Feed-Livestock Relationships: A Model for Analyzing Management Decisions

By R. J. McConnen, C. M. McCorkle, Jr., and D. D. Caton

Economists studying the operation of individual firms have relied in large part on the general theory of production economics as presented in standard references (3) (6). Usually some changes are required to increase the usefulness of the theory in formulating realistic and testable hypotheses. Generally it is assumed that the goal of the firm is one of maximizing profits. The objective of this paper is to develop a general model especially helpful for analyzing the operation of firms that produce livestock products. This is based on certain modifications and elaborations of the conventional theory of production. For the purpose of illustrating the use of the general model, a California feeder-steer operation is used. The general model should be helpful in evaluating many other types of livestock operations. The major modification is the separation of the livestock operation into production stages. "The technical definition of a stage is a matter of both convenience and logic, depending on the importance of the elemental operations and the way in which they fit in with the flow of products and materials. . . . Thus, a stage consists of all productive services—durable or nondurable—that cooperate in performing a single operation or a group of minor but closely related operations" (4, p. 545).

Managers of firms base their decisions on simultaneous consideration of several stages. The three major production stages for livestock firms are (1) feed procurement, (2) feed consumption, and (3) feed conversion. These stages taken together form a sequence in production. More detailed stages can be defined for certain problems. For instance, in the analysis of a feedlot operation, feed procurement might be broken down into feed acquisition, feed storage, feed processing, and feed transportation within the feedlot. These operations may be sufficiently important to justify the explicit definitions of stages for these operations.

1. Italic numbers in parentheses refer to Literature Cited, p. 47.

Feed procurement, stage 1, consists of those operations necessary to acquire the feed for the livestock firm and to have the feed ready for consumption. The feed may be procured by buying or raising, or both. The decision of how much feed to raise, if any, should be based on a comparison of the marginal cost of production and the purchase price per unit. In order to determine the "best" of alternative methods of feed procurement, it may be necessary to complete the analysis of all three stages before the choice among alternatives can be made. For example, the way in which the forage is grazed (method of consumption) may affect the rate of forage growth (a method of feed procurement). When the alternative methods of feed procurement would result in feeds of like quality and timing, the method which will procure the feed at the lowest cost would be the "best."

Feed consumption, stage 2, comprises all those operations necessary in order for the livestock to consume the feed offered them. The type of grazing management, the type of feed containers, and the timing of the feed operations are major factors affecting the quantity of feed offered to the animal and actually consumed. In a feedlot operation the type of feeding facilities will influence the portion of the feed offered that is consumed. Any investment that increases the ratio of feed consumed to feed offered represents the substitution of capital for feed.

Feed conversion, stage 3, is the process of conversion of the feed consumed by the livestock into a livestock product. The efficiency with which livestock convert the consumed feed into a livestock product will depend on the inherent productivity of each of the individual animals and the past and present environment of the animals. For any class of livestock the most efficient animals
may not always be the most profitable. If the price differential is great enough, a manager may find it more profitable to buy less efficient animals. The livestock product may be a final product which is sold, such as slaughter cattle or market milk. The livestock product could be an intermediate product for the firm such as replacement heifers or feeder cattle. The resulting product could also be a joint product, such as a group of calves, part of which are sold and part held over as replacement stock.

**Application of the General Model**

The general model presented in figure 1 was used as the basis for evaluating range fertilization for a ranch located in the Coastal Range of California. The ranch of 1,008 acres is all rangeland except a small area used in the farmstead. It has adequate stock water and no unusual factors that restrict its use as a livestock ranch. Five land classes based on soil capability ratings, are defined for this ranch (table 1). The land is fenced into pastures which correspond roughly to the land classes. Therefore, different stocking rates for different land classes can be maintained and fertilizer responses by pastures can be determined.

Without range fertilization, the owner usually purchases 350 head of 300 to 400 pound weaner calves in early spring whenever the feed is ready to be grazed (between 15 February and 15 March). He grazes the calves as long as they continue to gain, usually 100 to 120 days. The average gain per head for this period is two pounds per day.

