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ENERGY SUBSTITUTION POSSIBILITIES IN THE U.S. ECONOMY

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1. Introduction

The relationship of energy to other variables in the production process, particularly other inputs, has not been studied until recently as energy has been relatively cheap, available and of virtually no concern. In fact, the energy sector had been investigated in isolation from the rest of the U.S. economy as it was felt that energy had little impact on the economy. The situation at this time is different. Relative prices of energy have risen significantly since 1973 and it has been perceived that this price increase has had significant impact on the U.S. economy. That effect is just beginning to be quantified.

The effect increased energy prices have on the economy is difficult to quantify as it is not certain how energy consumption is linked to economic growth. One method that has been utilized to quantify the effect of higher energy prices on output (and thus the relationship of energy consumption and economic growth) is measurement of the elasticities of substitution between energy and other inputs. Economic theory indicates that if energy and other inputs, in particular capital, are complements, increases in energy prices will lead to decreases in capital investment and ultimately decreases in output and the rate of growth. If, however, energy and capital are substitutes it is not clear that growth will decrease. What is important, here, is the magnitude of the elasticity of substitution. For instance, if the elasticity of substitution is close to zero, substitution possibilities are limited and even though the inputs are substitutes, growth may decrease. If the elasticity of substitution between energy and other inputs is large then growth may not necessarily decrease as other inputs could easily take the place of energy in the production process.

The major objective of this paper is to determine what impact energy prices have had on input substitution. If a micro approach is taken, it is felt that the actual reaction of output cannot be determined, as many macroeconomic forces also affect output in the economy. Determining the relationship of inputs, or the response of inputs, to increased energy prices is the first step in determining the effect higher energy prices have on the economy.

Two models which estimate the relationship of inputs in the production process are presented in this paper using data for 20 manufacturing sectors and agriculture for 1947 to 1976.1/ The first model discussed is a static, non-linear model. In this model, a cost function approach is taken to estimate the elasticities of substitution between capital, labor, energy and intermediate materials. The cost function used is a generalized Box-Cox cost function introduced by Khaled (1978). The generalized Box-Cox cost function allows the estimation of elasticities of substitution, bias

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1/ The sectors analyzed are: (20) food products, (21) tobacco products, (22) textiles, (23) apparel, (24) lumber and wood products, (25) furniture, (26) paper products, (27) printing and publishing, (28) chemicals, (29) petroleum products, (30) rubber products, (31) leather products, (32) stone, clay, glass products, (33) primary metals, (34) fabricated metals, (35) machinery, (36) electronic equipment, (37) transportation equipment, (38) instruments, (39) miscellaneous manufacuturing and (40) agriculture.

The variables used are as follows: The capital variable is measured as gross book value, a stock variable. The labor variable includes production and nonproduction labor. The energy variable includes energy forms used for heat, light and power in the production process. Intermediate materials includes all raw and semi-finished materials purchased by firms. It does not include advertising, insurance or other overhead costs. See the Appendix to this for a more detailed description of the variables used as data.

to technical change and returns to scale. As will be discussed, all 21 sectors were not analyzed in this framework because of cost constraints.

The second model discussed is a dynamic, linear model. This dynamic model is from the time series genre called vector autoregressions. In this vector autoregressive model, dynamic own and cross elasticities of input demand are estimated for capital, labor, energy and materials. These dynamic elasticities describe how the economy shifts from a point on one isoquant to another due to changes in relative prices of inputs. Because the time series data is available by sector, it is desirable to use all of the data in obtaining sectoral estimates. The method of incorporating the pooled data in this model is less restrictive than standard pooling procedures. The method used is a Bayesian procedure which utilizes an exchangeability prior. This exchangeability prior is discussed in conjunction with the vector autoregressive model.

2.1 Model Specification of the Generalized Box-Cox Cost Function

The generalized Box-Cox functional form (GBC), although it is static, is used because it is the least restrictive of the functional forms for cost functions. It allows the simultaneous estimation of elasticities of substitution, returns to scale and bias to technical change. This flexibility is obtained with a highly non-linear form. It should be noted that with this flexibility, estimation becomes more difficult than with more linear functions.

The most general form of the generalized Box-Cox cost functions proposed by Khaled (1978) is:

$$(1)\quad C = \{1 + \lambda[\alpha_o + \Sigma_i \alpha_i \frac{(P_i^{\lambda/2} - 1)}{\lambda/2} + 1/2\, \Sigma_i \Sigma_j \gamma_{ij} \frac{(P_i^{\lambda/2} - 1)}{\lambda/2} \frac{(P_j^{\lambda/2} - 1)}{\lambda/2}]\}^{1/\lambda}$$

$$\cdot\ Y^{(\beta + \theta/2 \ln Y + \Sigma_i \phi_i \ln P_i)} \exp \{t(\tau_o + \Sigma_i \tau_i \ln P_i)\}$$

$$[i, j = K, L, E, M]$$

where

C is total cost

P_i are prices of labor (L), capital (K), energy (E) and intermediate materials (M)

In this specification these prices are assumed to be exogenous.

Y is output, it is also assumed to be exogenous.

t is time

This cost function places no a priori restrictions on scale economies, technical change bias, elasticities of substitution or elasticities of input demand. In addition, this form has as special cases the generalized Leontief, the generalized square root quadratic and the translog.

The cost function has as its dual a production function. This cost function is dual to the production function the same way the indirect utility function is dual to the direct utility function.[3/] That is, the use of the cost function provides a way of finding the

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[2/] This form is the same as that given in Berndt and Khaled (1978). Berndt and Khaled used the generalized Box-Cox function with annual data from 1947-1971 to estimate the relationship between capital, labor, energy and materials in U.S. manufacturing.

[3/] This analogy is made in Christensen, Jorgensen, Lau (1973, p. 28).

production parameters such as input substitution parameters, input demand parameters, returns to scale parameters and bias to technical change without actually knowing the structure of the production function.

The condition of linear homogeneity of prices must be met in order for this cost function to be dual to a well-behaved production function that is linearly homogeneous in inputs. Linear homogeneity of prices implies that a proportional increase in all input prices increases cost proportionately. Mathematically this condition is written:

$$C(\rho P_1,\ldots,\rho P_n,\ T) = \rho C(P_1,\ldots,\ P_n,\ T). \tag{2}$$

When linear price homogeneity is imposed on the generalized Box-Cox cost function this condition (2) implies the following restrictions:

$$\begin{aligned} &\Sigma_i\alpha_i = 1 + \lambda\alpha_0 \\ &\Sigma\gamma_{ij} = \frac{\lambda}{2}\ \alpha_i \\ &\Sigma_i\phi_i = 0 \text{ and } \Sigma_i\tau_i = 0. \end{aligned} \tag{3}$$

When these conditions are imposed the GBC function for each sector can be written:

$$\begin{aligned} C_r = &\{\frac{2}{\lambda}\ \Sigma_i\Sigma_j\gamma_{ijr}\ P_{ir}^{\lambda/2}\ P_{jr}^{\ \lambda/2}\}^{1/\lambda}\ Y_r^{\ (\beta_r + \frac{\theta}{2}\ \ln Y_r + \Sigma_i\phi_{ir}\ \ln P_{ir})} \\ &\cdot \exp\ \{t(\tau_{or} + \Sigma_i\tau_{ir}\ \ln P_{ir})\} \qquad [i,\ j = K,L,E,M;\ r = 1,2,\ldots m\ (\text{sector})]. \end{aligned} \tag{4}$$

When $\lambda = 1$, the generalized Box-Cox cost function in (4) is equivalent to the generalized Leontief cost function. When $\lambda = 2$, the GBC function is equivalent to the generalized square root quadratic, and when $\lambda \to 0$, the GBC is equivalent to the translog cost function.

The GBC function that appears in (4) is nonhomothetic with exponential nonneutral technical change. By imposing conditions on the parameters, other structural forms can be obtained. For instance, if $\phi_i = 0$ for all i is imposed, the production structure is homothetic. This structure can be further restricted if, in addition, $\theta = 0$ is imposed. In this case production is homogeneous of degree $\frac{1}{\beta}$. The most restrictive structure, homogeneity of degree one (constant returns to scale), is obtained if, in addition to the above two restrictions, $\beta = 1$ is imposed. Hicks neutral technical change can be imposed by constraining $\tau_i = 0$, for all i. If this condition is not imposed, technical change is factor i-saving if $\tau_i < 0$ or factor i-using if $\tau_i > 0$.

The more restrictive conditions will not be imposed on the GBC function to be estimated because the least restrictive form is _a priori_ the most justifiable. This least restrictive model (4) will be estimated with gross output and the four input prices; price of capital, energy, labor and intermediate inputs as explanatory variables.

Since P_m is included in the cost function in (4) it is not necessary to assume weak separability of P_K, P_L, P_E from P_M in the cost function (as is necessary if only P_K, P_L and P_E are included in the specification). It is necessary, however, to assume weak separability of the form:

$$(5) \quad C = C[P_K(P_{K1},\ldots,P_{KK}),\ P_L(P_{L1},\ldots,P_{LL}),\ P_E(P_{E1},\ldots,P_{EE}),\ P_M(P_{M1},\ldots,P_{MM})]$$

where

P_K, P_L, P_E and P_M are aggregate index prices of individual components (P_{Ki}), (P_{Li}), (P_{Ei}) and (P_{Mi}) respectively.[4]

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[4] See Berndt and Christensen (1973).

It is most desirable to test (5), but individual data are not readily available, making this test impossible. Weak separability of this type is, by necessity, assumed to exist.

Taking the derivative of the unit cost function (4) with respect to P_i, utilizing Shephard's lemma ($\partial C/\partial P_i = X_i$) and dividing by Y_r gives input-output equations

$$(6)\quad b_{ir} = \frac{X_{ir}}{Y_r} = \frac{\partial C/\partial P_{ir}}{\partial Y_r} = \{\frac{2}{\lambda} \Sigma_j \lambda_{ijr} (\frac{P_{jr}}{P_{ir}})^{\lambda/2}\}$$

$$\cdot\ Y_r^{\lambda[(\beta_r + \frac{\Theta_r}{2} \ln Y_r + \Sigma_i \phi_{ir} \ln P_{ir}) -1]}$$

$$\cdot\ \exp \{\lambda t(\tau_{or} + \Sigma_i \tau_{ir} \ln P_{ir})\} (\frac{c_r}{P_{ir}})^{1-\lambda} + (\phi_{ir} \ln Y_r + \tau_{ir} t) \frac{c_r}{P_{ir}}$$

$[r = 1, \ldots, m;\ i, j = K,L,E,M]$,

where X_{ir} refers to input i in sector r and c_r is unit cost C/Y. These equations for b_{ir} in (6) are the equations that are used in estimating the coefficients of the cost function for each sector using annual data from 1947 to 1976. Cost constraints made it necessary to limit the number of sectors being considered to five. These five sectors were chosen on the basis of two criteria. First, the sectors were ordered by output. The 10 sectors with the largest output were then ordered by their energy output ratio. The five sectors with the highest energy-output ratio were chosen This procedure was used so the largest producing sectors with the highest energy use would be selected. The five sectors selected for analysis were:[5]

Sector 26	Paper and Allied Products
Sector 28	Chemical and Allied Products

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[5] The two digit number refers to the 2-digit SIC code number except for agriculture.

Sector 29 Petroleum and Coal Products

Sector 33 Primary Metal Industries

Sector 40 Agriculture

Several regularity conditions must be met in order to have a well behaved cost function. These conditions are monotonicity of the cost function and concavity in input prices. Monotonicity or positivity, as it is sometimes called, is met if each estimated cost share is positive. If the cost function is concave in input prices its cost shares will also be concave in those input prices. Concavity is met if the Hessian matrix:

$$(7) \quad \left[\frac{\partial^2 C}{\partial P_i \partial P_j}\right]$$

is negative definite at each observation.[6/] These conditions must be checked at each observation to insure that the GBC is well behaved at all points of estimation.

Uzawa (1962) showed that the Allen partial elasticities of substitution (AES) can be obtained from the cost function. This relationship is

$$(8) \quad \sigma_{ij} = \frac{CC_{ij}}{C_i C_j} \qquad i,j = K,L,E,M$$

where subscripts refer to first and second order derivatives with respect to P_i and P_j.[7/]

[6/] Binswanger (1974b) points out that (7) can be translated into the matrix of Allen partial elasticities of substitution (AES)$[\sigma_{ij}]$.

[7/] Uzawa (1962) proved this for the homogeneous production function. This proof was extended to the nonhomogeneous production function by Binswanger (1974a, p. 378).

This relationship can be used to derive the elasticity of substitution for the GBC function:

$$(9)\qquad \sigma_{ij} = 1-\lambda + \gamma_{ij}\frac{(P_iP_j)^{\lambda/2}}{S_iS_j}\left[\frac{C}{Y^{(\beta+\frac{\theta}{2}\ln Y + \Sigma_i\phi_i \ln P_i)}\exp[t(\tau_0+\Sigma_i\tau_i \ln P_i)}\right]^{-\lambda}$$

$$+\frac{\lambda(\phi_j \ln Y + \tau_j t)}{S_j} + \lambda\left\{1 - \frac{\phi_j \ln Y + \tau_j t}{S_j}\right\}\frac{\phi_i \ln Y + \tau_i t}{S_i}$$

$$[i,j = K, L, E, M;\ i \neq j]$$

$$(10)\qquad \sigma_{ii} = 1-\lambda + \gamma_{ii}\frac{P_i^{\lambda}}{S_i^{2}}\left[\frac{C}{Y^{(\beta + \frac{\theta}{2}\ln Y + \Sigma_i\phi_i \ln P_i}\exp[t(\tau_0+\Sigma_i\tau_i \ln P_i)}\right]^{-\lambda}$$

$$+\lambda\left[\frac{\phi_i \ln Y + \tau_i t}{S_i}\right] + \lambda\left\{1 - \frac{\phi_i \ln Y + \tau_i t}{S_i}\right\}\left[\frac{\phi_i \ln Y + \tau_i t}{S_i}\right]$$

$$+\frac{\lambda}{2}\left\{1 - \frac{(\phi_i \ln Y + \tau_i t)}{S_i}\right\}\frac{1}{S_i} - \frac{1}{S_i}\ \text{[8]}\qquad [i = K, L, E, M]$$

where

S_i, S_j refer to the cost shares of i, j;

$$S_i = \frac{P_iX_i}{\Sigma_iP_iX_i}\qquad [i,j = K, L, E, M]$$

By symmetry $\sigma_{ij} = \sigma_{ji}$.

Allen (1938) shows that elasticities of input demand are related to elasticities of substitution by:

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[8] These elasticities were derived by Berndt and Khaled (1978, p. 8).

$$(11) \qquad \frac{\partial X_i}{\partial P_j}\frac{P_j}{X_i} = \varepsilon_{ij} = S_j\sigma_{ij} \qquad \forall\ i,j = K, L, E, M$$

X_i are inputs.

From (11) inferences about input demand relationships can be made from the sign of σ_{ij}. If $\sigma_{ij} > 0$, the inputs X_i and X_j are substitutes because S_j is always greater than zero, thus implying $\varepsilon_{ij} > 0$, i.e. X_i and X_j are substitutes. Conversely if $\sigma_{ij} < 0$, X_i and X_j are complements since because S_j always positive implies $\varepsilon_{ij}<0$. In this case another input, X_k, substitutes for X_i and X_j as the use of both inputs diminishes if the price of either one increases.

From the latter relationship, it is obvious that some qualification of relationships must be made in order to have stable input demand. Allen has established two such relationships. These are:

1) $\sigma_{ii} < 0$.[9/]

2) Not all σ_{ij} can be negative; in addition the positive σ_{ij}'s must be either greater in number or quantity than the negative σ_{ij}.[10/]

In addition to the elasticity of substitution, the elasticity of total factor productivity can be obtained from the cost function. The elasticity of total factor productivity is the percentage change in output

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[9/] Note that the own elasticity of substitution, σ_{ii}, is related to the elasticity of input demand by the relationship $\sigma_{ii} = \frac{1}{S_i}\varepsilon_{ii}$. The cost share, S_i, is always positive. The elasticity of input demand, ε_{ii}, is always negative, thus the own elasticity of substitution is negative.

[10/] Allen (1938, p. 504-508).

over time. If input levels are held constant the change in output over time is directly attributable to changes in factor productivity. Thus if output increases over time, holding inputs and all else constant, this increase in output is directly attributable to increased productivity of all factors of production. Ohta (1974) defines the primal elasticity of productivity ε_{ft} (where primal refers to the production function), in terms of the cost function elasticities as:

$$(12) \qquad \varepsilon_{ft} = \frac{\partial \ln f(X,t)}{\partial t} = \varepsilon_{cy}^{-1} \cdot \varepsilon_{ct}$$

where Y = f(X,t) is the production function.

The terms, ε_{cy}^{-1} and ε_{ct}, are the returns to scale and rate of total cost diminution respectively. The returns to scale, ε_{cy}^{-1}, defined mathematically to be

$$(13) \qquad \varepsilon_{cy}^{-1} = \frac{1}{(\partial \ln C/\partial \ln Y)}$$

is the change in output due to a change in cost. The rate of total cost diminution, ε_{ct}, is defined mathematically to be

$$(14) \qquad \varepsilon_{ct} = - \partial \ln C / \partial \ln t$$

ε_{ct} is the decrease in cost of production over time, holding inputs constant. If decreases in cost occur over time, this implies ε_{ct} is positive, and total factor productivity will be increased. In the empirical work Berndt and Khaled have done, they found that ε_{ct} is a relatively small number, usually less than 0.01 percent, thus decreases in cost are small and occur slowly over time.

In the context of the GBC function in (4), the expression for total factor productivity is:

$$(15) \qquad \varepsilon_{ft} = \frac{-(\tau_0 + \Sigma_i \tau_i \ln P_i)}{\beta + \theta \ln Y + \Sigma_i \phi_i \ln P_i} \quad . \underline{11/} \qquad [i = K, L, E, M]$$

In this expression both P_i and Y can affect factor productivity. If the various homothetic constraints (all $\phi_i = 0$, $\theta = 0$ and $\beta = 1$) and Hicks neutral technical change (all $\tau_i = 0$) are imposed, the effects of P_i and Y on total factor productivity are wiped out. This measurement will be especially useful in looking at the effect of increased energy prices on total factor productivity.

The basic assumptions discussed in this model involve simultaneous measurement of returns to scale, technical change bias, elasticities of substitution and functional form. Diamond and McFadden (1965) contended that the first three of these cannot be estimated simultaneously, unless sufficient structural form is given. Berndt and Khaled contended that the function has sufficient form to allow estimation of all of these phenomena. The returns to scale Berndt and Khaled obtained seemed surprisingly large to them. Berndt and Khaled stated that there is a rationale for the existences of scale economies on an aggregate analysis. Namely, in the imperfect market, any existing excess capacity in each firm is eventually incorporated into the production process. This could mean that increasing returns to scale are observed if the absorption of excess

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11/ This expression was derived in Berndt and Khaled (1978, p. 7) capacity is more highly correlated with output than with time.[12/] Berndt and Khaled feel that this argument may explain the large returns to scale they observed.

2.2 The Stochastic Specification of the Generalized Box-Cox Cost Function

The main step in the stochastic specification is determining the likelihood function of the nonlinear generalized Box-Cox cost function. The specification of this likelihood function follows Berndt and Khaled (1978).

The crucial assumption in specification of the likelihood function is that errors occur in cost minimization and inputs are chosen with the knowledge that errors in cost minimization have occurred. The observed sectoral input-output value, b^*_{irt}, is then written as

$$(16)\quad b^*_{irt} = \{\frac{2}{\lambda}\,\Sigma_j \gamma_{ijr}\,(\frac{P_{jr}}{P_{ir}})^{\lambda/2}\}Y_{rt}\;\lambda[(\beta_r + \frac{\Theta_r}{2}\ln Y_{rt} + \Sigma_i \phi_{ir}\ln P_{irt})-1]$$

$$\cdot\ \exp\,[\lambda t(\tau_0 + \Sigma_i \tau_{ir}\ln P_{ir})]$$

$$\cdot\ (\frac{c^*_{rt}}{P_{irt}})^{(1-\lambda)} + (\phi_{ir}\ln Y_{rt} + \tau_{ir}\,t)\,\frac{c^*_{rt}}{P_{irt}} + \varepsilon_{irt}$$

$[i,j = K, L, E, M;\ r = 1, \ldots, m \text{ and } t = 1, \ldots T.]$

b_{irt} from equation (6) is the equilibrium input-output value.

b^*_{irt} is the observed input output value.

12/ This argument was given in Berndt and Khaled (1978, p. 28).

c_{rt} is equilibrium unit cost and

c^*_{rt} is observed unit cost.

ε_{irt} is the error term. It is assumed that $\varepsilon_{irt} \sim N(0, \sigma_{ir})$.

Correlation across i is not assumed to be zero, that is $\varepsilon_r \varepsilon'_r = \Sigma_r$.

t refers to time.

Notice that errors enter b^*_{irt} through ε_{irt} and c^*_{rt}.[13/]

All of the parameters of the Box-Cox cost function (4) are contained in the input-output values (6). Thus estimation can be accomplished by simultaneous estimation of b^*_{ir}. The cost function does not have to be included in the estimation because b^*_{ir} contains all the parameters, adding c^*_r would be redundant.[14/] To do this simultaneous estimation, b^*_{ir} must be incorporated into the likelihood function. Since the b_{ir} are assumed to be multivariate normal, the form of the concentrated log-likelihood function for each sector is:

[13/] Berndt and Khaled (1978) note that $b^*_{irt} \neq b_{irt} + \varepsilon_{irt}$ as c^*_{rt} has replaced c_{rt} in b_{irt}. In addition, one should note that the stochastic specification in this model is completely different from the stochastic specification used in most translog models. In the stochastic specification of the GBC, error is assumed to exist both in cost minimization and in the optimal use of the inputs. In the stochastic specification of the translog errors are assumed to occur only in the cost share. Thus even if the GBC approaches the translog analytically as $\lambda \to 0$, the translog model as specified by Berndt and Wood (1975) is not equivalent to the translog case of the Berndt and Khaled (1978) model.

[14/] Berndt and Khaled (1978) note that no restrictions imposing $c^*_{rt} > c_{rt}$ are imposed as this would be extremely complex to do in practice.

$$(17) \quad \ln L_r = \text{constant} - \frac{T}{2} \ln |\hat{\Sigma}_r| + \sum_{t=1}^{T} \ln ||J_{rt}||.$$

where Σ_r is the covariance matrix measuring correlation among i equations, i = K, L, E, M, and $||J_{rt}||$ is the absolute value of the Jacobian discussed below.

An expression for the Jacobian must be obtained to estimate the concentrated log likelihood function. The input-output value b^*_{irt} in (16) must be rewritten in terms of ε_{irt}.

$$(18) \quad \varepsilon_{irt} = b^*_{irt} - \{\frac{2}{\lambda} \Sigma_j \gamma_{ijr} (P_{jrt}/P_{irt})^{\lambda/2}\}$$

$$Y_{rt}^{\lambda[(\beta_r + \frac{\theta r}{2} \ln Y_{rt} + \Sigma_i \phi_{ir} \ln P_{irt})-1)]}$$

$$\cdot \exp [\lambda t(\tau_{or} + \Sigma_i \tau_{ir} \ln P_{irt})] (\frac{C^*_{rt}}{P_{irt}})^{1-\lambda} + (\phi_{ir} \ln Y_{rt} + \tau_{ir} t) \frac{c^*_{rt}}{P_{irt}}$$

for i, j = K, L, E, M; r = 1, ..., m, t = 1, ..., T.

Recall that the Jacobian is the matrix of first partials whose typical element is

$$(19) \quad J_{rt} = [\frac{\partial \varepsilon_{irt}}{\partial b^*_{jrt}}].$$

The terms of the Jacobian are obtained by taking the partial derivative of ε_{irt} with respect to b^*_{irt} and b^*_{jrt}. This gives:

$$(20)\quad \frac{\partial \varepsilon_{irt}}{\partial b^{*}_{irt}} = 1 - (1-\lambda)\{\tfrac{2}{\lambda}\Sigma_i \Sigma_j \gamma_{ijr}\, P_{irt}^{\lambda/2}\, P_{jrt}^{\lambda/2}\}$$

$$\cdot\ Y_{rt}^{\lambda[(\beta_r + \frac{\theta r}{2}\ln Y_{rt} + \Sigma_i \phi_{ir}\ln P_{irt})-1]}$$

$$\cdot\ \exp \lambda t(\tau_{or} + \Sigma_i \tau_{ir}\ln P_{irt}) \cdot (c^{*}_{rt})^{-\lambda} + (\phi_{ir}\ln Y_{rt} + \tau_{ir}t)$$

$$= 1 - \{(1-\lambda)c_{rt}^{\lambda}\ (c^{*}_{rt})^{-\lambda} + (\phi_{ir}\ln Y_{rt} + \tau_{it}\ t)\} = 1 - B_{ir}$$

and

$$(21)\quad \frac{\partial \varepsilon_{irt}}{\partial b^{*}_{jrt}} = \frac{P_{jrt}}{P_{irt}}\,[(1-\lambda)\ \{\tfrac{2}{\lambda}\ \Sigma_i \Sigma_j \gamma_{ijr} P_{irt}^{\lambda/2}\ P_{jrt}^{\lambda/2}\}$$

$$\cdot\ Y_{rt}^{\lambda[(\beta_r + \frac{\theta r}{2}\ln Y_{rt} + \Sigma_i \phi_i \ln P_{irt}) - 1]}$$

$$\cdot\ \exp\,[\lambda t(\tau_{or} + \Sigma_i \tau_{ir}\ln P_{irt})]\ (c^{*}_{rt})^{-\lambda} + (\phi_{ir}\ln Y_{rt} + \tau_{ir}t)]$$

$$= \frac{P_{jrt}}{P_{irt}}\,\{(1-\lambda)c^{\lambda}_{rt}\ (c^{*}_{rt})^{-\lambda} + (\phi_{ir}\ln Y_{rt} + \tau_{ir}t)\}$$

$$= \frac{P_{jrt}}{P_{irt}}\,B_{ir}.$$

Substituting each of the respective terms into the Jacobian and taking the determinant gives

$$(22)\quad \det|J_{rt}| = 1 - \Sigma_i B_{ir} = 1 - (1-\lambda)\ (c_{rt})^{\lambda}\ (c^{*}_{rt})^{-\lambda} = 1 - (1-\lambda)\ \left(\frac{c_{rt}}{c^{*}_{rt}}\right)^{\lambda}$$

as the terms $\Sigma_i \phi_i$ and $\Sigma_i \tau_i$ equal 0 by the constraints given in (3).

The likelihood function is obtained by substituting (22) into (17). This gives:

$$(23)\quad \ln L_r = \text{constant} - \frac{T}{2}|\hat{\Sigma}_r| + \sum_{t=1}^{T} \ln\,[1-(1-\lambda)\;(\frac{c_{rt}}{c^*_{rt}})^{\lambda}]$$

where $\hat{\Sigma}_{ir} = \varepsilon_r\varepsilon_r'$ the covariance matrix discussed in (16).

In estimation, observed cost is substituted into (23) for c^*_{rt} and the cost function from (4) is also substituted into (23); this log-likelihood function is maximized over the 19 free parameters.

When the value $\lambda=0$ (translog case) is substituted into (23), notice that the $\det|J_{rt}| = 0$. The logarithm of zero is undefined, so the likelihood function is not defined at this point. The likelihood function is continuous at this point, but numerically the translog function cannot be estimated. Because the likelihood function is continuous, values of λ close to zero can be used to evaluate the likelihood function. Comparison with values of the likelihood function for other values of λ helps to determine if a global maximum exists at $\lambda = 0$. Now that the likelihood function has been derived, the stochastic specification is complete except for determining the variance of the estimate.

A property of maximum likelihood estimates is that the standard error of the estimates can be obtained from the inverse of the Hessian. This is intuitive as the Hessian or matrix of second derivatives gives the rate of change of the function. In one dimension, a large second derivative indicates the function has a steep peak. The inverse of this large second derivative is small meaning the maximum of the function occurs within a small area. This is proved rigorously in Dhrymes (1970, p. 122-3). Asymptotically it is proved that

$$(24) \qquad \sqrt{T}(\Phi - \Phi_0) \sim N(0, -\left[\frac{1}{T}\frac{\partial^2 L(P,\Phi)}{\partial\Phi\partial\Phi}\right]^{-1})$$

where Φ is the vector of parameters

P is the vector of variables

Φ_0 is the vector of true parameters

$L(P,\Phi)$ is the log-likelihood function

T is the number of observations.

2.3 Estimation Procedure for the Generalized Box-Cox Cost Function

To obtain estimates of the Generalized Box-Cox cost function, a program was written that utilizes ZXMIN.[15/] This program will numerically approximate both the gradient and Hessian of the likelihood function. Initially, ZXMIN was used in this manner, where the gradient was obtained by numerical methods. This, however, was costly and required many function evaluations for each iteration. To cut computer time, ZXMIN was altered so that it utilized the analytically derived gradient. The problem of the Hessian proved less solvable since the Hessian is a 19 x 19 symmetric matrix. The symmetry decreases the number of unique entries, however, 190 entries still exist to be calculated.[16/]

15/ ZXMIN is part of the IMSL library available through the University of Minnesota Computer Center. The negative of the log likelihood function was minimized.

16/ The equation to calculate the number of entries in a symmetric matrix is $\frac{n(n+1)}{2}$ where n = 19.

For this reason, a numerical approximation which updates the Hessian was used (the initial Hessian used is the identity matrix.)[17/] The problem with this large number of parameters is that it is difficult to obtain a good approximation of the Hessian. This presents a problem in obtaining good estimates of the standard errors.

There are other approximations of the standard error of the estimates that use only first derivatives. The method actually used in this thesis to obtain the estimates of the standard error is that suggested by Maddala (1977, p. 179). Maddala states that other methods of nonlinear estimation which also use first derivatives to approximate the Hessian such as the Davidon-Fletcher-Powell method (Maddala (1977, p. 173)) are numerical techniques that do not use the properties of the likelihood function. The method Maddala describes is a method of estimation first suggested by Berndt, Hall, Hall and Hausman (1974). In this procedure the Hessian is approximated by

$$(25) \qquad Q(\Phi) = \sum_{t=1}^{T} \left[\frac{\partial \ln f(P_t, \Phi)}{\partial \phi}\right]^2$$

where $\ln f(P_t, \Phi)$ is the log-likelihood function at each observation, $t=1,\ldots,T$.

The inverse of $Q(\Phi)$ gives the estimate of the variance of the estimates.

It was found that when the value of λ changed the value of the other parameters changed greatly. Thus final estimates for one value of λ did not serve as good initial estimates for another value of λ. So a fine

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[17/] The method utilized by ZXMIN is the Davidon-Fletcher-Powell (DFP) method of updating the Hessian.

grid search over λ again was not feasible because of cost constraints. To approximate a more coarse grid search, three values of λ were chosen. The values of λ are .5, 1 and 2. The function was estimated for each of these values of λ for each of the five sectors separately. The value of λ for which the likelihood function is largest will be discussed.

3.1 Model Specification for the Vector Autoregressive Model

The model described in this section, called a vector autoregression, differs from the static model described above in several ways. First, the vector autoregression is explicitly dynamic. The main interest in this study is to determine the reaction of the economy to increases in energy price; this is a dynamic reaction. This vector autoregression (VAR) is dynamic in that each of F variables is defined by a linear stochastic difference equation. Second, a cost function structure is not specified. The vector autoregressive model does not impose a particular theoretical structure. A minimum number of restrictions are imposed upon the VAR in the hope of allowing the underlying stochastic process to be captured. The restrictions imposed by the VAR framework are the choice of variables in the model and the length of the lag. These restrictions can be relaxed as more data become available. A Bayesian framework adds an additional restriction to this VAR model, the exchangeability prior.

The value of this procedure in the analysis of input substitution is not in the estimated coefficients or calculated elasticities of substitution but rather in the impulse response function (IRF) associated with the vector autoregression. The VAR is not a structural model, but rather a reduced form type of model, thus its coefficients cannot be related to structural phenomena and are therefore of little intuitive value. The IRF gives the response of all variables in the system to a shock in any one of the variables. This is valuable in determining how the use of capital, labor and materials reacts to an increase in the price of energy. The effect on output can also be determined. This IRF can be viewed as a dynamic elasticity of input.[18] A single number is no longer enough to capture the response of the system, instead a whole reaction path or series of elasticities over time is needed for each variable.

The system of stochastic difference equations with F variables is defined as a system of F equations with lagged values of all F variables as right hand side variables. This specification allows independent equation-by-equation estimation of this system because all right hand side variables are the same for each equation.[19] This system of equations for each of r sectors can be written:

$$(26) \qquad X_r(t) = \sum_{s=1}^{n} \gamma_r(s) X_r(t-s) + u_r(t) \qquad [t = 1, \ldots, T;\ n = 1, \ldots M]$$

where

$X_r(t)$ is an F x 1 vector of variables

$\gamma_r(s)$ is an F x F vector of coefficients

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[18] Pindyck and Rubinfeld (1976) define dynamic elasticities as $\frac{P_T}{Q_T} \cdot \frac{\Delta Q_{T+\tau}}{\Delta P_{T+\tau}}$ where τ refers to an increment in time. This is exactly the term in the impulse response function.

[19] See Johnston (1963, p. 240). As will be discussed in the estimation section, each equation is solved iteratively for all sectors; the sectoral estimates are not independent of each other for a given equation.

$u_r(t)$ is an F x 1 vector of innovations (shocks)

n is the number of lags used.

s is the particular lag

T is the number of time series observations

r is the sector.

