# An Approach to Production Response

By Richard H. Day

**Nonnormative models of aggregative agricultural production are frequently based on the correlation of output variables with prices (usually lagged) and outputs lagged over time. "Structural" models of this kind relate these correlations indirectly to the behavioral and technical structure that gives rise to them. The main purpose of this paper is to show how a knowledge of structure may be used to construct a model that simulates the aggregate actions of producers. The resulting model is useful in explaining past production patterns, in predicting future changes, and in assessing effects of alternative policies. A further purpose is to provide a theoretical understanding of the many forces that determine production through time. The author develops certain hypotheses of long standing in economic theory in an explicitly empirical and institutional context. He notes the following three as among the most important: (1) The neoclassical concept that firms maximize something when they determine output; (2) the Marshallian relation of investment to past profits and quasirents; and (3) the Schumpeterian relation between investment and technological change. To these he adds the use of behavioral constraints that arise from uncertainty and lack of knowledge. Some of the observations and principles described in the paper are empirical rediscoveries of certain theoretical principles cogently set forth by Schumpeter (*12*).[1] The method is also closely related at some points to the work of Leontief (*6, 7, 8*), Wood (*14*), Henderson (*5*), Georgescu-Roegen (*3*), Marshall (*9*), Walras (*13*), and Nerlove (*10*).** **The empirical research out of which the new approach took form owes much to staff members of the Farm Economics Division, Economic Research Service. Thanks are due the National Cotton Council for support in the preparation of this paper.**

PRODUCTION RESPONSE is the term that research workers use to describe the study of forces that have determined past production patterns and may govern future ones. As it includes forecasting the repercussion on production patterns of actual or potential policy actions, production response belongs to the descriptive and predictive branch rather than to the normative branch of economic science. The term "production response" has been used for many years in this broad context to distinguish its content from that of more narrowly oriented studies of supply response to price.

Recent articles by students of production response have emphasized the complex nature of production in agriculture (*11*). Many methods have been used, with varying degrees of success, to describe and analyze production response phenomena (*4, 6, 7*). In a forthcoming book based on my dissertation completed last year, I describe a new approach to this problem.[2] This approach uses a dynamic mathematical system called "recursive programming" to describe and interpret the production patterns of an agricultural region over time. Briefly stated, recursive programming consists of a sequential chain of recurring linear programming problems in which the structural components of each year's problem depend on the solution for the preceding year. It involves optimizing over a limited time horizon on the basis of knowledge from the experience of the past with regeneration and reformulation as each year's experience is accumulated. The model enables a wide variety of forces that are related to production changes to be analyzed together.

Observations and hypotheses about economic behavior from which the model grew are given in this article. It describes how the model accounts for some of the major economic forces related to changes in agricultural production. Many of these observations and hypotheses have played a major role in general economic theory and in discussions of agricultural economics. In

[1] Italic numbers in parentheses refer to Literature Cited, page 148.

[2] Day, R. H., Recursive Programming and Production Response. 1961. On file Harvard University, Cambridge. The book, which is based on the dissertation, is to be published in the series, "Contributions to Economic Analysis," North-Holland Publishing Company, Amsterdam—C, The Netherlands. Also described in (*4*) pp. 108–125.

this approach they are empirically verified by relating them to what actually occurs at a regional level as a result of farm producers' real economic decisions [3] and they are given formal mathematical expression so as to generalize our theoretical understanding of the economics of production.

The discussion is divided into four major sections. The first deals with the optimizing principle; the second describes certain aspects of the farmer's decision environment; the third describes how investment in variable, quasifixed and fixed production inputs appears to operate in agriculture, and how technological change occurs; and the fourth describes nonmathematically how a recursive programming model is based on the observation presented in the preceding sections. A publication describing the empirical results of this model, now in preparation, will include a discussion of techniques used in estimating the various structural parameters in the model.[4] Attention here is confined to its methodological properties.

## The Optimizing Principle

Let us begin with what is perhaps the most novel—and controversial—aspect of the method, the application of the optimizing principle to the description of actual economic behavior. Statistical methods for estimating supply relationships have typically disguised the role of this principle, and it has come to occupy an ambiguous position in both empirical and theoretical understanding.

### Do Farmers Maximize?

Inadequate knowledge of technical possibilities and of the complex forces affecting prices and costs of production surely make true optimal action as difficult to determine for farmers as for economists. But to explain actual planning behavior, it is virtually impossible to dispense with profit maximizing. For example, a small southern renter with a 15-acre operation and a 3-acre cotton allotment, who refuses to abandon farming even in the face of a substandard income, attempts to allocate his remaining acreage among such alternative crops as he believes to be economically best. To be sure he does this with little certainty that his expectations will be realized; but do it he must, as best he can.

The more one becomes acquainted with farmers of varying types and varying success in operation, the more one is impressed with the validity of the optimizing principle. It is not exercised as a tool of ideal optimal choice. Rather, it seems to be a homely guide to choosing among limited alternatives in an environment that is only partly the result of a farmer's own volition.

This empirical hypothesis that farmers attempt to plan as best they can is the foundation for explicitly using the optimizing principle in nonnormative analysis.[5]

### A Theoretical Imperative?

Spatial equilibrium or regional competition models of production are examples of explicit uses of the optimizing principle (*4*, pp. 203–227; *2*). In their current forms, however, they are both static and normative in nature. Both have limitations for the problem under consideration here but in application both can be alleviated to some extent. Thus, the results of such models can be viewed as long-run equilibria and may be used to project the direction in which production patterns are likely to move. Therefore, this kind of comparative static analysis is a useful tool for nonnormative analysis if the empirical hypothesis of optimizing behavior is accepted.