Stage 1 considers alternative quantities of feed offered the animals. The level of forage production per acre with no fertilization for each land class is based on estimates by range technicians and on the amount of livestock output produced. The forage response functions with fertilization were approximated on the basis of data from Soil Conservation Service range fertilization experiments conducted 15 miles from the ranch.3

<table>
<thead>
<tr>
<th>Land class</th>
<th>Area</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>46</td>
</tr>
<tr>
<td>2</td>
<td>86</td>
</tr>
<tr>
<td>3</td>
<td>505</td>
</tr>
<tr>
<td>4</td>
<td>348</td>
</tr>
<tr>
<td>5</td>
<td>23</td>
</tr>
<tr>
<td>Total</td>
<td>1,008</td>
</tr>
</tbody>
</table>

1 Recommendations for use of land class 4 indicate that this land class should not be fertilized. This is primarily because of the steep slopes associated with this class.
2 Area in this land class was assumed to be of no value for grazing purposes.

Since no alternative sources of feed are considered, the total feed offered to the animals in terms of dry matter can be varied only by varying the level of fertilization. In this case, the total product of Stage 1 can be expressed as:

\[
\sum_{i=1}^{4} (A_{c_i})Y_{ki} = \left(\sum_{i=1}^{4} (A_{c_i})F_{ki} + \sum_{i=1}^{4} (A_{c_i})f_{ki}\right)
\]

where \(A_{c_i}\) is the number of acres in the \(i\)-th land class and \(A_{c_i}\) is the number of acres fertilized in the \(i\)-th land class. In this case, the feed is offered to the livestock as standing forage.

Stage 2 comprises those operations in which the livestock consume the forage. It is generally accepted that not all the standing forage offered grazing livestock is consumed (5) (7) (10).

"The weight of clipped herbage gives an estimate of the total herbage offered to animals while the animal production gives an estimate of the value of the herbage eaten by the animals . . . ."

the levels of application of fertilizer. The approximated fertilizer response functions did not violate the experimental data and they seemed to give "reasonable" results except at very low levels of fertilization. The general net response function in terms of dry matter per acre for each land class is:

\[
F_{ki} = a_iX^{b_i}
\]

where \(X\) is the rate of fertilization (in increments of 100 pounds of 16-20-0 per acre). The individual parameters are: land class 1, \(f_{k1}=40 X^{.8}\); land class 2, \(f_{k2}=30 X^{.7}\); land class 3, \(f_{k3}=25 X^{.8}\). The total dry matter produced per acre for land class 1 is \(Y_{k1}=F_{k1}+F_{k2}\) where \(F_{k1}\) is the dry matter produced per acre with no fertilization, where \(F_{k1} = 1,800; F_{k2} = 1,400, F_{k3} = 1,000\) and \(F_{k4} = 703\).
The ratio of these two values is called the clipping-grazing ratio. German and Swiss agronomists often use this ratio as a criterion to judge the efficiency of pasture management systems. This is one measure of what Brown calls the difference between gross forage production and net forage production (1, p. 147).

The model used to represent the operation of stage 2 in this study is: 

\[ \hat{Y}_k = Y_k (1 - r) \]

where \( \hat{Y}_k \) is that amount of the standing forage per acre (\( Y_k \)) in terms of pounds of dry matter that is consumed by the livestock and \( r \) is the portion of the standing forage not consumed. It

\footnote{Five different consumption models were considered. One model \( \hat{Y}_k = Y_k \) assumes that all the feed offered livestock is consumed. This model is occasionally (and strongly) used by workers evaluating pasture improvements who assume all clipped forage will be consumed by livestock. The model \( \hat{Y}_k = Y_k - R \), where \( R \) is some fixed amount of forage left ungrazed, is often used where \( R \) is related to either conservation needs or cover required to insure adequate production in subsequent periods. The model \( \hat{Y}_k = Y_k (1 - r) \) was used for this paper. The model \( \hat{Y}_k = Y_k [1 - f(n)] \) recognizes that the portion of the standing forage not used, \( f(n) \), is a function of the stocking rate (\( n \)). The justification for this model is borne out by Gaalaas and Rogler (5). However, data in the present study were not adequate to make even a crude approximation of this function. The model \( \hat{Y}_k = Y_k [1 - f(n, t, w, \ldots)] \) considers the impact of stocking rate, topography, stock water availability, and other factors on the portion of standing forage not used. This model is perhaps the most realistic, but difficult to deal with in functional form. The impact of these factors on feed consumption is discussed in Stoddart and Smith (11, ch. IX, X, XI).}
was assumed, based on the findings of Reuss, that the portion of standing forage consumed was a constant of \(0.6 = (1-0.4)\) regardless of either the amount of forage growth or the level of grazing (10, p. 11).