Note that in this model (26), each of the F variables in the model is regressed on lags of all F variables in the model. This gives one equation per variable. It is assumed that $u_r(t)$ is uncorrelated over time. It is also assumed that $u_r(t)$ is uncorrelated with past values of X_r:

$$E[u_r(t) \mid X_r(t-1), X_r(t-2), \ldots, u_r(t-1), u_r(t-2), \ldots] = 0 \tag{27}$$

In the analysis of input substitution, the twenty manufacturing sectors and agriculture will be investigated with data from 1947 to 1976. These sectors are:

20) Food and Kindred Products
21) Tobacco Products
22) Textile Mill Products
23) Apparel, Other Textile Products
24) Lumber and Wood Products
25) Furniture and Fixtures
26) Paper and Allied Products
27) Printing and Publishing
28) Chemical and Allied Products
29) Petroleum and Coal Products
30) Rubber, Misc. Plastics Products
31) Leather, Leather Products
32) Stone, Clay, Glass Products
33) Primary Metal Industries
34) Fabricated Metal Products
35) Machinery, except Electrical
36) Electric, Electronic Equipment
37) Transportation Equipment
38) Instruments, Related Products
39) Miscellaneous Manufacturing
40) Agriculture

The particular variables to be used in the vector autoregression for each sector are:

Y = output in each sector

P_K = the real price of capital in each sector

P_L = the real price of labor in each sector

P_E = the real price of energy in each sector

P_M = the real price of intermediate materials in each sector

K = capital in each sector

L = labor in each sector

E = energy in each sector and

M = intermediate materials in each sector.

These variables are entered in logarithmic form so that the impulse response function can be interpreted as percentage change rather than change in level.

At this point it should be noted that the price of output is not included in the model. Because of the definition of the output variable (dollars of gross output) the implicit assumption is made that the price of output is equal to one. This implies that producers respond to the real prices but not the relative price of input to output in each sector as the price of output is fixed. The impulse response of output to a shock in input price is not the same as the impulse response of output to a shock in the relative price of input to output. This is just a slight change in interpretation which is only evident for certain sectors where the output is energy related. It should also be noted that the price deflator used, the wholesale price index, is most likely not very different from the price of output in each sector. Therefore, if the relative price of input to output had been used it would make very little difference. The fact that relative prices are not used matters very little, because it is not felt that the output response in this model would be representative of the actual response of output in this situation. To obtain a good reaction of output, the demand side of the model would need to be included, and perhaps a tie-in with other macro variables would be needed.

In this model X is a 9x1 vector These variables have been chosen because they are closely related to the production process. The same variables or a combination of these variables are used in the generalized Box-Cox cost function. Since the focus is on measuring input substitution, the variables selected are limited to these nine. This model can give the direct effect of energy price increases on inputs, as well as the indirect effect. It should be understood that the indirect effects discussed in this thesis are the "second round" effects of higher energy prices. For example, in the production of any good, energy price increases will directly increase the cost of production of that good. This is a "first round" effect. Higher energy prices will also eventually increase the cost of machinery used in the production of that good, because energy price increases have increased the cost of producing the machinery. The increases in the cost of the machinery (capital) and materials due to higher energy price (or the "second round" effects) are what are considered indirect effects in this thesis.

The lag length to be used in this model is one (n=1) with annual observations. Although it most likely takes more than one year for energy price changes to work through the economy, the small number of observations available (30) make it necessary to limit the lag length to one. As more data become available, it would be desirable to extend the length of the lag.

Equation (26) represents the characteristic functional form to be estimated. The complete model specification incorporates the exchangeability prior. The exchangeability prior provides a framework for incorporating all of the data in all sectors into the estimation of the parameters of each sector. This method of pooling the data is not as restrictive as many other methods of pooling. In particular, the exchangeability prior does not impose coefficients to be the same for all sectors. A digression motivating the exchangeability prior is necessary before the stochastic specification which utilizes it can be given.

3.2 The Exchangeability Prior

The notion of exchangeability was first introduced by de Finetti in 1937.[20] Lindley and Smith (1972) extended de Finetti's notion of exchangeability in the context in which it is used in this analysis. The data used in this analysis are sectoral time series data. An independence prior, applied to these data, would imply the belief that each sector is independent or completely unrelated. The exchangeability prior implies the belief that the sectors are related, and, in fact, that each sector's coefficients are drawn from the same probability distribution.[21]

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[20] For a brief description of de Finetti's hypothesis, see Leamer (1958, pp. 49-51).

[21] This word, exchangeable, comes from the fact that the distribution of a parameter from one sector is exchangeable with the distribution of the same parameter in another sector. This is true if the parameters come from the same distribution, but only true by coincidence if the parameters come from completely independent distributions.

The exchangeability prior, in essence, draws each sectoral estimate towards the mean of the distribution and decreases standard errors of the estimate.[22] So in cases where all sectoral coefficients are close, the exchangeability prior allows the use of all the data in estimating each sector's coefficients. This procedure strengthens the estimates of each sector. If each sector's coefficients are very different, this procedure will still pull the estimates slightly toward the mean.

The exchangeability prior is a priori considered appropriate in this analysis. Two justifications can be given. First, all but one of the sectors are manufacturing sectors. Thus, it can be reasoned that since these sectors are interrelated in their economic transactions the coefficients of the equations estimated for each sector are related. In the agricultural sector, transactions are also interrelated with the manufacturing sectors so it is reasonable to assume that the coefficients of the processes of the agricultural sector are related to the coefficients of the processes of the manufacturing sectors. A natural method for relating the coefficients in all sectors is to assume that the coefficients come from the same distribution. The second justification for the use of the exchangeability prior is that in this model the same equations are being estimated for each sector. That is, similar processes are being estimated. Leamer (1978, p. 271) suggests that when similar processes are used pooling is "intuitively sensible and necessary."

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[22] Leamer (1978) and Lindley and Smith (1972) give examples of this.

The exchangeability prior is not as restrictive as the more standard models that pool time series and cross sectional data.[23] The exchangeablity prior is less restrictive in that it does not restrict the coefficients for each sector to be the same. The exchangeability prior is akin to the random coefficients model when a nonBayesian method is chosen.[24]

The use of Bayesian methods versus classical methods has been extensively debated. To debate this question here would be fruitless. Instead, let it be said that several compelling reasons have led to the adoption of the Bayesian approach. The presence of lagged dependent variables in the linear, dynamic vector autoregression analysis led naturally to a Bayesian approach. Standard classical theory does not give adequate procedures for the analysis of lagged dependent (stochastic) variables, given the assumptions of the vector autoregressive model. In particular, the small sample properties, which are of interest with finite data, are not known. Within the Bayesian framework, however, the presence of stochastic explanatory variables is not troublesome as all data and coefficients are assumed to be stochastic. Also, the nature of the data, cross sections and time series (similar processes), lead to the exchangeability prior as noted above.

3.3 Stochastic Specification of the Vector Autoregressive Model Incorporating the Exchangeability Prior

The stochastic specification for the vector autoregressive model incorporates the Bayesian exchangeability prior. The basic model described here is that developed by Lindley and Smith (1972).

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[23] See Maddala (1971) for a discussion of these more standard models.

[24] For a discussion of random coefficients models see Lee and Griffiths (1979), Swamy (1970), Swamy (1971) and Zarembka (1968).

The Bayesian model has implicit in it three stages. The first stage makes explicit assumptions about the data. To see this in the VAR, first note that each equation in the VAR contains the same variables. This implies that each equation of the F equations can be estimated separately. To see this first rewrite (26) as:

$$X_r(t) = \delta_r W'_r(t) + u_r(t) \qquad r = 1, \ldots, m; \ t = 1, \ldots, T. \tag{28}$$

where

$X_r(t)$ is an F x 1 vector of dependent variables

$W_r(t)$ is an nF x 1 vector of lagged dependent variables

δ_r is an F x 1 vector of coefficients

n is the number of lags used

t refers to the time series observation

r is the sector.

Note that

$$W_r(t) = \begin{vmatrix} X_r(t-1) \\ X_r(t-n) \end{vmatrix} \tag{29}$$

in the model n = 1 and F = 9, so $W_r(t)$ is a 9 x 1 vector. This is done for convenience to give present and lagged variables different symbols.

Lindley and Smith assume the variables have a normal distribution. In the VAR this implies the following condition for each equation in each sector,

(30) $$X_{fr} \sim N(W_{fr}\delta_{fr}, \sigma^2_{fr} I) \qquad [f = 1, \ldots, F \text{ (variables)}; \; r = 1, \ldots, m]$$

where X_{fr} is a T x 1 vector

δ_{fr} is an F x 1 vector

W_{fr} is an T x F vector.

This is derived from equation (28), and the assumption that each variable, X_{fr} has a normal distribution with mean $W_{fr}\ \delta_{fr}$ and variance σ^2_{fr} I. Notice that (30) does not restrict σ^2_{fr} to be the same for each variable (equation).

The second stage is the exchangeability prior:

(31) $$\delta_{fr} \sim N(\delta_{fo}, \Omega)$$

where δ_{fo} is the mean of the distribution.

This prior integrates to one and thus by definition it is a proper prior.

The third stage expresses the prior knowledge of δ_{fo}. In general, very little is known about δ_{fo}, so a vague or diffuse prior is used. A uniform distribution is used in this model. Equations (30), (31) and the vague prior for δ_{fo}, give the stochastic specification for the VAR model.

3.4 The Estimation Procedure for the Vector Autoregressive Model

To obtain estimates for the VAR model discussed above, the posterior distribution of the parameters must be obtained. The posterior distribution is obtained by multiplying the likelihood function for X_{fr} by the prior for δ_{fr} in (31). If a quadratic loss function is used, the mean of the posterior distribution is the Bayesian point estimate.

To obtain this posterior distribution for this model with unknown variance, σ^2_{fr} and Ω, prior distributions for σ^2_{fr} and Ω must be specified. The priors suggested by Smith (1973) are:

(32) $$\frac{\lambda_{fr}\nu_{fr}}{\sigma^2_{fr}} \sim X^2_{\nu fr}$$ (σ^2_{fr} is distributed with a conjugate inverse χ^2 with ν degrees of freedom)

and Ω has a conjugate Wishart distribution with ρ degrees of freedom and mean matrix R. These priors were chosen because the variance generally has a χ^2 distribution. The conjugate Wishart distribution is the matrix extension of the χ^2 distribution.

The joint distribution for the vector autoregressive model is:

(33) $$P\ (\delta_{fr}, \delta_{fo}, \sigma^2_{fr}, \Omega^{-1}|X_{fr})\ \alpha$$

$$(\sigma^2_{fr})^{-\frac{1}{2}T} \exp\{-\frac{1}{2\sigma^2}\sum_{r=1}^{m} (X_{fr} - W_{fr}\delta_{fr})' (X_{fr} - W_{fr}\delta_{fr})\}$$

$$\cdot\ |\Omega|^{-\frac{1}{2}m} \exp\{-\tfrac{1}{2}\sum_{r=1}^{m} (\delta_{fr} - \delta_{fo})' \Omega^{-1} (\delta_{fr} - \delta_{fo})\}$$

$$\cdot\ |\Omega|^{-\frac{1}{2}(\rho-f-1)} \exp -\{(\tfrac{1}{2}tr\ \Omega^{-1}R)\}$$

$$[r = 1, \ldots . m]$$

This distribution is obtained by multiplication of the log-likelihood function for X_{fr} by the priors for σ^2_{fr} and Ω. There exists one of these joint distribution functions for each of the F variables in the model.

Integrating with respect to δ_{fo} gives the posterior distribution for δ_{fr}, σ^2_{fr} and Ω^{-1}.

(34)
$$P(\delta_{fr}, \sigma_{fr}^{2}, \Omega^{-1} | X_{fr}, \delta_{fo}) \propto$$

$$(\sigma_{fr}^{2})^{-\frac{1}{2}(T + \nu_{r} + 2)} \exp [-\tfrac{1}{2} \sigma_{fr}^{2} \sum_{r=1}^{m} m^{-1} \upsilon_{r} \lambda_{r} +$$

$$[(X_{fr} - W_{fr} \delta_{fr})'(X_{fr} - W_{fr} \delta_{fr})]$$

$$\cdot |\Omega|^{-\frac{1}{2}(m+\rho-f-2)} \exp [-\tfrac{1}{2} \text{ tr } \Omega^{-1} \{R + \sum_{r=1}^{m} (\delta_{fr} - \delta_{f\cdot})(\delta_{fr} - \delta_{f\cdot})'\}]$$

where $\delta_{f\cdot} = m^{-1} \sum_{r=1}^{m} \delta_{fr}$

Estimates of δ_{fr} are obtained from the mean of the marginal posterior distribution for δ_{fr}. To do this, Ω^{-1} and σ_{fr}^{2} must be integrated out. Lindley and Smith point out that this integration is quite difficult. They suggest instead that the mean can be approximated by the mode of the posterior distribution. The mode is the maximum value of the joint posterior distribution; it satisfies

(35)
$$\frac{\partial}{\partial \delta_{fr}} P(\delta_{fr}, \delta_{fo}, \Omega^{-1}, \sigma_{fr}^{2} | X_{fr}) = \frac{\partial}{\partial \delta_{fo}} P(\delta_{fr}, \delta_{fo}, \Omega^{-1}, \sigma_{fr}^{2} | X_{fr})$$

$$= \frac{\partial}{\partial \sigma_{fr}^{2}} P(\delta_{fr}, \delta_{fo}, \Omega^{-1}, \sigma_{fr}^{2} | X_{fr}) = \frac{\partial}{\partial \Omega^{-1}} P(\delta_{fr}, \delta_{fo}, \Omega^{-1}, \sigma_{fr}^{2} | X_{fr}) = 0.$$

Instead of a complicated integration, a much simpler differentiation will give reasonable approximations to the coefficients (the posterior mean).[25]

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[25] Lindley and Smith actually prove that the value of the posterior mode of the joint distribution is equal to the mode of the conditional distribution of the parameters, evaluated at the value of the nuisance parameters, σ_{fr}^{2} and Ω.

The modal estimates for the parameters are obtained from (34). These estimates are:

$$(36) \qquad \tilde{\delta}_{fr} = (\sigma_{fr}^{-2} W_{fr}' W_{fr} + \Omega_f^{-1})^{-1} (\sigma_{fr}^{-2} W'_{fr} X_{fr} + \Omega_f^{-1} \tilde{\delta}_{fo})$$

$$(37) \qquad \tilde{\delta}_{fo} = \frac{1}{m} \sum_{r=1}^{m} \tilde{\delta}_{fr} = \sum_{r_1=1}^{m} \{[\sum_{r_2=1}^{m} (X'_{fr_2} X_{fr_2} \sigma_{fr_2}^{-2} + \Omega^{-1})^{-1} X'_{fr_2} X_{fr_2} \sigma_{fr_2}^{-2}]^{-1}$$

$$\cdot (X'_{fr_1} X_{fr_1} \sigma_{fr_1}^{-2} + \Omega_f^{-1})^{-1} X'_{fr_1} X_{fr_1} \sigma_{fr_1}^{-2}\} \ \hat{\delta}_{fr}.$$

$$(38) \qquad \tilde{\sigma}_{fr}^{2} = \{\nu_{fr}\lambda_{fr} + (X_{fr} - W_{fr}\tilde{\delta}_{fr})' (X_{fr} - W_{fr}\tilde{\delta}_{fr})\}/(T+\nu_{fr} + 2)$$

$$(39) \qquad \tilde{\Omega}_f = \{R_f + \Sigma_{r=1}^{m} (\tilde{\delta}_{fr} - \tilde{\delta}_{fo}) (\tilde{\delta}_{fr} - \tilde{\delta}_{fo})'\}/(m+\rho-f-2)$$ [26]

[f = 1, ..., F (variables); $r_1 = r_2 = r = 1$, ...m (sectors)]

$\hat{\delta}_{fr}$ is the OLS estimate for δ_{fr}.

Lindley and Smith suggest that equations (36) - (39) be iterated until the estimates converge. To insure convergence, it is advisable to begin with estimates of σ_{fr}^2 and Ω_f that are close to the final estimates rather than a completely arbitrary number. Reasonable first estimates can be obtained by using:

$$(40) \qquad \hat{\Omega}_f = \frac{1}{m} \Sigma_{r_2=1}^{m} (\hat{\delta}_{fr_2} - \frac{1}{m} \Sigma_{r_1=1}^{m} \hat{\delta}_{fr_1})(\hat{\delta}_{fr_2} - \frac{1}{m} \Sigma_{r=1}^{m} \hat{\delta}_{fr_1})'$$

where $r = r_1 = r_2 = 1, \ldots m$ (sectors)

for Ω_f and

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[26] The estimates for $\tilde{\sigma}_{fr}^2$, $\tilde{\Omega}$ and $\tilde{\varepsilon}_{fr}$ are found in Lindley and Smith (1972, p. 10-15). The estimate for $\tilde{\delta}_{fo}$ is given in Smith (1972, p. 73).

(41) $$\hat{\sigma}^2_{fr} = \frac{1}{T}(X_{fr} - W_{fr}\hat{\delta}_{fr})' (X_{fr}-W_{fr}\hat{\delta}_{fr})$$

for σ_{fr}, where

(42) $$\hat{\delta}_{fr} = (W'_{fr} W_{fr})^{-1} (W'_{fr} X_{fr}),$$ the OLS sectoral estimates.

These estimates can then be used to obtain estimates of the coefficients. More explicitly, these initial estimates can be substituted into (36) to obtain improved estimates of $\tilde{\delta}_{fr}$, call these $\tilde{\delta}_{fr}^{(1)}$. The $\tilde{\delta}_{fr}(1)$ can be substituted into (38) and (37) to obtain $\tilde{\sigma}^{2(1)}_{fr}$ and $\tilde{\delta}_{fo}^{(1)}$; $\tilde{\delta}_{fr}^{(1)}$ and $\tilde{\delta}_{fo}^{(1)}$ can be substituted into (39) to obtain $\tilde{\Omega}_f^{(1)}$. This procedure can be repeated until the estimates converge, where convergence is defined as:

(43) $$[\tilde{\delta}_{fo}^{(n)} - \tilde{\delta}_{fo}^{(n-1)}]^2 < Z$$

where n is the number of the iteration and Z is appropriately small.

Smith (1972) suggested that approximations to vague priors can be obtained for σ^2_{fr} and Ω if ν_r and ρ, the degrees of freedom of the distributions, are small and R has small diagonal elements, i.e. values of the mean close to zero. In this case, the estimates of σ^2_{fr} and $\tilde{\Omega}_f$ in (38) and (39) are approximately equal to the estimates in the random coefficients model suggested by Lee and Griffiths (1979). This implies that if vague prior information for σ^2_{fr} and Ω is assumed, the Bayesian procedure will give estimates similar to nonBayesian estimates. On the other hand, if a tight prior is assumed (values for ν_r, ρ and R much larger), the Bayesian estimates will differ significantly from the non Bayesian estimates proposed by Lee and Griffiths.

In the estimation of the VAR, the priors described below were used. As Lindley and Smith suggested it was assumed that σ^2_{fr} (where f refers to equation, r refers to sector) has an conjugate inverse χ^2 distribution with mean ν and λ degrees of freedom. Following Lindley and Smith (1972) values of $\nu = 0$ and $\lambda = 0$ were used. This denotes vague information. It was also assumed that Ω has a conjugate Wishart distribution with matrix R and ρ degrees of freedom. For matrix R, the matrix:

$$(44) \qquad R = \begin{bmatrix} .02 & & & 0 \\ & .01 & & \\ & & .01 & \\ & & & \ddots & \\ & 0 & & & .01 \end{bmatrix}$$

was used. This matrix implies that a priori the constant has a larger variance than the other parameters. Again because the values are small this prior is one of vague knowledge. For degrees of freedom ρ, two values were used for the first equation; $\rho = 1$ and $\rho = 18$. The value $\rho = 1$ again assumes vague knowledge. The value $\rho = 18$ is obtained by assuming that the exchangeability prior gives slightly less information than is contained in the data. This assumption may be criticized as being too strong, however, upon examination of the equations estimated, it is apparent that even with a large value of ρ the estimates which use the exchangeability prior (EXC) are not greatly different from the OLS estimates. The conclusion, after viewing the results of the first equation with $\rho = 1$ and $\rho = 18$ was basically that the larger value of ρ would change the estimates slightly more, but not much more than the smaller value of ρ. Since the value of ρ did not make much difference, the larger value of ρ was chosen thereby claiming a strong prior assumption of exchangeability.

The estimates of the standard errors for each coefficient were obtained from the covariance matrix of the coefficient vector, In the Bayesian analysis this covariance matrix is the covariance of the posterior distribution. Recall in this model that it was impossible to obtain the mean of the posterior distribution, so the posterior mode was used as an approximation. Similarly, the variance of the posterior distribution is intractable.

Leamer (1978, p. 274) gives a conditional estimate of the variance of δ_{fr}:

$$(45) \quad \mathrm{Var}(\delta_{fr} \mid \delta^*_{fo}, X_{fr}, W_{fr}, \Omega_f) = [(W_{fr}{}^{1} W_{fr})\,\sigma_{fr}{}^{-2} + \Omega_f{}^{-1}]^{-1}$$

where δ^*_{fo} is the true parameter.[27]

It should be noted that if δ^*_{fo} is known, this variance is smaller than the case where δ^*_{fo} is not known. This estimate of the variance provides a lower bound.

3.5 Impulse Response Function

The impulse response function (IRF), also called the moving average representation, is the reaction of the system of variables to a shock in one variable in the system. The impulse response function was obtained by

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[27] Other authors including Swamy (1972) and Lindley and Smith (1972) give only the variance of the mean parameter.

solving $X_r(t)$ in terms of $u_r(t)$, where $u_r(t)$ are the innovations (shocks) to the system. The IRF is then written:

$$(46) \quad X_r(t) = \sum_{s=o}^{\infty} \theta_r(s)\ u(t-s)$$

where

$$(46a) \quad \sum_{s=o}^{\infty} \theta_r(s) L^s\ u_r(t) = [I - \sum_{s=o}^{\infty} \gamma_r(s) L^s]^{-1}\ u_r(t)$$

L^s here is the lag operator where $L^1 u_r(t) = u_r(t-1)$.

Equations (26) and (46), the autoregressive representation and the moving average representation respectively are different representations of the same system, thus $\gamma(s)$ from equation (26) is related to $\Theta(s)$ from equation (46) by equation (46a). The matrix $\Theta(s)$ then gives the response of the system. In particular, $\Theta_{rab}(1)$, an element of the matrix $\Theta(s)$, gives the response of the ath variable to a shock in the bth variable that occurred in period 1. The shock to the system is usually defined in terms of one standard error of the particular variable as variables typically differ greatly in size.

In practice the IRF is generated by first setting all variables in the system equal to zero. A shock to the system is then entered through the vector of innovations (shocks). The form of this vector is $u_{fr}(0) = (0, \ldots, 0, \sigma_f, 0, \ldots 0)$. This shock is then followed through the system as far as desired. The coefficients used in this system are the estimated coefficients. Error terms that enter the system in future periods are set to zero. The result of this process then gives the response of all variables to a shock in X_f.

The following example describes more clearly the method by which the impulse response function is obtained. Consider the two variable, one lag system:

(47) $Y_1(t) = \beta_{11}\ Y_1(t-1) + \beta_{12}\ Y_2(t-1) + \varepsilon_1(t)$

and

(48) $Y_2(t) = \beta_{21}\quad Y_1(t-1) + \beta_{22}\ Y_a(t-1) + \varepsilon_2(t)$

Now if $Y_1(t)$ is shocked, the vector of innovations $(\sigma_{11},0)$ is entered; all variables are set to zero. In the first period, the value of $Y_1(1)$ is σ_{11}, and the value of $Y_2(1)$ is 0. In the second and subsequent periods, the error terms are entered as (0,0). The original shock is followed through the system, thus $Y_1(2)$ is $\beta_{11}\sigma_{11}$ and $Y_2(2)$ is $\beta_{21}\sigma_{11}$. In the third period, the value for $Y_1(3)$ is $\beta_{11}^2\sigma_{11} + \beta_{12}\beta_{22}\sigma_{11}$ and the value for $Y_2(3)$ is $\beta_{21}\beta_{11}\sigma_{11} + \beta_{22}\beta_{21}\sigma_{11}$, and so on. The values for the β_{ii}'s are the estimated coefficients from the autoregressive representation given in equation (26). In this way the impulse response functions for the system are obtained for as many steps ahead as desired. Each variable in the system can be shocked separately in this way.

It should be noted that in the impulse response functon shocks to the system occur one at a time, independent of all other systematic effects. Realistically, these shocks occur simultaneously to more than one variable. However, the IRF allows isolation of each shock and its effect on the system. This is most useful in understanding the underlying working of the model.

Although the IRF can be viewed as another method of describing the system, there is some debate about its use in policy. Lucas (1972) argues that the IRF should not be used explicitly for policy implementation. His argument is that once policy is implemented the structure of the model is changed and thus the IRF changes. This is, the reaction of the model after policy is implemented will differ from the predicted reaction of the IRF. Others (Sims (1977)) argue that the IRF can be used, in some cases, for determining policy.

3.6 Orthogonal Decomposition of Variance as Used in the Impulse Response Function

The assumptions of the VAR model do not restrict the covariance of the residuals between equations of the system to be zero. As will be discussed in the estimation procedure, each equation can be estimated separately regardless of this assumption, as the variables in each equation are the same. However, it is desirable that shocks entered in obtaining the impulse response function have the same covariance structure as past shocks to the system. If the impulse response function does not take this covariance structure into account, the shocks to the system are of a type different from the previous shocks. So if this covariance structure is accounted for in the impulse response function, the shocks that affect the system will now be shocks similar to what the system has previously experienced thus the predicted effect should be better.

The way in which this covariance structure is accounted for in the impulse response function is a method of orthogonal decomposition of variance. The method of orthogonalization used is that suggested by Sims (1977) and it depends on a particular ordering of the variables. The actual ordering may have a different economic interpretation, but all orderings are statistically equivalent as the orthogonalization may have many representations.

An example will best illustrate the method of orthogonalization. Consider the two variable system introduced in equations (47) and (48). The covariance matrix for the residuals $\varepsilon_1(t)$ and $\varepsilon_2(t)$ is:

$$(49) \qquad E[\varepsilon\varepsilon'] = \begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix}$$

Equation (49) illustrates that the covariance of residuals is nonzero Now define a new set of errors, $u_1(t)$ and $u_2(t)$

where

$$(50) \qquad \varepsilon_1(t) = u_1(t).$$

Let $u_2(t)$ be the part of $\varepsilon_2(t)$ that is orthogonal to $\varepsilon_1(t)$ and thus $u_1(t)$:

$$(51) \qquad \varepsilon_2(t) = A\varepsilon_1(t) + u_2(t)$$

where

$$(52) \qquad E[u_2(t) \cdot \varepsilon_1(t)] = 0$$

Equation (51) can be recognized as a regression equation of $\varepsilon_1(t)$ on $\varepsilon_2(t)$ where A is the regression coefficient. A is defined by

$$(53) \qquad A = \frac{\sigma_{12}}{\sigma_{11}}$$

The new errors are now orthogonalized or have zero covariance, so

$$(54) \qquad E[uu'] = D = \begin{bmatrix} d_1 & 0 \\ 0 & d_2 \end{bmatrix}$$

where d_1 and d_2 are positive numbers. The system (47) and (48) can be rewritten in terms of the new errors as

(55) $$Y_1(t) = \beta_{11}Y_1(t-1) + \beta_{12}Y_2(t-1) + u_1(t)$$

and

(56) $$Y_2(t) = \beta_{21}Y_1(t-1) + \beta_{22}Y_2(t-1) + A\varepsilon_1(t) + u_2(t)$$

Substituting for $\varepsilon_1(t)$ from (47) into (56) gives:

(57) $$Y_2(t) - \beta_{21}Y_1(t-1) + \beta_{22}Y_2(t-1) = A[Y_1(t) - \beta_{11}Y(t-1) - \beta_{12}Y_2(t-1)] + u_2(t)$$

then

(58) $$Y_2(t) = AY_1(t) + (\beta_{21} - A\beta_{11})\,Y_1(t-1) + (\beta_{22} - A\beta_{12})Y_2(t-1) + u_2(t)$$

The new system is now defined by equations (55) and (58). Notice the new coefficients for $Y_1(t-1)$ and $Y_2(t-1)$ in (58); the correlation of residuals is accounted for in these new coefficients. Also notice that this redefined system $Y_2(t)$ is correlated with contemporaneous $Y_1(t)$ as $Y_1(t)$ appears in the equation for $Y_2(t)$ (equation 58) but notice $Y_2(t)$ does not appear in the equation for $Y_1(t)$ (equation 55). If the system had been ordered in the opposite way so that

(59) $$u_2(t) = \varepsilon_2(t)$$

and

(60) $$\varepsilon_1(t) = A\varepsilon_2(t) + u_1(t)$$

where

(61) $E[u_1(t) \cdot \varepsilon_2(t)] = 0$

then $Y_1(t)$ would still be correlated with contemporaneous $Y_2(t)$ but $Y_1(t)$ would have not appeared in the equation for $Y_2(t)$. This is how the ordering of the variables enters the orthogonal decomposition.

All orderings of variables are equally consistent with the data in that each of the models [e.g. defined by equations (55) and (58) and the model associated with assumptions (59), (60) and (61)] will fit equally well in terms of sum of squared residuals. (Sum of squared residuals will be the same for all orderings). There is a difference in interpretation associated with each ordering. In the case of the VAR discussed above, 9 variables are contained in the model, so there are 9 factorial (over 300,000) orderings possible. Considering each of these orderings is too costly and time consuming.

Because one of the main considerations in this thesis is the response of the economy to an increase in the price of energy, an ordering with the price of energy first is chosen, the other variables are entered as they are entered in the OLS and exchangeability models. This gives the ordering:

(1) Price of energy

(2) Output

(3) Capital

(4) Price of Capital

(5) Labor

(6) Price of Labor

(7) Energy

(8) Intermediate Materials

(9) Price of Intermediate Materials

This ordering captures the contemporaneous correlation of all variables to the innovation in the price of energy. In fact, it should be noted that in the orthogonalized system, the first response in the impulse response function will be the contemporaneous covariance between the residuals of the original, non-orthogonalized system. In the impulse response function for the price of energy, the first period response for all variables is the nonzero contemporaneous correlation between residuals. In the impulse response function for the other variables, for example,labor, the first period response will be the nonzero contemporaneous correlation between residuals for the variables following labor in the ordering, but the first period response for the variables preceding labor in the ordering will be zero because of the orthogonalization.

4.1 Results of the Generalized Box-Cox Cost Function

The computational costs associated with the generalized Box-Cox cost function (GBC) were high. For this reason, as was mentioned in Section 2.3, only three values of λ (λ = .5, 1 and 2) were investigated for each of the five sectors:

(26) Paper and Allied Products

(28) Chemical and Allied Products

(29) Petroleum and Coal Products

(33) Primary Metal Industries

(40) Agriculture

By considering only three values for λ, a rough grid search can be done. It was not possible to pinpoint the exact value for λ for which the log-likelihood function is maximized but the general area of the maximum could be found. The results show that for four of the five sectors, paper, petroleum and coal, primary metals and agriculture, the log-likelihood function was largest when $\lambda = 1$. For sector 28, chemical products, the log-likelihood function was largest when $\lambda = 2$. Recall the GBC function is equivalent to the generalized Leontief cost function when $\lambda = 1$ and the generalized square root quadratic when $\lambda = 2$. The results will be discussed for the values of λ associated with the largest value of the log-likelihood function.

The functional form of the GBC function used in the estimation procedure allows both nonhomothetic production and nonneutral technical change. A nonhomothetic, nonneutral production structure allows utilization of the least restrictive form of the GBC function under these least restrictive assumptions. Table 1 gives the estimated coefficients for the GBC function under these assumptions. A comment should be made about the size of the coefficients in Table 1 as size varies considerably over sectors. The likelihood function is such that parameter size makes no difference because it is essentially a sum of ratios with the parameters appearing in both the numerator and denominator. In those sectors where the coefficients were very large several initial points were used; in each case the program arrived at the same estimates, indicating some degree of consistency in the results.