Statistical models of supply, on the other hand, do not describe maximizing behavior directly. The results of these models can be evaluated *statistically* in a relatively straightforward way, an advantage not currently shared by programming models of regional competition. The *economic* evaluation of such results poses a more difficult problem. It is here that practitioners of the statistical arts are typically vague. Some argue that the statistical method relieves their studies of certain aggregation problems (a subject we shall re-

[3] Some of these hypotheses have been regarded as valid for normative or prescriptive analysis and have been utilized for this purpose (*4*, pp. 203–227) (see Day, footnote 2). Our application is to nonnormative or descriptive and predictive economics.

[4] For preliminary empirical results, estimation techniques, and mathematical analysis, see Day, footnote 2.

[5] This hypothesis has been tested both formally and informally, as have those described later. It was tested formally in detailed discussions about planning procedures with a randomly selected group of farmers in the Mississippi Delta. I rely also upon informal conversations with Iowa farmers over a period of many years. Both sources support the validity of the optimizing principle as it is described in the paragraphs preceding, and in those that follow.

turn to below) and that "aggregative behavior" differs, after all, from the behavior of individual firms (*11*). But when it comes to economic evaluation, they often rely upon the optimizing principle. Thus, negative price elasticities of supply are usually regarded as unmeaningful, positive ones as valid. This conclusion rests on the idea that it does not make sense for producers to expand production of a commodity when its expected price is falling. That is, such an inverse supply response does not conform to our idea of what farmers need to do in order to maximize their incomes.[6]

A reference to the optimizing principle must be made in order to evaluate the economic meaningfulness of statistically fitted supply models. Thus its explicit application to predictive and descriptive analysis is not as novel as might first appear.

Models that explicitly employ this principle are usually called "programming" or "mathematical programming" models. They include linear, nonlinear, stochastic, and dynamic programming models, and also the method described here, recursive programming. Applied to agricultural production response, recursive programming can express the hypothesis that farmers make their production plans in order to maximize their incomes. But they do this with due regard for the uncertain nature of output, prices, production costs, and yields of their various enterprises. This "due regard" becomes the heart of the problem once the optimizing principle is accepted—that is, the question is not whether or not farmers optimize, but rather how the circumstances that condition their optimizing activity can best be described.

Although strong empirical and theoretical arguments favor the explicit use of the optimizing principle, one can be less emphatic in specifying just what is maximized. Current applications of recursive programming use expected gross profits, that is, expected gross income less current production expenses, including the opportunity cost of operator labor. Since any income or profit variable is closely related to this "gross profit," it is useful as a first approximation. Initial results using this variable suggest that it is substantially valid.

[6] This is true in a static sense only. It can be shown by means of recursive programming theory (*4*, pp. 108–125) that such inverse supply relations can, and frequently do, logically result from the optimizing principle placed in a dynamic context. The latter result is described below.

But it would be advisable to test other maximizing variables. Possibilities that seem particularly relevant would be the total discounted gross profit over several time periods, an income measure from which the opportunity cost of nonwage labor has not been deducted, or a measure from which allowances for depreciation and other "fixed" costs have been deducted.

It would follow from the pure economic theory of the agricultural firm-household that a utility function should be maximized in which household as well as production activities are included. But the overwhelming difficulties of applying this approach for empirical analysis at an aggregative level appear—at least at present—to preclude its consideration.

### Planning Over Time

Production plans for a given production period depend upon both the past and the future. Past decisions result in the current stock of resources and assets that form the basis on which current plans for the future are contingent. On the other hand, current plans also include anticipation of actions to be taken in future periods. The number of future periods for which the producer accounts in his current plans is commonly called the *time horizon* of the production plan. Because of the extreme uncertainties in agriculture, this horizon is typically short, and not owing to myopia, but rather to the rapidly decreasing certainty of anticipated actions being realized, as plans are extended further into the future.

The theory of planning over time was introduced into the history of economic theory comparatively recently, but it has undergone an extensive and highly esoteric development in dynamic programming literature. The application of this theory to the aggregative empirical problem under consideration here is beset with difficulties. This is particularly so in the choice of a time horizon and the weighting of anticipated decision criteria at the regional level, to represent accurately the time horizons and weights that are formulated at the firm level. Further research may show, however, that it is useful to attack these difficulties and to account explicitly for time horizons and their accompanying uncertainty discounts.

In either case, the important distinction between planning over a fixed time horizon (dynamic programming) and the sequential generation of production plans (recursive programming) should be noted. The former determines plans that are to be executed in the current period, and anticipates plans that are *currently* designed for future execution. The realities of economic life, however, force changes in even the best laid plans. So, with the passage of time, future plans are revised in light of current and temporary conditions. A model that reflects that actual course of economic planning as it affects actual production decisions must provide for the continual regeneration of the planning process. Recursive programming models do this, even when they include planning over time, and consequently must be distinguished from dynamic programming models.[7]

## The Decision Milieu

We must now turn to consideration of the environmental forces that influence maximizing behavior. If farmers' activities are to be described adequately, and more important, if the aggregate results of their decisions are to be explained and predicted with useful accuracy, it is necessary to account for the salient features of their economic environment. This section discusses three important aspects of the farmer's decision milieu: (1) Market structure, (2) uncertainty, and (3) management leadership.

### Market Structure and Aggregation

Agriculture is one of the very few of the industrial and commercial sectors in the United States that resemble the market structure assumed by classical theory of perfect competition. For most major farm commodities, large numbers of producers grow and market substantially homogeneous goods. None has the power to influence appreciably the market prices for outputs. Moreover, producers rarely exercise any real control over the prices of productive inputs. Indeed, it is largely these phenomena of an atomistic industry that have enabled the econometrician to apply his methods to agriculture with considerable success; and it greatly enhances the explicit application of the optimizing principle for explaining actual behavior.

Because farmers cannot influence prices through independent action, they plan output given certain price expectations. Price is a predetermined variable in the decision process. The many theorectical and empirical problems of oligopolistic and oligopsonistic behavior are not confronted. The output decisions of each producer can more hopefully be aggregated into a single regional decision problem.

