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# Nonlinear Programming of Field and Plant Vegetable Processing Activities

By Robert H. Reed and James N. Boles

Empirical studies of plant costs and efficiency always have stressed the importance of both size of plant and length of operating season on the level and shape of the economies of scale curve as well as the technical organization of production implied in it. Because most processing firms are integrated systems of product assembly, processing, and distribution, the minimum-cost combination of plant size and hours operated must be considered in terms of total operation, not of any single "stage" or component. This paper presents a method for determining optimum combinations of hours of operation and size of plant (as measured by rates of output) for two components, field and plant operations, of an integrated system of preparing lima beans for freezing. Though the analysis is oriented to only two "stages" and a particular product, the authors are hopeful that it may point the way toward extension to additional "stages" and other products.

**C**OST SYNTHESIS is an effective economic tool for reflecting in-plant cost functions, technical relationships, and operating characteristics of agricultural processing operations.<sup>1</sup> Through this procedure, data on elemental input-output and plant records are used to develop cost-output relationships among individual operating stages with alternative production techniques. Comparison of such stage-cost functions provides the basis for selecting least-cost operating techniques and for the development of generalized plant-cost functions that are closely analogous to the long-run cost or planning functions of economic theory.

<sup>1</sup> For detailed presentation of economic-engineering techniques in cost measurement, refer to B. C. French, L. L. Sammet, and R. G. Bressler, Jr., "Economic Efficiency in Plant Operations with Special Reference to the Marketing of California Pears," *Hilgardia*, Vol. 24, No. 19, July 1956, pp. 543-721. Also, Sammet, L. L., "Economic and Engineering Factors in Agricultural Processing Plant Design" (unpublished Ph. D. thesis, Department of Agricultural Economics, University of California, Berkeley, 1959), 434 pp.

As long-run total and unit plant costs vary with both length of season and capacity output rates, optimum plant adjustment is determined by both variables simultaneously. This may be made clear by noting several general characteristics of long-run average cost behavior.

First, with any given pattern of daily operating hours and length of operating period, total hours per period are fixed and so differences in total planned volume per period requires changes in plant capacity (measured as an output rate). Consequently, each point on a particular long-run average cost curve represents unit cost with a different plant. With a given length of operating period, and over a wide range of output scale, unit costs as represented by such a curve decrease as plant capacity increases. The decrease results from more effective utilization of supervisory and other partially fixed labor inputs and the substitution of various cost-reducing techniques in the larger plants.

Second, plant capacity rates necessary to achieve any given season volume decrease as hours of operation per season increase. As capacity decreases, investment cost and the corresponding annual fixed costs are smaller, with the result that unit fixed costs decline. But variable costs tend to rise and some of the cost advantages of increased scale are lost. Thus, efficient plant organization calls for balancing the net cost effects of scale of plant and operating hours.<sup>2</sup>

Most processing firms are an integrated system of product procurement, processing, and distribution. With the technique of cost synthesis the analyst can develop a separate cost function for each "stage" or component operation of the integrated system. But he must consider minimum-cost combination of rates and hours in terms of the total operation, not of any single component.

<sup>2</sup> An example and discussion of the problem of finding optimum combinations of plant size and time of operation for any given season volume is given in: French, Sammet, and Bressler, *op. cit.*, pp. 684-704.

If output rates for individual stages can be varied and part of the product stored between stages, the problem may usually be treated as a constrained minimum problem whereby a nonlinear objective function is minimized, subject to a series of linear restraints. A detailed development of the nonlinear programming problem in which the restraining functions are linear is given by Dorfman, Samuelson, and Solow.<sup>3</sup> These authors point out that, though "no sure fire practical method for solving nonlinear programming problems has yet been found, . . . special approximative methods can be tailor-made to solve individual problems as they arise."