As mentioned in Section 2.3, ZXMIN uses the Davidon-Fletcher-Powell (DFP) method to update the Hessian [see Maddala (1977), p. 173 for description). In some cases, numerical problems

Table 1. Estimates of the Generalized Box-Cox Cost Function.
(asymptotic standard errors in parentheses)

	Paper Products $\lambda=1.$	Chemical Products $\lambda=2.$	Petroleum Products $\lambda=1.$	Primary Metals $\lambda=1.$	Agriculture $\lambda=1.$
FREE PARAMETERS					
γ_{KK}	2.353	-28441.2	-1.497	-821193.4	50.798
	(1.427)	(78820.7)	(2.064)	(6286819.)	(723.1)
γ_{KL}	.850	247902.0	2.035	41928270.	88.65
	(1.746)	(628981.0)	(2.505)	(130977800.)	(1160.8)
γ_{KE}	-.063	32173.3	.316	2676448.	5.291
	(.336)	(81166.3)	(.484)	(10400250.)	(82.57)
γ_{KM}	-3.308	-92619.8	1.669	-20257670.	-85.35
	(2.044)	(221449.2)	(2.485)	(63640630.)	(1165.8)
γ_{LL}	(9.337)	276409.3	-1.711	52858850.	4917.5
	(6.99)	(741912.0)	(3.190)	(161599200.)	(67809.8)
γ_{LE}	1.989	262006.6	1.275	-9760260.	365.67
	(1.628)	(676588.2)	(2.191)	(48965390.)	(4971.1)
γ_{LM}	-.410	78155.9	5.011	4293391.	-2345.4
	(2.121)	(305289.7)	(7.711)	(58018270.)	(32479.2)
γ_{EE}	-1.039	85009.6	-.955	-2051875.	-3.376
	(.861)	(223567.)	(1.457)	(18227630.)	(66.65)
γ_{EM}	2.317	106841.8	1.148	18870570.	-163.2
	(1.635)	(286987.4)	(1.956)	(69321840.)	(2286.3)
γ_{MM}	8.984	241221.7	-.458	8353974.	1546.7
	(7.629)	(623946.3)	(3.197)	(83392860.)	(21520.1)
β	-.442	-1.338	-.357	-5.419	-1.065
	(.301)	(.455)	(.555)	(1.199)	(4.455)
Θ	.263	.358	.253	1.052	.253
	(.061)	(.087)	(.106)	(.221)	(.728)
ϕ_K	.011	-.007	-.025	-.020	.0023
	(.012)	(.005)	(.007)	(.025)	(.0147)
ϕ_L	-.039	-.034	-.055	-.062	-.1017
	(.016)	(.018)	(.014)	(.048)	(.0463)
ϕ_E	-.029	-.043	-.018	-.0069	-.0097
	(.005)	(.009)	(.005)	(.044)	(.0054)
τ_O	-.004	.003	-.0037	-.00046	-.0197
	(.002)	(.005)	(.003)	(.0065)	(.0013)
τ_K	.00009	.00004	.002	.00046	.0038
	(.0006)	(.0002)	(.0003)	(.00121)	(.0010)
τ_L	-.0002	-.0041	-.00006	-.0017	-.0127
	.0005	(.0024)	(.0008)	(.0046)	(.0044)
τ_E	.0023	.0034	.0013	.0007	.0006
	(.0003)	(.0008)	(.0003)	(.0028)	(.0004)
CONSTRAINED PARAMETERS					
ϕ_M	.057	.084	.098	.0889	.1091
τ_M	-.00219	-.00066	-.00324	.00054	.0083
Value of the log-likelihood function	503.42	464.71	457.72	388.62	377.31

were encountered when the Hessian, obtained from the DFP method, was inverted. For this reason, the method of scoring suggested by Maddala was used to update the Hessian when rounding error was encountered. It was found that this method often worked quite well in moving the estimates to a point where the likelihood function was maximized. This estimate of the Hessian, Q, given in equation (25) in Section 2.3 was used as an approximation to the true Hessian. The inverse of Q, then gave the estimate of the variance. The asymptotic standard errors, obtained from the inverse of Q, are reported in Table 1. Maddala noted that Q is only asymptotically equivalent to the Hessian but in this case, with sample size 30, the approximation may be inaccurate. Given this qualification, the estimates of the standard errors are generally large for the γ_{ij}'s. The standard error estimates are much smaller for the last nine parameters.

Although only an approximation to the standard error could be obtained, the fit of the GBC function is good. Figures 1 and 2 show the fit of the GBC estimates for $\lambda = 1$ for the sectors with the highest and lowest values of the log-likelihood function. The actual and estimated values for 1947, 1957, 1967 and 1976 for each of K/Y, L/Y, E/Y and M/Y are shown.[28/] It can be seen in both Figures 1 and 2 that the actual and estimated values are close. Figure 1 is associated with the function which has the least good fit, so it forms an approximate lower bound on goodness of fit. One of the regularity conditions discussed in section 2 is that the input-output values be positive. This condition is met for all sectors for all input-output values.

[28/] The years 1947, 1957, 1967 and 1976 were chosen for these figures and all discussion of elasticities, etc. before analysis was begun, in order to decrease the magnitude of results and yet still give an overview of the time period.

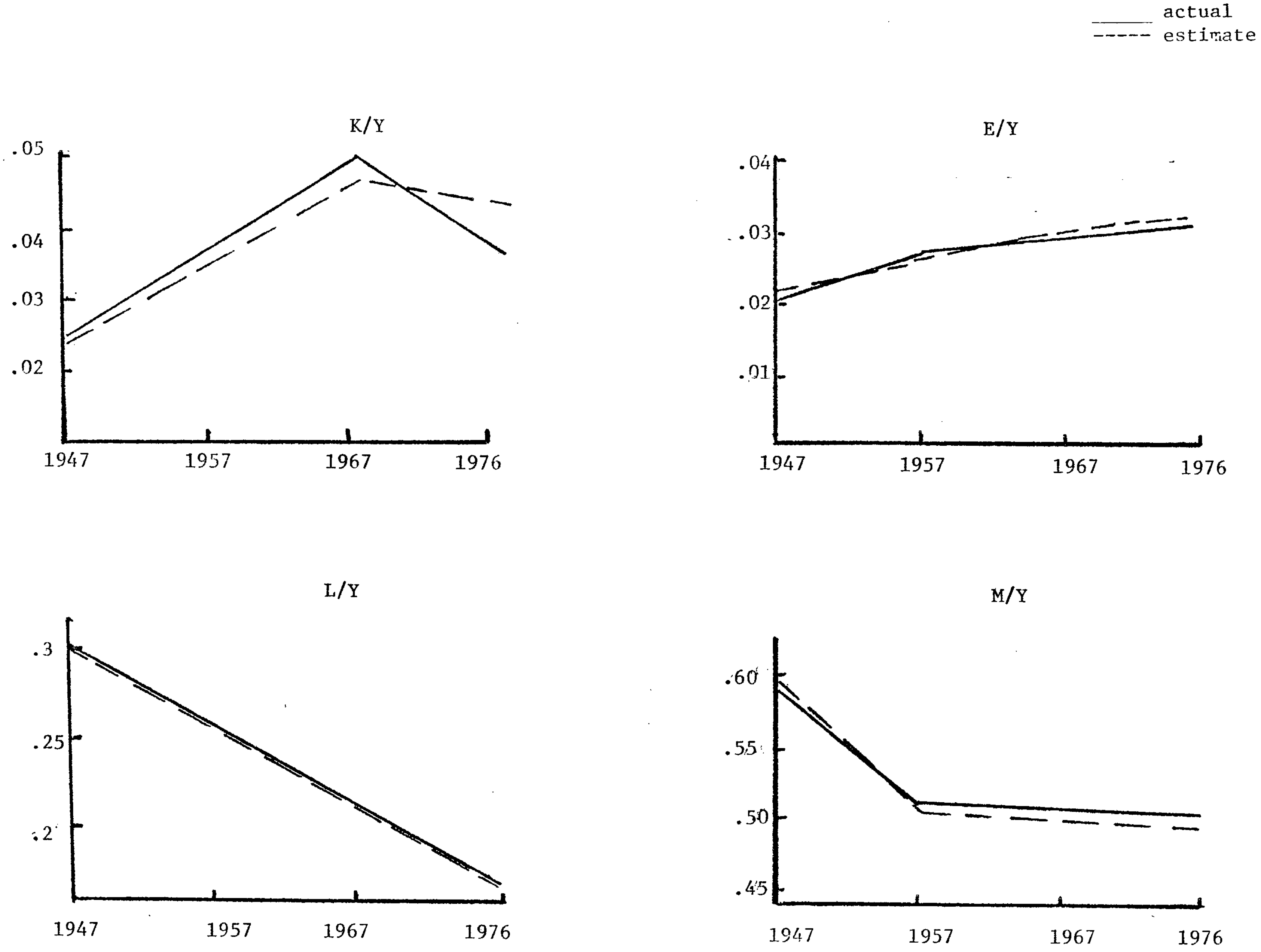

Figure 1. Estimated and Actual Input-Output Ratios for Paper Products for 1947, 1957, 1967, and 1976.

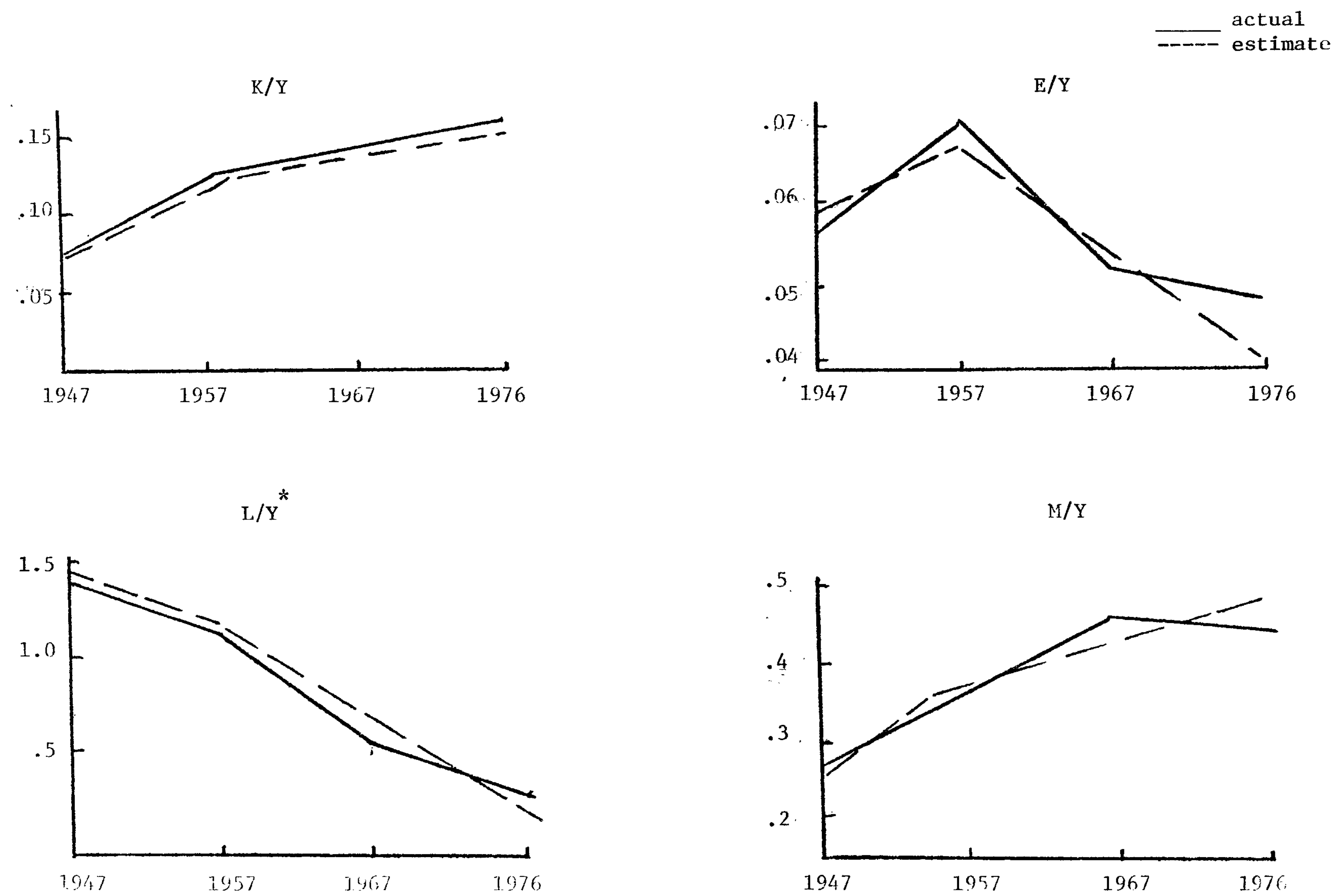

Figure 2. Estimated and Actual Input-Output Ratios for Agriculture for 1947, 1957, 1967 and 1976.

[*] The valuation of L/Y in 1947 is greater than one due to the use of the manufacturing wage (≡ opportunity cost of farmer) in obtaining the labor quantity index as discussed in Appendix I. As noted there, this was used in lieu of the agricultural wage as this latter wage reported is that earned only by hired labor and does not represent income earned by farm owners. The use of the manufacturing wage in determining L caused this disparity in L/Y in only a small number of years.

4.2 Elasticities of Substitution

Before the estimated elasticities of substitution of this particular GBC model can be discussed, a discussion of substitution of inputs is needed. The production process determines which input substitutes for another when input prices change. The time period over which changes in input substitution are allowed will also affect the ultimate input response.

Consider a firm that produces one good (for simplicity), that uses capital, labor, energy and intermediate materials in its production process. A given amount of energy is required to run the machinery (capital). In addition, energy is used to heat and light the plant. Labor is only used to operate the machinery, however labor could be utilized to manually perform some parts of the production process. Now suppose the price of energy rises and the prices of all other inputs remain the same. In the time period immediately following the energy price increase, the firm will try to use less of the higher priced energy input. The production process is fairly rigid, but substitution possibilities do exist. For instance, the production process can be altered slightly to incorporate more labor. The plant can also be insulated and unnecessary lights can be turned off. This involves substitution of labor and capital for energy. The response in this short run time period is one of conservation of energy in the least costly ways possible.

In a slightly longer time horizon, as the machinery wears out, it can be replaced with new capital that uses energy more efficiently, but requires more labor to operate it. The same amount of output can then be obtained with less energy usage than was previously possible. In this longer run time period, capital and labor both substitute for energy.

In the still longer run, technological advances make an entirely new production process available which uses even less energy than was used with the more energy-efficient capital discussed above. This new production process requires the use of less labor, less than was used before the original energy price increase. Thus in this longest run time horizon, capital substitutes for energy, whereas labor is complementary with energy usage.

It should be noted that the input substitutions described above depend on the assumptions of the production process. Other production processes which are less rigid in the short run or which have energy a substitute for labor and a complement with capital in the longest run are also possible. This discussion primarily illustrates the difference in input substitution possibilities over time.

In the GBC function discussed in this section, input substitution is assumed to take place in the same year as energy prices increase. Therefore the GBC function does not measure the long run elasticities, however, the vector autoregressive (VAR) model discussed in the following section does allow investigation of the longer run elasticities.

The main interest in the GBC model lies in the elasticities of substitution. These elasticities of substitution are given for 1947, 1957, 1967 and 1976 in Table 2. The elasticities of substitution are found by substituting the maximum likelihood estimates for the parameters from Table 1 and the estimated input-output values, cost shares and total cost into equations (9) and (10) in Section 2.1.

One of the concavity requirements is that the own elasticities of substitution be negative. From Table 2, it can be seen that this condition is met for petroleum products for all four years. Two of the sectors, primary metals and agriculture meet this condition for most years, but σ_{MM} is positive in agriculture in 1947 and σ_{LL} is positive in agriculture in 1967. In primary metals σ_{KK} is positive for all years except 1976. Chemical products has many positive own elasticities of substitution; σ_{LL} is positive for all four years, σ_{KK} is positive for four years and σ_{EE} and σ_{MM} are positive in one year. Food products appears to have the worst problems as σ_{KK} and σ_{LL} are both positive for all four years and σ_{MM} is positive for three of the four years. Failure in meeting this condition indicates that the GBC function is not well behaved in the area of the estimates. In particular, as the price of the input increases more of it is demanded if this own elasticity of substitution (or input demand) is positive.

A more stringent regularity condition regarding curvature of the cost function is that the matrix of second derivatives of the cost function (or equivalently the matrix of elasticities of substitution) be negative semidefinite.[29/] This condition was checked for the same four years. This condition of negative semidefiniteness was not met except for agriculture in 1967. Failure to meet this condition indicates that the cost function does not have the correct concave curvature required for well behaved cost minimization. Failure to meet this condition could have several causes.

[29/] These two conditions were proven equivalent by Binswanger 1974b.

A possible cause of the curvature violation is the data. If the parameter estimates of the functional form are sensitive to the data, small changes in the variables can lead either to meeting regularity conditions or curvature violation. In particular data used for the capital variable may contribute to problems in estimation which manifest themselves in curvature violation. For a general discussion of the data and data problems see Appendix I. Another cause of this curvature violation could be misspecification. Curvature violation may occur if the functional form used is incorrect. Other measures such as goodness of fit should be used in determining if this is the case. In this particular model, the fit of the equations estimated is good. However, the estimates of the γ_{ij} have very large standard errors. Recall from section 2.1 equations (9) and (10), that the γ_{ij} are important in the estimation of the σ_{ij} and thus the regularity condition. Large standard errors of the γ_{ij} indicate that the estimates of the true parameters are not tight estimates and the estimated parameters could lie anywhere in a large confidence region around the true parameters. It is, therefore, probable that the true parameters would meet the regularity conditions but because of estimation problems evidenced by large standard errors of the γ_{ij}, the estimated parameters do not meet the regularity condition.

As the coefficients differed from sector to sector and from one value of λ to another, the elasticities of substitution also varied, as can be seen from Table 2. These results from Table 2 are discussed below.

Capital and labor are substitutes in all sectors. The size of the elasticity decreased over time in paper products and primary metals. In both sectors the elasticity of substitution was greater than one, indicating the substitution possibilities between K and L were significant. The fact that the elasticity was decreasing over time indicates that the production

Table 2. Estimates of the Elasticities of Substitution.

Elasticity	Year	Paper Products $\lambda=1$	Chemical Products $\lambda=2$	Petroleum and Coal $\lambda=1$	Primary Metals $\lambda=1$	Agriculture $\lambda=1$
σ_{KL}	1947	11.36	32.28	.83	24.68	.68
	1957	4.43	17.16	.96	6.54	.25
	1967	4.02	17.73	3.41	6.40	.45
	1976	3.53	19.42	5.02	5.26	.69
σ_{KE}	1947	11.26	50.14	-7.18	-.58	.64
	1957	2.10	-.91	-9.30	.36	.31
	1967	1.93	-11.48	-3.38	.61	.59
	1976	.95	-1.21	-1.14	.78	.87
σ_{KM}	1947	-6.23	-8.94	1.45	-9.61	-.89
	1957	-2.79	-3.49	.99	-2.28	1.01
	1967	-2.07	-1.66	1.21	-1.70	.98
	1976	-1.59	-1.39	1.11	-1.10	.89
σ_{LE}	1947	-.55	36.99	.39	-5.81	.31
	1957	-1.78	2.89	1.90	-6.64	-.51
	1967	-2.32	3.22	.86	-9.18	-1.03
	1976	-.65	5.73	.65	-8.59	-1.22
σ_{LM}	1947	-.16	-.28	1.19	.66	-.33
	1957	-.05	.73	1.39	1.14	1.08
	1967	.003	1.56	1.69	1.25	1.37
	1976	-.09	1.91	1.03	1.08	.98
σ_{EM}	1947	2.59	16.58	1.79	3.03	-.93
	1957	2.39	9.50	1.29	4.06	1.01
	1967	2.69	11.99	1.21	4.53	1.24
	1976	2.29	2.61	1.05	3.69	.95
σ_{KK}	1947	37.07	66.87	-29.43	117.24	-8.44
	1957	9.30	-2.45	-12.80	1.61	-4.44
	1967	.10	5.51	-17.49	2.17	-4.64
	1976	.59	.29	-20.95	-1.12	-4.51
σ_{LL}	1947	1.44	3.31	-7.08	-.11	-.23
	1957	.94	2.38	-11.07	-1.17	-.94
	1967	1.37	7.06	-13.79	-.99	-1.62
	1976	1.71	7.65	-21.21	-1.02	.18
σ_{EE}	1947	-16.96	-346.61	-52.26	-6.47	-3.93
	1957	-7.29	-1.70	-29.34	-8.83	-4.23
	1967	-1.13	47.12	-18.03	-10.99	-4.16
	1976	-12.82	-11.85	-27.56	-10.93	-4.05
σ_{MM}	1947	.15	.24	-.24	-.45	2.81
	1957	.12	-.45	-.19	-.79	-1.22
	1967	.04	-1.09	-.25	-.76	-1.71
	1976	-.01	-1.11	-.14	-.60	-1.17

process was becoming more rigid as time passed, possibly due to a decline in excess capacity. In petroleum and coal products, the elasticity between K and L increased. In 1947 and 1957, this elasticity was less than one indicating that substitution possibilities were limited. In 1967 and 1976, however, this elasticity became greater than one which indicated greatly increased substitution possibilities. In chemical products and agriculture the elasticity initially decreased but increased in 1976. There is a difference in these two sectors. In chemical products, the elasticity was very large indicating that substitution between K and L was quite easily accomplished, even in the years when the elasticity was decreasing. In agriculture, the elasticity was less than one, so substitution possibilities were limited. During this time, labor was being rapidly replaced by capital in agriculture, so this empirical result is weak.

Capital and energy were found to be complements in two of the five sectors, chemical products and coal and petroleum products. This elasticity became larger from 1957 to 1967 in chemicals but decreased in 1976. This change implied that from 1957 to 1967 the complementary structure of the production process strengthened, implying that small increases in energy price would lead to greatly reduced capital use. This rigidity diminished in 1976. In petroleum and coal, the complementary relationship between K and E in the production process diminished some (became smaller in absolute value) in 1967 and even more so in 1976, coming very close to 1. Capital and energy in paper products, primary metal and agriculture were substitutes. In paper products this elasticity is large, but in 1976 this elasticity of substitution fell to less than one. So although the inputs, K and E were substitutes in 1976 in paper products, the substitution possibilities were limited. In both primary metals and agriculture, the elasticity decreased in 1957 and 1967, but it increased in 1976. In both cases this elasticity was less than one, so substitution possibilities were limited.

Capital and materials were complements in paper products, chemical products and primary metals. In all of these sectors, this negative elasticity decreased in each period. This indicates that the complementary nature of the inputs was decreasing; it did not become insignificant, however, in any of these sectors. In petroleum and coal products, capital and materials were substitutes. This relationship was fairly stable, remaining close to 1 over the entire period. In agriculture, capital and materials were complements in 1947, but in following years the relationship changed to substitutes with the elasticity decreased from 1957 to 1967 but increased in 1976. In all years, this elasticity remained close to 1, indicating somewhat limited substitutition between K and M.

The elasticity of substitution between labor and energy was negative (complements) in all except chemical products and petroleum products. In paper products and primary metals this negative elasticity became larger, in absolute value, from 1947 to 1967 but decreased in 1976. This indicates that the relationship between L and E was becoming more rigid until 1967, when the rigidity decreased in 1976, perhaps in response to higher energy prices. There was a difference in paper products and primary metals, in that in 1976 the elasticity was less than 1 in paper products indicating weak complementary relations between L and E. In primary metals, however, this elasticity was quite large, so although the rigidity between L and E may have decreased, rigidity still remained. In agriculture, L and E were substitutes in 1947 but the relationship changed to complements in 1957 and remained so in 1967 and 1976, becoming larger in each successive year, indicating that rigidity in L-E use was increasing. However, the elasticity was close to one indicating the complementary relationship was a weak one. In chemical products and petroleum products, labor and energy were substitutes. In chemicals, this elasticity decreased from 1947 to 1967 but increased in 1976. This elasticity was very large in all years, so the flexibility in L - E use remained strong. The opposite was true in petroleum products. In this sector, the elasticity increased from 1947 to 1957 but decreased thereafter. In addition the elasticity remained small which indicates that the relationship of substitutes was a weak one.

Labor and materials were substitutes for all but paper products. In primary metals and agriculture, this elasticity increased from 1947 to 1967 but decreased in 1976. The elasticity remained close to 1 in all years. In chemicals, this positive elasticity became larger in each successive year and it became greater than one in 1967. Petroleum and coal products showed the most variability, decreasing from 1967 to 1976. The elasticities were close to 1, except in 1967, so the substitute relationship remained fairly constant. In paper products, labor and materials were complements except in 1967. The elasticity was this sector is very close to zero barely a complementary relationship.

Finally, energy and materials were substitutes in all sectors for all years except agriculture in 1947. Petroleum and coal products showed a decrease in the size of this elasticity in each year but it remained greater than one. This elasticity was large initially but changes over time indicate that the substitute relationship was weakening somewhat. Primary metals and agriculture showed an increase from 1947 to 1967 but decreased in 1976. The difference is that in primary metals this elasticity was larger than 1 in all years whereas in agricultrue it was close to 1, becoming less than 1 in 1976. Thus the substitute relationship was stronger in primary metals than agriculture. In paper products and chemicals, this elasticity decreased from 1947 to 1957, increased in 1967 but decreased again in 1976. This movement is larger in chemical products indicating the substitute relationship was weakening over time but remained strong. In paper products, the size changed very little. In 1976, paper products and chemicals were of the same ending magnitude in the substitution elasticity between E and M.

In summary, paper products had strong substitution occurring between capital-labor and energy-materials throughout the period. Capital and energy were also substitutes throughout, however, this relationship weakened over time. Capital and materials were strong complements in 1947, but this relationship became more flexible by 1976. Labor and energy were also complements, but this relationship was relatively weak; in 1976, it was less than one (in absolute value). Labor and materials were extremely weak complements, barely different from zero. The substitution relationships decreased over time, but they remained strong. In addition the complementary relationships became less rigid. The labor-energy complementarity decreased significantly by 1976. So in this sector it appeared that there was ample room for adjustment of inputs to higher energy prices. In particular, capital and materials would replace energy as energy use decreased.

In chemical products, capital and labor were very strong substitutes. Labor-energy, labor-materials and energy-materials were also strong substitutes, although these relationships appear to be weaker than that of capital-labor. The positive elasticities of substitution in this sector, particularly in 1947 were extremely large. Elasticities of this size are questionable and could be the result of data and estimation problems. Conclusions based on these elasticities must therefore be used very cautiously. The positive elasticities in 1976 were more reasonable in size. Capital and energy were complements as were capital and materials. The capital-energy relationship appeared to change greatly in both sign and size. In 1976, the complementary relationship was down close to one. The capital-materials relationship was not as variable. It, too, was close to one in 1976. The chemical products sector, like the paper products sector, showed great possibility for adjusting to higher energy prices. In particular, labor and materials would replace energy as its use is decreased. Capital use in this sector would decrease as energy use decreased.

In petroleum and coal, capital and labor were substitutes, with the relationship growing stronger from 1947 to 1976. This trend was different than the trend for capital-labor in the other sectors in that it grew stronger rather than weaker over time. Capital-materials, labor-energy, labor-materials and energy-materials all had substitution relationships. These relationships, however, had somewhat limited but important substitution possibilities, as all of them were close to or less than one. Capital and energy, in this sector, were weak complements. This was the only complementary relationship among the inputs in this sector. As energy prices increase, labor and materials would be used to replace energy as its use decreased. Capital usage would decrease with decreased energy usage.

In primary metals, capital-labor and energy-materials were strong substitutes, while capital-energy and labor-materials were weak substitutes. The capital-energy elasticity was less than one. Capital and materials displayed a weak complementary relationship. Labor and energy also had a much stronger complementary relationship. This labor-energy complementarity was the strongest complementary relationship estimated in all sectors. As energy prices increase, capital and materials would replace energy. Since the materials-energy relationship was stronger than the capital-energy relationship, it would be likely that more materials than capital would be substituted for energy. Since labor and energy were strong complements, labor use would decrease significantly when energy use decreased.

In agriculture, capital-labor, capital-energy, capital-materials, labor-materials and energy-materials all shared weak substitution relationships. Labor-energy was the only complementary relationship in agriculture. Given these relationships, capital and materials would replace energy when its price rose, labor use would decrease with decreased energy use. The capital variable in agriculture is unlike that used in the other sectors discussed in this analysis. Land is included in capital along with structures and equipment; in the other sectors, land is not included. It is possible that inclusion of the land variable in capital diminishes estimated substitution possibilities that exist between capital and energy as well as capital and labor and possibly the other inputs. Historically capital has been a strong substitute for labor; the estimates of the GBC function did not reflect this substitution. It is therefore likely that the other estimated elasticities of substitution in agriculture have been biased downward (to the point of being negative for σ_{LE}) by including land in the capital variable.

4.3 Elasticities of Input Demand

The elasticities of input demand are obtainable from the elasticities of substitution by multiplying by the appropriate cost share.[30] The size of the elasticity of input demand indicates the percentage change in input use that is associated with a percentage change in the price of that or other inputs. The elasticities of input demand are given in Table 3 for the λ's discussed in the text for 1947, 1957, 1967 and 1976.

The elasticities of input demand contain information that is similar to that of the elasticities of substitution, in that the sign is the same. However, from footnote 30, it is apparent that this elasticity of input demand scales the elasticity of substitution by the cost share of the input. So there is a different interpretation to the elasticity of input demand. In particular, the elasticity of input demand is a percentage change in quantity due to a one percent change in price whereas the elasticity of substitution is a change in input quantity ratios due to a change in input price ratios.

The own elasticities of input demand have the same sign as the own elasticities of substitution, so there are as many positive signs in the elasticities of input demand as there are for the elasticities of substitution. Most of the negative own elasticities of input demand were inelastic indicating that all inputs were relatively insensitive to an increase in their own price. The input which was most responsive to a change in its own price varied in each sector from year to year. For instance, capital was more responsive than labor in petroleum and coal products and agriculture in 1947 and 1976 ($\varepsilon_{KK} > \varepsilon_{LL}$), but labor was more responsive than capital in 1957 and 1967 ($\varepsilon_{LL} > \varepsilon_{KK}$). The responsiveness of materials and energy also

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[30] Note that although $\sigma_{ij} = \sigma_{ji}$, $\varepsilon_{ij} \neq \varepsilon_{ji}$. Note that $\varepsilon_{ij} = \sigma_{ij} S_j$, now $\sigma_{ij} = \sigma_{ji}$ but $S_i \neq S_j$ thus $\varepsilon_{ij} \neq \varepsilon_{ji}$ where S_j is the cost share of input j.

Table 3. Estimates of Elasticities of Input Demand

Elasticity	Year	Paper Products $\lambda = 1$	Chemical Products $\lambda = 2$	Petroleum and Coal $\lambda = 1$	Primary Metals $\lambda = 1$	Agri-culture $\lambda = 1$
ε_{KK}	1947	.55	1.06	-.76	1.84	-.36
	1957	.35	-.11	-.34	.08	-.53
	1967	.005	.24	-.76	.11	-.64
	1976	.03	.02	-1.03	-.07	-.69
ε_{KL}	1947	2.09	4.98	.09	4.80	.55
	1957	.94	3.02	.07	1.49	.22
	1967	.83	2.51	.22	1.30	.24
	1976	.64	2.20	.21	.95	.15
ε_{KE}	1947	.31	.82	-.11	-.05	.04
	1957	.06	-.03	-.16	.02	.02
	1967	.05	-.31	-.08	.02	.03
	1976	.04	-.05	-.31	.04	.04
ε_{KM}	1947	-3.54	-4.50	1.11	-5.69	-.24
	1957	-1.50	-1.52	.78	-1.19	.33
	1967	-1.06	.69	.91	-.93	.41
	1976	-.83	-.63	.92	-.65	.44
ε_{LK}	1947	.17	.51	.02	.39	.03
	1957	.16	.76	.03	.33	.03
	1967	.18	.78	.15	.32	.06
	1976	.19	.97	.25	.33	.11
ε_{LL}	1947	.27	.50	-.74	-.02	-.18
	1957	.20	.42	-.82	-.27	-.83
	1967	.28	.99	-.89	-.20	-.88
	1976	.31	.87	-.89	-.19	.04
ε_{LE}	1947	-.02	.61	.006	-.43	.02
	1957	-.05	.09	.03	-.34	-.03
	1967	-.06	.09	.02	-.34	-.06
	1976	-.03	.26	.02	-.42	-.05
ε_{LM}	1947	-.09	-.14	.92	.39	-.09
	1957	-.03	.32	1.11	.59	.36
	1967	.002	.65	1.26	.68	.56
	1976	-.04	.86	.86	.63	.49

(-continued on next page-)

Table 3. - continued....

Elasticity	Year	Paper Products $\lambda = 1$	Chemical Products $\lambda = 2$	Petroleum and coal $\lambda = 1$	Primary Metals $\lambda = 1$	Agri-culture $\lambda = 1$
ε_{EK}	1947	.16	.79	-.18	-.009	.03
	1957	.08	-.04	-.25	.02	.04
	1967	.09	-.51	-.15	.03	.08
	1976	.05	-.06	-.05	.05	.13
ε_{EL}	1947	-.10	5.72	.04	-1.13	.24
	1957	-.38	.51	.14	-1.52	-.45
	1967	-.48	.45	.06	-1.86	-.56
	1976	-.12	.65	.03	-1.56	-.28
ε_{EE}	1947	-.47	-5.68	-.83	-.54	-.27
	1957	-.22	-.06	-.51	-.46	.32
	1967	-.03	1.26	-.41	-.40	-.22
	1976	-.65	-.53	-.74	-.54	-.18
ε_{EM}	1947	1.47	8.35	1.37	1.79	-.25
	1957	1.29	4.13	1.03	2.12	.33
	1967	1.38	5.02	.91	2.46	.51
	1976	1.19	3.45	.87	2.17	.47
ε_{MK}	1947	-.09	-.14	.04	-.15	-.04
	1957	-.11	-.15	.03	-.11	.12
	1967	-.09	-.07	.05	-.08	.14
	1976	-.09	-.07	.05	-.07	.14
ε_{ML}	1947	-.03	-.04	.13	.13	-.27
	1957	-.01	.13	.10	.26	.95
	1967	.0007	.22	.11	.25	.74
	1976	-.02	.22	.04	.19	.22
ε_{ME}	1947	.07	.27	.03	.25	-.06
	1957	.07	.31	.02	.21	.08
	1967	.08	.32	.03	.17	.07
	1976	.11	.34	.03	.18	.04
ε_{MM}	1947	.09	.12	-.18	-.27	.75
	1957	.06	-.19	-.15	-.41	-.40
	1967	.02	-.46	-.19	-.42	-.70
	1976	-.01	-.50	-.13	-.35	-.59

changed from year to year relative to the other inputs. In all sectors except chemical products and agriculture, energy (ε_{EE}) became more responsive to a change in its own price in 1976 than it was in 1967. This indicates that although energy demand remained inelastic as energy prices rose significantly, demand appeared to become more responsive than was the case before the oil price rise of 1974.