An aggregate composed of farming units with identical linear technical possibilities and with price expectations and supplies of quasi-fixed and fixed factors that differ only by factors of proportionality could be exactly described by a single regional decision model.[8] This concept becomes the prototype for the explicit application of the optimizing principle. Recursive programming models must be applied to geographical or type-of-farm aggregates that are homogeneous enough to avoid serious distortion. A hypothesis that causes a considerable degree of effective homogeneity among farmers in a given area is described below. Before turning to its consideration, however, let us first examine some aspects of market structure that lead to "imperfections" in atomistic competition.[9]

### Uncertainty and Imperfect Knowledge

As suggested above, farmers are well aware that their incomes seldom turn out to be those planned, that price expectations are seldom realized. Their best expectation of output prices is uncertain. Contrary to this fact, the neoclassical analysts of competition assumed something called "perfect knowledge," an assumption that even though demand schedules were not known, the equilibrium prices that *would be* obtained were known exactly. This theoretical acrobatic was

[7] How both planning over time and the sequential generation of planning can be treated together by means of recursive programming has been shown elsewhere (*4*, pp. 108–125; and *6*. See also Day, footnote 2).

[8] By identical linear technical possibilities we mean that all farmers' production structures can be presented by identical process analyses or linear programming models. We mean by a factor of proportionality that all farmers use the same quasi-fixed factors and that the stocks of these factors which a given farmer possesses vary from those of another farmer by some percentage constant for all factors.

[9] An atomistic market is one in which all members produce a homogeneous product, and in which none can influence prices in his own favor by his own production and marketing decisions. Perfect competition implies much more; cf. Chamberlin (*1* p. 6).

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needed because the neoclassical economists had no well-developed tools of dynamic analysis. Such an assumption was required to obtain equilibrium solutions to static market models. Thus the mathematics of perfect competition lack the requirements to describe actual planning behavior. Applications of conventional linear programming to long-run normative analysis must logically rely on a similar assumption that long-run equilibrium prices are known with perfect certainty. If this were not so, their results could hardly be regarded as long-run normative solutions.

Recursive programming gives the optimizing principle a different context by adopting the hypothesis that farmers' plans are made on the basis of expectations derived from past experience, which at best will only be partially fulfilled, and further, that these plans are influenced by *imperfect* knowledge of demand.[10]

The first part of this hypothesis is common to nearly all empirical expectation models that use past prices discounted in some way for uncertainty. In a linear programming model with a finite number of alternatives, predictions of aggregative behavior could be grossly unstable if only this part of the hypothesis were included. In some agricultural regions certain rather specialized alternatives, even at *discounted prices*, are relatively more profitable than others, and usually by substantial margins. Consequently, unless explicit upper and lower bounds are set to reflect uncertainty, a programming model will predict much greater specialization for a given year than is commonly observed in many areas of the country. These bounds have the effect of limiting the concentration of the most profitable crop on a farm or in a region.

Intense regional specialization in one year, however, would cause a fall in prices received (in the absence of supports) through its effect on aggregate production. The most profitable crop for the next year might now appear to differ from the most profitable one for the first year, and intense specialization might then occur in a different crop. In short, such a model would predict violent yearly oscillations in production patterns. Since agricultural production as measured by acreage does not oscillate violently from year to year, something in addition to price discounting must be included in a production-response model of the linear programming genre. As upper and lower bounds in a linear model will limit the production changes implied by a given set of net returns, we regard the formation of such bounds or constraints as a part of the decision process. How or what is the information by which such contraints are formed constitutes the problem.[11] It is in the imperfect knowledge of demand that we find the required ingredient.

Informal and formal discussions with Iowa farmers and with sharecroppers, small farmers, and large plantation operators in Mississippi suggest that farmers everywhere realize that output and price are related and that an aggregate shift in production from one commodity to another or in the general level of output has, in the absence of price supports, a definite and inverse relation to prices. A farmer knows also that what is good for him is commonly good for many other farmers in the same producing region. Therefore, he knows that the intensity of a relatively profitable alternative may be increased by many farmers and that the net result may be to damage his profit position if he specializes too extensively. This *imperfect* knowledge of the effect of aggregative supplies on price leads to restraint in changing output patterns in response to relative discounted price expectations. Such restraints can be explicitly provided for in current recursive programming models of production by means of dynamic upper and lower bounds. These bounds are called flexibility constraints. They are described below.

**Management Leadership and Innovation**

A third important characteristic of the decision process in agriculture that has particular relevance for the direct application of the optimizing principle to nonnormative analysis is the role played by the efficient producers and innovators of a given

[10] For many years, statistical supply models have accounted for price uncertainty by using various expectation models. Cf. Nerlove (*10*). The attempt here is to introduce this approach in an optimizing model of production. Various models of stochastic programming have also attempted to account for uncertainty, as in a different context have various models of portfolio selection. For the most part, these have been normative in nature.

[11] Henderson (*5*) was the first to recognize the importance of such restraints. He used them as "catchall" bounds to describe the constraining influence of uncertainty, lack of knowledge, and quasi-fixed factors, whereas we treat these forces separately and in an explicitly dynamic context.

area. Most counties in any important agricultural region contain some farms that are significantly more efficient and prosperous than others, partly because of superior natural resources, but largely owing to superior management. These efficiently operated farms tend to become planning models for those producers who are only modestly gifted in the art of farm management.

The managers of these efficient operations are frequently in close touch with agricultural experiment stations and other sources of expert knowedge of advanced farming techniques. Ordinarily, they are not the first to apply a new production technique and are thus to be distinguished from innovators. But they are commonly among the first to adopt a successful innovation and, conversely, they seldom adopt an unsuccessful one. Most producers are slower to see the economic advantage of a given innovation. They are more cautious in the application of a new technique and they require more time to become convinced of its economic advantage. The same observation can be made of the process of altering livestock and cropping patterns.