In this paper we demonstrate a method for determining optimum combinations of hours of operation and rates of output for two components of an integrated system.<sup>4</sup> The example involves field and plant operations in preparing lima beans for freezing, using data developed in a 1958 study in California. The field operations consist of vining or shelling the beans and transporting them to the receiving station of the plant. At the freezing plant, the product is pumped, flumed, or otherwise conveyed from the receiving station through a series of in-plant operations. Substantial investment is required at both field and plant locations.<sup>5</sup>

The following assumptions and constraints reflect conditions consistent with actual experience: (1) A maximum of 8 hours of storage (with ice) is allowed between vining and in-plant processing, and is assumed to have no measurable effect on quality; (2) the daily operating hours and rates of output of vining and plant operations are such that the *total* volume vined or shelled per day equals the *total* daily volume processed; and (3)

in recognition of time lost daily in lunch periods, changing shifts, cleanup, equipment servicing, re- periods, and other delays, a maximum of 16 operating hours per day is assumed.

Economic-engineering methods were used to synthesize cost functions for field and plant activities representing total and average planning costs for each activity. Specific equations are given by Reed.<sup>6</sup> These are expressed here solely in terms of rates of output (R) and hours operated per season (H) by substituting, in more general equations, particular values for the variables representing distance of haul, percentage manual grade-out, and proportions packed in retail, institutional, and bulk containers. That is:

$$\begin{aligned} (1) \quad TSC_1 &= \$3,929 + \$2,633R_1 + \$0.3691H_1 + \\ &\quad \$A_1R_1H_1 \\ (2) \quad TSC_2 &= \$15,353 + \$1,870R_2 + \$27.6177H_2 + \\ &\quad \$A_2R_2H_2 \end{aligned}$$

where

$TSC_1$  is total annual costs of vining and hauling.

$TSC_2$  is total annual costs of in-plant processing.

$R_1$  and  $R_2$  are hourly rates of vining and in-plant processing, respectively, in 1,000-pound units.

$H_1$  and  $H_2$  are hours of vining and in-plant processing per season.

$Q = R_1H_1 = R_2H_2$  is total annual volume processed, in 1,000 pounds packed-weight equivalent.

$A_1$  and  $A_2$  are constants whose values depend on those specified for the variables: distance of haul; percentage manual grade-out; and proportions packed in the various size containers.

Daily cost equations may be derived by dividing the coefficients by the number of days operated per season. As  $R_1H_1 = R_2H_2 = Q$  by constraint (2) above—with  $Q$  expressed in packed-weight equivalent—the above equations can be written solely in terms of (H's) and (Q).

For any length of season (number of days operated), values of  $H_1$  and  $H_2$  and  $R_1$  and  $R_2$  can be found that minimize the combined costs of field and plant operations. The solution presented below assumes a 40-day operating season. (Solutions

<sup>3</sup> Robert Dorfman, Paul A. Samuelson, and Robert M. Solow, *Linear Programming and Economic Analysis* (New York: McGraw-Hill, Inc., 1958), 527 pp. Also, H. W. Kuhn and A. W. Tucker, "Nonlinear Programming," *Proceedings of the Second Berkeley Symposium on Mathematical Statistics and Probability*, ed. J. Neyman (Berkeley: University of California Press, 1951), pp. 481-492.

<sup>4</sup> Though oriented to two "stages" and a particular product, the analysis may be extended to additional stages, and to other products, involving similar considerations.

<sup>5</sup> A detailed analysis of lima bean assembly and processing operations and costs is given in: Robert H. Reed, *Economic Efficiency in Assembly and Processing Lima Beans for Freezing*, California AES Mimeographed Report No. 219 (Berkeley, 1959), 106 pp.