In developing the model it was further assumed that (1) no interaction occurs between the different stages, (2) there is no grazing before the time the rangeland reaches grazing readiness, and (3) there is no residual response to fertilization. The simplifying assumptions were incorporated because of data limitations rather than because of any constraints imposed by the general model.

Stage 3 is concerned with the conversion into a livestock product of feed consumed. For the purposes of this paper, the livestock product is assumed to be a final product, all of which is sold.

Much research has been done to determine how feed is converted into a livestock product. Winchester and Hendricks (13) have developed the functional relationship of feed conversion to livestock product for beef calves:

\[ Z = 0.0553 \left( W^{2/3} \right) (1 + 0.805 \, g), \]

where \(Z\) is the daily energy intake in terms of pounds of total digestible nutrients, \(g\) is the average daily gain or loss in body weight, \(W\) is the average weight for the period being equal to \(W_i + \frac{D_g}{2}\) where \(W_i\) is the initial weight, and \(D\) is the number of days in the feeding period. The use of a single aggregate measure of feed value such as total digestible nutrients assumes away many of the problems of feed conversion. However, since only similar feeds are considered in this paper, and since these feeds provide a fairly well balanced ration during the time they are used, the dangers of using a single aggregate as an indicator of feed value are lessened.

The portion of feed offered that is consumed by the livestock is expressed in terms of pounds of dry matter, whereas the feed unit in the conversion function is expressed in terms of pounds of total digestible nutrients. On the basis of secondary information (8) and actual livestock production, it is estimated that 55 percent of the dry matter is total digestible nutrients. The percentage of total digestible nutrients is identified as \(K\) in the subsequent discussion. It is assumed that this percentage remains constant regardless of the level of fertilization.

It was assumed that all calves purchased will, on the average, perform in accordance with the specification \(Z = 0.0553 \left( W^{2/3} \right) (1 + 0.805 \, g)\).

Introducing stages into objective features of the firm.—Once the nature of the three operating stages has been defined, they can be brought together. It is assumed that the goal of maximizing profits can be satisfied by maximizing gross ranch profit (GRP) defined as total revenue (TR) minus total variable costs (TVC).

\[
\text{GRP} = \text{TR} - \text{TVC} \quad \text{where} \quad \text{TR} = (P \cdot Q) \cdot (N) \cdot (W_e) \quad \text{and} \quad \text{TVC} = P_1 \cdot N \cdot W_i + \sum_{i=1}^{4} [(A \cdot e_i) \cdot X_i \cdot P_x + (A \cdot e_i) \cdot A]
\]

where \(P_0\) is the price per pound received from the calves when sold, \(N\) is the number of calves (it is assumed there is no death loss and that \(N\) is a variable), \(W_e\) is the average ending weight, \(P_1\) is the initial price per pound paid for calves, \(W_i\) is the average initial weight. \(A \cdot e_i\) the number of acres fertilized in the \(i\)-th land class (a variable), \(X_i\) is the rate of fertilization in the \(i\)-th land class (a variable), \(P_x\) is the cost of fertilizer per hundred pounds and \(A\) is the application cost per acre which is assumed to be constant. Substituting relationships previously defined, gross ranch profit is defined as:

\[
\text{GRP} = P_0 \cdot N \left\{ \frac{[(1 - r) \cdot K] \cdot \left[ \sum_{i=1}^{4} F_{i1} \cdot (A \cdot e_i) + \sum_{i=1}^{4} a_i \cdot x^{b_1} \cdot (A \cdot e_i) \right]}{N \cdot D \cdot (a + a_k)} + \frac{D_g}{2} \right\}^{3/2}
\]

There is evidence that subjects this assumption to some question. The percentage of total digestible nutrients will also be affected by the method of utilization and the level of fertilization. However, adequate data were not available to indicate the nature of these relationships.