For these reasons, response to changing economic conditions is commonly distributed through time. This is true of the response to both changing technical possibilities (innovations) and other changing sources of profits. The proper application of the optimizing principle must therefore be made in the context of this dynamic adjustment process.

The systematic search for optimum production plans seems to be an activity confined to a relatively small number of producers, and the resulting decisions guide the great body of farmers (in the way just described) in their own decision-making activities. The important implication of these observations is that the aggregation problem should be greatly reduced; in a given region, economic variables should behave as if they were the result of a much more homogeneous group of producers than is actually the case. Empirical results obtained appear to confirm the validity of this hypothesis.

## The Dynamics of Input Utilization

An observation that must occur to any student familiar with the broad spectrum of econometric production investigations, is that the production economists seem to have been concerned almost exclusively with details of structure and not with aggregative implications. The macroeconomists, however, have too frequently confined their research to the behavior of aggregative variables without examining the structure by which the movements of such variables are conditioned. It is true, they have attempted to derive structure indirectly by use of "structural" statistical models. But efforts in this direction have not supplanted the need for direct exploration of the dynamic structural nature of the farm economy.

The awareness of this impasse undoubtedly explains the increasing effort in recent years to dig beneath the surface of aggregative variables and to explore the basic input-output relationships and their relationship to the movements of aggregate production data over time. Technical description of input-output relationships has reached a high state of development and we have more in the way of meaningful hypotheses here than elsewhere. Budgeting analyses, statistical production function analyses, and more recently input-output and process analyses now provide a rich source for analyzing the ultimate conditions upon which rest the dynamics of production.

Therefore, I shall emphasize the relationships between the *technical* structure of production—as revealed through analyses of the kinds mentioned—and the dynamics of investment in the productive inputs themselves. For this purpose, it is convenient to follow the common classification of production inputs; inputs that are variable factors, such as labor services, fertilizer, and other materials normally used up in one production period, and inputs that are quasifixed and fixed factors not used up in one production period. When farmers depend to some degree upon the market for money capital, these two categories correspond roughly to the two major types of money loans connected with production, production loans and investment loans.[12]

### Variable Factors

The demands for variable inputs, such as fertilizer, labor, and other production materials, are related to the technical structure of production and the prices of both outputs and inputs. The former determines the quantities of inputs required to produce various quantities of outputs. The latter

[12] Glenn Johnson, Earl Heady, Clifford Hildreth, and Marc Nerlove are among those who have attempted to recognize explicitly the role of quasifixed and fixed factors in agricultural production response.

determines the profits from alternative production plans. However, the actual demand for a given input or, in other words, the planned utilization of a given input, also depends upon the supplies that are expected to be available of *all* variable, quasi-fixed, and fixed inputs. This has been amply demonstrated by past responses of farmers to restrictions in the supply of land imposed by acreage controls. For example, the increased intensity of fertilizer use appears to have been accelerated by the effects of a declining supply of land available for production of controlled commodities. To explain the relationship of production to the corresponding use of variable factors, the economist must understand what limits the supply of productive inputs at any given time.

It has been argued that the supply of variable inputs does not affect production, that is, in the short-run—the "run" in which all plans are actually consummated—these inputs have an unlimited availability. Further, it is sometimes supposed that they have an unlimited availability at *existing prices*. But farmers are too well acquainted with their local factor markets to suppose that this is the case. They know from experience that at least some inputs are in restricted supply within the range of prices they can pay.

Just as they limit the production of profitable outputs because of known but uncertain relationships between aggregate supply and output prices, so they limit their demand for inputs because of correspondingly known but somewhat less certain interaction between input supplies and input prices. A Mississippi Delta farmer explained to me that he was investing in cottonpickers, even though at existing prices hand labor was cheaper, because he knew that the local supply of labor was falling and to induce labor from surrounding areas to pick his cotton he would undoubtedly need to pay an uneconomic price.

Demands for inputs are dampened by the effects of such considerations, or we might say the supplies of inputs expected at prices farmers are willing to pay are limited. To the extent that purchases of variable inputs are financed from production loans, this dampening takes the form of credit rationing. Farmers on the other hand are led to ration their demand for money capital, while local banks on the other hand are led to ration their supply of credit.

Migration of agricultural laborers to urban areas, particularly since World War II, has caused the supply of available labor services to decline. The constraining effect of this migration depends upon the rate of adoption of labor-saving technology. If the rate of adoption is sufficiently high, the supply of labor is in effect unlimited and no effective labor constraint arises. But labor may constrain output in periods of low investment in labor-saving technology. For our purposes, we need not commit ourselves to the inclusion of one effect or the other. Rather we attempt to include the supply of labor through time as a part of the dynamic structure of production and then derive the actual times and places and hypothetical conditions in which one or the other of the possible effects can be observed.

A third force leading to short-run constraint in the supply of a variable input arises in a period of rapid expansion in the demand for that input. Its production follows a growth process of its own. Available supplies of such a factor are limited by this process. This kind of input supply restraint is particularly evident in periods of technical innovation and adoption when the supply of laborers with certain skills is limited by the rate at which these skills can be disseminated or, for example, when the supply of a new type of fertilizer or insecticide is restricted by the rate at which capital formation takes place in the (possibly new) industry producing it.

I have mentioned three forces that may lead to restriction of variable input supplies in the short-run. One is the effective supply of a factor within a price range a farmer is willing and able to pay. The second is the absolute decline in physical capital from which production services as variable inputs can flow. The third, which is logically similar to the second, is the possibility that the rate of growth in supply of a variable input may be restricted because of the process by which the industry producing it expands. In connection with the last two points, we have discussed the relation of *physical* capital to the supply of variable inputs, but we have not yet discussed *money* capital, and we now turn our attention to this subject.