<sup>6</sup> Reed, *op. cit.*, pp. 78-80.

for seasons of different length are presented later (the paper.) Carrying through the substitutions noted, the *daily* cost equations for vining and in-plant processing are as follows:

$$(3) \text{ TDC}_1 = C_1 = \$98.225 + \frac{\$65.825Q}{H_1} + \$0.3691H_1 + a_1Q$$

$$(4) \text{ TDC}_2 = C_2 = \$383.825 + \frac{\$46.750Q}{H_2} + \$27.6177H_2 + a_2Q$$

The optimum combination of daily hours and output rates is obtained by minimizing  $C = C_1 + C_2$ , subject to the constraints of a maximum of 16 hours operation per day and a maximum temporary storage period of 8 hours per day.<sup>7</sup>

Thus:

$$H_1 - H_2 \leq 8$$

$$H_2 - H_1 \leq 8$$

$$H_1 \leq 16$$

$$H_2 \leq 16$$

These constraints are graphically depicted in Figure 1. Every point on the graph corresponds to a pair of values for  $H_1$  and  $H_2$ . Any point inside or on the boundary lines of the figure (OABCDE) corresponds to combinations of  $H_1$  and  $H_2$  that simultaneously satisfy all the constraints.

The solution to this problem is simplified by temporarily assuming there are no effective restrictions on daily hours of operation and that all variables except output and total cost are independent of hours operated per day. Then  $H_1$  and  $H_2$  are free to vary up to 24 hours per day with no increase in cost rates and equations  $C_1$  and  $C_2$  may be minimized separately, that is:

<sup>7</sup> As the unit cost of temporary storage operations averages less than five-tenths of a mill per pound, the total daily cost function ( $C = C_1 + C_2$ ) was not adjusted for these costs for each of the constraints on  $H_1$  and  $H_2$ . Adjustment of the daily cost functions to account for variations in temporary storage costs as the constraints vary would have no significant effect on the solution obtained. Where temporary storage costs are important, however, costs should be adjusted to reflect such variation, or included in the analysis as an additional "stage" or component cost function.

$$\frac{dC_1}{dH_1} = \frac{-65.825Q}{H_1^2} + 0.3691 = 0$$

$$\frac{dC_2}{dH_2} = \frac{-46.75Q}{H_2^2} + 27.6177 = 0$$

$$H_1 = 13.354(Q)^{1/2}$$

$$H_2 = 1.301(Q)^{1/2}$$

Thus, the locus of cost-minimizing combinations of  $H_1$  and  $H_2$  for different values of  $Q$  is given by the equation:

$$(5) \quad H_1 = 10.264H_2$$

If total daily volume ( $Q$ ) is allowed to increase from zero along the "expansion path" defined by equation (5), the extent of movement is constrained by the limitation  $H_1 - H_2 = 8$ . The values for  $H_1$  and  $H_2$ —and consequently, for  $R_1$ ,  $R_2$ , and  $Q$ —for which this constraint first becomes binding, are found by solving the pair of equations:

$$H_1 - H_2 = 8$$

$$H_1 = 10.624H_2$$

These equations imply that  $H_1 = 8.86$ ,  $H_2 = 0.86$ , and  $Q = 0.440$ , which define point F in figure 1. Plants of such low capacity are below the range found in actual processing operations.

For larger volumes of daily output ( $Q$ ), the constrained cost-minimizing expansion path follows the line  $H_1 - H_2 = 8$ . The next step then is to minimize the function  $C = C_1 + C_2$  subject to the linear restraint  $H_1 - H_2 = 8$ . To do so, let  $\theta = C - \lambda(H_1 - H_2 - 8)$ , where  $\lambda$  is a Lagrange multiplier.

$$\frac{\partial \theta}{\partial H_1} = \frac{-65.825Q}{H_1^2} + 0.3601 - \lambda = 0$$

$$\frac{\partial \theta}{\partial H_2} = \frac{-46.75Q}{H_2^2} + 27.6177 + \lambda = 0$$

Adding these equations and clearing of fractions results in

$$(6) \quad -65.825QH_2^2 + 27.9868H_1^2H_2^2 - 46.75QH_1^2 = 0$$

This equation can be solved explicitly for  $H_1$  and  $H_2$  noting that  $H_1 = H_2 + 8$ . This gives a 4th degree equation, however, and it is easier to specify