The elasticities of input demand show the same relationship as the elasticities of substitution, but as discussed above the quantity of the elasticity of input demand represents how much input quantity changes in response to a price change, rather than the ease of substitution between inputs. Capital and labor were substitutes in all sectors. In addition, the response of capital to a change in the price of labor was always greater than the response of labor to a change in the price of capital ($\varepsilon_{KL} > \varepsilon_{LK}$). This was most likely due to the fact that the cost share of labor in production was much larger than the cost share of capital. (See Appendix I, Table 13 for cost shares data) For this reason, changes in labor cost would have a greater impact on total cost of production than changes in capital cost, so it is reasonable that capital would adjust more readily to changes in labor cost than labor to changes in capital cost. Capital cost increases would be absorbed. The elasticities ε_{KL} and ε_{LK} were generally inelastic, with ε_{KL} in petroleum and coal products being very close to zero. There were exceptions, however, notably ε_{KL} in chemical products and primary metals; both sectors had large elastic responses, decreasing from 1947 to 1976.

In 1947, capital was more responsive to a change in the price of energy than energy use was to a change in the price of capital in all sectors except petroleum and coal products. ($\varepsilon_{KE} > \varepsilon_{EK}$). In 1957, this relationship changed ($\varepsilon_{EK} > \varepsilon_{KE}$) except for primary metals. This latter relationship held for all sectors in 1967 and 1976. This change in relationship was most likely caused by the similarity in size of the capital and energy cost shares. (See Appendix I, Table 1 for these cost shares) These elasticities were of the same sign as the elasticities of substitution between capital and energy and they were very inelastic. Recall that σ_{KE} in Table 2 is very close to one in 1976 in all sectors, regardless of sign.

Capital was more responsive to a change in the price of materials than materials was to a change in the price of capital ($\varepsilon_{KM} > \varepsilon_{MK}$). This is similar to the capital-labor relationship. In this case, because materials have a dominant cost share, (see Appendix I, Table 1), it is likely that capital is quite sensitive to a change in materials price for this reason. The elasticity, ε_{MK} was quite inelastic for all sectors and years, but ε_{KM} was quite elastic in paper products, chemical products and primary metals, ε_{KM} became inelastic in these sectors in 1976. These ε_{KM} were negative in these sectors, recall that σ_{KM} was very large and negative in these sectors also. The ε_{KM} were inelastic (and positive) for petroleum and coal products and agriculture. A small substitution relationship between capital and materials was evidenced by σ_{KM} in these sectors also.

The relationship between labor and energy was complementary for paper products, primary metals and agriculture in all years examined except for agriculture in 1947. The relationship was one of substitutes for chemical products and petroleum and coal products. These relationships were also present in σ_{LE}. In this relationship, it was found that energy responds more to labor price changes than labor responds to energy price changes ($\varepsilon_{EL} > \varepsilon_{LE}$). Since the cost share of labor was much larger than the energy cost share, (see Appendix I, Table 1) this result seems reasonable. In paper products and petroleum and coal, this relationship was inelastic, but became more elastic in 1957 and 1967. In 1976, however, these elasticities decreased in size. Recall that σ_{LE} followed this same pattern; in these sectors it was also close to one. In chemical products, primary metals and agriculture, this elasticity became more inelastic over time, although primary metals remained relatively elastic. In 1976, this elasticity increased in chemical products and primary metals.

The relationships between materials and energy and materials and labor were similar in that materials substituted for both labor and energy in all sectors except paper products. In addition, both energy and labor responded more to a change in the price of materials than materials responded to a change in the price of either energy or labor ($\varepsilon_{EM} > \varepsilon_{ME}$ and $\varepsilon_{LM} > \varepsilon_{ML}$). The materials cost share was larger than both labor and energy (as shown in Appendix I, Table 1), so this result is similar to those results discussed above. These elasticities between labor and materials, although inelastic, became more elastic in each year considered in chemical products. In the other four sectors this elasticity, also inelastic, became more elastic from 1947 to 1967 but less elastic in 1976. Recall that σ_{LM} was one of the smaller elasticities of substitution. Notice that ε_{EM} was substantially elastic in all except the agricultural sector, whereas ε_{ME} was very inelastic in all sectors. The σ_{EM} elasticity was also very large and positive indicating substantial substitution possibilities between energy and materials except in agriculture. In general ε_{EM} decreased in size from 1977 to 1976, ε_{ME} changed relatively little over the time period considered.

4.4 Summary of Results for Elasticities of Substitution and Input Demand

In summary, these elasticities of substitution and input demand had the correct negative sign for own elasticities in petroleum and coal, primarily metals and agriculture. In paper products, capital, labor and materials had mostly positive signs and in chemical products, capital and labor also had mostly positive signs. These incorrect signs are related to the curvature violations discussed above.

Capital and labor were strong substitutes in all five sectors, evidenced by large elasticities of substitution. The elasticities of input demand ε_{KL} and ε_{LK} were mostly inelastic except for ε_{KL} for chemicals and primary metals.

Capital and energy were substitutes in paper products, primary metals and agriculture and complements in chemical products and petroleum and coal. The elasticities of input demand ε_{KE} and ε_{EK} were all very inelastic.

Capital and materials were strong complements in paper products, chemical products and primary metals. The elasticities of input demand, ε_{KM} were relatively elastic except in 1976, although ε_{MK} for these sectors was inelastic. Capital and materials were substitutes in petroleum and coal, elasticities of substitution in these sectors were close to one. The elasticities of input demand ε_{KM}, were also close to one. This is because the cost share of materials in these sectors was only slightly less than one. (Note that $\varepsilon_{ii} = \sigma_{ii}$ if and only if the cost share of i is 1, or only one input is used in production, this is rarely the case). On the other hand, ε_{MK} was very inelastic.

Labor and energy were complements in paper products, primary metals and agriculture and substitutes in chemical products and petroleum and coal. The elasticities of input demand, ε_{LE} and ε_{EL}, were both inelastic.

Labor and materials were substitutes in chemical products, petroleum and coal, primary metals and agriculture. The elasticities of input demand were very inelastic in these sectors. Labor and materials were complements in paper products, the elasticities of input demand were also much less than one in this sector.

Energy and materials were strong substitutes in all sectors. In addition, elasticities of input demand, ε_{EM} were quite elastic in paper products, chemical products and primary metals. These elasticities of input demand were very close to one in petroleum and coal and agriculture. Elasticities of input demand, ε_{ME} were very inelastic.

The magnitudes of the cross elasticities varied from year to year. This was most likely because the elasticities of substitution varied substantially over time; recall that the elasticities of input demand are derived from the elasticities of substitution as in footnote 30. A pattern that was sometimes present was that the elasticities became more inelastic from 1947 to 1967 but more elastic from 1967 to 1976; ε_{KK}, ε_{KE}, ε_{KM}, ε_{LK}, ε_{EE}, ε_{LK}, ε_{LE}, and ε_{EL} displayed this pattern for some sectors. It was also common to have the elasticities either increase or decrease over the whole time period.

The response of energy to an increase in its own price was greater in 1976. It is interesting that this elasticity increased in 1976 when energy prices were higher, after a long period of gradually declining real energy prices. This greater response of energy to an increase in its own price in 1976 may have resulted in the increased response of other variables to that same energy price increase, as the response of other variables was often more responsive to an energy price increase in 1976.

The elasticities of substitution and input demand changed significantly when different values of λ are considered. Since the log-likelihood function for $\lambda = 1$ is maximized for paper products, petroleum and coal products, primary metals and agriculture and for $\lambda = 2$ for chemical products, these elasticities represent the statistically optimal estimates, given the constraints of the estimation.

It should also be noted that the estimates of the elasticities of substitution and input demand are based on estimates of the γ_{ij} which have large standard errors; for this reason they must be qualified.

The variability of results from sector to sector must be acknowledged. Each sector reacted differently in both size and sign. Also, the direction of change of elasticities from time period to time period varied across sectors. This result indicates that each sector must be examined independently to see what the effects of increased energy prices on a particular sector will be. This result has important policy implications. First, there is reason for optimism in the diversity of reactions in all sectors. In three of the five sectors capital and energy are substitutes (although limited) and in only two of the five sectors are they complements. So even if energy price increases affect some sectors adversely, not all will be equally affected. Second, this diversity indicates that broadsweeping general energy policy is not appropriate, unless the policy allows each sector to determine how much government aid that sector requires. This can only be accomplished by use of market mechanisms which allow each sector to adjust as is required for profit maximization.

4.5 Comparison of Elasticities with the Berndt-Khaled Results

There are many differences in the signs of these elasticities compared to those found by Berndt and Khaled (1978). There are some elasticities which have the same sign. (Recall that Berndt-Khaled used the K, L, E, M model for aggregate U.S. manufacturing data for 1947-1971.) These include σ_{KL} and σ_{EM}, which are positive in the results of both models. However, Berndt and Khaled have estimated a σ_{KL} which is closer to one. The estimates for σ_{KL} in Table 2 are much larger than the Berndt-Khaled estimates, except for agriculture, which is less than one. The Berndt-Khaled estimate for σ_{EM} is much smaller than one and thus smaller than the estimates in Table 2. For σ_{KM}, three of the five sectors in this model have negative signs which agree in sign with the Berndt-Khaled results. Berndt and Khaled's estimate for σ_{KM} is much smaller than the estimates in this model.

Both σ_{KE} and σ_{LE} have two sectors in this model which agree in sign with the Berndt-Khaled results. The Berndt-Khaled estimate for σ_{KE} is negative and much larger than the negative estimates for chemical products and petroleum and coal products. The estimate for σ_{LE} for chemical products is larger than the Berndt-Khaled estimate, but the σ_{LE} estimate for coal and petroleum products in this model is much smaller than the Berndt-Khaled estimate. The other three sectors in each case disagree in sign with the Berndt-Khaled estimates for σ_{KE} and σ_{LE}. The elasticities for σ_{KE} with the sign opposite the Berndt-Khaled estimate (in paper products, primary metals and agriculture) are positive, but close to one. Thus, substitution between capital and energy in these three sectors exists but is limited.

The elasticity, σ_{LM}, is the same sign as the Berndt-Khaled estimate for only one of the five sectors. The negative elasticity estimated for paper products is very close in size to the negative elasticity Berndt and Khaled estimate. The elasticities for the other four sectors are positive thus opposite in sign from the Berndt-Khaled estimates. These positive

σ_{KM} are close to one so substitution here is also limited.

The differences in these results have certain implications. The most important differences are in the estimates for σ_{KE} and σ_{LE}. In particular, Berndt and Khaled estimate a large negative relationship between capital and energy which implies that capital use will decrease significantly with increased energy prices. The results from this model indicate that capital use will decrease slightly when energy prices increase in two of the sectors analyzed. Capital use will increase slightly in the other three sectors. The results of this model do not imply that growth will be hampered if energy prices are increased; the results of Berndt and Khaled indicate that growth will most likely be dampened by higher energy prices.

Similarly, Berndt and Khaled estimate a strong positive relationship between labor and energy whereas in this model this relationship is estimated to be positive in only two sectors, in the other three sectors the relationship is estimated to be negative. The Berndt-Khaled result implies that labor use will increase with increased energy prices as labor replaces energy. This indicates that unemployment problems will be eased somewhat despite energy price increases. The results of this model imply that the unemployment problem will be exacerbated by energy price increases. Divergence in the results of these models indicates that the estimates of the elasticities are sensitive to the data. The estimates of the parameters of the GBC function which are used in the derivation of the various elasticities have large standard errors so that they cannot refute the estimates of Berndt and Khaled which have much smaller standard errors. Further study is needed to determine if these differences in results are due to disaggregation, time period covered or capital (or other) variable differences.

4.6 Returns to Scale, Rate of Total Cost Diminution, Total Factor Productivity, and Bias to Technical Change

The returns to scale and rate of total cost diminution discussed in section 2.1 defined in equation (13) and (14) respectively are given in Table 4, for 1947, 1957, 1967 and 1976. These two terms are multiplied to give the primal rate of total factor productivity which is given in Table 4 also discussed in section 2.1 and defined in equation (15).

In all sectors, the returns to scale were greater than 1 in 1947. The returns to scale generally decreased between 1947 and 1976 although in agriculture, returns to scale increased in 1957 and decrease thereafter. In petroleum and coal and primary metals, returns to scale became slightly less than one in 1967 and significantly less than one in 1976. In 1976, the returns to scale in paper products also became less than but close to one. Returns to scale remained larger than one in chemical products and agriculture for all years considered. If the argument is true, that these large increasing returns are due to utilizing excess capacity it indicates that excess capacity in firms was increasing over time.[31] In particular, excess capacity was not utilized in 1976 to the extent it was utilized in 1947, thus returns to scale decreased in 1976.

The estimates of rate of total cost diminution in all sectors were small except in agriculture. Although the rate increased in most sectors between 1947 and 1967, it decreased again in 1976. In any case, rate of total cost diminution remained small in the other four sectors. These results

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[31] Berndt and Khaled (1978) make this argument as they obtained similar results for returns to scale and total cost diminution.

Table 4. Estimates of Returns to Scale, Rate of Total Cost Diminution and Rate of Total Factor Productivity.

		Paper Products $\lambda=1$	Chemical Products $\lambda=2$	Petroleum and Coal $\lambda=1$	Primary Metals $\lambda=1$	Agri-culture $\lambda=1$
ε_{cy}^{-1}	1947	1.33	2.18	1.19	3.12	1.97
	1957	1.16	1.56	1.04	1.44	2.35
	1967	1.03	1.20	.99	.94	2.09
	1976	.98	1.08	.85	.87	1.93
ε_{ct}^{*}	1947	.002	-.007	.004	-.0003	.015
	1957	.004	-.005	.004	.00005	.017
	1967	.004	-.003	.004	.0005	.019
	1976	.003	-.005	.003	.0002	.019
ε_{ft}^{*}	1947	.003	-.015	.004	-.001	.029
	1957	.004	-.007	.005	.00008	.040
	1967	.005	-.004	.004	.0004	.041
	1976	.003	-.005	.002	.0002	.038

ε_{cy}^{-1} is returns to scale.

ε_{ct}^{*} is rate of total cost diminution.

ε_{ft}^{*} is rate of total factor productivity.

indicate that increases in factor productivity were due more to increasing returns to scale and less to technical change. This result may be explained by the correlation of cost, output capacity and time. Agriculture showed significant cost decreases over time as well as large scale economies and productivity increases.

If excess capacity becomes utilized more fully, costs will decrease. If decreases in costs (increasing returns to scale) are more highly correlated with output than with time, the result will be large returns to scale and a small rate of total cost diminution.

The annual rate of total factor productivity is also shown in Table 4. Total factor productivity was positive for all sectors except chemical proucts which showed a .5% decrease in productivity in 1976. The other sectors showed positive factor productivity. Annual productivity increased from 1947 to 1967 but decreased thereafter in 1976 for paper products and agriculture. In petroleum and coal the decrease was in 1967. Primary metals showed the smallest productivity increases; in 1976, factor productivity increased only .02%. The largest productivity increase, 4.1%, occurred in 1967 in agriculture, but it is large in agriculture in all years. These estimates of total factor productivity are similar in size to the estimates Berndt and Khaled obtain. The 1976 decrease in the rate of factor productivity increases substantiately the current claim that productivity is increasing less rapidly in the U.S. manufacturing in current years, productivity in agriculture remains high.

From the derivation of total factor productivity in section 2.1, it is observed that prices will have an effect on total factor productivity. The way these various input prices affect total factor productivity can be obtained by taking the partial derivative of total factor productivity with respect to the appropriate price [equation (25) in section 2.1]. The effects of the various price changes on total factor productivity are given in Table 5 for 1947, 1957, 1967 and 1976. In all sectors, capital price increases caused productivity in all sectors to decrease in all years considered, but with very small increases in paper and chemical products. Labor price increases, however, did not decrease factor productivity in any sector or year. These decreases or increases in productivity remained fairly constant from year to year. Material price increases decreased total factor productivity in paper products, primary metals and agriculture in every year. In petroleum and coal, material price increases did not decrease total factor productivity. In chemical products, material price increases did not decrease factor productivity in 1947 and 1957 but they did decrease factor productivity in 1967 and 1976.

The main interest in this paper is the effect that energy price increases have on total factor productivity. From Table 5, it can be seen that energy price increases would decrease total factor productivity with some sectors more seriously affected than others. This result, coupled with the negative elasticity of substitution between energy and capital in chemical products and petroleum and coal indicate that energy price increases would have large negative effects on output in these sectors. It should be noted that the effect on total factor productivity of increases in the price of capital, labor and energy are directly related to the technical bias, discussed below.

The coefficients τ_K, τ_L, τ_E and τ_M in Table 1 give the bias to technical change. The coefficients show that technical change was capital using in all sectors. All sectors showed labor saving and energy using biases. Primary metals and agriculture showed

Table 5. Changes in Total Factor Productivity Due to Changes in Input Price.

Change in Total Factor Productivity Due to a 1% Increase in:	Year	Paper Products $\lambda=1$	Chemical Products $\lambda=2$	Petroleum and Coal $\lambda=1$	Primary Metals $\lambda=1$	Agri-culture $\lambda=1$
P_K	1947	-.0003	-.0005	-.0025	-.0024	-.0135
	1957	-.0001	-.0001	-.0028	-.0006	-.0096
	1967	-.0001	-.0001	-.0019	-.0004	-.0083
	1976	-.0001	-.0001	-.0012	-.0003	-.0074
P_L	1947	.0057	.0137	.0006	.0087	.0537
	1957	.0036	.0074	.0004	.0029	.0498
	1967	.0027	.0047	.0003	.0016	.0355
	1976	.0025	.0043	.0001	.0014	.0309
P_E	1947	-.0021	-.0061	-.0013	-.0017	-.0007
	1957	-.0021	-.0050	-.0012	-.0008	-.0006
	1967	-.0022	-.0043	-.0013	-.0006	-.0006
	1976	-.0013	-.0027	-.0008	-.0004	-.0006
P_M	1947	-.0002	.0016	.0036	-.0015	-.0219
	1957	-.0003	.00009	.0029	-.0007	-.0308
	1967	-.0003	-.00028	.0029	-.0005	-.0267
	1976	-.0002	-.00015	.0026	-.0005	-.0226

technical change as materials using but paper products, chemical products and petroleum and coal showed a materials saving bias. These results are very similar to those of Berndt and Khaled. The energy using bias is expected as energy prices were decreasing over most of the time period considered. The recent energy price increases may eventually change this bias, however, the static nature of the GBC model makes this difficult to capture as the bias to technical change is constant over the whole time period considered, because it was necessary to assume the parameters are constant over the whole time period. In relation to total factor productivity notice that capital and energy were factor using and negatively affected total factor productivity when prices were increased. Labor was factor saving, so increases in its price did not decrease factor productivity.

Again it should be stressed that the estimates of the γ_{ij} parameters have large standard errors. The estimates of the other nine parameters, however, have smaller standard errors. This implies that estimates of bias to technical change, returns to scale, total cost diminution and total factor productivity are fairly reliable. The estimates of the elasticities of substitution and input demand, however, are not as reliable. The results and implication of the results must again be qualified by this fact.

5.1 Results of the Vector Autoregressive Model

The general conclusion that can be drawn from the coefficients of the vector autoregressive model (VAR) is that the estimates associated with the exchangeability prior (EXC) generally do not differ very much from the OLS estimates, meaning that the coefficients across sectors are not very similar.32/ This indicates that if the coefficients come from the

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32/ These coefficients will not be presented in this paper because it would entail the presentation of 378 equations. Interested readers are directed to the dissertation of the author, appendix VI.

same distribution that it has a large variance. This is not to say that the exchangeability prior is inappropriate, but rather that the stochastic processes underlying each sector are different.[33]

A test for each equation was performed, using the OLS estimates, to see if a standard pooled time series-cross section analysis would be appropriate. The null hypothesis in this test is that the coefficients for each variable are the same across sectors. This hypothesis was rejected for all equations at the .01 percent level. The test statistics are given in Table 6. Rejection of the null hypothesis implies that the standard pooling models are inappropriate. Since the exchangeability model does not restrict coefficients across sectors to be the same, this test says nothing about the exchangeability prior. This test also implies that reaction from sector to sector differs and a sectoral analysis is necessary to determine what the effect will be. More will be said on this point in a latter part of this paper.

The exchangeability estimation procedure allows the data to determine how similar coefficients will be across sectors. There are two different phenomena operating in this procedure. The first is the method in which the data enter the estimates (eqns. (36)-(39)). If the variance within the sector is small relative to the variance across sectors, the exchangeability estimates (EXC) will be closer to the OLS estimates, although it will still be pulled towards the mean. If the variance within one sector is large relative to the variance across sectors, the EXC estimates will be pulled

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[33] These estimates for the OLS coefficients were obtained from a program written by Thomas Doan, Dept. of Economics, Univ. of Minnesota called Regression Analysis of Time Series (RATS).

The exchangeability estimates were obtained with the use of this program which is able to utilize a subroutine written by the user. The subroutine followed the estimation technique discussed in section 3.4.

The impulse response function using the orthogonal decomposition discussed in section 3.6 was also done by RATS, using the exchangeability estimates.

Table 6. Test Statistics for Pooling Data

Equation for	F Statistic*
RY	3.083
K	1.769
RPK	2.431
L	3.038
RPL	1.603
E	2.276
RPE	4.441
IM	2.969
RPIM	1.265

$$*H_o: \delta_{11} = \delta_{21} = \cdots = \delta_{21\ 1}$$

$$\delta_{12} = \delta_{22} = \cdots = \delta_{22\ 2}$$

$$\vdots$$

$$\delta_{1\ 10} = \delta_{2\ 10} = \cdots\ \delta_{21\ 10}$$

$$F = \frac{SSR_R - SSR_u/(10M - 10)}{SSR_u/(\sum_{r=1}^{m} T_r - 10M)}$$

SRR_R = Sum of Squared Residuals from restricted regression (regression using all observations).

SSR_u = Sum of Squared Residuals from unrestricted regressions (regression of each sector's observations, summed)

T_r = number of observations in sector r

M = Number of sector (21).

$F_{200,399}$ (.01) = 1.0.

more heavily toward the mean estimates. Second, the priors assumed for the means and variances of the different parameters can also pull the estimates toward the OLS or mean estimates. In this model, a relatively strong exchangeability prior was assumed (recall $\rho = 18$ was used as a prior). Thus even with a strong exchangeability prior, the data carried more weight and suggested that sectoral parameters are very different. This result substantiates the result of the GBC analysis which suggests that estimates differ across sectors. This result also implies that a standard pooled time series, cross sectional approach that restricts all sectors to have the same coefficients or an aggregate analysis is inappropriate.

Although the OLS and EXC estimates do not differ greatly, the use of the exchangeability prior is still justified. Of course, the actual justification for this exchangeability procedure is the a priori belief that the exchangeability prior is appropriate for this data set because the cross-sectioned units are interrelated through economic interactions and similar processes are estimated in each sector. However, if this reason for using the procedure is put aside, other reasons exist for using this procedure. First, the underlying Bayesian model, which led to the exchangeability prior for this problem demands explicit statement of the prior. Note that the exchangeability procedure is similar to ridge regression.[34/] One problem with ridge regression, that is circumvented by the exchangeability procedure, is that picking a particular value, K, to add to the X'X matrix is often not justifiable. The value chosen, in a standard analysis is justified implicitly by the analyst. In the Bayesian framework, however, the prior is described explicitly, resulting in a certain value to be added to the X'X matrix. The

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[34/] In the model $Y = XB + U$, the ridge estimator is: $(X'X + kI)^{-1}(X'Y)$ or more generally $(X'X + kQ)^{-1}(X'Y)$, see Hoerl and Kennard (1970).

reviewer of the Bayesian work may not agree with the prior, but because the prior is stated explicitly, the results can be viewed in light of the assumptions underlying the work. The reviewer of the standard (non-Bayesian) work, does not know the implicit assumptions of the analyst, which enter in ridge regression via k. If the reviewer takes issue with the results (of the standard genre), it is not clear if the issue with the results per se or with the implicit (unstated) assumptions of the analyst. Unstated influences or arbitrariness are not a problem in the Bayesian context.

Another reason for using the exchangeability procedure is given by Lindley and Smith (1972, p. 17). They state that "least squares have a tendency to produce regression estimates which are too large in absolute value, of incorrect sign and unstable with respect to small changes in the data." The reason for this is that OLS estimates are sensitive to outliers; with limited data this is a problem. The exchangeability estimates (EXC) tend to draw the sectoral coefficients to the mean, thereby stabilizing them, avoiding some of the problems of the OLS estimates.

It should be noted that the standard errors of the coefficients in the exchangeability model are significantly smaller than those of the OLS model. The standard errors of the EXC should be smaller than those of the OLS model, as discussed in section 3.4. However, a large difference occurs because the standard error estimates of the exchangeability model are conditional on the true mean parameter and thus provide a lower bound for the estimates of the standard errors. Since the true mean is not known, the variance of the EXC coefficients would be larger. In this case, because the OLS and EXC coefficients are so similar, the standard errors of the EXC coefficients would be closer to (but less than) the OLS standard errors than the conditional standard errors given. The OLS standard errors form an upper bound.

One final point can be made about the VAR model. Generally, in the vector autoregressive specification the assumption of no serial correlation is made. This is legitimate because the autoregressive representation of the stochastic process (equation (26) in section 3.1 for this model) will have no serial correlation if enough lags are included in the specification. The reason being that as more lags are included, more of the nonrandom element of the stochastic process is explained. If enough lags are included, the only thing which is not explained is the random element, which is not serially correlated. In this model, only one lag is used because of data constraints, so it is possible that serial correlation exists. The Durbin-Watson statistic is not a legitimate test when lagged dependent (stochastic) variables are used as explanatory variables, so another test must be used.[35] Therefore, in this model another test was used to determine if serial correlation was a problem. The test used was: the residuals from the OLS regression, u_t, were run on the lagged residuals from the same regression, u_{t-1} (i.e. the regression $u_t = w\, u_{t-1} + \varepsilon_t$ was run). If the autocorrelation coefficient w, from this regression, is close to one, autocorrelation is a problem. The results from this test, autocorrelation coefficients for all equations for all sectors are given in Table 7. The largest autocorrelation coefficient reported is .37. Most coefficients are much smaller than this, so autocorrelation is judged not to be a very serious problem in this model.

The coefficients of the vector autoregressive model hold very little intuitive meaning because the equations are not structural or behavioral equations, but rather are more mechanical, like reduced form equations. In addition, the coefficients of these equations do not describe how the system interacts. The system interaction, in response to a shock in one of the system variables which is what is of interest, is described by the impulse response function. Since the autoregressive coefficients hold little economic content, they will not be discussed further. The discussion of the results of this model will center on the impulse response function. Recall that the IRF can be derived from the coefficients of the VAR as shown in equation (46) in section 3.5.

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[35] The Durbin-Watson test applies only to nonstochastic variables. See Johnston (1972, p. 305).

Table 7. Autocorrelation Coefficients.

	EQUATION FOR								
	RY	K	RPK	L	RPL	E	RPE	IM	RPIM
Food Products	-.077	-.049	-.233	-.296	-.057	-.214	-.125	.177	.038
Tobacco Products	-.107	-.035	.100	-.181	-.073	-.188	-.169	-.032	-.235
Textile Products	-.032	.018	-.013	-.082	.041	-.026	-.072	-.087	-.086
Apparel	.019	-.014	-.135	-.064	.165	-.003	-.023	.237	-.173
Lumber Products	.098	.039	-.257	.194	-.317	.143	.226	.121	-.057
Furniture & Fixtures	.143	.033	-.026	.158	-.161	.061	.296	.294	-.127
Paper Products	.124	-.026	-.060	.170	.095	-.186	.176	.211	-.133
Printing & Publishing	-.284	-.106	-.336	-.249	-.081	-.218	.011	-.376	.142
Chemical Products	-.178	.009	.249	.016	.050	.001	.183	-.091	-.209
Petroleum & Coal	-.132	-.031	-.257	-.151	.045	-.209	.118	-.122	-.190
Rubber Products	-.017	-.275	-.017	.133	.087	.135	.233	.008	.042
Leather Products	-.047	.179	-.288	-.046	.032	-.338	-.233	-.120	-.271
Stone, Clay & Glass	-.007	-.303	-.121	.012	.041	-.003	.190	.022	-.120
Primary Metals	-.075	-.245	-.015	-.040	-.307	-.174	.148	-.075	-.208
Fabricated Metals	-.026	.176	-.109	-.081	.033	-.276	-.157	-.199	-.137
Machinery	-.112	-.054	-.266	-.041	-.017	-.006	.010	.028	-.269
Electronic Equipment	-.105	-.073	-.086	-.111	-.112	-.133	-.097	-.242	-.184
Transportation	-.106	-.250	-.327	-.057	-.203	-.050	.186	-.206	-.214
Instruments	-.062	-.046	.151	-.029	-.069	-.122	.169	.095	-.058
Misc. Manuf.	.231	.046	-.224	.171	-.047	.109	.064	.182	-.165
Agriculture	-.278	-.057	.279	-.075	-.148	-.251	.012	-.185	-.173

As noted in section 3.6, the impulse response function considered is orthogonalized to account for covariance of the residuals of the equations using a particular ordering.[36/] If each variable is shocked in each sector the result is 189 separate impulse response functions (IRF), each IRF showing the effect on the nine variables in the system. This is too much information to discuss in any detail, so rather than presenting all the results, emphasis will be placed on those impulse response functions related to energy price increases. It is also possible to investigate indirect effects of energy price increases on the price of other inputs, discussed in section 3.1. This will also be done.

5.2 First Period Responses of Inputs to Changes in Their Own Price

Before discussing the impulse response functions in detail, a general comment can be made. The reaction measured by the impulse response function is dynamic in that the path of changes in quantity response to initial price increase is observed. The first period response in this orthogonalized impulse response function is the covariance of residuals of the equations. This is discussed in some detail in section 3.6. However, often in subsequent periods the sign of the reaction changes, implying that it takes several years for reactions to shocks in variables to work through the economy. This dynamic system cannot be captured by a static model.

The variables used in this analysis are entered as natural logarithms so that the interpretation of the impulse response function is percentage change in all variables in the system in response to a 1% shock (change)

[36/] Recall that the ordering used is: RPE, RY, K, RPK, L, RPL, E, IM, RPIM.

in a particular variable.[37/] This is also the definition of an elasticity. This shock is carried out for ten steps. The first response is very similar to a static elasticity. The first period response (step 0) of energy to an increase in the price of energy is no-zero.[38/] The following responses (step 1-9) of the IRF are the succeeding percentage change in variables of the system in response to the orginal shock. For this reason the impulse response functions are labelled dynamic elasticities.[39/] Because of this elasticity interpretation, the size of the response as well as the sign have some economic interpretation.

First, consider the response of the input variables to a change in their own price. These first period responses to the price of energy, price

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[37/] The impulse response function was normalized so it could be interpreted in this way. Originally the shock was one standard deviation of the variable as illustrated in the example using equations (47) and (48).

[38/] Because of the ordering, variables appearing first are correlated with all variables after it in the ordering. Variables appearing at the end of the ordering have zero covariance with preceding variables. This is illustrated by the IRF in the Appendix.

[39/] Pindyck and Rubinfeld (1976) define dynamic elasticities as $\frac{P_T}{Q_T} \cdot \frac{\Delta Q_{T+\tau}}{\Delta P_T}$. These are exactly the terms that are in the impulse response function, where τ indicates an increment in time.

of capital, price of labor and price of materials are given in Figures 3, 4, 5 and 6 respectively. Each figure shows the response of all 21 sectors.[40/]

The bar graph in Figure 3 represents the percentage response in the same period to a one percent change in energy price. The actual numbers are given in Appendix II, Tables 1-21, as the zero step forecast. Because of the order of orthogonalization, the zero step responses of the other inputs, capital, labor and materials, to an increase in their own price are zero. Figures 4, 5 and 6, therefore, represent the percentage response of capital, labor and materials in the period immediately following a one percent increase in their own price.

The response of energy to a one percent increase in the price of energy is negative for all sectors. Most researchers find that in aggregate analyses, the own price elasticity for energy is inelastic. Figure 3 shows that most sectors do indeed show an inelastic response to energy price increases, although five sectors show an elastic response and two sectors show unit elasticities. Energy has the smallest factor share of all inputs (see Appendix I, Table 1) The majority of these sectors with elastic or unit elasticity responses: apparel, printing and publishing, rubber and plastics, and leather products are involved in the production of nondurable goods. This larger response indicates that energy use in these nondurable goods production is more sensitive to energy price increases than energy use is in most of the durable goods sectors. It also seems that the production process

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[40/] Recall the sector number refers to the 2-digit SIC code except for agriculture. The sectors are: (20) Food and Kindred Products; (21) Tobacco Products; (22) Textile Mill Products; (23) Apparel, Other Textile Products; (24) Lumber and Wood Products; (25) Furniture and Fixtures; (26) Paper and Allied Products; (27) Printing and Publishing; (28) Chemical and Allied Products; (29) Petroleum and Coal Products; (30) Rubber, Misc. Plastics Products; (31) Leather, Leather Products; (32) Stone, Clay and Glass Products; (33) Primary Metal Industries; (34) Fabricated Metal Products; (35) Machinery, except Electrical; (36) Electric, Electronic Equipment; (37) Transportation Equipment; (38) Instruments Related Products; (39) Miscellaneous Manufacturing; (40) Agriculture.

is slightly more flexible in these nondurable goods sectors than in the other nondurable goods sectors not mentioned above. The remaining three sectors, which show elastic responses to energy price increases: lumber products, furniture and fixtures and miscellaneous manufacturing, produce durable goods, two are related to lumber. This result indicates that these are the only durable goods sectors with enough flexibility sufficient for energy use to decrease significantly with increased energy prices.

Figure 4 gives the first period percentage response of capital in all sectors to a one percent change in the price of capital. The bare majority of responses in this figure are negative, with nine positive responses. All responses seem to be quite small (less than .5 in absolute value) which indicates that the capital variable is relatively unresponsive. This is possibly due to the different nature of the capital input. Capital is different from labor in that once capital is purchased, it is held until it becomes obsolete or no longer functional. Labor and other inputs on the other hand, can be increased or decreased with ease. For a more complete discussion of the capital variable see Appendix I.