The process of investment cannot be ignored even in considering the short-run production plan of the firm. This is because a farmer must allocate any money capital he may acquire between the purchase of variable and quasifixed and fixed inputs—that is, between short- and long-run investments. Here again some argue that this area may be ignored because the supply of money capital is practically unlimited. My conversations with farmers, however, have suggested that, even production loans are often hard to come by.

In one such conversation a person told me he had farmed during the immediate postwar period. He was denied further advances from the local bank after a single failure to repay the year's production loan. It would be desirable, therefore, to include in a production response model a dynamic variable describing available supplies of money capital. Such a variable would act as an upper bound, changing through time, on the total regional effective demand for money to invest in productive inputs. It would be related to such variables as past years' regional income and interest rates. Current recursive programming models do not in fact include such a relation, but the methodology can accommodate it. Further efforts should be made to remove this inadequacy.

### Quasifixed and Fixed Factors

In a developed region the supply of land is limited and can be treated realistically as a fixed factor, but in an underdeveloped region investment in land need not be considered as differing essentially from investment in machinery and buildings and other less durable inputs. Even in a developed region, land can be regarded as a quasifixed factor. Erosion and nutrient leaching diminish its productivity, which can be maintained only through investment in various conservation improvements. This is analogous to depreciation on less durable inputs. Consequently, in what follows, we may restrict attention to quasifixed factors.[13]

Quasifixed factors (durable production inputs) are not entirely used up in the production period but yield a flow of services. The quantity of services flowing from a given machine is limited by the machine's capacity. The capacity of a quasifixed factor is particularly difficult to define or to measure. A given factor in many instances can be more or less intensively utilized, but often it is possible to define a rough measure that is useful for empirical investigation. The capacity of plowing services for a four-row tractor, for example, may be regarded as the number of acres it can plow under normal conditions in an average working day.

The demand for services of quasifixed factors, and consequently the demand and supply of quasifixed factors themselves, is governed essentially by the same forces as those which govern the demand and supply for variable factors. The amounts of their services required for various levels of output, together with prices, determine the demands for such inputs, and the three forces and money capital discussed in the preceding section operate in much the same way. However, the durability of these factors requires some additional consideration.

As quasifixed factors can yield services not only in the current production period but also in subsequent periods, decisions to invest in them must account not only for current but for anticipated future production plans. Since future prices (and Government-control policies) are even more uncertain than those for the imminent production period, the number of periods in the future that need to be accounted for is usually few. The problem of choosing a finite time horizon for economic analysis, even one of relatively short length that would adequately represent the effects at a regional level of farmer's individual decision processes, may be even more difficult than measuring the capacity of durable inputs. However, there is a second approach that has proved useful in current regional applications of recursive programming. It enables us to account for investment in quasifixed factors without explicitly accounting for a time horizon longer than a single time period.

Suppose a given process of production is particularly profitable, but that the capacity of a certain quasi-fixed factor whose services are required by the process is limited within a given region. For example, suppose harvesting soybeans with self-propelled combines is highly profitable but only a few are owned by farmers in the region. The maximum number of machines required to harvest the desired acreage of the crop will not be purchased immediately, partly for the reasons already indicated, but mainly because farmers do not want to have too large a stock of this capital good in succeeding years if soybean prices should

[13] Note also that in an agriculturally developed region, urban encroachment will force treatment of land not as a fixed, but as a quasifixed factor. Current models have not yet accounted for this phenomenon.

fall. If the process continues to be profitable, increasing numbers of machines will be purchased.

It has been observed empirically (*14*) that, in the face of continuing profitableness over a long period of time, the purchases of a given capital good will grow at something like a geometric rate over time. In agriculture this rate will depend upon the rapidity with which farmers can accustom themselves to the new machines, and to the growth process of the supplying industry. This may be called the *maximum potential growth principle.* It defines a maximum limit by which investment in quasifixed factors may be constrained in a given region. This maximum limit, as indicated, need not actually be attained. Actual investment in a particular input may fall below the rate regarded as possible under this principle, and frequently it will do so.

Other forces, such as a limited supply of complementary variable, quasi-fixed, and fixed factors, may constrain production. Consequently, at some time, or perhaps always, investment may fall below the potential rate. Thus, within the framework of these constraints, actual regional investment can be determined. As before, we do not commit ourselves to one or another of the possible effects of the limitation on the supply of quasifixed factors. Instead, we ask under what conditions they will or will not affect the output of a given commodity.

The optimizing principle applied at the regional level implies that the stocks of quasifixed factors predicted for the region can be distributed among farmers in an optimal manner. In many regions, this condition is brought about approximately by an active custom market for the services of quasifixed factors. To the extent that such markets do not bring about a free movement of such resources, the model will be biased.

### Technological Change

Technological change can be conveniently broken into three components—invention, innovation, and diffusion or rate of adoption. The first of these three components is of crucial importance in the growth process, and it is particularly intractable to existing tools of economic analysis. But for the problem of aggregative production analysis, it need not be explicitly considered. Only in the hands of an innovator does an invention enter the production process. Consequently, treatment of technological change begins with the second component.

It is almost as difficult to predict innovations as inventions. This need not detain us, however, since the number of innovators in a given region is typically very small relative to total number of producers. For this reason, initial impact on production is small.[14] This aspect of the problem is described earlier in this paper.

To study the effects of technological change on production, we begin with a historical period in which a major innovation, such as use of the cottonpicker or production of a new commodity, is introduced, and regard it as a *fait accompli.* That is, we treat innovation as a condition of the production structure historically given. We now turn to consideration of the third component.

In a developed region (or one subject to rigid production controls) in which the total outputs of its several commodities are either stable or declining, the adoption of a new innovation may not cause an expansion of production. The effects of adoption operate primarily on the use of other techniques of production now obsolete. Substitution takes place between new and old techniques, the old being gradually abandoned and replaced with the new, typically at an accelerated rate over time.