For the purposes of this paper, certain of the variables are assumed to have constant values as follows: \(P_0 = 25\) cents per pound, \(P_1 = 25\) cents per pound, \(W_i = 350\) pounds, \(P_x = \$4.50\) per hundredweight, and \(A = \$1\) per acre.
In this equation, $D$ (days in the grazing period) is equal to 110 for land that is not fertilized and to 130 for land that is fertilized. Since

$$W = \left[ \frac{Z}{a + akg} \right]^{3/2}$$

Therefore

$$W_e = W + \frac{D_g}{2} = \left[ \frac{Z}{a + akg} \right]^{3/2} + \frac{D_g}{2}$$

$Z$ equals the total pounds of consumed total digestible nutrients divided by the total number of calf days and can be expressed as:

$$Z = \frac{(1-r)K}{N D} \left[ \sum_{i=1}^{4} Y_{ki} A_{c1} \right]$$

$$= \frac{(1-r)K}{N D} \left[ \sum_{i=1}^{4} F_{ki} (A_{c1}) + \sum_{i=1}^{4} (a_i X^h) (A_{c1}) \right]$$

Where $A_{c1}$ is the number of acres in the $i$-th land class and $A_{c1}$ is the number of acres in the $i$-th land class that are fertilized. $\sum_{i=1}^{4} \lambda_i (A_{c1} - A_{c1})$ are constraints which say the acres in the $i$-th land class that are fertilized must be equal to the number of acres in that class.

**Choosing the “best” combination of inputs.—**

The optimum combination of inputs is defined as that combination which maximizes gross ranch profits consistent with the constraints imposed on the acreage in each land class. The inputs which were permitted to vary in this case are the number of calves, the average gain per day per head, and the level of fertilization for each land class. The usual next step would be to find the derivatives of the GRP with respect to those inputs which are allowed to vary and solve for the values of these inputs which would maximize the objective function. Certain secondary conditions would also have to be met. In this particular case, the calculus was not used to find the optimum values for the variable inputs. This occurred because some of the first order derivatives exist as fifth degree equations. It would have been possible to use a digital computer to approximate an optimum solution by numerical exploration of the model. However, this was not done. The GRP function was used to generate the gross ranch profit for a finite number of combinations of the inputs which were allowed to vary. These combinations of the inputs and the resulting gross ranch profits were used as a basis for defining the activities in a linear programming model.

The programming model, with gross ranch profit defined as the objective function, was used as a device for choosing the “best” of input combinations which were considered. Because the inputs were assumed to be variable with infinite density in the GRP function, it would be possible to find the optimum combination of inputs if the calculus could be used. In the programming model, only a finite number of combinations of inputs were considered. Therefore, the programming solution can only be regarded as an approximation of the optimum solution and is referred to as the “best” rather than optimum combination of inputs.

As the programming model used (table 2) is of standard design, it will not be explained in great detail. Activities $P_1$ through $P_5$ are disposal activities. Activity $P_6$ indicates that one acre of land class 1 is fertilized at the rate of 200 pounds of 16-20-0, stocked at the rate of 1.7 head per acre, requires $159 working capital to purchase the needed cattle and to purchase and apply the fertilizer, results in an average gain per head per day of 1.4 pounds, and gives $72 in gross ranch profits. Activities $P_7$ through $P_{21}$ are constructed in the same way as $P_6$. The details of the activities are given in table 3. A working capital constraint of $50,000 was used.

A portion of the final programming tableau is presented in table 4. The items in the $P_1$ column indicate which activities occur in the final program. The values in the $P_6$ column indicate the levels at which the activities in the final program are operated. The $Z_i$ element in the $P_6$ column

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1 Linear programming studies have been used previously to analyze the problem of integrating crop and livestock enterprise. For example, see Peterson (9, pp. 546-554) and Swanson (12, pp. 1240-1259).
(\$24,625). The \(Z_1-C_1\) values listed under the disposal activities \(P_1\) through \(P_5\) indicate the marginal value of product of the four land classes and working capital. There were \$8,573 of the \$50,000 available working capital which were not used. Therefore, the marginal value of product of added working capital is zero. Only land classes 1 and 2 would be fertilized and at the heavier rate, in the final program. The final program called for the heavier rate of gain and the lower stocking rate. The gross ranch profit with no fertilization would be \$19,363 and require \$30,069 in working capital.