The first period response of labor to the increase in the price of labor is given for all sectors in Figure 5. It can be noted that all but six sectors show that labor use increases as the price of labor increases. One explanation for this positive response is that as prices (and wages) increase, an upswing in the business cycle is often being experienced, thus increased output and increased employment are also reported. This effect outweighs the decrease in employment that might otherwise result from increased wages. Only transportation showed an elastic response.

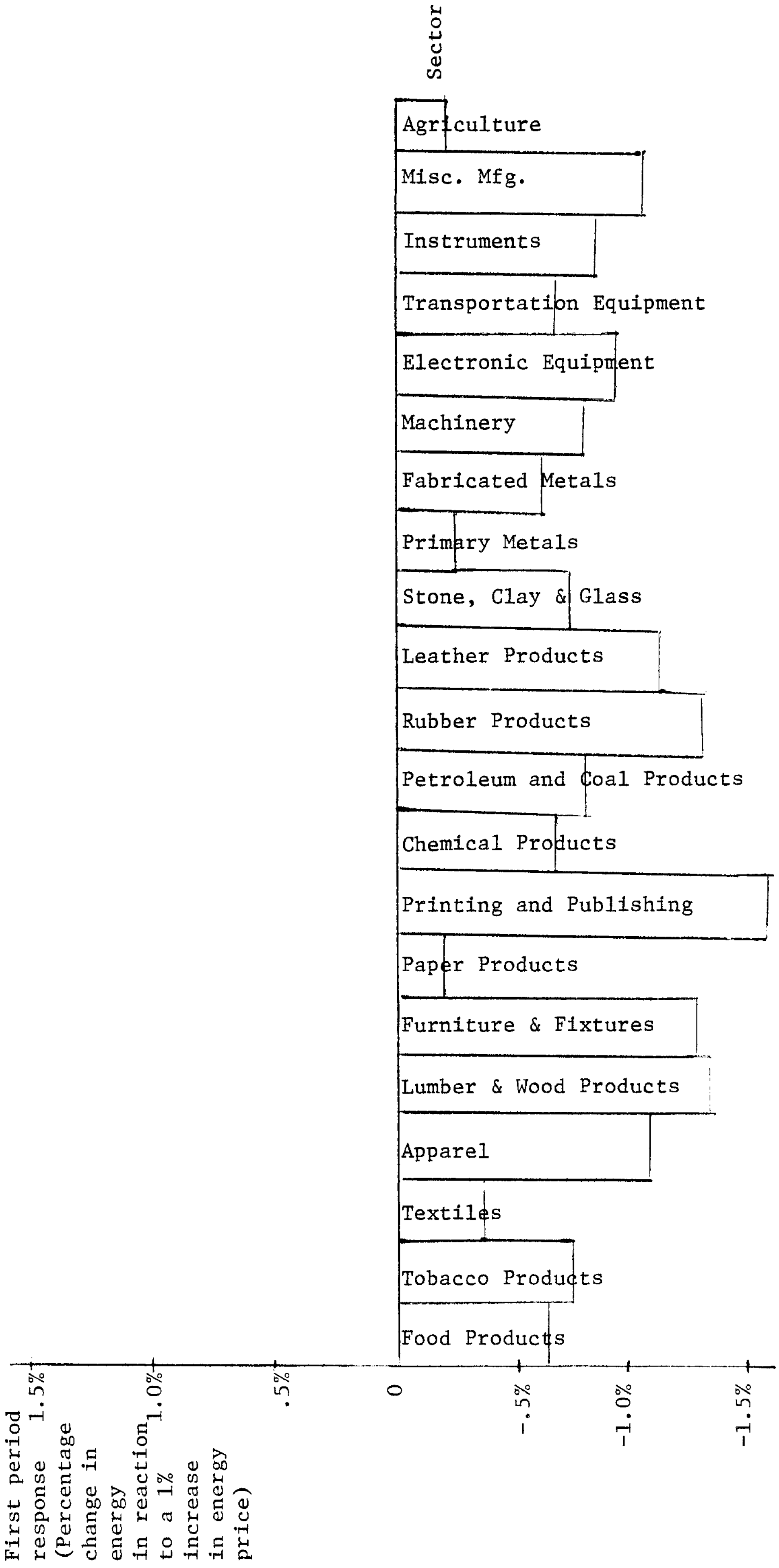

Figure 3. First Period Response of Energy to a 1% increase in Energy Price.

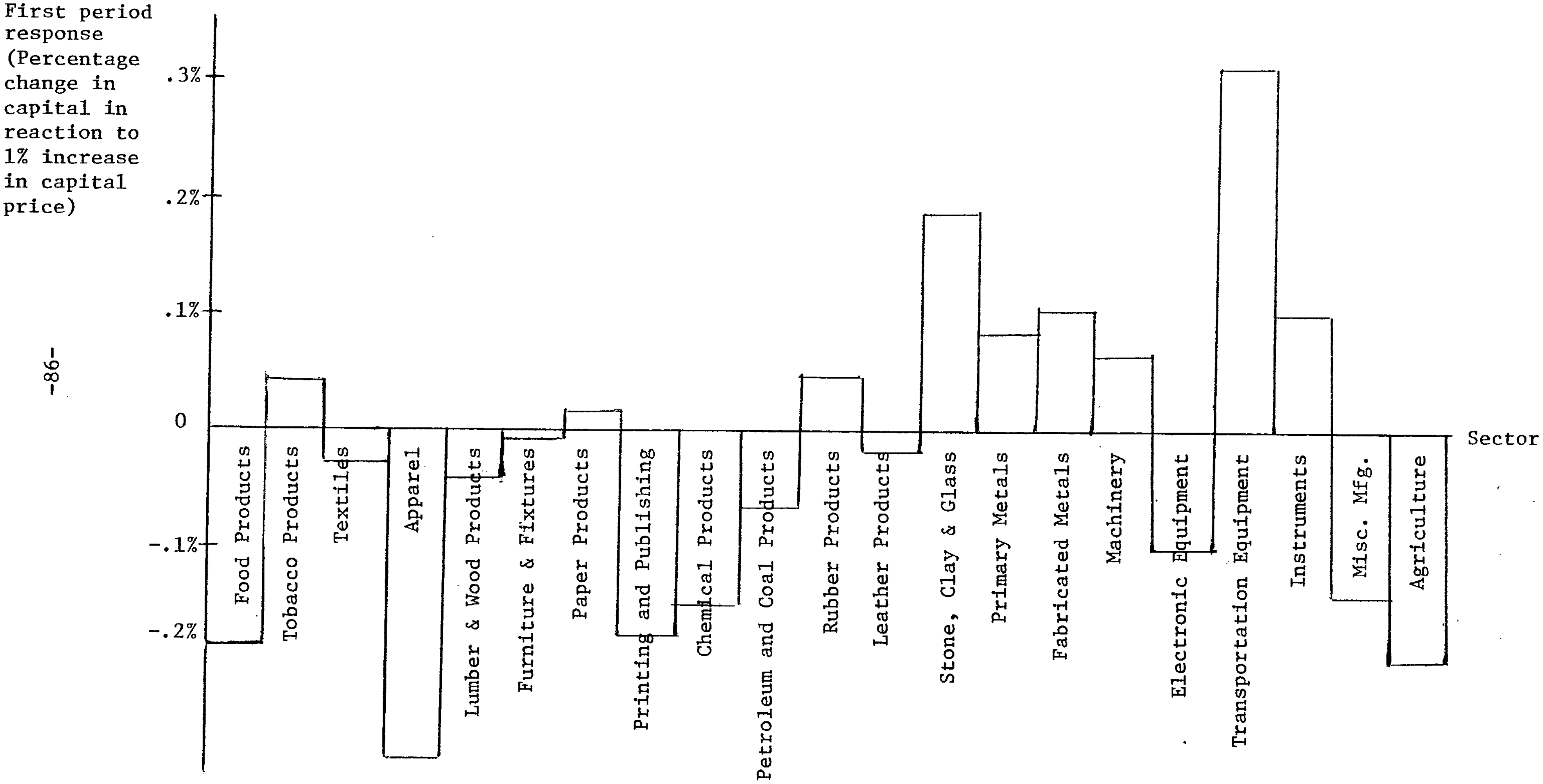

Figure 4. First Period Response of Capital to a 1% Increase in Capital Price.

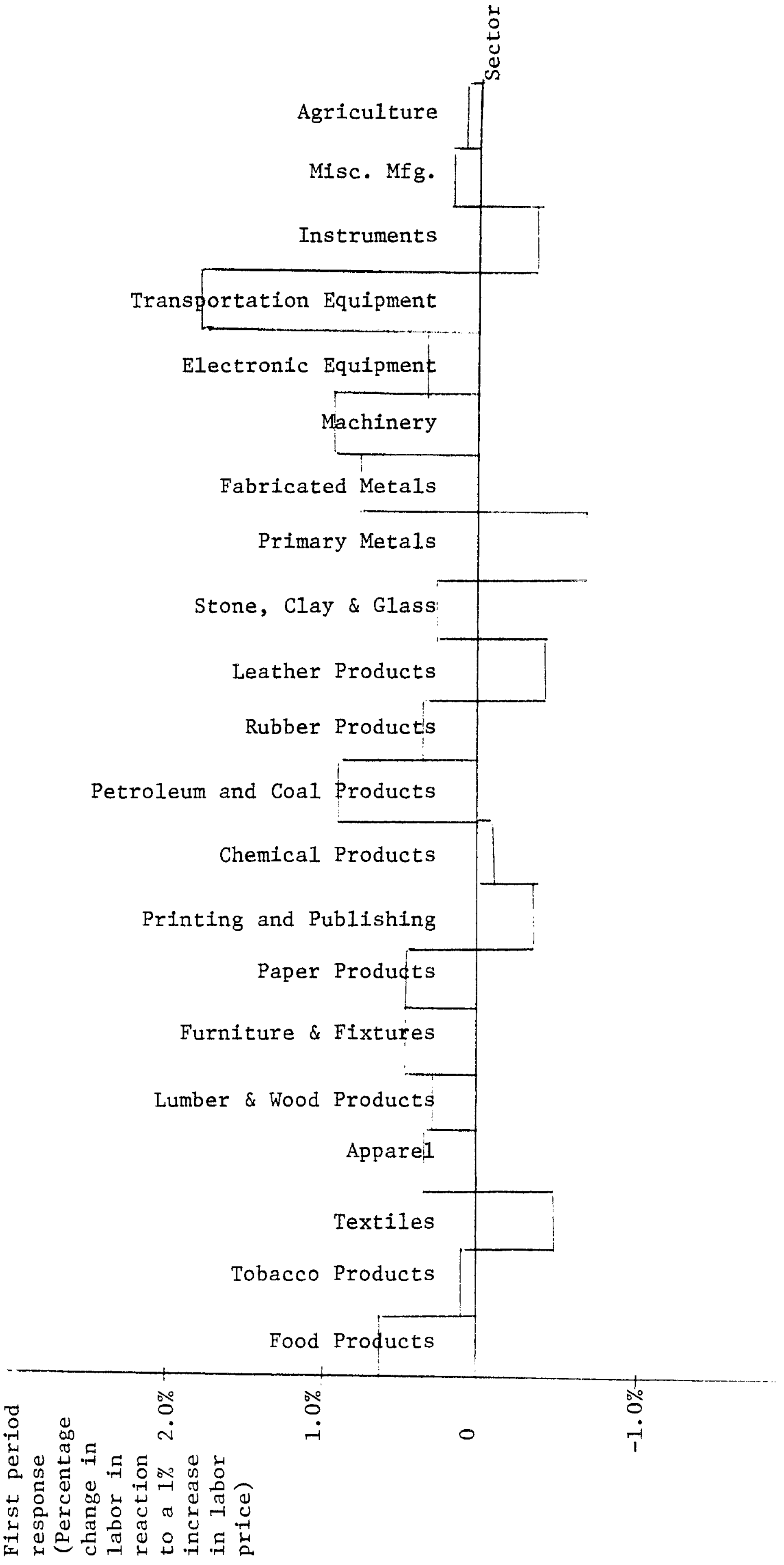

Figure 5. First Period Response of Labor to a 1% Increase in Labor Price.

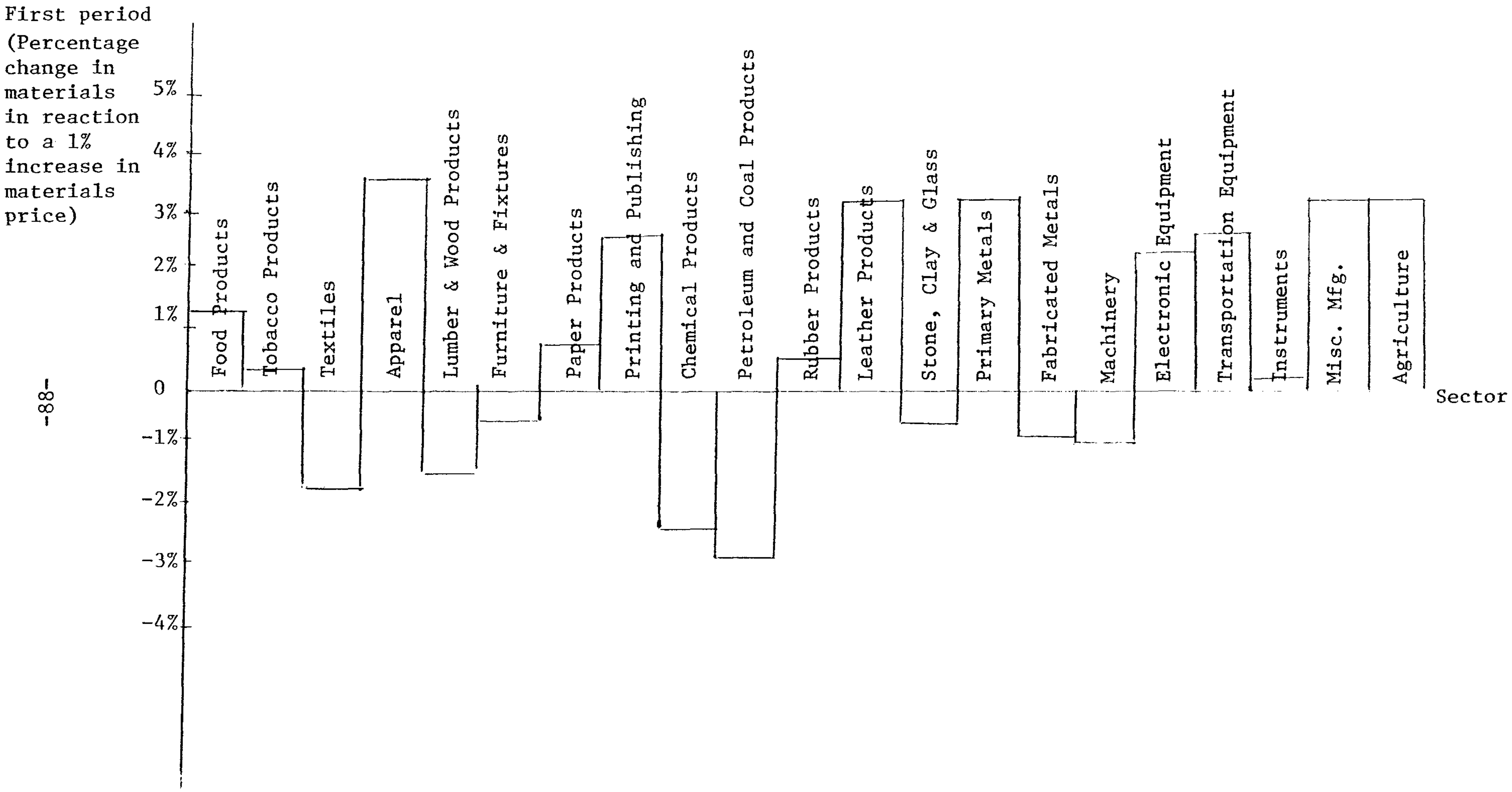

Figure 6. First Period Response of Materials to 1% Increase in Price of Materials

From Figure 6, it is apparent that the first period response of materials is generally elastic and quite variable. In eight sectors the response is negative. In four sectors the response is positive but less than one. However, in nine remaining sectors, the response is positive and greater than one. Materials has the largest factor share, much larger than any other input, thus changes in its price have a greater effect on cost than do changes in the price of other inputs, which explains the large response. It is possible that the positive response exhibited by the majority of sectors is also explained by the large materials cost share in production. In particular, it is possible that, like labor, materials prices may be most likely to increase during an upswing in the economy, when output is also increasing. Since materials compose such a large portion of output, it is likely that overall materials use would increase as output increased, despite an initial decrease in materials use that would have most likely occurred in reaction to increased materials prices.

5.3 Dynamic Responses to Changes in the Price of Energy: Direct Effects

The results discussed thus far refer only to the first period response to a shock in the four price variables of the system. It is also important to look at the dynamic reaction as this gives more insight into the workings of the system. In this paper only the dynamic response of the system of variables to a shock in the price of energy will be discussed. The response of a system to a shock in the price of energy is of particular interest because it shows the simultaneous response of the system over time to an energy price increase. Also, the sectoral breakdown clearly indicates that each sector responds in a unique way as is evidenced by the size and sign of the reaction.[41]

The responses of inputs and output to an increase in the price of energy will be discussed as these responses show the direct effect of an energy price increase. In addition, the response of the prices of capital and intermediate materials to a shock in the price of energy will be discussed as these responses may show some of the indirect effects of energy price increases.

The impulse response functions given in Appendix II have printed responses from steps 0 to 9. However, in any forecast or simulation such as the impulse response function, the variance of the forecast increases with time. Thus the variance of the fourth step is larger than the variance of the zero step and the variance of the ninth step is considerably larger than the variance of the fourth step and so on. The reason for this is that the first prediction is based on actual data; forecasts in subsequent periods are based on forecasts from previous periods which have variance. These variances thus grow with each consecutive forecast considered. In considering forecasts or simulations based on annual data, the greatest accuracy is in the first few simulations as the simulations are also annual. If quarterly or monthly data is used, projections in later periods are more meaningful as the projections are on a quarterly or monthly basis. For

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[41] The impulse response functions for the response to an increase in the price of energy are shown in Appendix II. Impulse response functions that are of particular interest are shown graphically.

this reason the discussion of the impulse response function is limited to steps 0 through 4, although steps through step 9 are reported. It should be noted that even in this shortened projection period considered, the zero and first steps are the most reliable.

As noted above, one interesting result of this model is that the response of each sector is unique. This has specific policy implications, as will be discussed in a latter section, but it presents difficulty in the discussion of the results. Since the capital response is perhaps one of the most important responses to increased energy prices, the sectors have been divided into three groups based on the reaction of the capital variable. A representative sector or two will be discussed in each group, reactions which greatly differ from the representative sectors will be noted.

Figures 7-11 illustrate graphically the percentage response of output, capital, labor, energy and materials to a one percent increase in energy price. In step zero, the response shown by Y, K, L, E and M occurs in the same period (year) as the increase in energy prices. In the following periods, the reactions of these variables to the same increase in energy price are shown for consecutive years. In essence these diagrams show how this energy price increase moves through the system. In terms of an explicit interpretation, consider Figure 7. In step 0, the graph shows that energy use decreased by about .09 by one percent in the same period energy price increased by one percent. Output decreased by .15 percent in that same year while capital increased by .19 percent, labor use decreased by .15 percent and materials use decreased by .37 percent. The year following the energy price increase (step 1), energy use rose .03 percent from the level used the previous year (step 0). Capital increased .25 percent from the level of the previous year, and so on. In interpreting these graphs, there are several things to note. First, reactions of variables are to original one percent increase in the price of energy (step 0), the price of energy does not increase in subsequent years (steps 1-4). Second, the effects are cumulative in the sense that each step shows the change in that variable from the level of the previous year, not from step 0 (the year of the energy price increase). Finally, note that the scales of these figures differ; this is due to the diverse and unique reactions of each sector to a 1% energy price increased.

In the first group, capital shows a sustained positive response except for sector 35, machinery except electrical, in which the reaction of capital is negative for one period. The sectors which belong to this group are: (20) food and kindred products; (21) tobacco products; (22) textile products; (26) paper and allied products; (35) machinery except electrical; (36) electric, electronic equipment; (38) instruments and related products; (39) miscellaneous manufacturing; and (40) agriculture. Within this group, there are two basic responses in the direction of the positive response: positive with an increasing rate and positive with a decreasing rate. The positive, increasing rate response is the smallest group and is composed of paper products, machinery, and miscellaneous manufacturing. The other sectors belong to the positive, decreasing rate group. A representative sector from each of these subgroups will be discussed. These are sector 26, paper and allied products for the positive, increasing rate group and sector 40, agriculture for the positive, decreasing rate group.

In the second group, the change in capital is initially positive, but negative thereafter. The sectors in this group are: (23) apparel; (24) lumber and wood products; (25) furniture and fixtures; (27) printing and publishing; (28) chemical and allied products; (29) petroleum and coal products; (31) leather and leather products; (32) stone, clay and glass products; (33) primary metals products; (34) fabricated metal products, and (37) transportation equipment. In this group, two representative sectors; (28) chemical products and (34) fabricated metals, will be discussed. Although the capital reaction is the same in both these sectors, the reactions of other variables differ.

The third group has only one member, sector (30), rubber and miscellaneous plastic products. In this group the reaction of capital is negative and remains negative. Because this sector's reaction is uniquely different from the other sectors in that its initial response is negative, it warrants discussion.

Capital initially increases in response to a 1% increase in the price energy in the paper products sector. This is illustrated in Figure 7. Energy use decreases in this period. This combination of reactions indicates that the new capital purchases are energy saving, however, this capital may be in the form of insulation as the decrease in energy use is small. In this same period labor, materials and output decrease. In the next period (step 1) capital use increases from the previous level. In this period, the use of other inputs: energy, labor and materials, as well as output also increase. Since energy use and output both increase it is difficult to say if this new capital is energy efficient. In the following period (step 2), capital use continues to increase, but the rate of increase is less than the previous period. All

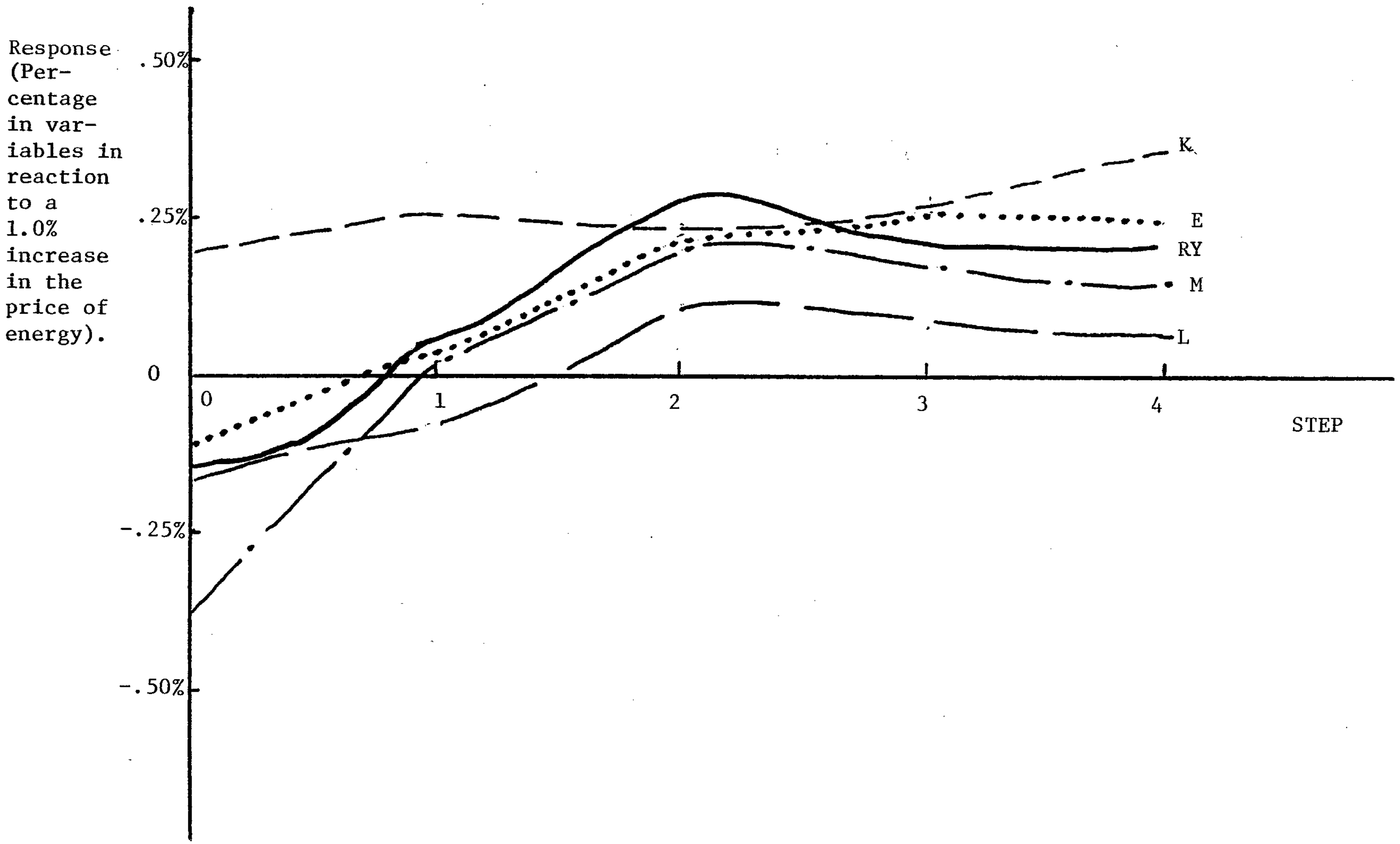

Figure 7. Impulse Response Functions for Paper Products: Responses to a 1% Increase in Energy Price.

of the other inputs continue to be used at an increasing rate, but use of energy, labor and materials peak in this period. The percentage increase in output in this period is greater than the increases in inputs, which indicates that although inputs increase significantly proportional increase in output is greater. In subsequent periods (steps 3 and 4), capital stock increases at an increasing rate however the use of energy, although positive, is less than in the previous period. It is possible that the spurt in the capital stock is energy saving, although energy use has not decreased substantially. The rate of increase in use of the other inputs, labor and materials has also decreased from the previous period but the change in use is still positive. The change in output follows this same path with the rate of increase this period less than the previous period but the net change is still positive. The other sectors in this subgroup: machinery and miscellaneous manufacturing show similar patterns. See Appendix II, Tables 16 and 20. Miscellaneous manufacturing shows definite decreases in energy consumption but machinery shows more of a levelling off of energy use. If indeed the second spurt of investment is for energy efficient capital, the most effective energy saving capital is purchased in the miscellaneous manufacturing sector as change in energy use remain negative in this sector. The least effective capital is purchased by the machinery sector as the rate of change in energy in this sector increases over time. Paper products is in between these, because rate of change in energy use, although positive decreases over time.

In the second subgroup of this first group, the agricultural sector is the representative sector discussed. The response of capital to a 1% increase in energy price in this sector is also positive but increases over time occur at a decreasing rate. This is illustrated in Figure 8. In the same period as the energy price increase (step 0) capital increases but energy use decreases. The other input variables, labor and materials, as well as output, also decrease in this period. Since capital in this sector increases, in this period, while energy use decreases, it seems that the capital purchased is somewhat energy saving. It is also likely that this capital is labor

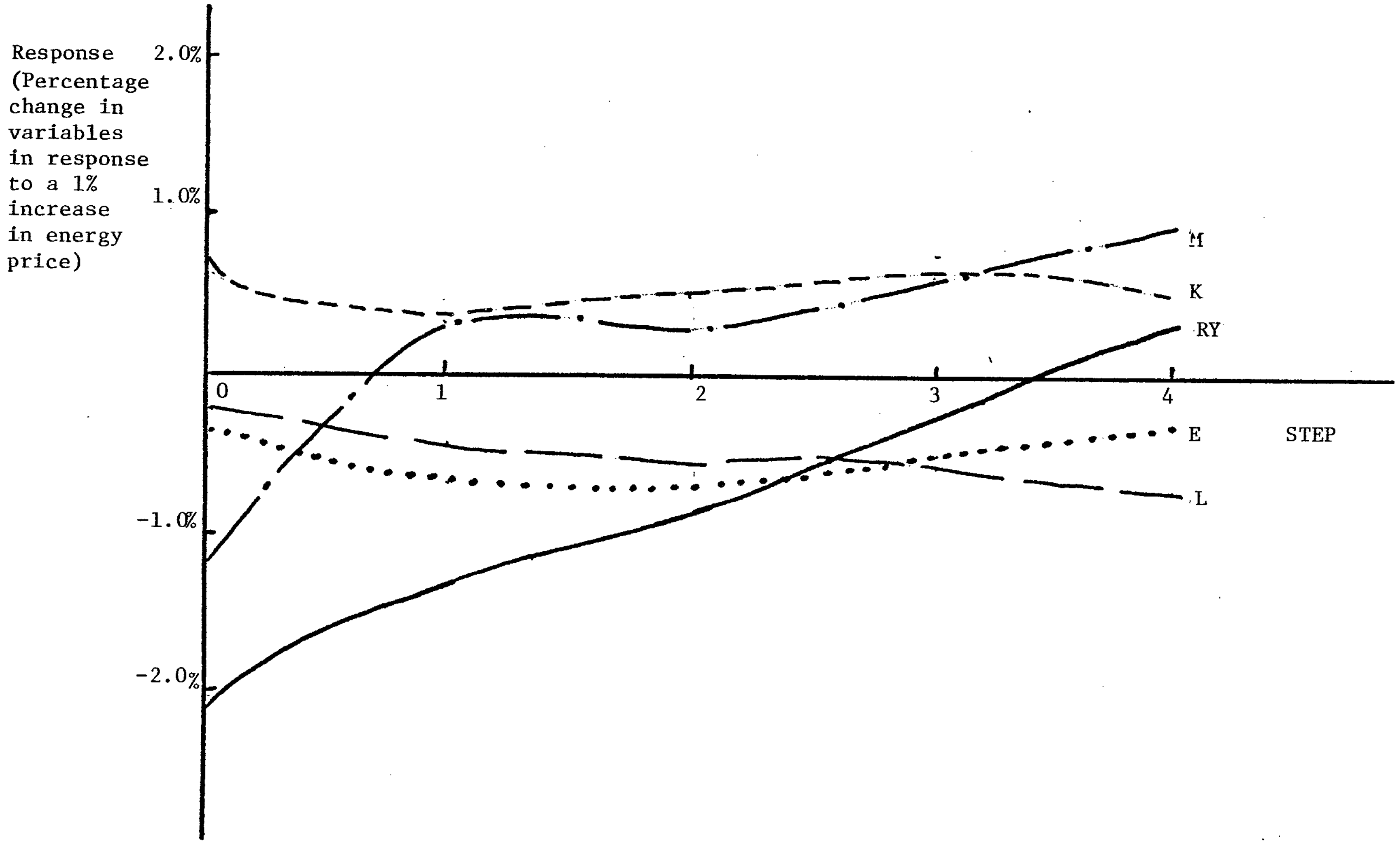

Figure 8. Impulse Response Functions for Agriculture: Responses to a 1% Increase in Energy Price.

saving. In the second period (step 1), the capital stock increases at a decreasing rate, labor and energy continue to decrease in this period while materials use increases. Output decreases in this period but the decrease is less than in the previous period. This pattern continues for the following periods for all variables (steps 2,3 and 4) except that the change in energy, although it remains negative, decreases at a slower rate from steps 2 to 4. Output increases in step 4, probably in response to the sustained increase in materials use. This pattern of input use indicates that capital being purchased is somewhat energy and labor saving.

In other sectors in this subgroup, electric equipment and instruments also show negative change in energy use, in neither case is the change a sustained decrease in use, but rather the decrease in use slows at some period in the projection interval considered; to see this, consider Tables 17 and 19 in Appendix II. The other three sectors in this subgroup: food products, tobacco products and textile products show energy use eventually increasing, after an initial decrease. In this subgroup then it seems that the capital purchases do incorporate some moderate energy saving equipment, as decreases in energy use occur over several initial periods. Since energy use does eventually increase in some sectors in this subgroup, long-run energy efficient capital purchases are not evident. It could be that, in these sectors in this subgroup where energy use does eventually increase, sustained capital purchases are primarily for capital that is geared at energy saving such as insulation, but is not geared at changing the entire production process.

Of the second group discussed above, the chemical products sector is analyzed first. Figure 9 shows that in chemical products capital increases slightly in response to a 1% energy price increase. Energy use decreases in this first period also. In the next period (step 1), the capital variable decreases slightly. Energy and labor use are still negative but the use of materials increases. In this sector materials have the largest factor share; thus it is possible that increases in materials use means output need not decrease even though the use of other inputs decreases. Output in this period has increased. In the next period (step 2) capital continues to decrease, but the other inputs including energy increase. It is possible that in this period, capital is being used more intensively, as is evidenced by the increased use of the other inputs. Capital is measured as a stock rather than the flow of capital actually used each year, so actual capital utilized is not measured.[42/] In the last period (step 4), the change in energy use is negative the change in capital is still negative. The use of the other inputs increases, but at a decreasing rate. Thus the increased utilization of capital and increased input usage is still evident. These responses indicate that initial capital purchases are geared primarily at conservation, such as greater purchases of insulation, new energy saving lighting fixtures. This type of capital use could account for the small response of capital. Since energy use also decreases, conservation is evident. This impulse response function seems to capture relatively short run responses to the energy price increase. Longer run responses such as the purchase of new energy efficient capital are not picked up.