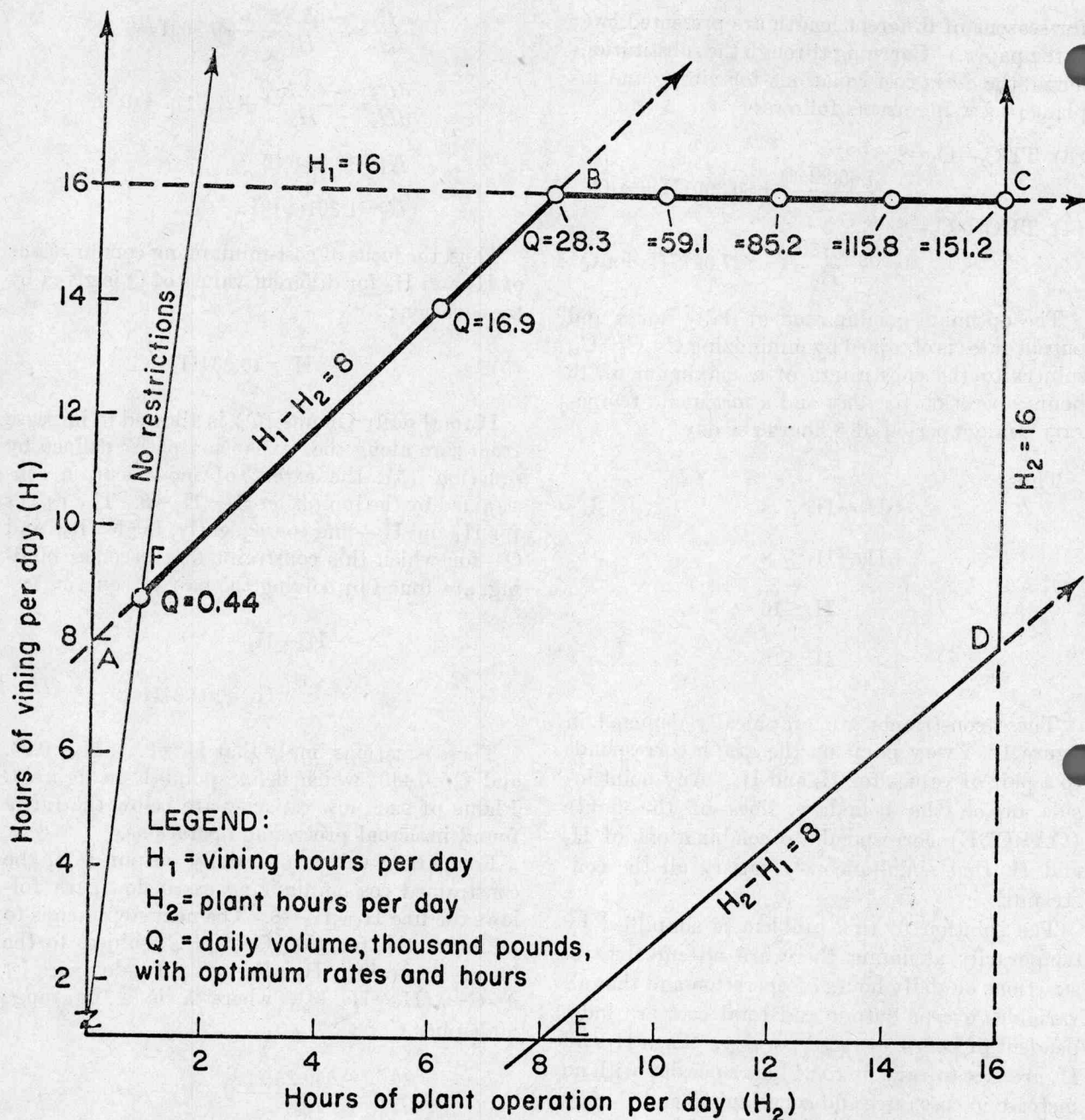


FIGURE 1.—Feasible and optimal combinations of daily hours of field and plant operations for frozen lima bean processing, California.

a point on the line ( $H_1 - H_2 = 8$ ) and use the above equation to find the corresponding value of  $Q$ . For example, if  $H_1$  is set at 14 hours and  $H_2$  at 6 hours, the above equation implies that  $Q = 16.9$ .