### Conclusion

The schematic diagram of three stages outlined in figure 1 was used as a basis for developing a model of a particular livestock firm. Only three inputs, the number of livestock, the average daily gain, and the level of fertilization per acre for three land classes were allowed to vary. The "best" of alternative combinations of these three inputs were chosen with the use of a programming model.

Maximization of the GRP function was not a straightforward exercise in the calculus. The breakdown of the operations of the livestock firm into the three operating stages was the basis for conceptualizing the problem. While this type of conceptualization is of much greater value to the researcher than to the actual manager, it does provide a clearer understanding of the important relationships between variables affecting the profits of a livestock operation. Managers of livestock firms may benefit directly as the result of improved research into the economic structure of their operations. Alternative combinations of the variables considered significant can be synthesized. Combinations can be used that are both different and more numerous than those alternative combinations found in either operating livestock firms or experimental work. The development of the three stages, and their subsequent integration suggests areas in which experimentation may have considerable economic relevance.

In the development of the three stages, it was necessary to make numerous assumptions about the nature of the stages and the nature of the relationship between them because of lack of adequate data. When the assumptions become uncomfortable for the researcher to live with (in this study, many of them were of this nature), a plea must be made for more knowledge. It was found, however, that much more useful information was available than is often utilized in economic studies involving physical production problems of this kind. For instance, in the illustration used, the form in which experimental data on range fertilization was made available to the rancher would not permit a systematic consideration of the impact of different levels of stocking and fertilization on profits. In order to accomplish this, considerable secondary information must be combined with the experimental data in an appropriate economic model.

A simple livestock firm and a limited number of alternative input combinations were used to demonstrate the use of a schematic model. Addi
TABLE 4.—A portion of the final programming tableau for a California range feeder-steer operation

<table>
<thead>
<tr>
<th>Activity</th>
<th>Rate of stocking per acre (n)</th>
<th>Gain per head per day (g)</th>
<th>Rate of fertilization (X)</th>
</tr>
</thead>
<tbody>
<tr>
<td>P_6</td>
<td>1.70</td>
<td>1.4</td>
<td>200</td>
</tr>
<tr>
<td>P_7</td>
<td>.99</td>
<td>1.4</td>
<td>200</td>
</tr>
<tr>
<td>P_8</td>
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<td>1.4</td>
<td>200</td>
</tr>
<tr>
<td>P_9</td>
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<td>2.0</td>
<td>200</td>
</tr>
<tr>
<td>P_10</td>
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<tr>
<td>P_11</td>
<td>.37</td>
<td>2.0</td>
<td>200</td>
</tr>
<tr>
<td>P_12</td>
<td>.47</td>
<td>1.4</td>
<td>400</td>
</tr>
<tr>
<td>P_13</td>
<td>1.26</td>
<td>1.4</td>
<td>400</td>
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<td>1.4</td>
<td>400</td>
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<td>P_15</td>
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<td>400</td>
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<td>P_16</td>
<td>.97</td>
<td>2.0</td>
<td>400</td>
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<td>P_17</td>
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<td>2.0</td>
<td>400</td>
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<td>P_18</td>
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</tr>
<tr>
<td>P_20</td>
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<td>400</td>
</tr>
<tr>
<td>P_21</td>
<td>.31</td>
<td>2.0</td>
<td>400</td>
</tr>
</tbody>
</table>

The same technique could be useful in analyzing dairy operations, feed lot operations and "hog factory" operations. In some of these cases it may be helpful to specify more detailed stages. Certain research projects may deal with either specific stages or with the entire sequence. Since any livestock firm consists of a series of operating stages, ignoring either their existence or their interaction results too often in the solutions of many managerial problems by implicitly assuming them away.

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