[42/] Measures of capital are generally in stock terms. In particular, in this paper, capital is measured as the stock of capital or book value. For a more detailed description of the capital variable see Appendix I. In general, flow measures of capital are difficult to obtain. Diversity of capital is one reason this type of measure is difficult to obtain.

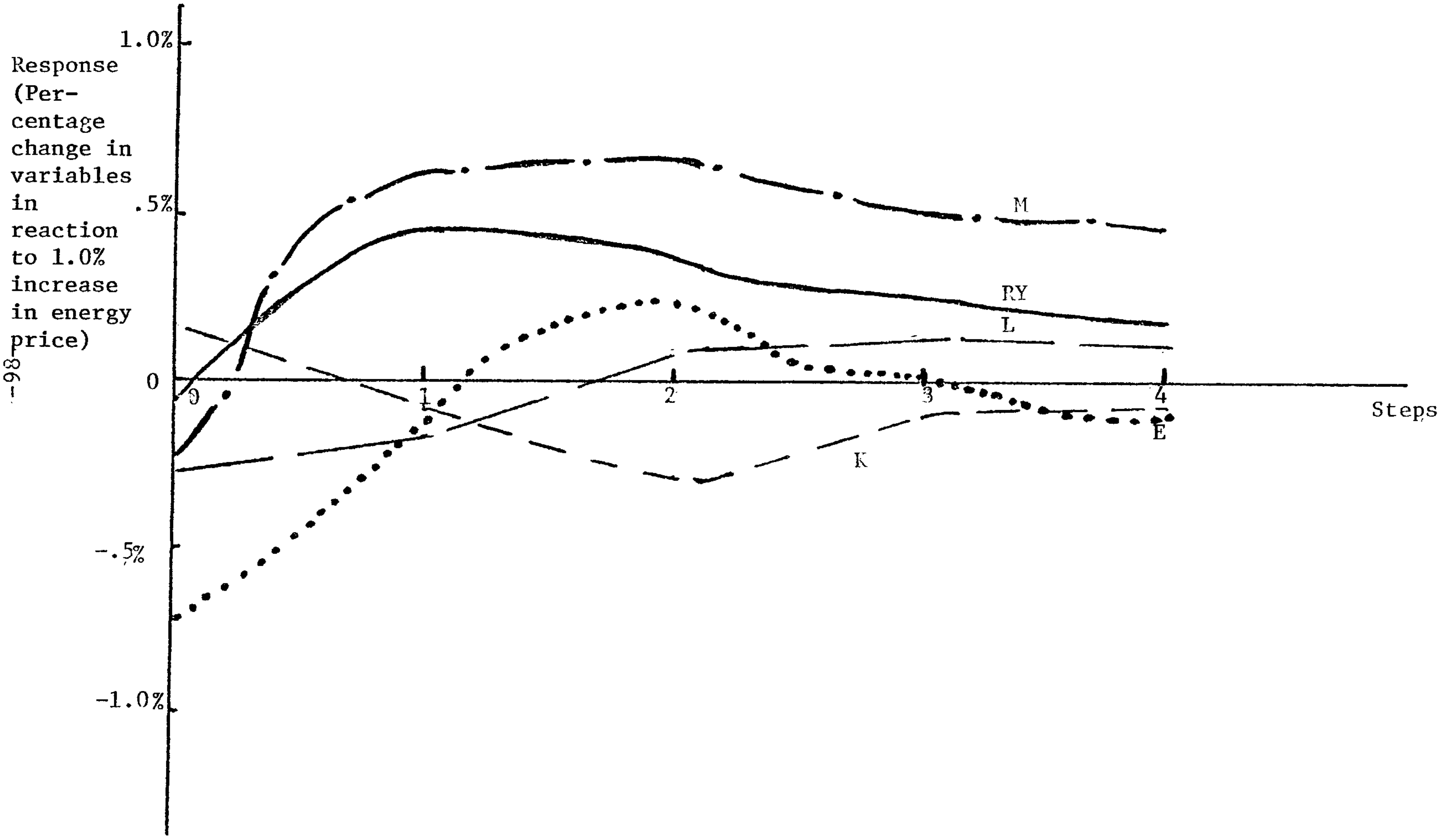

Figure 9. Impulse Response Functions for Chemical Products: Responses to a 1% Increase in Energy Price.

Fabricated metal products is in the second group with chemical products, but it differs in several respects from the response of chemical products, as illustrated in Figure 10. In fabricated metal products, the change in capital is also initially positive (step 0) in response to a 1% increase in energy price. The use of all other inputs and output decrease in this time period. Capital in the next period (step 1) increases, but the rate of increase has slowed. The use of all inputs except energy increases. Output also increases. In the following period (step 2) capital continues to increase, but the rate of increase has fallen still further. The use of all other inputs is positive, even energy usage increases. This pattern of input usage indicates that initial capital expenditures are geared at conservation as energy usage has decreased as capital expenditures have increased. However, as output increases it is apparent that the new capital is not energy efficient as energy use eventually increases. In the next period (step 3), capital use decreases, but the use of other inputs increases. The rate of increase in the use of other factors is less than in the previous periods. In the next period (step 4), the use of energy and labor continue to increase, but the use of capital, materials and output decreases. In this sector, it appears that energy is conserved, initially with the use of some additional capital. In subsequent periods as materials and output increase, energy use increase, possibly because energy comprises a small percentage of total costs; thus other factors have a greater impact on energy usage. It does not appear that capital accumulation involves long-run energy efficient processes as capital accumulation diminishes and energy use increases.

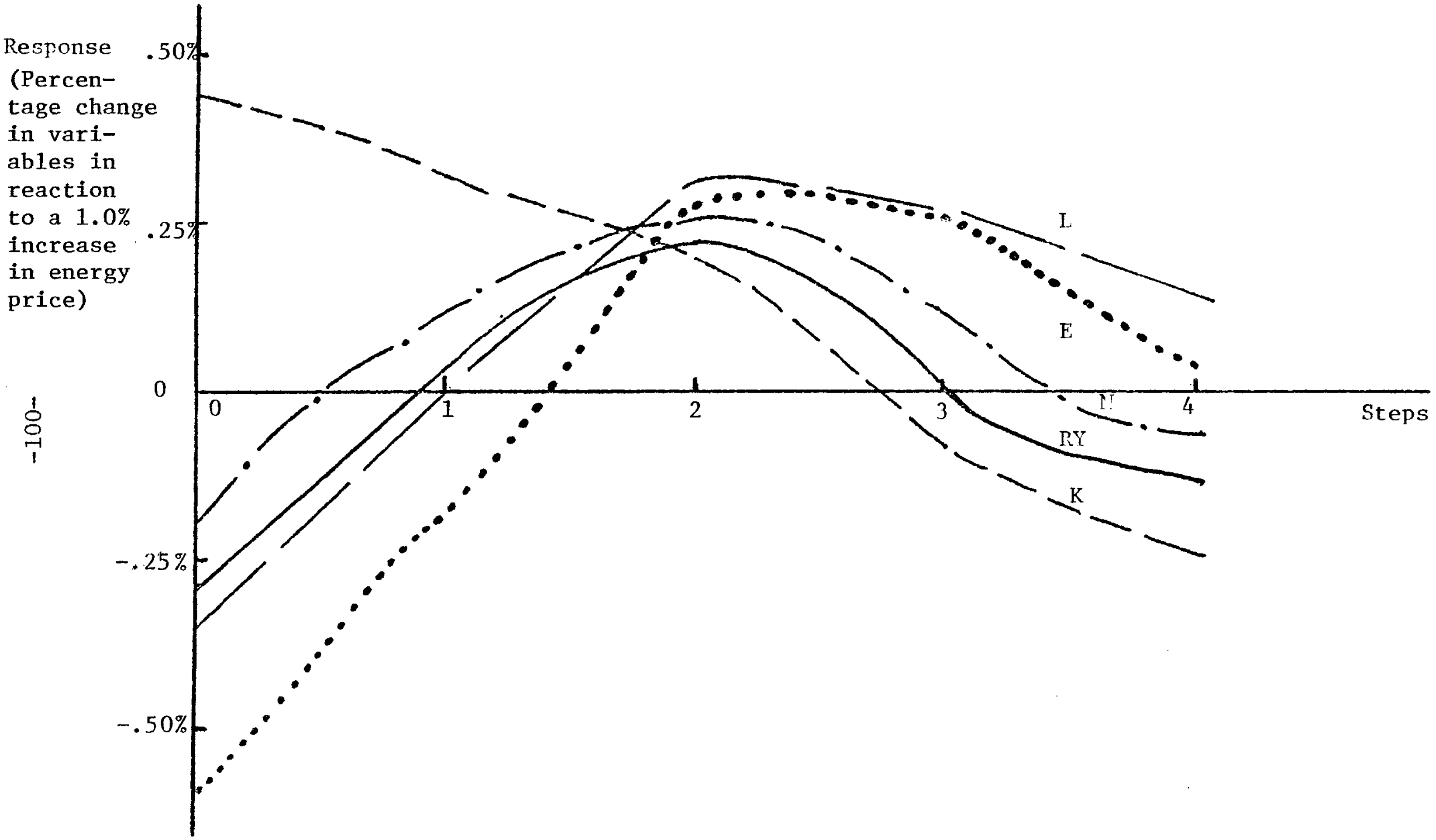

Figure 10. Impulse Response Functions for Fabricated Metal Products; Responses to a 1% Increase in Energy Price.

Of the other sectors in this group: wood products; furniture and fixtures; leather products; stone, clay and glass products; and transportation equipment show energy use (sustained negative change in use) comparable to the chemical products sector. Apparel, printing and publishing, petroleum products and primary metals show the same pattern of usage of energy (initial negative change and then positive change in use) as the fabricated metal products sector. In all of these sectors capital first increases and decreases in subsequent periods, energy responds as noted. Because capital use increases only slightly in most of these sectors, it seems that capital purchases go toward energy conserving type capital, with short payback periods, such as insulation. In sectors where energy use remains negative, this capital is effective. In sectors where energy use eventually increases, it seems that other factors have greater influence on energy usage than the price of energy.

In the third group is the rubber, miscellaneous plastics products, shown in Figure 11. This sector alone shows an initial decrease in capital in response to a 1% increase in energy price. In that same period all other inputs, labor, energy, materials as well as output decrease. In fact, the change in all inputs, capital, labor, energy and materials, as well as output remains negative for the entire projection period. Capital continues to decrease more in each period whereas with labor, energy, materials and output although change in usage remains negative, the rate of decline in usage slows. Although energy use decreases in this sector, the decrease in other inputs indicates that there is a general decline in this sector.

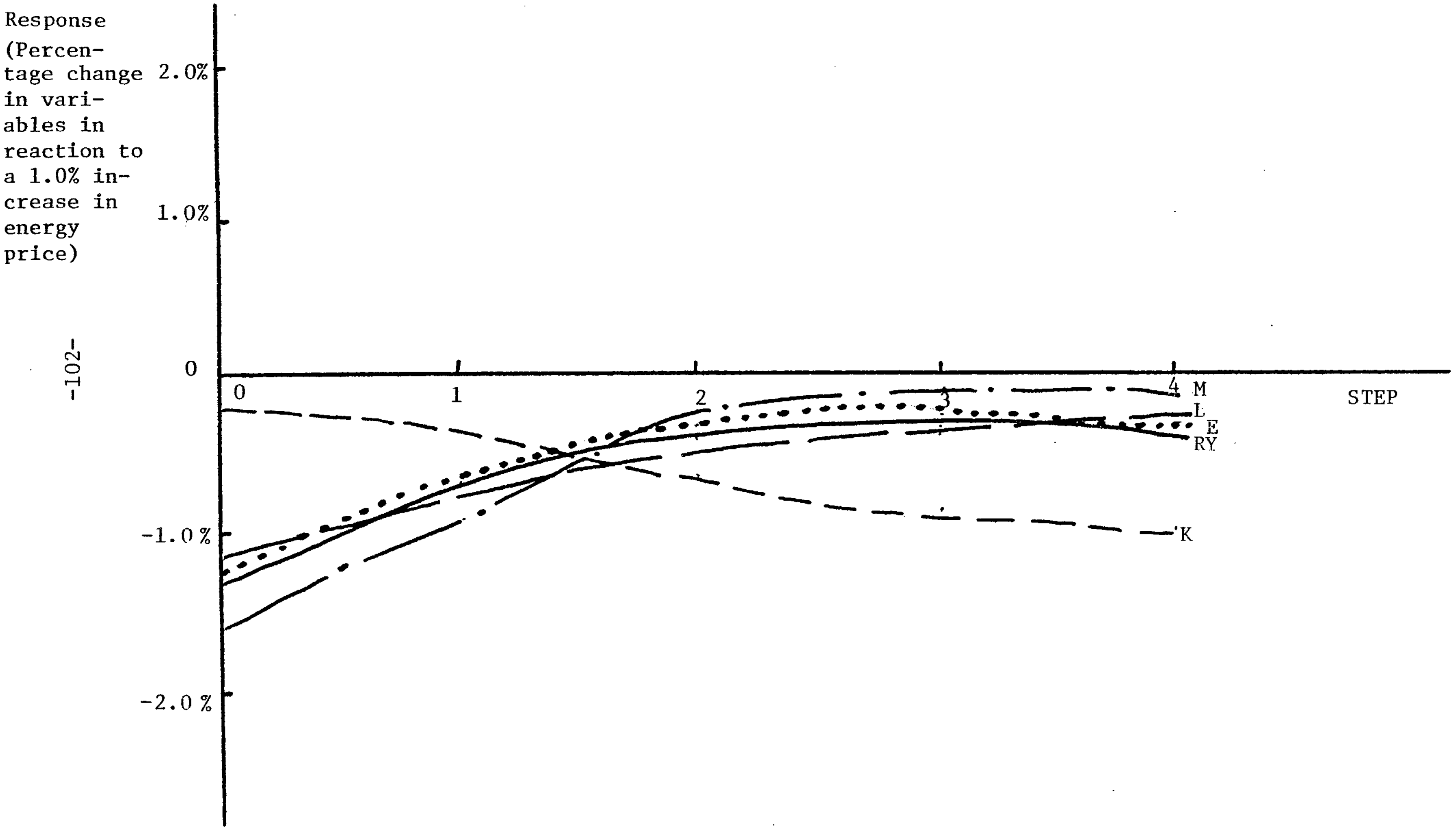

Figure 11. Impulse Response Functions for Rubber Products: Responses to a 1% Increase in Energy Price.

No new capital is purchased, and output falls. Since no new capital is purchased, it is unlikely that energy saving capital is adopted. This sector, and thus this group, seems to be the most adversely affected by an increase in energy price.

The size of the responses of variables to an increase in the price of energy varies greatly from sector to sector. In general, the response of capital is initially small, but it becomes larger (more elastic) in the long run. This response is small because energy has a small factor share and a one percent increase in its price is likely to have only a small effect on capital purchases. The size of the reaction of labor is generally less than one in absolute value (inelastic response) for the whole time span considered. This indicates that labor's response to the change in the price of energy is proportionately small, again most likely because energy comprises such a small proportion of total cost. The reaction of energy is more elastic than labor, however, it, too, tends to be small. This reaction indicates that changes in the use of energy are more responsive than changes in labor, meaning that energy use can be adjusted more easily in the short run than labor use. In general, energy use seems to be more responsive (more elastic) in later steps, indicating that energy use is more responsive in the long run than in the short run. The differing responses of energy across sectors are partially explained by differences in capital across sectors. Differing production processes will also elicit differing energy reactions across sectors. Some sectors would be expected to be more able to respond to energy price changes. The reaction of intermediate materials seems to vary the most and have the largest reactions. This is due to the wide variety of materials used across sectors. The response of output is generally small, but is large for some sectors. This again reflects the varying production processes across sectors.

The relationships of the inputs in this model are harder to summarize than in the static model. In the static model all reaction is assumed to occur simultaneously in each year, whereas in this model the path of adjustment is observable. As reaction of inputs occurs over time, the relationships may change, perhaps due to accommodations to the price change in the production procedures. This response cannot be detected in the static model.

As noted above, the reactions of the twenty-one sectors to an increase in the price of energy have been divided into three groups. In the first group capital purchases increase in reaction to higher energy prices; in the majority of sectors in that group energy use decreases, labor use decreases, materials use and output increase. The reactions in this sector indicate that some moderate energy saving capital is purchased, particularly in those sectors where energy use is negative. These sectors are: food products, electronic equipment, instruments, miscellaneous manufacturing and agriculture. The energy saving capital can include direct conservation types of capital such as insulation or more efficient furnaces for plant heating. It also possibly can include some moderate energy saving changes in the production process. This more capital intensive energy saving production process also uses more material and less labor in the majority of sectors.

In the second group capital purchases initially increase but decrease thereafter. The use of labor, energy and materials decrease in most of the sectors in this group, although energy use subsequently increases in apparel, printing and publishing, petroleum and coal products, primary metals and fabricated metals. These responses indicate that it is likely that some additional capital is purchased for insulation, more efficient lighting, etc. Because the capital response subsequently decreases it indicates that short term capital measures directed at energy conservation are more likely than adaptation of a new energy efficient production process which would seem to require sustained, increasing capital purchases over a long period of time.

In the third group, rubber and miscellaneous plastics products, capital purchases, as well as use of other inputs and output, decrease. It is apparent that no new capital of any type is purchased in this sector. Energy saving occurs in response to an increase in the price of energy, however, the other inputs do not adjust so output decreases.

These results are somewhat difficult to compare to the results of Berndt-Khaled (1978), or Berndt-Wood (1975) (a study which used the KLEM model but with a translog cost function) because of their diversity. The main result of these authors is that capital and energy are strong complements. This indicates that as energy prices increase, energy use will decrease and capital purchases or investment will decrease because of the complementarity between energy and capital. This will most likely lead to decreases in the capital stock and possibly output. The dynamic results of this model indicate this type of behavior only in rubber and plastics products, group III. The majority of sectors, however, display different behavior. In particular, the sectors in group I: food products, tobacco products, textile products, paper and allied products, machinery, electronic equipment, instruments, miscellaneous manufacturing and agriculture, show new capital purchases in all time periods considered. This new capital could be either a new moderately energy efficient production process or geared more directly at energy conservation. The second group: apparel, wood products, furniture and fixtures, printing and publishing, chemical and allied products, petroleum and coal products, leather products, stone, clay and glass products, primary metals, fabricated metals and transportation equipment, show initial increases in capital purchases, but eventual decreases in capital. This indicates capital purchases geared at energy conservation.

These responses indicate that, generally, some new capital is purchased in all sectors except rubber products, although the type of capital purchased does vary by group. In most sectors, however, there is no evidence that energy use continually decreases, so it is unlikely that substantative changes in production processes occur, except possibly in miscellaneous manufacturing. In this sense the results of this model agree with the results of Berndt-Khaled and Berndt-Wood. Adjustments in production processes involve basic changes in the system, and energy price increases to date may not have been sufficient to stimulate this sort of reaction. One reason that this may be the case is that the energy cost share in all sectors is very small.

The level of energy prices over the time period studied varies only slightly. Real energy prices decreased, in general until 1974, when they rose sharply; the rate of energy price increase decreased somewhat in 1975 and 1976. Other variables in the system reacted strongly in these last 3 years of data. These last 3 years of data thus reflect these greatly increased prices. Since OLS estimates are pulled towards outliers such as these last three years of data, they and the exchangeability estimates may weight these latter observations quite heavily. Thus, the last few years of data may have strongly influenced the results of the estimates and the impulse response function. Since the effect of increased energy prices is what is being measured, this weighting may not be undesirable, but it could account for the difference in the reaction of the capital variable in this model from other models. The studies previously mentioned did not use post-1971 data, so they had no opportunity to capture these effects. Data differences could also account for differences in results between this model and that of Berndt and Khaled.

As discussed above, the reaction of each sector to an energy price increase is unique. Representative sectors were used to facilitate discussion, however, examination of Tables 1-21 in Appendix II confirms the uniqueness of each sector. Past studies investigating this problem have considered only a highly aggregate model. The diversity of results across sectors in the VAR model indicates that aggregation may not be appropriate, therefore conclusions based on a highly aggregate model may be involved. Aggregation generally assumes that each unit in the aggregation has the same structural form. In the case of Berndt-Khaled and Berndt-Wood, it is assumed that the same cost structure underlies each sector. This is a strong assumption but one that is made out of necessity. However, if policy implications are to be drawn from the results, the aggregation problem becomes important. The same criticism can be leveled against this VAR model in that it is assumed that each industry in a sector has the same underlying stochastic process. In fact each industry within each sector may react differently to shocks in the system. The level of aggregation in this model is lower, so it might be argued that more similar industries are being aggregated.

5.3 Indirect Effects of Energy Price Increases

Most studies dealing with energy consider only the effect of direct energy use in response to energy price increases. Direct energy use can be defined as plant use of energy for power, heat and light. The concept of indirect usage of energy, as used in this paper, is the energy used in the production of capital and intermediate materials that are then used in the production process. It is likely that capital produced with higher priced energy will be more expensive, similarly for materials. This higher priced energy should be reflected in a higher price for capital and materials, so there will be a double kick from energy price increases.

To see one aspect of the indirect effects of higher energy prices, the response of capital and materials prices in reaction to a shock in energy prices can be examined. The impulse response functions in Tables 1-21 in Appendix II show an initial increase in the price of capital in only 11 of the 21 sectors. Only 16 sectors show an initial or eventual positive response.[43/] The reaction of the price of materials is similar, 11 with an initial price increase; 17 with an initial or eventual price increase.[44/] The reason that all sectors do not show some increase in these prices of capital or materials is most likely that these indirect effects may take longer to show up in the data because of differing production processes. As noted, the data showed large energy price increases in and following 1974, but before that energy prices were fairly stable. As more data become available, it can be better determined how long these indirect effects take to work through the economy.

[43/] Sectors: (21) tobacco, (23) apparel, (24) lumber and wood, (25) furniture and fixtures, (26) paper products, (27) printing and publishing, (28) chemical products, (29) petroleum and coal products, (30) rubber products, (33) primary metals, (37) transportation equipment show an initial positive response; in addition, sectors (20) food products, (22) textile products, (32) stone, clay, glass, (36) electronic equipment and (40) agriculture show an eventual increase in price of capital, RPK.

[44/] Sectors (23) apparel, (24) wood products, (25) furniture and fixtures, (26) paper products, (30) rubber products, (33) primary metals, (34) fabricated metals, (35) machinery, (36) electronics, (37) transportation equipment and (38) instruments show an initial positive response; Sectors (21) tobacco, (22) textiles, (27) printing and publishing, (29) petroleum and coal products, (31) leather products, and (40) agriculture show an eventual increase in price of materials, RPIM.

6. Comparison of the Models

In this section the results of the generalized Box-Cox cost function will be compared to the results of the vector autoregressive model. The results of the VAR model are difficult to compare to the GBC model because VAR model is dynamic, where as the GBC model is static. To facilitate this comparison, the results of the vector autoregression that are related to the direct effect of energy price increases will be compared to the results of the GBC model.

Some similarities exist between the two models. The similarities are mainly in the size of the elasticities in the models. It was found in the GBC function that most own and cross elasticities of input demand were inelastic. The same was found, in general, for the elasticities of inputs that responded to increases in price in the VAR. There were a few exceptions. For instance, in the response to the price of energy, initial energy responses were elastic in some sectors. The five sectors considered in the GBC analysis showed an initial inelastic response of energy to energy price increase. It was also found that the reaction of inputs to a change in the price of materials was fairly elastic compared to the response of inputs to changes in other input prices in the VAR. In the GBC function, elasticities of demand for materials were generally inelastic.

In the GBC analysis, the elasticities of substitution between capital and energy, σ_{KE}, were positive in three sectors: paper products, primary metals and agriculture. In these same sectors, the elasticities, σ_{LE}, were negative. The own elasticities were also negative. These results imply that energy price increases will cause capital to substitute for energy. As energy use decreases, labor use will also decrease because of the complementarity between energy and labor. Thus, output will become more capital intensive. The substitution between capital and energy in these three sectors is limited, so it is possible that output will decrease as capital purchases may not sufficiently replace energy use. If the relationship between energy and capital had been more flexible (σ_{KE} larger), it is possible that output would not decrease, as investment would have significantly increased capital. The complementarity between labor and energy implies that energy price increases will have a negative effect on employment in these three sectors, which is not what Berndt and White (1978) found in the aggregate.

In chemical products and coal and petroleum products, σ_{KE} is negative and σ_{LE} is positive. These results suggest that energy price increases will make existing capital undesirable as it is energy inefficient. This will lead to decreases in investment which over time would decrease the capital stock. Decreases in the capital stock could lead to decreases in output and growth. So in these two sectors problems of adjustment to higher energy prices could exist. This scenario is suggested by Berndt (1978). Recall, the results of these sectors are similar to those of Berndt and Khaled. Because only five sectors were analyzed, it cannot be determined if the majority of sectoral results would be more similar to the majority of sectors analyzed in this thesis or to the Berndt and Khaled results.

The VAR results show several different responses to an increase in the price of energy. These responses have been divided into three groups. In the first group capital showed continued increases, meaning that it was possible that moderately energy efficient production processes along with energy saving capital, such as insulation were being purchased. The second group showed an initial increase in capital but a subsequent decrease. This pattern of capital suggests that this capital was primarily of the energy saving type and did not include new energy efficient production processes as the capital purchases were not long and sustained as would be expected for this change in production process. The third group, consisting of only one sector, rubber products, showed no increase in capital. In this group, other inputs did not show any adjustment to the increases in the price of energy. Only group I could show long run adaptation of extensively energy efficient processes as only this group shows sustained capital investment. Of this group, only miscellaneous manufacturing showed this as a possibility as energy use in this sector continually fell, whereas it eventually increased in the other sectors in this group.

In general, the results of the GBC model which show K-E substitutability are similar to the results of group I of the VAR which show sustained capital purchases. Two of the three sectors of the GBC model which show this K-E relationship are in group I of the VAR; these sectors are paper products and agriculture. The results of the GBC model which show K-E complementarity are similar to the results of group III which shows sustained decreases in capital. Group III contains only rubber products which was not analyzed in the GBC model, thus no sector analyzed in the GBC model is included in group III of the VAR. Group II of the VAR is more difficult to compare to the GBC results as its first period reactions make it similar to those sectors analyzed with the GBC which show K-E substitutability. Later period reactions show decreases in capital which makes group II of the VAR comparable to those sectors of the GBC model which show K-E complementarity. The three remaining sectors analyzed with the GBC model are contained in group II of the VAR. Of these sectors primary metals shows K-E substitutability in the GBC analysis and chemical products and petroleum and coal products show K-E complementarity.

As noted above, the size of the K-E relationships regardless of sign decrease from 1947 to 1976 and in fact those that have positive sign all became less than one in 1976, indicating limited substitution. It should be pointed out that those sectors with the largest positive σ_{KE} values are contained in group I of the VAR, these sectors being paper products and agriculture. This does indicate a degree of similarity between the results of the two models. It should also be stated that the limited K-E substitution which occurs in these two sectors is somewhat similar to the capital increases which occur in group I of the VAR in that some adjustment is made in the production process to accommodate the decreased use of energy. However, since elasticities of substitution are less than one in the sectors of the GBC model and, as previously noted, only miscellaneous manufacturing shows the possibility of making long run adjustments, it seems that the accommodation of capital to increased energy prices is limited.

The remaining sectors of the GBC model, chemical products, petroleum and coal products ($\sigma_{KE} < 0$), and primary metals ($\sigma_{KE} > 0$) fall into group II of the VAR model. These elasticities of substitution between K and E, associated with the GBC model for these sectors, although different in sign, are not that different in size in 1976. Notably for chemicals and petroleum products, σ_{KE} is slightly greater than one (in absolute value) and for primary metals σ_{KE} is slightly less than one, so the degree of complementarity and substitutability is small. The VAR results for group II indicate that some initial capital purchases are most likely for insulation and other energy saving capital; the relatively small σ_{KE} for primary metals could possibly reflect this limited adjustment. Group II then shows subsequent decreases in capital which indicates that capital. changes are probably not made in the production process, thus adjustment is limited in this group. This basic result agrees in size, at least, with the elasticities of substitution between K and E of these three sectors. No sector in the GBC analysis shows the strong rigidity in the adjustment process that is shown by group III of the VAR; consistently large negative σ_{KE} would reflect this rigidity.

Again, it should be stressed that the differences in the adjustment mechanism in the two models make strict comparison difficult. In the GBC model it is assumed that adjustment to changes in prices occur within the same year. In the VAR model it is assumed that adjustment occurs over a period of several years. The impulse response function can capture the adjustment path over many years, whereas static elasticities of substitution cannot. In any case, the difference in adjustment time results in the estimation of different phenomena. It is not surprising, therefore, that the results are different. Even in the case where the GBC and VAR models give similar results, the phenomena being measured are still different.

The exchangeability prior incorporated into the VAR model might strongly affect the coefficients if the parameters for each sector were very similar. In this application the sectoral parameters appeared to be quite distinct so the exchangeability prior barely altered the coefficients, so the fact that the exchangeability prior is used in one model (VAR) and not the other (GBC), is not significant.

Only five sectors were estimated with the GBC function because of the computational cost. The cost of estimating all 21 sectors using the VAR was small. This indicates one advantage of the VAR model. The incorporation of the exchangeability prior is done easily with the linear VAR, whereas it is impossible with the nonlinear GBC. The exchangeability prior is useful as it provides a way of explicitly incorporating the analyst's prior intuitions into the estimation, although in the VAR it changes estimates very little. The adjustment to energy price increases is a dynamic adjustment so it is important that the model be able to capture the dynamic aspects of the process. The VAR provides a way of giving a system a dynamic nature. Other structural models of input demand that are dynamic in nature are still being developed.[45] Models of this type involve dynamic optimization over time; because the models are usually highly nonlinear, they are estimated by maximum likelihood estimation. Estimation of this type of model is more difficult than estimation of the VAR.

7. Policy Implications

In determining an appropriate set of policies the policy goals must first be stated explicitly. In this case, it seems desirable for society to decrease the use of energy, given the decreased energy supplies and increased dependence of the United States on foreign countries for its oil supplies. Another goal to be considered along with this energy conservation goal is sustained increases in output, as this assures economic well-being for the nation in general. The results in this thesis do not indicate that these goals are incompatible for the majority of sectors in the economy. The problem is devising a method of achieving these goals.

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[45] See Sargent (1978).

Before a method is recommended to achieve these goals consider the policy discussions of other analysts. Berndt and Wood (1975) warn that giving tax incentives to stimulate investment is inappropriate if capital and energy are complements as increased capital leads to increased energy use thus defeating the goal of energy conservation. It should also be noted that even if capital and energy are not complements that this method may not be efficient as tax incentives will go to all sectors more or less equally, and not be directed at those sectors which are the largest or the most inefficient energy users

Again before policy recommendations can be made it sould be understood that the results of these models should not be taken as proved facts but rather as a first attempt to obtain sectoral estimates of the relationships of inputs in the production process, where energy is taken as a separate input. It should also be stressed that the most robust result over both models is the large difference in estimated coefficients and thus relationships across sectors. This result has an important policy implication, namely that whatever policy option is used must be flexible enough to allow each sector to determine how much it will react to that policy option. Any policy alternative which requires all sectors to utilize it equally will be inefficient in light of these results.

Given the goals of energy conservation and sustained growth and the differences among sectors, one policy recommendation is a BTU tax. This alternative is efficient in two ways. First, taxing fuels on a BTU basis taxes fuels equally on the basis of heat content, this leads to end use efficiency (adaptation of the "best" fuel for a particular task). Second, taxing fuel on a BTU basis allows producers to decide how much to decrease consumption of each fuel. This alternative will thus allow adjustment of fuels within a sector and adjustment of total energy usage for a particular sector.

This policy element induces energy conservation as a BTU tax increases the price of all fuels. Since this alternative works through the market system it allows producers to determine their profit maximizing input usage accordingly, thus two of the policy criteria are met. As for the last goal of sustaining growth in output, the results of this thesis indicate that it is likely growth will continue despite energy price increases, but further analysis is needed to determine accurately if this is the case.

The policy alternative has only been discussed qualitatively, before enactment it must be quantified. This step goes beyond the scope of this paper. Quantification would require that simulations be run with the VAR model by shocking the energy price variable at different levels until the desired effects are achieved. To quantify the BTU tax with the GBC model, the GBC model would have to be incorporated into a larger macro model as was done by Hudson and Jorgenson (1974) because the GBC model structure alone does not contain the mechanism for determining these macro effects. Simulations would then have to be run on this larger macro model using various BTU taxes, the tax which best achieves the required result is then chosen. Before quantification of the BTU tax is done using the VAR model it is highly recommended that improvements in the capital variable be made. However, since this task is more easily accomplished than constructing a macro model for the GBC function, the use of VAR model is suggested.

The use of the BTU tax will create a pool of funds. It is suggested that these funds be used for the development of alternative sources of energy as well as more efficient production processes. Although the results of the models in this paper suggest that some increase in capital takes place after an energy price increase, that increase is not large enough to substantially change the capital stock or production process. It is important, therefore, that research on both alternative energy sources and new production processes proceed for time lags between development and implementation will most likely be long.

As in any policy decision, carrying out the policy is not without problems. Since this policy works through the market place, these problems are hopefully minimized. It should be stressed that this policy is equitable and efficient in achieving energy conservation and therefore seems appropriate at this time.

8. Conclusions

One of the main objectives of this paper is to determine the relationship of the use of energy to the use of other inputs in U. S. manufacturing and agriculture, on a sector by sector basis. In the past, studies have looked in detail at the relationship between capital and labor. More recently, there have been other studies that have looked at the relationship of energy to capital, labor and materials, but these have been of an aggregate nature. The relationship of inputs is important in determining how inputs will adjust to higher energy prices. It was felt that a more disaggregate analysis, in which twenty (two-digit SIC code) manufacturing industries and agriculture were analyzed separately would indicate the relationship of inputs in each sector as well as which sectors would have the most problem adjusting to higher energy prices. It was also felt that the study would be strengthened by extending the analysis to include post-1974 data, a time period in which energy prices were rising.