When total production of some outputs is expanding, an innovation may not at first be observed to replace old techniques. If production is expanding faster than the rate of adoption of the new technique, even the old techniques will expand for a time until the rate of adoption of the former surpasses that of output growth when the same substitution effects as before will be observed. It is also possible that the entire expansion of output will occur via the new techniques and that the old methods will remain relatively unchanged for a time, or perhaps begin an immediate decline. The actual paths of these variables are determined by all forces acting on investment and production in the region.

In order to understand the relation of technological change to production, it is evident that the

[14] This is not to imply that the economics of innovation should not be studied. We can proceed with an analysis even though adequate models of innovation do not yet exist. The effects become increasingly important as diffusion takes place.

process of diffusion needs to be understood. The decision to invest in a machine not previously utilized, or to produce a commodity not previously cultivated, is governed by much the same forces as determine investment in available and quasifixed factors already commonly used throughout a region. Consequently, all those forces already discussed can be expected to operate in this sphere. Indeed, I shall take as the fundamental hypothesis about the process of technological change that it needs to be treated as an integral part of production and investment planning, and that the analysis of variable and quasifixed factors applies with equal validity to its analysis.

But innovations are characteristically new and unfamiliar, and this distinction between them and the older techniques leads to an additional force acting on investment and production that cannot be ignored. This force is the progress of technical know-how and confidence in the advantage of the innovation which must accompany its diffusion. Diffusion is limited by this progress, and unless it is accounted for explicitly, investment in new techniques is frequently and significantly overpredicted.

Confidence, familiarity, and knowledge are acquired more gradually by the great body of producers than by the innovators or rapid adopters, so that innovation is frequently observed to follow something resembling a geometric or compound interest growth curve—if its high profitableness continues. Furthermore, the rate of its growth appears to depend upon its newness. Thus a radically different innovation is likely to be adopted more slowly, all other things equal, than a relatively minor change in an already familiar technique. For example, the diffusion of the use of liquid nitrogen occurred at a vastly greater rate in many areas than the earlier diffusion of the use of solid nitrogen fertilizers.

As the magnitude of additional profits induced by adoption (marginal returns to investment) typically declines, and as diffusion continues, the availability of complementary factors may become limited, and uncertainty as to continued profitableness may grow. After diffusion has continued for a time, therefore, it may be expected to exhibit a declining rate. This explains the familiar S-shaped curve observed in many diffusion or rate of adoption studies.

## The Method of Recursive Programming

How can these salient traits of production response be formalized and given empirical content? This leads to the construction of a recursive programming model for a particular economic area and thence to the mathematical and economic theory of recursive programming systems in general. Both aspects of this demonstration were developed elsewhere (*4*, pp. 108–125).[15] Here the discussion is limited to a description of how recursive programming simulates the economics of production as conceived in the preceding sections.

As in all econometric investigations, a particular model is a judicious compromise between the conceptual understanding of an economic process and the possibility of representing and testing this conceptual understanding with real data. It is one thing, therefore, to be satisfied with a model and another to be satisfied with the general theoretical system which the model represents. To distinguish these two aspects, the description of current models is followed by some further remarks on the pure theory of recursive programming. This will enable us to see how our understanding of production response may be formalized, even though current econometric models may not describe that understanding completely.

### The Processes of Production

The technical production processes available in a region in a given year are represented by their respective input and output coefficients. These coefficients measure the yields expected per unit of the process and the physical costs in terms of quantities of variable factors used up and the services of quasifixed factors employed per unit of the process. The processes are defined for machine combinations of a given technological stage, by soil class, and by a number of discrete fertilizer intensities.

Thus, separate processes are defined for each machine—soil class—fertilizer level combination for each crop.[16] Each process is measured in terms of acres. Each output coefficient is measured in yield per acre in suitable units such as bushels, hundredweight, and so on. Each variable input coefficient is measured in per-acre requirements in

[15] See Day, footnote 2.

[16] The fact that a nonlinear relation exists between the yield of a crop and the quantity of fertilizer used leads to no difficulty. (Day, pp. 93–100, see footnote 2.)

such units as hours or pounds. Each quasifixed or fixed factor input coefficient is measured in acres or production units.[17]

The complementarity of various farm enterprises can be accounted for in the construction of these processes. Thus production of feed for livestock could be treated as an activity generating a supply of feed that could be sold or used as an input in a livestock-producing process. In the Delta Model[18] corn production for draft animal feed was treated as complementary to processes producing cotton by means of mule-powered field operations.

### Net Returns

Expected net returns are measured in terms of expected gross returns per acre minus expected variable costs, assuming that these are evaluated at yields and input coefficients for normal weather. Thus, one obtains a measure of net returns expected with average weather. To account for the fact that farmers do not know the prices they will receive at the culmination of the crop year, a weighted function of preceding years' prices might be used. Current models use the simplest such expectation model, that is, prices lagged one year. This gives what net returns would have been in the preceding year if normal weather had prevailed as the measure of expected net returns per unit of each process for the given year.[19]

It should be noted that this approach regards production response as determined directly by net returns, and only indirectly by prices. All output prices and all input prices make their contributions indirectly through the actual decision variables. This not only conforms to the general theory of production but also to the facts of actual decision making.

### The Objective Function

The objective function represents what farmers in a region are attempting to maximize. This function was discussed earlier in this paper and the characteristics of the decision process that appear to validate its application at the regional level were described. This function is the sum of the net returns expected per unit (acre) of each process multiplied by its corresponding process level. The aggregate of farmers' individual decisions is regarded as a set of regional process levels which maximizes the regional expected total net returns as just defined, subject to certain crucial limitations.

