075 (43,058) | 43,058 |
| 5.05 | 32,688 (30,753) | 30,731 | 35,837 (34, 827) | 34,827 | 39,271 (38,922) | 38,922 | 43,039 (43,018) | 43,018 |
| 6.06 | 32,680 (30,728) | 30,703 | 35,821 (34,794) | 34,794 | 39,246 (38,886) | 38,886 | 43,003 (42,977) | 42,977 |
| 7.07 | 32,672 (30,703) | 30,675 | 35,806 (34,762) | 34,762 | 39,222 (38,850) | 38,850 | 42,968 (42,937) | 42,937 |
| 8.08 | 32,664 (30,678) | 30,647 | 35,791 (34, 730) | 34,730 | 39,198 (38,814) | 38,814 | 42,932 (42,897) | 42,897 |
| 9.09 | 32,656 (30,654) | 30,619 | 35,776 (34,698) | 34,698 | 39,173 (38,777) | 38,777 | 42,896 (42,857) | 42,857 |
| 10.00 | 32,649 (30,631) | 30,593 | 35,762 (34,669) | 34,669 | 39,152 (38,745) | 38,745 | 42,864 (42,820) | 42,820 |

Note: Values in parentheses represent the value of the options for the model with a fire risk. Payoff values are the same for both models.

**Table 3. Value of the Options for Various Stock Volumes and Stand Ages (in dollars)**

| Carbon Price (metric ton) | 137.05 Cubic Meters | 186.89 Cubic Meters | 267.87 Cubic Meters | 336.40 Cubic Meters | 392.47 Cubic Meters |
|---|---|---|---|---|---|
| | 37.75 Years | 45.64 Years | 60.24 Years | 75.53 Years | 91.20 Years |
| 0.10 | 45,273 | 61,740 | 88,490 | 111,120 | 129,650 |
| 1.01 | 45,235 | 61,680 | 88,410 | 111,030 | 129,530 |
| 2.02 | 45,193 | 61,620 | 88,320 | 110,920 | 129,400 |
| 3.03 | 45,150 | 61,570 | 88,240 | 110,810 | 129,280 |
| 4.04 | 45,108 | 61,510 | 88,150 | 110,700 | 129,150 |
| 5.05 | 45,066 | 61,450 | 88,070 | 110,590 | 129,020 |
| 6.06 | 45,023 | 61,390 | 87,980 | 110,480 | 128,890 |
| 7.07 | 44,981 | 61,330 | 87,890 | 110,370 | 128,760 |
| 8.08 | 44,938 | 61,270 | 87,810 | 110,260 | 128,640 |
| 9.09 | 44,896 | 61,210 | 87,720 | 110,160 | 128,510 |
| 10.00 | 44,858 | 61,160 | 87,650 | 110,060 | 128,390 |

Note: Values of the options also illustrate the immediate payoff amounts.

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