Movement along the line  $H_1 - H_2 = 8$  can proceed with increasing daily volume,  $Q$ , until  $H_1$  reaches the specified maximum of 16 hours. At this point (point B, figure 1), the value of  $H_1$  is 16 and the value of  $H_2$  is 8. With these values of  $H_1$  and

$H_2$ , the daily volume  $Q$  implied by equation (6) is 28.3 and the expansion path becomes the horizontal line  $H_1 = 16$ . With  $H_1$  fixed at 16 hours per day, C can be minimized for  $Q$  greater than 28.3 by minimizing  $C_2$  independently, or as derived above,  $H_2 = 1.301(Q)^{1/2}$ . For example, if  $Q = 85.2$ , the cost-minimizing value of  $H_2$  is 12 hours.

As total daily volume increases further,  $H_2$  can expand to its absolute limit of 16 hours (point C,

TABLE 1.—*Minimum-cost combinations of hours operated and rates of output for field and plant operations in processing lima beans for freezing, three lengths of operating season, California, 1958*

Hours operated per day		Output per hour		Daily volume  (Q)	Season volume  (q)
Field (H <sub>1</sub> )	Plant (H <sub>2</sub> )	Field (R <sub>1</sub> )	Plant (R <sub>2</sub> )		
30-DAY SEASON					
<i>Hours</i>	<i>Hours</i>	<i>Pounds</i>	<i>Pounds</i>	<i>Thousands pounds</i>	<i>Million pounds</i>
16	8	1, 328	2, 657	21. 254	0. 637
16	10	2, 769	4, 430	44. 304	1. 329
16	12	3, 993	5, 324	63. 890	1. 917
16	14	5, 428	6, 203	86. 848	2. 605
16	16	7, 089	7, 089	113. 418	3. 403
16	16	9, 375	9, 375	150. 000	4. 500
16	16	12, 500	12, 500	200. 000	6. 000
16	16	18, 750	18, 750	300. 000	9. 000
16	16	25, 000	25, 000	400. 000	12. 000
40-DAY SEASON					
16	8	1, 771	3, 542	28. 338	1. 134
16	10	3, 692	5, 907	59. 072	2. 363
16	12	5, 324	7, 099	85. 186	3. 407
16	14	7, 237	8, 271	115. 797	4. 632
16	16	9, 452	9, 452	151. 224	6. 049
16	16	12, 500	12, 500	200. 000	8. 000
16	16	18, 750	18, 750	300. 000	12. 000
16	16	25, 000	25, 000	400. 000	16. 000
50-DAY SEASON					
16	8	2, 214	4, 428	35. 423	1. 771
16	10	4, 615	7, 384	73. 840	3. 692
16	12	6, 655	8, 874	106. 483	5. 324
16	14	9, 046	10, 339	144. 746	7. 237
16	16	11, 814	11, 184	189. 030	9. 452
16	16	12, 500	12, 500	200. 000	10. 000
16	16	18, 750	18, 750	300. 000	15. 000
16	16	25, 000	25, 000	400. 000	20. 000

figure 1), which corresponds to  $Q=151.2$ . No further adjustment of  $H_1$  and  $H_2$  is possible as  $Q$  expands and increasing daily volume beyond  $Q=151.2$  can only be achieved with proportional increases in the hourly output rates (size of vining and plant facilities).

The number of days operated per season has an important effect on the least-cost combination of daily operating hours and size of facilities. The effect on rates and hours is directly proportional to the length of season. With a 30-day operating season, for example, the total daily volume ( $Q$ ) corresponding to point C of figure 7 is 113.4, exactly three-fourths of the value for  $Q$  with a 40-

day operating season. Table 1 gives selected values for combinations of hours and rates of field and plant operations, both daily and seasonal, for operating seasons of 30, 40, and 50 days.

The above example suggests that total combined costs of field and plant operations are lowest when storage time and field hours operated per day are maximized. The relatively low level of economies of scale found in the field operations suggests the same conclusion. In situations where assembly and storage costs are more important components of the total cost picture, a complete analysis would require their explicit consideration.