As we have seen, numerous constraints prevent the farmers in a region from achieving an ideal optimum in this sense. The way in which the objective function is used as a more homely guide to maximizing is now briefly described.[20]

### Flexibility Constraints

We have shown that upper and lower bounds on the acreage of a given crop changing through time, are both a theoretical and a practical description of the way in which production changes are stabilized in the face of uncertainty. Such constraints limit flexibility in selecting process levels and thus account for the cautious way in which profit maximizing actually takes place. The maximum acreage allowed in the region for all processes producing the same commodity is some function of the actual total acreages of the crop in the past years and perhaps of other variables. The same is true of the minimum acreages allowed. I have used for such a relationship a constant percentage increase (or decrease) over the preceding year's actual acreage.[21] I have called these constant percentages flexibility coefficients, because they define the degree of flexibility in cropping changes that a given region can potentially manifest.

The total acreage of each commodity has an upper and lower bound for each year, defined by the preceding year's actual acreage, and by its upper and lower flexibility coefficients. More general formulations are currently being explored.[22]

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[17] Capacity utilization per acre is as difficult to define rigorously as capacity itself. Current results, however, suggest that this approach provides a useful empirical approximation.

[18] See Day, footnote 2.

[19] It is hoped that more realistic net returns expectation models will be included in future studies.

[20] We have already mentioned how planning over time may be included. The present remarks are limited to current model structuring.

[21] In this respect, my work is directly related to Henderson's (*5*).

[22] W. Neill Schaller, Farm Economics Division, Economic Research Service, in his Ph. D. thesis, "A Recursive Programming Analysis of Regional Production Response," recently submitted to the University of California, has constructed a recursive programming model in which the flexibility constraints include certain nonlinear functions of past acreages, the acreages of a given crop placed under controls, and the total available cropland. Further explorations, including yield variability, price or profit variability, other uncertainty measures, and "noneconomic" variables are planned. The crucial role which these constraints play and the complicated behavior phenomena they describe suggest that much research on their nature is needed.

This construction leads to distributed lags in price-adjustments. Last year's acreages account for the effects of all preceding production decisions, and therefore represent the accumulated effects of all preceding prices. Moreover, the effects on production of price changes in any given period are distributed over time because of the influence of the flexibility and capacity constraints. This observation applies to all production and investment variables included in the model.

## Constraints on the Utilization of Variable, Quasifixed, and Fixed Factors

Maximum potential investment in a given technical stage or combination of quasifixed factors is assumed to bear some fixed relations to the preceding year's actual capacity of that stage. The constant percentages describing this relation are called investment coefficients and define the maximum rate at which growth in the particular capacity can occur. These coefficients represent the combined effects of growth in the off-farm industries producing the several machines involved, the learning process required by farmers for the adoption of methods previously not used by them, and finally, the uncertainty attached to the continued profitableness of investment in the particular technology stage.[23]

This hypothesis is analogous to the accelerator theory of investment frequently used in macroeconomic models. But it is a more flexible hypothesis, as actual investment is determined by the flexibility constraints as well as by these "maximal potential growth" constraints. Investment occurs at its maximum potential rate (the full acceleration case) only if limited profit maximizing justifies it. Actual investment may not reach this "acceleration" rate. In fact, disinvestment, or a decline in capacity utilization frequently occurs.[24]

[23] Note that this type of uncertainty differs from that described by the flexibility coefficients. This one applies to a given technique of production, of which there may be several for a given crop, while flexibility constraints apply to the total acreage of a crop.

[24] Unusual movements of capital into a region are not accounted for in this model, if by unusual we mean an amount exceeding the maximum predicted under the maximum potential growth principle. However, I have sketched out an interregional model that would predict the flow of capital among regions (*4*, pp. 108–125).

The limitations on the supplies of labor, fertilizer, and other variable factors at prices farmers are willing to pay have been accounted for by trends in past supplies, which determine upper bounds changing through time for the utilization of these factors during any one time period. Consequently, unlike the capacities of quasifixed factors, these factors are exogenous variables in the model.[25]

The only inputs regarded as fixed are the total acreages of the several soil classes or land types in the region available for production. An endogenous growth mechanism similar to that for quasifixed factors could be formulated for regions in which resources of this kind are underdeveloped.

Actual planned demand (utilization) for variable and fixed factors, investment in the several quasifixed factors, the planned level of each production process, is determined by the maximizing decisions. Thus, actual input utilization may or may not be determined at maximum potential levels.

The same distributed lag phenomenon described by the flexibility constraints applies to input utilization of all kinds. Current planned utilization depends not only upon last year's net returns but on all preceding years' net returns and so on all preceding years' output and input prices. It can further be shown that investment is greatest in those inputs whose marginal net-revenue or quasirents are highest. Consequently, the model is a formal description of Marshall's famous quasirent theory of investment and also a dynamic version of the closely related marginal productivity theory of investment.

### Technological Change

Innovation as described can be introduced into the model by means of new production processes and new historically known or estimated capacities actually utilized during the innovation period. The new capacities, together with known (or estimated) capacities of old factors, form a new set of initial conditions for the model. The rate of adoption of these new processes is determined by exactly the same maximum potential growth principle described for old technologies.

[25] Current research is directed partly toward developing an endogenous mechanism for labor and fertilizer supplies.

The estimation of investment coefficients for newly innovated processes or new flexibility coefficients for newly innovated commodities is more difficult and less accurate than for those for older methods. This problem can be solved only by a deeper understanding of the forces determining investment patterns. Each innovation is in some respects unique, and it is doubtful whether empirical prediction of adoption rates can ever be more than approximated. However, it is likely that a detailed knowledge of past rates of adoption for a variety of new technologies can be used to estimate probable maximal potential growth for current or future innovations.

### The Dynamics of the Current Model Structure

Two viewpoints can be taken in summarizing the model structure. The first regards the model as a chain of recursively dependent linear programming problems. The second regards it as a set of dynamic inequalities or *difference* inequations augmented by a dynamic "potential" function. We shall look at each view in turn.