Two models were specified to measure the relationships between inputs, a static highly structured model given in equation (4) and a dynamic more loosely structured model given in equation (26). The time period used is 1947-1976. Data were obtained for 21 sectors, including 20 manufacturing sectors and agriculture. Unfortunately, the static model is highly nonlinear and costly to estimate on a sector by sector basis, so the number of sectors analyzed was reduced to five.

The static generalized Box-Cox model allows nonhomothetic production and non-neutral technical change. In addition, elasticities of substitution and input demand can be estimated. Returns to scale, rate of total cost diminution and total factor productivity can be obtained from the estimated coefficients. It was shown in equation (15) that total factor productivity is related to output and input price.

The dynamic model is a time series model that uses a Bayesian approach. The prior used is an exchangeability prior which assumes that the sectors are all structurally related rather than completely independent. This relationship of the sectors is incorporated by assuming that each coefficient for each sector, comes from the same distribution, rather than independent distributions. The variables chosen in this model are related to the production process and include output, inputs and input prices. It was realized that this micro approach could not give accurate reactions of output as this approach concentrates only on the production side and does not consider the demand side or the macro affects on output.

In estimating the generalized Box-Cox cost function, it was found that $\lambda = 1$, the generalized Leontief case, gave the largest value of the log-likelihood function in four of the five sectors. In the other sector, $\lambda = 2$, the generalized square root quadratic case, gave the largest value of the log-likelihood function. The results of the GBC model for the values of λ associated with the largest log-likelihood function values indicated that capital and energy were substitutes in three of the five sectors considered. These sectors were:

(26) Paper and Allied Products
(33) Primary Metal Industries
(40) Agriculture

The substitution relationship between energy and capital was limited (σ_{KE} less than one) in these sectors. In these same sectors, labor and energy were complements. The implication of these results is that as energy prices increase, capital would replace energy and labor, although the increase in capital may be small. This would eventually increase unemployment as labor use decreases with energy use. Although investment may increase, it may not be large enough to offset decreases in energy and labor, so output could decrease. In the other two sectors analyzed, problems could arise because in these two sectors:

(28) Chemical and Allied Products, and

(29) Petroleum and Coal Products,

capital and energy were complements and labor and energy were substitutes. This implies eventual decreases in investment which could lead to decreases in the capital stock and output, since output is capital intensive. The complementarity here was not large, and it was decreasing over time thus if policy that encourages the use of energy efficient capital is instituted, output and growth need not decrease.

It was also found that returns to scale were large and rates of total cost diminution were low. This gives low estimates of growth rates of total factor productivity. Total factor productivity was decreased if energy prices were increased. Finally, technical change was found to be capital and energy using and labor saving for all sectors; technical change was materials using for paper products, primary metals and agriculture and materials saving for chemicals and petroleum and coal products.

The results of this model must be qualified, however, as the standard errors of the coefficients important in determining the elasticities of substitution were very large. This model is unrestrictive, which is good in that it does not force the analyst to make *a priori* structural restrictions, but allows the data to do this. There is a tradeoff, in that this unrestrictive model has many free parameters, 19 to be specific. Since there are only 30 annual observations, this leaves 11 degrees of freedom. It is asking much of the data to estimate all of these parameters accurately. These problems with the estimation emphasize that the results of the GBC model must be qualified on these several grounds.

The results of the vector autoregressive model also indicate diversity. Group I which includes sectors (20) food products, (21) tobacco products, (22) textile products, (26) paper products, (35) machinery, (36) electronics, (38) instruments, (39) miscellaneous manufacturing and (40) agriculture shows sustained capital purchases when energy prices increase. This indicates that some moderate changes are being made in the production process towards greater energy efficiency. However, in most of these sectors energy consumption generally increases so it is believed that changes are not long run in nature (ie. involving technical innovation in production process, etc.). Group II which includes sectors (23) apparel, (24) wood products, (25) furniture and fixtures, (27) printing and publishing, (28) chemicals, (29) petroleum and coal products, (31) leather products, (32) stone, clay and glass products, (33) primary metals, (34) fabricated metals, and (37) transportion equipment shows an increase in capital and then a decrease in response to higher energy prices. This has been interpreted to be capital purchases of insulation and other direct conservation devices, but no substantive change in the production process. Group III which includes only sector (30) rubberproducts shows no increase in capital but rather a decrease in response to increased energy prices. This signifies no adjustment is made in this sector as energy becomes more costly.

Indirect effects can also be captured with the VAR. Those effects are contained in the reactions of the price of capital and materials because energy price increases will affect the production of capital equipment and materials. Higher energy prices will eventually increase the price of capital and materials which use this higher priced energy. Sixteen of the 21 sectors show an increase in the price of capital, due to a shock in the price of energy. In 17 sectors, materials price increases in response to higher energy prices. All sectors do not show an increase. This is possibly due to the fact that these indirect effects take even longer to work through the economy. Since energy prices have only increased since 1974, it is likely that these reactions have not appeared in the data yet.

The standard errors of the VAR estimates are small compared to the standard errors of the GBC function. It is possible to do a sectoral analysis incorporating the exchangeability prior with the VAR, but not with the GBC. The exchangeability prior draws each sector's coefficients slightly toward the mean. With the VAR, it was found that the coefficients for each sector are very different, this strengthens the need for a sectoral analysis Because the coefficients for each sector are very different, it is important that policy based on these results does not restrict all sectors to adopt identical policies. Policy should be flexible enough to allow each sector to determine its own product output and input use, and yet achieve the goals it sets forth, such as energy conservation and sustained output.

The GBC function is too complicated to incorporate the exchangeability prior. In particular, in the highly nonlinear framework of the GBC function, estimation poses problems. Incorporation of the exchangeability prior in the GBC function mandates simultaneous estimation of all sectors. This gives too many free parameters in the estimation process and is thus impossible to implement. It would be useful, however, to incorporate the exchangeability prior if a simpler structural model, such as the translog, were used. The estimation procedure would be similar to the estimation of the VAR, as the translog can be estimated using linear equations. This would allow incorporation of all data in the estimation of each sector, yet it is not as restrictive as most pooling procedures.

The results of both models indicate that for the most part some substitution of capital for energy occurs in the majority of sectors. This substitution seems to be an immediate response, the installation of insulation, etc. or short run in nature, such as some alterations in the production process. The substitution is not strong enough in either model to indicate that far reaching changes in the production process have occurred, such a change would indicate a long-run adjustment to higher energy prices.

This lack of long-run response may be due to the fact that although energy prices rose significantly in 1974, the rate of increase thereafter was much smaller until 1979. It is possible that an initial conservation response was observed, but longer run responses were not needed, because energy has a small factor share and increases in its price were not sufficient to stimulate changes in the production process. Another likely reason why long run responses have not shown up in the data yet is only three years of data with increased energy prices were available. Long-run reactions occur over a long period of time thus most likely require more time to exhibit themselves in the data. Only studies using subsequent data can substantiate this point.

Since long-run reactions did not occur, a policy measure to conserve energy, sustain growth and allow for sectoral differences was proposed. This policy alternative was a BTU tax. This tax would meet the goals in an efficient, equitable way. In addition, the funds collected from this tax can be used to develop new energy sources and new energy-efficient production processes.

REFERENCES

Allen, R. G. D. (1938): Mathematical Analysis for Economists, Macmillan, London.

Berndt, E.R. (1978): "Aggregate Energy, Efficiency and Productivity Measurement," Working Paper No. 25, University of British Columbia Programme in Natural Resource Economics, Vancouver, June.

Berndt, E.R. and L. R. Christensen (1973): "The Internal Structure of Functional Relationships: Separability, Substitution and Aggregation," Review of Economic Studies, 40, pp. 403-410.

Berndt, E. R., B. H. Hall, R. E. Hall and J. A. Hausman (1974): "Estimation and Inference in Nonlinear Structural Models," Annals of Economic and Social Measurement, 3, pp. 653-665.

Berndt, E. R. and M. S. Khaled (1978): "Parametric Productivity Measurement and Choice Among Flexible Functional Forms," Discussion Paper No. 78-28, University of British Columbia Department of Economics, Vancouver, August.

Berndt, E. R. and C. M. White (1978): "Income Redistribution and Employment Effects of Rising Energy Prices," Working Paper No. 30 in University of British Columbia Programme in Natural Resource Economics, Vancouver, December.

Berndt, E. R. and D. O. Wood (1975): "Technology, Prices and the Derived Demand for Energy," Review of Economics and Statistics, 56, pp. 259-268, August.

Berndt, E. R. and D. O. Wood (1979): "Engineering and Econometric Interpretations of Energy-Capital Complementarity," American Economic Review, 69, pp. 342-354, June.

Binswanger, H. P. (1974a): "A Cost Function Approach to the Measurement of Elasticities of Factor Demand and Elasticities of Substitution," American Journal of Agricultural Economics, 5, pp. 377-387, May.

Binswanger, H. P. (1974b): "The measurement of Technical-Change Biases with Many Factors of Production," American Economic Review, 56, pp. 964-976, December

Binswanger, H. P., and V. W. Ruttan, et.al. (1978): Induced Innovation Technology, Institutions and Development, Johns Hopkins Press, Baltimore.

Christensen, L. R. and W. H. Greene (1976): "Economics of Scale in U.S. Electric Power Generation," Journal of Political Economy, 84, pp. 655-676, August.

Christensen, L. R. and D. W. Jorgenson (1969): "The Measurement of U.S. Real Capital Input, 1929-1967," Review of Income and Wealth, 15, pp. 293-320.

Christensen, L.R., D. W. Jorgenson, and L. J. Lau (1971): "Conjugate Duality and the Transcendental Logarithmic Function," Econometrica, 39, pp. 255-256, July.

Christensen, L.R., D. W. Jorgenson and L. J. Lau (1973): "Transcendental Logarithmic Production Frontiers," Review of Economics and Statistics, 55, pp. 28-45, February.

Dhrymes, P. J. (1970): Econometrics, Statistical Foundations and Applications, Harper and Row, New York.

Diamond, P.A. and D. McFadden (1965): "Identification of the Elasticity of Substitution and the Bias of Technical Change: An Impossibility Theorem," Berkeley: University of California Institute of Business and Economic Research, Working Paper No. 62, March. (Revised and retitled, "Measurement of the Elasticity of Factor Substitution and Bias of Technical Change," in M. Fuss and D. McFadden, editors Production Economics: A Dual Approach to Theory and Applications, North-Holland, Amsterdam.

Diewert, W. E. (1971): "An Application of the Shephard Duality Theorem: A Generalized Leontief Production Function," Journal of Political Economy, 79, pp. 481-506, May/June.

Diewert, W. E. (1973): "Functional Forms for Profit and Transformation Functions," Journal of Economic Theory, 63, pp. 284-316.

Diewert, W. E. (1974): "Applications of Duality Theory," in Frontiers of Quantitative Economics, M.D. Intrilligator and D. A. Kendrick, editors, North-Holland, Amsterdam.

Dresch, F. W. (1938): "Index Numbers and the General Economic Equilibrium," Bulletin of the American Mathematical Society, 44, pp. 134-141, February.

Dresch, F.W. (1945-46): "Continuous Index Numbers and Quantitative Study of the General Economy," Proceedings of the First Berkeley Symposium in Mathematical Statistics and Probability, Vol. 1, ed. J. Neyman, pp. 203-221.

Fuss, M. A. (1977): "The Demand for Energy in Canadian Manufacturing," Journal of Econometrics, 5, pp. 89-116.

Griffin, J. M. and P. R. Gregory (1976): "An Intercountry Translog Model of Energy Substitution Responses," American Economic Review, 66, pp. 845-857, December.

Hausman, J. A. (1975): "An Instrumental Variable Approach to Full Information Estimators for Linear and Certain Nonlinear Econometric Models," Econometrica, 43, pp. 727-738, July.

Hitch, C.R., editor (1977): Modeling Energy-Economy Interactions: Five Approaches, Baltimore, Johns Hopkins University Press for Resources for the Future.

Hoerl, A.E. and R. W. Kennard (1970): "Ridge Regression: Biased Estimation for Nonorthogonal Problems," Technometrics, 12, pp. 55-67.

Hudson, E. A. and D.W. Jorgenson (1974): "U.S. Energy Policy and Economic Growth, 1975-2000," Bell Journal of Economics and Management Science, 5, pp. 461-514, Autumn.

Humphrey, D. B. and J. R. Moroney (1975): "Substitution Among Capital, Labor and Natural Resource Products in American Manufacturing," Journal of Political Economy, 83, pp. 57-82.

Hulten, C. R. (1973) "Divisia Index Numbers," Econometrica, 41, pp. 1017-1024, November.

Jaszi, G., C.F. Schwartz, L. Grose and I. Rottenberg (1970): Readings in Concepts and Methods of National Income Statistics, U.S. Department of Commerce, NTIS No. PB 194 900.

Johnston, J. (1963): Econometric Methods, McGraw-Hill, New York.

Jorgenson, D.W. and J. A. Stephenson (1967): "Investment Behavior in U.S. Manufacturing, 1947-1960," Econometrica, 35, pp. 169-220, April.

Khaled, M. S. (1978): "Quadratic Utility, Labor Supply and Commodity Demand," Princeton: Princeton University Industrial Relations Section, Working Paper No . 67, April.

Leamer, E.E. (1978): Specification Searches, John Wiley, New York.

Lee, L.R. and W. E.Griffiths (1979): "The Prior Likelihood and Best Linear Unbiased Prediction in Stochastic Coefficient Linear Models," University of Minnesota, Center for Economic Research, Discussion Paper No. 79-107, January.

Lindley, D.V. and A. F. M. Smith (1972): "Bayes Estimates for the Linear Model," with Discussion , Journal of the Royal Statistical Society, Series B, 34, pp. 1-41.

Maddala, G. S. (1971): "The Use of Variance Components in Pooling Cross Section and Time Series Data," Econometrica, 39, pp. 341-358, March.

Maddala, G.S. (1977): Econometrics, McGraw-Hill, New York.

Malinvaud, E. (1970): Statistical Methods of Econometrics, Rand McNally, Chicago.

Ohta, M. (1974): "A Note on the Duality Between Production and Cost Functions: Rate of Returns to Scale and Rate of Technical Progress," Economic Studies Quarterly, 15, pp. 73-77, March.

Pindyck, R.S. and D. Rubinfeld (1976): Econometric Models and Economic Forecasts, McGraw-Hill, New York.

Richter, M. K. (1966): "Invariance Axioms and Economic Indexes," Econometrica, 34, pp. 739-755, October.

Roy, R. (1946): "Balance Des Comptes et Relations Change," Econometrica, 14, pp. 257-284, October.

Ruble, W. L. (1968): "Improving the Computation of Simultaneous Stochastic Linear Equation Estimates," unpublished Ph.D. dissertation, Michigan State University.

Ruggles, R. and N. Ruggles (1970): The Design of Economic Accounts, National Bureau of Economic Research, New York.

Sargent, T. J. (1978): "Estimation of Dynamic Labor Demand Schedules Under Rational Expectations," Journal of Political Economy, 86, pp. 1009-1044, December.

Shephard, R. W. (1970): Theory of Cost and Production Functions, Princeton University Press, Princeton, New Jersey.

Sims, C. A. (1977): "Macroeconomics and Reality," University of Minnesota, Center for Economic Research, Discussion Paper No. 77-91, December.

Smith, A.F.M. (1973): "A General Bayesian Linear Model," Journal of the Royal Statistical Society, Series B, 35, pp. 67-75.

Stigler, G. J. (1963): Capital and Rates of Return in Manufacturing Industries, Princeton University Press, Princeton, New Jersey, for National Bureau of Economic Research.

Swamy, P.A.V.B. (1970): "Efficient Inference in a Random Coefficient Regression Model," Econometrica, 38, pp. 311-323.

Swamy, P.A.V.B. (1971): Statistical Inference in Random Coefficient Models, Springer-Verlag, New York.

United States Department of Agriculture (1961), Farm Cost Situation, November.

United States Department of Agriculture (1974): Energy and U.S. Agriculture Data Base: 1974, in conjunction with the Federal Energy Administration.

United States Department of Commerce, Bureau of Economic Analysis (1976): "National Income and Product Tables," Survey of Current Business, 56, No. 4, pp. 11-18, April.

Uzawa, H. (1962): "Production Functions with Constant Elasticities of Substitution," Review of Economic Studies, 30, pp. 291-299, October.

Zarembka, P. (1974): Frontiers in Econometrics, Academic Press, New York.

Zellner, A. (1971): An Introduction to Bayesian Inference in Econometrics, John Wiley, New York.

DATA REFERENCES

United States Department of Agriculture, _Agricultural Statistics_, 1948-1977, annual issues.

United States Department of Commerce, _Annual Survey of Manufactures: General Statistics_, 1947-1976, annual issues.

United States Department of Commerce, _Fuels and Electricity Consumed_, annual issues, 1974, 1975 and 1976.

United States Department of Commerce, Bureau of Economic Analysis, _National Income and Product Accounts of the United States_, 1977.

United States Department of Labor, Bureau of Labor Statistics, _Wholesale Prices and Price Indexes_, annual issues 1948-1977.

Appendix I. Description of Variables Used as Data.

The time period used for both models is 1947 to 1976. Twenty manufacturing sectors (2 digit SIC code) and the agricultural sector are analyzed. The main sources of data used are the Annual Survey of Manufactures (ASM), 1974-1976; Agricultural Statistics (AS), 1948-1977 and Wholesale Prices and Price Indexes (WPPI), 1948-1977. Several series are also obtained from the National Income and Product Accounts, 1977 (NIPA) which is published by the Bureau of Economic Analysis (BEA). More complete citations are given in the Data References (p. 128).

The Variables

RY: Real Output

The sectoral nature of the data leads to the use of gross sectoral output in each sector. This is distinguished from gross output in the GNP accounts which is a measure of value added. The data series used for the manufacturing sectors was the value of shipments. This is a series reported in the ASM. For the agricultural sector, a similar concept was used; this variable is defined to be value added plus the cost of materials. Value added is the gross output variable for the farm sector reported in the NIPA. Cost of materials is reported in AS as expenditures of farmers. Both series are in current dollars. These two series were summed to obtain the output variable. The output variables for both manufacturing and agriculture were divided by the wholesale price index (WPI), 1967=100, to obtain output in constant 1967 dollars.

K: Capital

The capital variable used for the manufacturing sector was gross book value. This series is published in ASM. Included in the value of gross book value are buildings, structures, machinery and equipment for which depreciation reserves are maintained. Inventories, land and mineral rights are not included. This value represents the actual cost of the assets at the time of acquisition. It also includes all costs that make capital usable, such as repair costs. If the plant changes ownership, the assets are revalued at their appraised or purchase price.

A similar capital variable, farm assets, was used for the farm sector. This capital variable, farm assets, unlike gross book value, contains the value of land, which is an important agricultural input. The source of this series is AS.

Gross book value was not reported by the ASM for a few years in all sectors. For those years for which it was not reported, gross book value was approximated by an equation given in Christensen and Jorgenson (1969):

$$(1) \quad K_{it} = I_{it} + (1-u_i)K_{it-1}$$

where K_{it} is current capital stock (gross book value)

K_{it-1} is the capital stock (gross book value) in the previous year

I_{it} is investment in period t

u_i is the depreciation rate and

i refers to the sector.

If K_{it}, K_{it-1} and I_{it} are known and substituted into equation (1), u_i can be solved for in each sector. This was done for years when these variables were known. Then for the several years K_{it} was not available u_i was estimated by interpolating between the nearest available observations. Since it was found the u_i did not change much over the period considered, this seemed to be an adequate procedure.

It should be acknowledged that there are some problems with the use of gross book value as the measure of capital. Humphrey and Moroney (1975, p. 67) point out three shortcomings for cross-sectional use of this data at a point in time.

1) capital stocks are accumulated in different industries at different average prices,

2) there probably exist unknown interindustry differences in the percent of gross book value reported which are economically useful, and

3) there are interindustry differences in capacity utilization rates of reported book values.

However, because other better measures of capital require extremely detailed data, the gross book value measure was used, acknowledging its shortcomings.

For use in the regressions the capital stock measures for manufacturing and farming (gross book value and farm assets respectively) were deflated by the wholesale price index, 1967=100. This gives gross book value or farm assets in constant 1967 dollars. This capital variable was scaled by a constant. This constant is:

$$(2) \quad \frac{TCK\ (1967)}{K\ (1967)} = \text{price of capital in 1967,}$$

where TCK is total cost of capital, discussed below.

Both of these variables in (2) are in dollars, so this is a pure number. This gives the capital variable:

$$(3) \quad K_t \times \frac{TCK\ (1967)}{K\ (1967)}$$

Notice for 1967 the capital stock measures equals the actual nominal capital stock in 1967. This scaling was done to obtain the quantity of capital in terms of cost in 1967 as the capital variable. Thus, this capital variable is comparable to the other input quantities which are also measured in terms of cost in 1967 dollars.

TCK: Total Cost of Capital*

Ruggles and Ruggles (1970, p. 57) suggest that "an estimate of the contribution of capital (i.e. the capital charge) could be based on capital consumption together with a proper imputed interest charge." Ruggles and Ruggles argue against using measured profits in capital cost for several reasons. They state that: "merely because capital is employed in a highly profitable industry, it does not necessarily follow that the contribution of capital is high. Monopoly for example, may be highly profitable and provide a return to the enterprise over and above the contribution of either capital or labor. Profit may also arise from entrepreneurial skill or exploitation of labor, which results in underpayment of these factors of production." 1/

The Ruggles and Ruggles suggestion of using capital consumption plus the proper imputed interest charge for cost of capital was taken. Capital consumption allowance reported by the BEA in <u>NIPA</u> on a sectoral basis was used for the capital consumption variable. This variables measures depreciation

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1/ Ruggles, R. and N. Ruggles (1970, p. 56).

* The cost of capital concept utilized in this thesis is the service flow of capital.

on plant and equipment. The proper imputed interest that should be added to capital consumption is the opportunity cost of holding that capital. The opportunity cost is the interest foregone because capital is held rather than invested in interest bearing assets.[2] To do this, the market value of the plant and equipment of the firm is needed, as well as the proper imputed interest rate on an aggregate sectoral basis.[3]

Since the preferred measures were not available, a proxy variable was used. The proxy variable is the net interest variable also reported by the BEA in the NIPA. Net interest measures the interest paid by the firm minus the interest received by the firm.

RPK: Real Price of Capital

The price of capital was obtained by dividing current dollar capital costs in each industry by that industry's current dollar capital stock, gross book value for the manufacturing sector or farm assets for the agricultural sector. This resultant quotient is a pure number which reflects the price of capital. This number was put into index form by dividing the price in each sector by that sector's 1967 price. This index was then divided by the WPI, 1967 = 100, to give the price of capital index in 1967 dollars.

2/ This is the correct imputation according to Ruggles and Ruggles.

3/ No data for value of plant and equipment at replacement cost could be found.

L: Labor

The labor variables available for manufacturing in the ASM are: total number of employees (production plus nonproduction workers), number of production employees, manhours of production employees and total compensation of employees. The number of nonproduction employees was obtained by subtracting number of production employees from total number of employees. The manhours of nonproduction employees are not reported most likely because these nonproduction employees (supervisor and higher positions) are salaried. Since salaried employees are often paid on the basis of a 40 hour week, this assumption was made. It was also assumed that these employees work a 50 week year. The number of nonproduction employees was then multiplied by 2000 (40x50) to give manhours of nonproduction employees for that year. The production and nonproduction manhours were summed to give total manhours. Total manhours were then multiplied by the educational adjustment index published by Berndt and White (1978). This index adjusts manufacturing labor for the change in education and thus the quality of labor input.

For agriculture, total manhours used in the farm enterprise reported in AS was used. This series includes hired labor, family labor and operator labor.

The labor variable discussed above is in physical units, manhours. To convert this input variable to the same 1967 dollar units the other input variables are in, the variable was multiplied by wages in 1967. The units of the quantity of labor input variable in (5) were then in terms of cost in 1967 dollars. Notice in 1967 this quantity is equal to total cost of labor.

TCL: Total Cost of Labor

Total employee compensation was the total cost variable used for the manufacturing sectors. This series is published in the ASM.

For the agricultural sector, total cost was obtained by multiplying price of labor (described below) by the labor variable (described above).

The total cost of labor for the twenty manufacturing sectors and agriculture were deflated by the wholesale price index, 1967=100, to obtain total cost in constant 1967 dollars.

RPL: Real Price of Labor

The price of labor for the manufacturing sectors was obtained by dividing total cost of labor by the quantity of labor (in man-hours). The price of labor was then in terms of $/man-hour. To put this price into index form to make it comparable to the price of other inputs, the wage rate in each year was divided by the wage in 1967. This was done for each sector. This wage index was then deflated by the WPI, 1967 = 100; this gave the wage index in constant 1967 dollars.

For the agricultural sector, total cost of labor was computed as the opportunity cost of agricultural labor. Since the most likely alternative is employment in manufacturing, the opportunity cost used is the wage rate of production employees in manufacturing. This series was derived from the data published by the ASM and published in Berndt and White (1978). The opportunity cost was used because the only reported agricultural wage is that paid to hired workers. This wage does not reflect the wage of the farmer-operator or his family, which is substantially higher than the wage of hired workers. The reported agricultural wage also does not include other elements of employee compensation. This wage rate was put into index form by dividing wage in each period by the wage in 1967. This wage index was divided by the WPI, 1967=100, to give the wage index in constant 1967 dollars. The wage rate, $/man-hour, was multiplied by man-hours to give total cost of labor in agriculture which was discussed above.

E: Energy

For the manufacturing sectors the energy variable was derived from the total cost of energy and the energy price index. Both total cost of energy and the energy price index are described below. The energy types included in this variable include: coal, coke, residual fuel oil, distillate fuel oil, natural gas and electricity used for heat, light and power in manufacturing. The units of the quantity of energy variable are in cost in 1967 dollars.

For the agricultural sector, quantities of energy were estimated as follows:

The U.S. Department of Agriculture has published energy use in agriculture for 1974 (USDA, 1974). The USDA has also calculated energy use for 1975 and 1976. These latter figures were obtained through personal correspondence. The construction of the energy variable was based on several assumptions which were utilized by the USDA in constructing the 1974 energy data base. First the petroleum fuels and coal were linked to the number of acres harvested. Gallons of fuel per acre (or tons of coal/acre) in 1974 determined the amount of petroleum fuel used per acre for the time period 1947-1973. These liquid petroleum fuels are: gasoline, diesel fuel, fuel oil and LP gas. In addition, over the past twenty years, there has been a trend toward using diesel fuel in place of gasoline. This trend was accounted for in the construction of the variable in the following way. Percentages of diesel fuel and gasoline used in 1952-1961 were found in Farm Cost Situation (USDA, Nov. 1961). The relative percentage of diesel fuel and gasoline used in 1974 were calculated from the 1974 data. The trend was continued from 1961 to 1974 assuming a constant rate of growth. It was also assumed in 1947 the relative percentage of gasoline to diesel fuel was 99:1, (this ratio was 94:6 in 1952) from 1952 to 1947 a constant rate of decrease in diesel fuel usage was assumed. The gallons per acre obtained above for gasoline and diesel fuel were then altered by the relative percentages of these fuels used.

Another key assumption to the development of the energy data in agriculture was the linking of natural gas usage to acres irrigated. Natural gas use in agriculture is primarily for crop drying and irrigation. A correspondent at the USDA suggested that irrigated acres would give the best trend for natural gas usage. The 1974 value of cubic feet of natural gas/irrigated acre was obtained from the 1974 energy data. Irrigated acreage is a series given in AS. This ratio of cubic feet of natural gas/irrigated acre in 1974 was multiplied by the irrigated acreage in each year to obtain natural gas usage in that year.

Electricity data were available from the Rural Electrification Administration (REA) published in the AS. These data contain home use, so home use must be subtracted out. Home usage of electricity was reported as 12% of total usage in 1974 (the figure obtained for 1974 indicated moderate electricity use in the farm household). It was assumed that home consumption was a constant 12% of reported usage. This amount was subtracted from the amount reported by the REA.

The fuel estimates discussed are in the physical units in which they are most commonly purchased (e.g. gallons, cubic feet, tons or kwh.). If these energy data were to be aggregated to form a physical unit measure, these energy series would have to be converted to BTU's. However, all other inputs are on a dollar basis. To obtain a dollar measure, the physical energy units were multiplied by their respective prices and summed to find total cost of energy in agriculture.[4/] These physical energy units were also used in the calculation of a Lespeyres energy price index (described below). The total cost of energy was divided by the price index for energy to give the energy variable in terms of cost in 1967 dollars.

RPE: Real Price of Energy

A price index for energy in each manufacturing sector was not available. Therefore, a Lespeyres price index (1967=100) was constructed for each sector using energy prices reported in the <u>WPPI</u> put out by the Bureau of Labor Statistics. These prices were the same for all manufacturing sectors; quantities differed from sector to sector. Quantities are reported in the <u>ASM</u> in 5 year intervals. For the agricultural sector, the prices were taken from the prices paid by farmers reported in the <u>AS</u>; if unavailable in this source, the wholesale fuel price was used. This is reasonable because for those periods when the same fuel price was available in both sources, the difference was small.

The Lespeyres price index used quantity weights from 1967 in proportion to use in 1967. These price indexes were deflated by the WPI, (1967=100), putting the price of energy in constant 1967 dollars.

[4/] Berndt (1978) argues for an energy index based on price not energy content as price reflects energy content as well as other properties of the fuel.

TCE: Total Cost of Energy

The total cost of energy for use in heat, power and light in manufacturing used in this paper, is a series reported annually by the ASM.

The total cost of energy for agriculture was obtained by multiplying prices of the fuels reported in the AS (or if not available, the wholesale price of fuel, as discussed under the price of energy) by the physical units described above.

The total cost in current dollars in both manufacturing and agriculture was then deflated by the WPI, 1967=100, to give total cost of energy in 1967 dollars.

IM: Intermediate Materials

A measure of the quantity of intermediate materials was obtained by dividing the total cost of intermediate materials by the price index in manufacturing and farm sectors (described below). Intermediate materials in manufacturing include all raw, semifinished goods, parts, containers, scrap and supplies put into production for operation or repair. It excludes advertising, insurance, research, development, consulting services of other establishments and other overhead costs.

For agriculture, these intermediate materials include feed purchased, feeder livestock purchased, seed and fertilizer purchased and miscellaneous inputs.

TCM: Total Cost of Materials

Total cost of intermediate materials is published by the ASM for the 2-digit SIC code manufacturing sectors for 1960-1976. It is reported for 4-digit SIC codes for the remaining years. These 4-digit SIC code sectors were summed to form the 2-digit SIC code sectors. These materials costs include the cost of energy which was subtracted from the total materials cost.

For the agricultural sector, the cost of intermediate materials costs was obtained from expenses for intermediate materials of farmers. These expenditures are published annually in the AS.

These total costs of materials were originally in current dollars for all sectors. These total costs for intermediate materials were converted to constant 1967 dollars by dividing by the WPI, 1967=100.

RPIM: Real Price of Intermediate Materials

For the price of intermediate materials for manufacturing, the wholesale price index for intermediate materials, supplies and components was used. This price index is published annually by the WPPI. There existed only one of these wholesale price indexes for intermediate materials; this index was used for all sectors. It would have been preferable to have an individual price series for each sector, however, no such series were available.

For agriculture, the prices paid by farmers for all enterprise related commodities was used. This index includes elements of energy and capital, however, a comprehensive index with only the selected intermediate inputs does not exist. This series is published in AS annually.

It should be noted that all series obtained from the ASM are not available for 1948. These series include: value of shipments, gross book value, number of total employees, number of production employees, production manhours, employees compensation, total cost of energy and total cost of materials. The 1948 data were obtained by interpolation assuming a constant rate of growth between 1947 and 1949.

The units of all like variables are the same. That is, gross output and total cost are always in constant 1967 dollars. Price is always a price index in constant 1967 dollars. Quantity is always a dollar value, the actual units are cost in 1967 dollars. In variables where quantity originated in physical terms such as labor, these physical units (such as manhours) were changed to cost in 1967 dollars. This was done because all inputs are not available in physical units and it is useful to have all inputs in the same units when interpreting results. Total cost is in constant 1967 dollars.

Interested readers should see the thesis of the author for the data used in the estimation of the GBC and VAR models.

Appendix II. Impulse Response Function for Increase in the Price of Energy.

TABLE 1. DYNAMIC ELASTICITIES

PERCENTAGE IMPULSE RESPONSE TO ORTHOGONALIZED ONE PERCENT SHOCK IN

PRICE OF ENERGY,

FOOD AND KINDRED PRODUCTS.

STEP	RPE20	RY20	K20	RPK20	L20	RPL20	E20	IM20	RPIM20
0.	1.0000	.1708	.5125	-.0778	-.0638	.3063	-.6057	.0954	-.0114
1.	.1755	-.0012	.3954	-.1335	.0720	.0671	.1108	-.0585	-.0468
2.	-.2295	-.1274	.2544	.0944	-.1179	-.0320	.0674	-.0635	-.0816
3.	-.2305	-.1049	.2001	.1816	-.2105	.0143	-.0803	.0138	-.0974
4.	-.1330	-.0316	.1568	.1829	-.1813	.0514	-.1230	.0812	-.0892
5.	-.0378	.0212	.1015	.1569	-.1177	.0460	-.0968	.0996	-.0616
6.	.0387	.0412	.0528	.1203	-.0653	.0244	-.0529	.0834	-.0320
7.	.0844	.0385	.0185	.0871	-.0348	.0028	-.0161	.0580	-.0126
8.	.1022	.0254	-.0065	.0670	-.0243	-.0155	.0025	.0374	-.0045
9.	.1081	.0103	-.0278	.0584	-.0259	-.0305	.0022	.0239	-.0032

TABLE 2. DYNAMIC ELASTICITIES

PERCENTAGE IMPULSE RESPONSE TO ORTHOGONALIZED ONE PERCENT SHOCK IN

PRICE OF ENERGY,

TOBACCO PRODUCTS.

STEP	RPE21	RY21	K21	RPK21	L21	RPL21	E21	IM21	RPIM21
0.	1.0000	.3202	.4954	.5126	-.0879	.2489	-.6848	.4645	-.0566
1.	.6225	.1711	.3755	-.3031	-.0334	-.0696	-.0107	.3269	-.0088
2.	.6361	.1463	.2813	-1.0979	-.0272	-.1440	.2398	.2305	.0278
3.	.4649	.1043	.1580	-1.3980	-.0683	-.1182	.3213	.1679	.0035
4.	.1531	.0427	.0116	-1.1070	-.0859	-.0742	.3508	.1136	-.0404
5.	-.1083	-.0202	-.1218	-.4837	-.0733	-.0291	.3284	.0570	-.0747
6.	-.2348	-.0796	-.2211	.2046	-.0454	.0015	.2644	-.0065	-.0900
7.	-.2196	-.1300	-.2819	.7769	-.0152	.0071	.1826	-.0739	-.0851
8.	-.0967	-.1634	-.3096	1.1335	.0074	-.0085	.1006	-.1337	-.0652
9.	.0738	-.1758	-.3150	1.2554	.0167	-.0339	.0288	-.1729	-.0401

TABLE 3. DYNAMIC ELASTICITIES

PERCENTAGE IMPULSE RESPONSE TO ORTHOGONALIZED ONE PERCENT SHOCK IN

PRICE OF ENERGY

TEXTILE PRODUCTS

STEP	RPE22	RY22	K22	RPK22	L22	RPL22	E22	IM22	RPIM22
0.	1.0000	-.0775	.5792	-.0936	-.0349	.3023	-.4055	-.2577	-.0690
1.	.1526	.8083	.5046	.1360	.7667	.0769	.3533	.6655	-.0627
2.	-.1262	.3088	.3751	.1573	.4328	-.0121	.4450	.3225	-.0873
3.	.0134	.0194	.1982	.0612	.1976	-.0420	.4531	.0084	-.0274
4.	.1752	-.1348	.0202	-.0092	.0887	-.0767	.3789	-.1810	-.0061
5.	.3180	-.2002	-.1403	-.0646	.0335	-.1194	.2616	-.2475	.0031
6.	.4196	-.2199	-.2728	-.1267	.0144	-.1602	.1453	-.2548	.0050
7.	.4854	-.2276	-.3761	-.1952	.0135	-.1938	.0381	-.2476	.0012
8.	.5260	-.2337	-.4537	-.2647	.0228	-.2197	-.0590	-.2410	-.0052
9.	.5473	-.2416	-.5103	-.3298	.0376	-.2392	-.1460	-.2399	-.0121

TABLE 4. DYNAMIC ELASTICITIES

PERCENTAGE IMPULSE RESPONSE TO ORTHOGONALIZED ONE PERCENT SHOCK IN

PRICE OF ENERGY,

APPAREL AND OTHER TEXTILE PRODUCTS.

STEP	RPE23	RY23	K23	RPK23	L23	RPL23	E23	IM23	RPIM23
0.	1.0000	.2171	.2661	.0994	.1977	.2469	-1.0564	.0887	.0326
1.	.2533	.1040	.2049	.7146	.3319	-.0057	-.9503	-.1199	-.0623
2.	.0788	-.3312	-.1291	.7687	.1079	-.1778	-.2100	-.5507	-.0114
3.	.1358	-.3603	-.3466	.5168	-.0580	-.1917	.1706	-.4272	.0245
4.	.2248	-.2187	-.4160	.3134	-.0793	-.1666	.0651	-.1978	.0293
5.	.2989	-.1858	-.4190	.2390	-.0592	-.1715	-.2649	-.1769	.0214
6.	.3494	-.2682	-.4222	.2654	-.0573	-.2014	-.5196	-.3098	.0141
7.	.3851	-.3707	-.4502	.3403	-.0732	-.2338	-.6028	-.4510	.0104
8.	.4205	-.4410	-.5020	.4280	-.0901	-.2601	-.5825	-.5388	.0088
9.	.4657	-.4859	-.5707	.5169	-.1011	-.2851	-.5547	-.5920	.0073

TABLE 5. DYNAMIC ELASTICITIES

PERCENTAGE IMPULSE RESPONSE TO ORTHOGONALIZED ONE PERCENT SHOCK IN

PRICE OF ENERGY

LUMBER AND WOOD PRODUCTS

STEP	RPE24	RY24	K24	RPK24	L24	RPL24	E24	IM24	RPIM24
0.	1.0000	.0557	.3133	.2580	-.1870	.0440	-1.4696	.3160	.0051
1.	.6043	1.1636	.1503	.1204	.7317	-.2205	-.6401	1.2547	.0058
2.	.5623	.6502	-.2396	.7490	.7105	-.5341	-.5106	.7516	.0448
3.	.8537	-.1031	-.6670	.9814	.2880	-.6512	-.7223	-.0223	.0907
4.	1.1002	-.2651	-.9115	.7635	.1940	-.6633	-.9516	-.2387	.1079
5.	1.1591	-.1723	-1.0800	.5121	.3651	-.7456	-1.1964	-.1779	.1031
6.	1.1521	-.2910	-1.3046	.4163	.4631	-.9064	-1.4992	-.2969	.1010
7.	1.1874	-.6296	-1.5799	.3636	.3927	-1.0618	-1.8379	-.6301	.1084
8.	1.2476	-.9724	-1.8377	.2292	.2737	-1.1714	-2.1653	-.9768	.1166
9.	1.2741	-1.2272	-2.0481	.0202	.1958	-1.2533	-2.4598	-1.2401	.1186

TABLE 6. DYNAMIC ELASTICITIES

PERCENTAGE IMPULSE RESPONSE TO ORTHOGONALIZED ONE PERCENT SHOCK IN

PRICE OF ENERGY

FURNITURE AND FIXTURES

STEP	RPE25	RY25	K25	RPK25	L25	RPL25	E25	IM25	RPIM25
0.	1.0000	-.1766	.0985	.3350	-.2639	-.0355	-1.3382	-.1276	.1385
1.	.4160	.0873	-.0514	.0941	.0941	-.2700	-.5728	.1713	.0556
2.	.2017	-.3297	-.3194	.1735	-.1450	-.4272	-.2668	-.2569	.0701
3.	.4127	-.5942	-.4806	.1853	-.3777	-.3897	-.3864	-.5206	.0651
4.	.5837	-.5194	-.5903	.2136	-.3054	-.3705	-.4862	-.4308	.0511
5.	.5758	-.5570	-.7681	.3145	-.2822	-.4536	-.5058	-.4703	.0611
6.	.6015	-.7522	-.9660	.3508	-.4033	-.5320	-.6105	-.6739	.0833
7.	.7013	-.9016	-1.1313	.3324	-.5116	-.5665	-.7949	-.8167	.0959
8.	.7730	-.9821	-1.2844	.3387	-.5585	-.6033	-.9611	-.8824	.1039
9.	.7993	-1.0869	-1.4463	.3699	-.6174	-.6559	-1.1021	-.9776	.1166

TABLE 7. DYNAMIC ELASTICITIES:

PERCENTAGE IMPULSE RESPONSE TO ORTHOGONALIZED ONE PERCENT SHOCK IN

PRICE OF ENERGY,

PAPER AND ALLIED PRODUCTS.

STEP	RPE26	RY26	K26	RPK26	L26	RPL26	E26	IM26	RPIM26
0.	1.0000	-.1501	.1958	.1088	-.1554	.1085	-.0991	-.3702	.1448
1.	.3872	.0538	.2568	.2400	-.0758	.0996	.0306	.0244	.0335
2.	.1015	.2793	.2495	.1420	.1061	.0571	.2460	.2281	.0409
3.	-.0035	.2421	.2886	.1314	.0848	.0725	.2831	.1318	.0461
4.	-.0209	.2094	.3626	.1320	.0785	.1324	.2447	.1682	.0341
5.	-.0771	.2389	.4030	.1212	.1138	.1582	.2625	.2017	.0212
6.	-.1250	.2572	.4095	.1046	.1354	.1533	.2988	.2167	.0168
7.	-.1285	.2416	.4082	.0905	.1295	.1474	.3050	.1989	.0149
8.	-.1101	.2216	.4044	.0777	.1186	.1448	.2923	.1804	.0111
9.	-.0927	.2115	.3924	.0641	.1128	.1380	.2822	.1736	.0069

TABLE 8. DYNAMIC ELASTICITIES:

PERCENTAGE IMPULSE RESPONSE TO ORTHOGONALIZED ONE PERCENT SHOCK IN

PRICE OF ENERGY,

PRINTING AND PUBLISHING.

STEP	RPE27	RY27	K27	RPK27	L27	RPL27	E27	IM27	RPIM27
0.	1.0000	.1095	.3660	.2819	.0577	.2728	-1.5076	.1279	-.1217
1.	.1737	-.2774	.0044	.0248	.1122	-.1845	-.5249	-.7490	-.0609
2.	.0536	-.4750	-.1801	-.0869	.1498	-.4059	.2340	-.8591	-.0799
3.	.1677	-.5229	-.2949	-.0796	.0374	-.3783	.6154	-.7943	-.0797
4.	.2865	-.4860	-.4340	-.0393	-.0774	-.3122	.4539	-.6402	-.0851
5.	.3654	-.5273	-.6622	.0505	-.1609	-.3437	.0191	-.6184	-.0684
6.	.4640	-.6491	-.9324	.1400	-.2144	-.4548	-.4180	-.7105	-.0292
7.	.6077	-.7872	-1.1859	.2005	-.2561	-.5766	-.7380	-.8283	.0127
8.	.7769	-.9107	-1.4191	.2447	-.2988	-.6831	-.9667	-.9350	.0420
9.	.9526	-1.0387	-1.6692	.2995	-.3467	-.7947	-1.1731	-1.0534	.0595

TABLE 9. DYNAMIC ELASTICITIES;

PERCENTAGE IMPULSE RESPONSE TO ORTHOGONALIZED ONE PERCENT SHOCK IN

PRICE OF ENERGY,

CHEMICAL AND ALLIED PRODUCTS.

STEP	RPE28	RY28	K28	RPK28	L28	RPL28	E28	IM28	RPIM28
0.	1.0000	-.0966	.1931	.1837	-.2900	.2067	-.6565	-.2317	-.0142
1.	.2803	.4160	-.0271	-.1710	-.1370	.0039	-.0752	.5099	-.0513
2.	.0412	.3927	-.1904	-.5019	.0598	-.2447	.1480	.6458	-.0364
3.	.2189	.2087	-.1626	-.4351	.1005	-.2707	.0260	.5109	-.0340
4.	.4459	.1758	-.0892	-.1963	.0848	-.1840	-.0639	.4305	-.0529
5.	.5141	.2814	-.1178	-.0542	.1061	-.1757	.0646	.5081	-.0629
6.	.5336	.3712	-.2197	-.0131	.1520	-.2555	.2849	.6296	-.0516
7.	.6467	.3976	-.3069	.0510	.1752	-.3330	.4531	.7077	-.0341
8.	.8500	.4263	-.3638	.1787	.1682	-.3743	.5808	.7775	-.0236
9.	1.0742	.5088	-.4313	.3177	.1562	-.4169	.7459	.9034	-.0183

TABLE 10. DYNAMIC ELASTICITIES;

PERCENTAGE IMPULSE RESPONSE TO ORTHOGONALIZED ONE PERCENT SHOCK IN

PRICE OF ENERGY,

PETROLEUM AND COAL PRODUCTS.

STEP	RPE29	RY29	K29	RPK29	L29	RPL29	E29	IM29	RPIM29
0.	1.0000	.0534	.5122	2.2009	-.2741	.4488	-.7461	.1099	.0487
1.	.4441	.1867	.1272	2.1771	-.4421	.1183	-.6383	.2592	-.0161
2.	.2201	.5397	-.0663	1.3655	-.2374	-.0043	.2058	.5093	.0209
3.	.2084	.5955	-.0694	1.0415	-.0730	-.0050	.5396	.5565	.0024
4.	.2488	.6325	-.0639	1.1554	-.0695	-.0094	.4361	.6564	-.0169
5.	.2995	.7889	-.0946	1.1183	-.0442	-.0333	.4699	.8386	-.0078
6.	.3802	.9514	-.1114	1.0478	.0047	-.0472	.5407	1.0092	.0032
7.	.4792	1.0971	-.1133	1.0990	.0235	-.0536	.5429	1.1717	.0104
8.	.5793	1.2589	-.1157	1.2020	.0264	-.0603	.5268	1.3476	.0209
9.	.6766	1.4317	-.1165	1.3196	.0300	-.0648	.5372	1.5279	.0331

TABLE 11. DYNAMIC ELASTICITIES;

PERCENTAGE IMPULSE RESPONSE TO ORTHOGONALIZED ONE PERCENT SHOCK IN

PRICE OF ENERGY,

RUBBER, MISCELLANEOUS PLASTIC PRODUCTS.

STEP	RPE30	RY30	K30	RPK30	L30	RPL30	E30	IM30	RPIM30
0.	1.0000	-1.2966	-.2251	.5416	-1.1467	-.0299	-1.2283	-1.6541	.1079
1.	.7895	-.7098	-.3773	.2088	-.7638	-.1107	-.7015	-.9164	.0480
2.	.5527	-.3965	-.6815	.2596	-.5043	-.3036	-.3606	-.3387	.0746
3.	.5734	-.3594	-.8926	.3803	-.4116	-.3574	-.2800	-.1389	.0947
4.	.6783	-.3704	-1.0038	.3147	-.3292	-.3451	-.3437	-.1481	.0923
5.	.6737	-.4473	-1.0982	.1155	-.3238	-.3588	-.5083	-.2386	.0830
6.	.5985	-.6087	-1.1766	-.0736	-.4152	-.3694	-.7287	-.3856	.0653
7.	.5220	-.7869	-1.2199	-.2152	-.5303	-.3529	-.9418	-.5650	.0402
8.	.4456	-.9365	-1.2386	-.3267	-.6335	-.3241	-1.1204	-.7361	.0166
9.	.3622	-1.0523	-1.2448	-.4055	-.7258	-.2929	-1.2616	-.8736	-.0011

TABLE 12. DYNAMIC ELASTICITIES;

PERCENTAGE IMPULSE RESPONSE TO ORTHOGONALIZED ONE PERCENT SHOCK IN

PRICE OF ENERGY,

LEATHER, LEATHER PRODUCTS.

STEP	RPE31	RY31	K31	RPK31	L31	RPL31	E31	IM31	RPIM31
0.	1.0000	-.2830	.1514	-.2734	-.0023	-.1038	-1.1962	-.3136	-.0681
1.	.6518	-.0393	.0603	-.8289	-.0395	-.1799	-.4507	.0797	.0333
2.	.1504	.2129	.1293	-1.0671	.1518	-.1477	-.0482	.5367	-.1089
3.	-.0323	.0138	.0443	-.5671	.1175	-.2099	-.1461	.3616	-.1630
4.	.1487	-.1882	-.0484	-.3941	.0331	-.1915	-.1953	-.0357	-.0820
5.	.2381	-.1565	-.0614	-.6177	.0101	-.0968	-.1059	-.1139	-.0201
6.	.1095	-.0360	-.0436	-.6782	.0572	-.0478	-.0525	.0545	-.0372
7.	-.0159	-.0219	-.0579	-.4480	.0902	-.0749	-.0997	.1090	-.0563
8.	-.0064	-.0699	-.0774	-.2786	.0862	-.0940	-.1290	.0036	-.0277
9.	.0265	-.0533	-.0547	-.3304	.0864	-.0583	-.0695	-.0513	.0089

TABLE 13. DYNAMIC ELASTICITIES;

PERCENTAGE IMPULSE RESPONSE TO ORTHOGONALIZED ONE PERCENT SHOCK IN PRICE OF ENERGY,

STONE, CLAY AND GLASS PRODUCTS.

STEP	RPE32	RY32	K32	RPK32	L32	RPL32	E32	IM32	RPIM32
0.	1.0000	-.3325	.1494	-.0736	-.4085	.0376	-.6435	-.3264	-.0162
1.	.5435	.1610	.0025	-.4004	.0117	-.0312	-.2765	.2530	-.1121
2.	-.0116	.1241	-.3018	-.3059	.3445	-.2849	.2317	.3523	-.1359
3.	.1898	-.2999	-.6201	.0559	.0913	-.3579	-.0267	-.0548	-.1108
4.	.6448	-.3973	-.8134	.2135	-.1033	-.2998	-.3552	-.2923	-.0776
5.	.7580	-.3204	-.9646	.0553	-.0437	-.3260	-.3257	-.3424	-.0584
6.	.7098	-.4030	-1.1482	-.0579	-.0026	-.4398	-.2597	-.4284	-.0722
7.	.8114	-.5671	-1.3461	-.0040	-.0655	-.5337	-.3571	-.5549	-.0859
8.	1.0073	-.6587	-1.5400	.0443	-.1149	-.5958	-.4821	-.6466	-.0886
9.	1.1504	-.7164	-1.7512	.0116	-.1109	-.6767	-.5398	-.7180	-.0955

TABLE 14. DYNAMIC ELASTICITIES;

PERCENTAGE IMPULSE RESPONSE TO ORTHOGONALIZED ONE PERCENT SHOCK IN PRICE OF ENERGY,

PRIMARY METAL INDUSTRIES.

STEP	RPE33	RY33	K33	RPK33	L33	RPL33	E33	IM33	RPIM33
0.	1.0000	-.2820	.0649	.4667	-.7762	.2229	-.2009	-.2677	.1748
1.	1.3320	-.1400	-.0035	.0259	-.7199	.3783	-.6707	-.0257	.0593
2.	.8805	.5211	-.5066	-.3816	-.1470	.0244	.1495	.6423	.0478
3.	.7716	.4114	-.8598	-.6977	-.0141	-.3037	.6095	.5434	.0562
4.	.8467	-.0270	-.9817	-.8136	-.2043	-.4215	.5084	.2394	.0040
5.	.8695	-.2333	-1.1240	-.8469	-.3290	-.4740	.4607	.1537	-.0613
6.	.8458	-.2155	-1.3743	-.8225	-.2890	-.5685	.6724	.1852	-.0891
7.	.8603	-.2179	-1.6475	-.7191	-.2250	-.6827	.8922	.1417	-.0874
8.	.9257	-.3135	-1.8808	-.5727	-.2255	-.7716	.9488	.0152	-.0821
9.	1.0013	-.4268	-2.0879	-.4466	-.2598	-.8360	.9028	-.1141	-.0798

TABLE 15. DYNAMIC ELASTICITIES:

PERCENTAGE IMPULSE RESPONSE TO ORTHOGONALIZED ONE PERCENT SHOCK IN

PRICE OF ENERGY,

FABRICATED METAL PRODUCTS.

STEP	RPE34	RY34	K34	RPK34	L34	RPL34	E34	IM34	RPIM34
0.	1.0000	-.2975	.3944	-.5021	-.3270	-.0977	-.6142	-.1853	.0218
1.	.5483	.0234	.3167	-.8182	.0146	-.2359	-.1660	.0749	-.0671
2.	.3342	.1640	.1543	-.7854	.2996	-.4043	.2878	.2441	-.0662
3.	.3975	.0208	-.0599	-.5024	.2587	-.4640	.2555	.1022	-.0701
4.	.5959	-.1367	-.2589	-.2886	.1262	-.4776	.0428	-.0907	-.0514
5.	.7859	-.2172	-.4368	-.2079	.0554	-.5151	-.1193	-.2056	-.0216
6.	.9337	-.2744	-.6147	-.1850	.0359	-.5897	-.2199	-.2843	.0043
7.	1.0719	-.3606	-.8031	-.1659	.0130	-.6818	-.3230	-.3850	.0238
8.	1.2260	-.4784	-1.0016	-.1483	-.0321	-.7772	-.4587	-.5156	.0397
9.	1.3955	-.6105	-1.2093	-.1443	-.0878	-.8768	-.6184	-.6602	.0541

TABLE 16. DYNAMIC ELASTICITIES:

PERCENTAGE IMPULSE RESPONSE TO ORTHOGONALIZED ONE PERCENT SHOCK IN

PRICE OF ENERGY,

MACHINERY, EXCEPT ELECTRICAL.

STEP	RPE35	RY35	K35	RPK35	L35	RPL35	E35	IM35	RPIM35
0.	1.0000	-.4450	.3860	-.0438	-.8228	.2543	-.8022	-.6241	.0181
1.	.3409	.0648	.0813	-.5172	-.0856	-.1011	-.0532	-.0125	.0190
2.	.0498	.3628	-.0066	-.4429	.2562	-.1971	.1880	.4913	-.0072
3.	.0226	.4312	.0165	-.2181	.3377	-.1627	.2255	.6079	-.0317
4.	.0818	.3845	.0407	-.0645	.3256	-.1279	.2297	.5098	-.0381
5.	.1438	.3090	.0335	-.0066	.2908	-.1250	.2292	.3790	-.0334
6.	.1916	.2488	.0020	-.0021	.2602	-.1421	.2197	.2923	-.0273
7.	.2300	.2161	-.0403	-.0178	.2470	-.1656	.2044	.2522	-.0235
8.	.2636	.2044	-.0844	-.0409	.2521	-.1902	.1899	.2370	-.0215
9.	.2937	.2011	-.1260	-.0683	.2678	-.2146	.1788	.2279	-.0206

TABLE 17. DYNAMIC ELASTICITIES:

PERCENTAGE IMPULSE RESPONSE TO ORTHOGONALIZED ONE PERCENT SHOCK IN

PRICE OF ENERGY,

ELECTRIC, ELECTRONIC EQUIPMENT.

STEP	RPE36	RY36	K36	RPK36	L36	RPL36	E36	IM36	RPIM36
0.	1.0000	-.3825	.4028	-.7432	-.5444	.3123	-.8898	-.7517	-.0228
1.	.2510	-.2173	.2915	-.8144	-.1997	-.0971	-.5304	-.4976	-.0442
2.	-.2265	-.2750	.1734	-.3676	.1311	-.3138	-.1797	-.4473	-.0455
3.	-.1154	-.5892	.1234	.0703	-.1803	-.2245	-.2471	-.6828	-.0383
4.	.1469	-.7737	.0354	.1177	-.5895	-.1182	-.4310	-.8061	-.0374
5.	.2251	-.7137	-.1218	-.0722	-.6252	-.1539	-.4584	-.7192	-.0358
6.	.1766	-.5913	-.2752	-.2110	-.4134	-.2483	-.3881	-.6189	-.0251
7.	.1669	-.5596	-.3635	-.2280	-.2700	-.2893	-.3768	-.6391	-.0084
8.	.2229	-.6022	-.3943	-.2186	-.2028	-.2717	-.4499	-.7283	.0057
9.	.2719	-.6383	-.4048	-.2519	-.3421	-.2485	-.5314	-.7864	.0133

TABLE 18. DYNAMIC ELASTICITIES:

PERCENTAGE IMPULSE RESPONSE TO ORTHOGONALIZED ONE PERCENT SHOCK IN

PRICE OF ENERGY,

TRANSPORTATION EQUIPMENT.

STEP	RPE37	RY37	K37	RPK37	L37	RPL37	E37	IM37	RPIM37
0.	1.0000	-.0391	.1383	.0883	-.7623	.2025	-.7928	-.0477	.1074
1.	.3461	.2893	.0610	-1.2385	-.4820	.1128	-.3754	.5217	.0239
2.	-.0529	-.1504	-.2690	-.6819	-.3084	-.2020	-.3138	.2616	.0047
3.	.0138	-.3739	-.4593	.0695	-.3892	-.2679	-.4893	.0790	-.0090
4.	.1481	-.2336	-.4396	.1475	-.4218	-.1495	-.5590	.1381	-.0322
5.	.1174	-.0719	-.3490	-.0949	-.2978	-.0623	-.4652	.1732	-.0476
6.	-.0004	-.0638	-.2726	-.1977	-.1365	-.0546	-.3415	.0656	-.0465
7.	-.0844	-.1254	-.1983	-.1433	-.0424	-.0584	-.2687	-.0858	-.0366
8.	-.1227	-.1518	-.1030	-.1021	-.0119	-.0334	-.2295	-.1777	-.0274
9.	-.1575	-.1363	.0039	-.1309	-.0029	.0055	-.1901	-.2034	-.0215

TABLE 19. DYNAMIC ELASTICITIES:

PERCENTAGE IMPULSE RESPONSE TO ORTHOGONALIZED ONE PERCENT SHOCK IN

PRICE OF ENERGY,

INSTRUMENTS, RELATED PRODUCTS.

STEP	RPE38	RY38	K38	RPK38	L38	RPL38	E38	IM38	RPIM38
0.	1.0000	-.4506	.3785	-1.2808	-.7962	.1559	-.8310	-.7238	.0913
1.	.2969	-.1029	.3435	-1.4642	-.2879	.0062	-.3930	-.2525	-.0049
2.	-.0042	-.0657	.2728	-.8258	-.0317	-.1272	-.1289	-.0910	-.0415
3.	-.0057	-.1066	.2024	-.6774	-.0347	-.1485	-.0459	-.0433	-.0441
4.	.0867	-.0924	.1615	-.7417	-.0463	-.1207	-.0388	-.0075	-.0421
5.	.1373	-.0685	.1174	-.7665	-.0089	-.1205	-.0330	.0033	-.0368
6.	.1609	-.0690	.0608	-.7397	.0356	-.1431	-.0283	-.0115	-.0276
7.	.1901	-.0906	-.0030	-.7164	.0595	-.1655	-.0404	-.0408	-.0177
8.	.2259	-.1222	-.0698	-.7132	.0661	-.1824	-.0706	-.0775	-.0091
9.	.2588	-.1595	-.1390	-.7180	.0648	-.1974	-.1122	-.1198	-.0020

TABLE 20. DYNAMIC ELASTICITIES;

PERCENTAGE IMPULSE RESPONSE TO ORTHOGONALIZED ONE PERCENT SHOCK IN

PRICE OF ENERGY,

MISCELLANEOUS MANUFACTURING.

STEP	RPE39	RY39	K39	RPK39	L39	RPL39	E39	IM39	RPIM39
0.	1.0000	.1312	.4933	-.1779	-.3146	.3020	-1.0104	.0496	.1326
1.	.3929	-.6228	.5456	-1.2646	-.5672	.0546	-.9360	-.2325	.0998
2.	-.1002	-.0126	.4829	-1.2292	-.2481	.0567	-.6758	.2113	.0955
3.	-.2037	.5095	.4980	-.9779	.0319	.2166	-.2561	.8334	.1036
4.	-.1676	.8440	.6219	-1.0260	.2284	.4301	-.0936	1.2794	.0775
5.	-.2135	1.0286	.7741	-1.2032	.3575	.5699	-.0333	1.4407	.0404
6.	-.3259	1.1477	.8998	-1.2664	.4546	.6243	.1143	1.4635	.0156
7.	-.4120	1.2363	.9957	-1.1887	.5343	.6373	.3441	1.4693	.0033
8.	-.4390	1.2821	1.0711	-1.0542	.5887	.6341	.5629	1.4631	-.0049
9.	-.4285	1.2699	1.1242	-.9136	.6072	.6122	.7179	1.4101	-.0123

TABLE 21. DYNAMIC ELASTICITIES;

PERCENTAGE IMPULSE RESPONSE TO ORTHOGONALIZED ONE PERCENT SHOCK IN

PRICE OF ENERGY,

AGRICULTURE.

STEP	RPE40	RY40	K40	RPK40	L40	RPL40	E40	IM40	RPIM40
0.	1.0000	-2.1187	.6126	-.3611	-.2097	.1189	-.3520	-1.0856	-.4057
1.	.2059	-1.3379	.4635	-.1964	-.4368	.4282	-.7052	.4412	-.3977
2.	-.3917	-.8136	.4698	-.2600	-.5241	.6426	-.7163	.4132	-.2885
3.	-.5010	-.2483	.5482	-.2378	-.6094	.8129	-.5227	.5728	-.1593
4.	-.4568	.2604	.4409	-.0817	-.7083	.7760	-.4034	.9033	-.0806
5.	-.4199	.5835	.2737	.0193	-.7545	.5727	-.2767	.9488	-.0127
6.	-.3254	.7468	.2218	.0137	-.7277	.3438	-.0826	.8138	.0625
7.	-.1714	.7812	.2635	-.0203	-.6563	.1596	.0875	.6884	.1178
8.	-.0282	.6958	.3206	-.0329	-.5707	.0355	.1677	.5741	.1369
9.	.0641	.5287	.3683	-.0246	-.4903	-.0220	.1730	.4477	.1270

Appendix III. Factor Cost Shares for 21 Sectors.

Table 22. Factor Cost Shares for 21 Sectors of the U.S. Economy, 1947, 1957, 1967 and 1976.*

Sector No.	Sector Name	Year	Cost Shares			
			Capital K	Labor L	Energy E	Materials M
20	Food and Kindred Products	1947	.014	.148	.011	.826
		1957	.021	.174	.012	.794
		1967	.024	.145	.010	.821
		1976	.025	.115	.014	.845
21	Tobacco Products	1947	.011	.097	.003	.889
		1957	.020	.104	.003	.873
		1967	.030	.112	.004	.854
		1976	.095	.119	.009	.777
22	Textile Mill Products	1947	.014	.334	.020	.632
		1957	.030	.789	.019	.662
		1967	.040	.261	.017	.682
		1976	.038	.240	.032	.690
23	Apparel, Other Textile Products	1947	.006	.340	.004	.651
		1957	.008	.334	.005	.654
		1967	.012	.326	.006	.656
		1976	.016	.315	.008	.660
24	Lumber and Wood Products	1947	.027	.342	.017	.613
		1957	.050	.301	.019	.630
		1967	.061	.291	.021	.627
		1976	.073	.235	.025	.668
25	Furniture and Fixtures	1947	.024	.417	.011	.548
		1957	.018	.375	.010	.596
		1967	.024	.375	.010	.591
		1976	.026	.343	.015	.616
26	Paper and Allied Products	1947	.020	.230	.056	.714
		1957	.047	.259	.038	.656
		1967	.061	.265	.034	.640
		1976	.059	.211	.066	.664
27	Printing and Publishing	1947	.017	.520	.008	.455
		1957	.028	.488	.008	.477
		1967	.036	.470	.008	.486
		1976	.044	.433	.012	.511

Table 22. Factor Cost Shares for 21 Sectors of the U.S. Economy, 1947, 1957, 1967 and 1976. (cont.)

Sector No.	Sector Name	Year	Cost Shares			
			Capital K	Labor L	Energy E	Materials M
28	Chemicals and Allied Products	1947	.024	.233	.036	.707
		1957	.067	.255	.054	.625
		1967	.073	.236	.044	.647
		1976	.075	.173	.075	.677
29	Petroleum and Coal Products	1947	.019	.107	.014	.860
		1957	.029	.080	.019	.873
		1967	.050	.064	.024	.861
		1976	.035	.033	.028	.904
30	Rubber, Misc. Plastics Products	1947	.027	.338	.020	.615
		1957	.045	.308	.017	.630
		1967	.046	.339	.018	.597
		1976	.049	.783	.031	.636
31	Leather, Leather Products	1947	.006	.300	.007	.687
		1957	.012	.359	.009	.620
		1967	.019	.358	.008	.615
		1976	.020	.322	.013	.646
32	Stone, Clay and Glass Products	1947	.031	.457	.097	.415
		1957	.061	.380	.074	.485
		1967	.076	.355	.059	.511
		1976	.065	.313	.096	.526
33	Primary Metal Industries	1947	.021	.233	.085	.661
		1957	.055	.270	.060	.615
		1967	.067	.248	.041	.644
		1976	.059	.208	.066	.667
34	Fabricated Metal Products	1947	.020	.396	.016	.569
		1957	.028	.369	.014	.589
		1967	.032	.347	.013	.608
		1976	.038	.304	.019	.638
35	Machinery, except Electrical	1947	.021	.466	.014	.499
		1957	.038	.446	.013	.503
		1967	.049	.382	.010	.560
		1976	.057	.332	.013	.597

Table 22. Factor Cost Shares for 21 Sectors of the U.S. Economy, 1947, 1957, 1967 and 1976. (cont.)

Sector No.	Sector Name	Year	Cost Shares			
			Capital K	Labor L	Energy E	Materials M
36	Electric, Electronic Equipment	1947	.016	.467	.013	.504
		1957	.034	.431	.011	.525
		1967	.038	.385	.009	.569
		1976	.070	.344	.014	.572
37	Transportation Equipment	1947	.014	.307	.010	.669
		1957	.031	.379	.010	.581
		1967	.057	.257	.006	.679
		1976	.038	.226	.010	.725
38	Instruments, Related Products	1947	.012	.463	.009	.516
		1957	.037	.482	.009	.472
		1967	.070	.409	.008	.513
		1976	.030	.411	.014	.545
39	Miscellaneous Manufacturing	1947	.023	.495	.014	.468
		1957	.024	.504	.015	.456
		1967	.014	.430	.009	.548
		1976	.041	.343	.016	.600
40	Agriculture	1947	.036	.688	.055	.220
		1957	.086	.625	.055	.235
		1967	.128	.440	.046	.386
		1976	.159	.352	.051	.439

*The cost share for each input is defined to be $\frac{P_i X_i}{\Sigma P_i X_i}$ where i = K,L,E,M. The data used for P_i and X_i described in Appendix I were subsituted into this expression to obtain the cost share data in this table.