First, the constraints for any one year are a function of the immediately preceding year's actual process levels, and the net returns are a function of the immediately preceding year's net returns. Thus for a given year, a linear programming problem exists which can be solved by the usual methods. One begins the model solution with a first or base year for which total acreages and input capacities are known (or estimated). These initial conditions provide the materials for a linear programming problem for the year following the base year. The solution gives the process levels and investment patterns for this second year and these in turn provide the information necessary to generate the next succeeding year's constraints. The process can be repeated for each year of the period under study. Because each year's programming problem depends upon the solution for the preceding year, we call the entire sequence of interdependent problems a recursive programming model.

Looked at in another way, the set of flexibility, investment, and fixed and variable factor constraints defines a set of dynamic inequalities in which the process levels of a given year are related to those of the preceding year by their various flexibility and technical coefficients and the exogenously determined variables. This results in a system of simultaneous difference inequations. Each such inequation defines a maximum positive or negative change in total acreage of a given crop, a maximum amount of investment, or a maximum amount of variable or fixed input utilization. Each of these is a potentially constraining factor, but none necessarily holds. Thus in itself, this set of inequations represents a variety of possible paths which production, investment, input utilization, and marginal returns may follow through time.

Such freedom exists in certain physical systems, also. The actual course of such systems is determined by a "potential" equation which must be satisfied at each point in time. The optimizing principle in our system is the potential equation which resolves our system of possible changes into a given system of actual or predicted changes. By means of it, the system of possible paths is resolved into a particular path.

From the philosophical viewpoint, the model has the following meaning. If free men with choices among alternative paths of action through time choose so as to do their best, as they currently view the best, their actions are predictable. Free choice is not inconsistent with determinate action. This is a commonplace fact. Faced with innumerable possibilities for the morrow, we really do something and do it as best we can, given our current values, myopically as they inevitably must be conceived.

### Some Practical Implications

With a bit of reflection, some practical results are immediately forthcoming from the model presented. First the system of potential constraints may be resolved differently at different points of time. That is, during various periods of time, different sets of dynamic inequations will be equated. Or, to express the matter in another way, different potential constraints will be effective. This means, that at some times, certain input supplies may constrain production of a given crop while at other times, uncertainty, or the dominance of superior alternatives, may determine production. A different kind of model that accommodates only one or the other of several forces, such as certain inputs, or certain prices, might describe production response in one time span but not in others. This approach recognized this possibility at the outset and provides freedom for the model to determine which among a variety of forces will actually influence production at a particular time and under particular economic conditions.

Second, as the response of a given crop to its own price is conditioned by all other input and output prices in addition to the possible constraining effects of uncertainty and input supplies—the effects of which may change through time—we can anticipate that price elasticities of supplies will vary considerably from period to period. Further, they are undoubtedly not invariant to short-run changes in price or other variables and consequently are not valid for estimating short-run effects of price changes. In fact, the short-run (as well as the long-run) supply curves for a given commodity are likely to be highly irregular step functions, meaning that a percentage change in output in response to a percentage change in price will vary considerably at different price levels.

Third, the frequently observed phenomenon of increased production of a commodity whose price is falling follows naturally from our considerations. Relative net returns to the finite number of production alternatives considered by farmers in their plans take discrete "jumps" as prices vary. This allows a given commodity to retain its attractiveness even as its own price falls. For the converse reason, production of a crop may contract even though its price is rising!

We reach the somewhat ironical conclusion that not only is such a phenomenon consistent with sequential optimizing behavior, but that if demand is stable it may be expected to occur frequently. When statistical models of price response are rejected because of this "inverse response relation," the "common sense" application of the optimizing principle leads to the wrong conclusion for the right reason!

Finally, it can be shown that investment and rate of adoption patterns may frequently be determined by forces other than their own maximum potential growth, particularly when they contribute to a high proportion of output. Uncertainty, if not declining profitableness, will set in eventually, and will determine investment patterns to a considerable degree. Thus the familiar S-like curve is traced through time and derived from the various response forces. It is not assumed as a trend.

**On the Pure Theory of Recursive Programming**

Certainly, the model just described is an imperfect description of actual production response in agriculture. The current model structure has many limitations, despite the fact that empirical tests indicate its practical usefulness.[26] It is the result of the "judicious compromise" between theoretical understanding and operational possibility described earlier. Further research will remove some of the limitations. Detailed comparison of the model estimates with actual data and other models of production response will be helpful. In any event, it is clear that recursive programming models contribute to theory and understanding the decision process in atomistic markets.

From this viewpoint, primary characteristics of the method are, first, its formal description of optimizing over a limited time horizon on the basis of knowledge gained from producers' past experience, and second, the sequential regeneration of the planning problem. A recursive programming problem is not solved by a single decision that claims to determine what action will be optimal in each planning period within the time horizon, as do current versions of "dynamic programming." Instead, it recognizes that plans for the future must be changed during each suceeding planning period to account for the actual history of economic variables.

The solution of such a system does not determine an optimal path leading to well-conceived terminal objectives (*6*). Rather, it determines a path in which each step is the result of an attempt at optimal action, but in which the terminal objectives toward which the step is taken are also changing as a result of an actual history that can never be fully determined by any one optimal plan. This model's decision process never ends. Just like world decision problems of any kind, it must continually be reformulated to account for newly acquired information.

Recursive programming reflects an essential aspect of the real economic world. Further cultivation of its particular empirical structure may be expected to bear fruitful results, both for the-

[26] In the sense that the percentage variation in actual crop acreages for a 20-year period explained by the model is quite high: about 85 to 95 percent for five major crops produced in the Mississippi Delta.

oretical understanding of production response and for predicting response to specific forces acting on production.

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