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Are Yearly Variations in Crop Yield Really Random?

By Richard J. Foote and Louis H. Bean

The study reported here is part of a research project, under the Research and Marketing Act of 1946, entitled Anticipating Year-to-Year Changes in Market Supplies Due to Changes in Yields Per Acre. One of the purposes is to learn whether the available records on crop yields per acre contain variations from year to year, or patterns of variations over periods of years, that might be usable in anticipating changes in per acre yields for a year or more in advance. Louis H. Bean here restates his views on the evidence of trends and patterns in crop yields and weather and as a first step in determining the reality of such trends and patterns, Richard J. Foote reports on the application of selected statistical tests to the fluctuations in corn yields and in a constructed series, concluding with a suggestion for a more appropriate statistical test.

Evidence of Trends and Patterns in Crop Yields and Weather 1

I N CROP YIELDS AND WEATHER 2 I pointed to the existence of both weather and cropyield patterns "that persist between one decade and another, between alternate decades or even longer intervals." A number of illustrations were presented "as preliminary evidence of the existence of patterns or yield variations over a period of several years that tend to be repeated 9 to perhaps 11 years later and that undoubtedly are caused by combinations of weather factors that also tend to take on pattern variations over several seasons, to be repeated 9 to 11 years later. The individual patterns in yearly yields and in seasonal weather may embrace 3 or 4 years, or as many as 10, and possibly more. These longer patterns, both by months and seasons, are here emphasized because of the greater improbability that they represent merely chance variations. If they are not due to chance variations but to terrestrial or extraterrestrial weather-making forces, it is highly important that these forces be surmised and discovered for otherwise successful season-to-season weather and crop forecasting will not be possible."

"The search for methods of long-range seasonto-season weather and crop forecasting is being stimulated by the U. S. Department of Agriculture, the Weather Bureau now in the Department of Commerce, and other agencies. Progress to date has been recorded in the 1941 yearbook of the Department of Agriculture.³ As yet there is no direct approach known to scientists in this field of forecasting enabling them to forecast for more than a few days in advance." Writing today we might possibly change the last few words of this statement to "a few weeks in advance."

This conclusion followed: "Meteorologists believe the most rapid and certain progress can be made by studying atmospheric phenomena directly, and discovering and measuring the physical factors that cause both persistent tendencies and changes in weather over long as well as short periods. The approach presented in this publication is statistical, but its object is the same as that of the physical studies of the meteorologists—to hasten the day when season-to-season long-range weather and crop-yield forecasting will be possible. If statistics can demonstrate the existence of weather patterns and can determine what they are, that in itself will be a long step toward the solution of the ultimate problem."

Since reporting on these findings, I have probed more extensively and have observed additional features with regard to the repetitive character of both trends and patterns in crop and weather data, here and abroad. Much of this probing has been done as a hobby without benefit of an official research project and is, therefore, not ready for formal presentation. But these recent probings confirm the observations made during the 1930's and the research suggestions offered in 1942. I would now repeat them with greater emphasis.

¹ By Louis H. Bean.

² BEAN, LOUIS H. CROP YIELDS AND WEATHER. U. S. Dept. Agr. Misc. Pub. 471. 1942. pp. 1-5.

³ United States Department of Agriculture. CLIMATE AND MAN. Yearbook of Agriculture. 1941.

In spite of the fact that we live in a universe of law and order, fluctuations in both weather and crop yields, whether short- or long-range, are almost universally looked upon as matters of chance. Practically all statistical studies that have raised the question of regularity in fluctuations in crops and weather conclude negatively; that is, they find that fluctuations of crops and weather are essentially similar to what might be expected in series of random numbers.

My observations with regard to the existence of trends and patterns in both monthly and yearly data make it difficult to accept this common attitude. In fact I doubt that the commonly used statistical tests of whether a time series is random are adequate for our problem.

This doubt is fortified by the probing we asked Richard Foote to undertake and the results given in the preliminary report beginning on this page. He tested, as others have done, whether the fluctuations in the yield per acre of corn in the United States conform to what might be expected of a random series. He finds, as others have done, that they do. He then applies the same tests to a constructed series that contains several repetitions of a trend, a 13-year pattern, and an extraneous element added at 10-year intervals. The constructed series contained 80 observations, to correspond to the yearly range of available crop and weather data —say 1870 to 1950. Foote finds that this series, too, according to the available tests, does not differ significantly from what one might expect of a random series.

Obviously we need other tests that will be more efficient in separating that which is random from that which is orderly. To indicate the nature of the tests that seem to be called for, Foote performed an additional analysis of fluctuations in corn yields. On the basis of my manipulations of that series, in which a graphic procedure for isolating trends and patterns in four contiguous periods is used, he finds the six correlation coefficients that are possible with four series not statistically significant. (They range between 0.08 and 0.63.) But after the elimination of flexible trends, separately determined for each of the four periods, he reports that the resulting correlation coefficients range between 0.78 and 0.94—all "highly significant based on the usual tables." Obviously the significance of these repeating long-range patterns

depends on the flexibility and "reasonableness" of the trends. The trends themselves that emerge in the graphic search for correlated patterns also show a high degree of similarity. Foote concludes that the additional tests we need should deal with the significance of correlations between observed patterns after allowing for the characteristics of the trends that are used to reveal them.

Application of Certain Statistical Tests to Corn Yields, 1866-1949 4

Frequency Distributions

The first step in this work was to ascertain whether corn yields are normally distributed.

Table 1 shows the frequency distributions by classes for average corn yields in the United States as reported, 1866-1940, and for specified States after adjustment has been made for estimated effects of the use of hybrid seed, 1866-1943. In each case, data were tabulated for four subperiods of approximately equal length. Except for a slight rising trend in some areas, the distribution for the subperiods did not differ significantly from the distribution for the entire period. It appeared that combining the subperiods would result in no great loss of information. In some instances, data in this table have been condensed from the original tabulations.

Table 1.—All Corn: Frequency distributions for yields per harvested acre, by class intervals, United States and specified States

Class interval	United States ¹	Illi- nois	Iowa	Wis- consin	Mis- souri
Bushels/acre	Frequency				
5.0- 9.9		District of			2
10.0-14.9			and the Paris	With water	1
15.0-19.9	4		2	1	6
20.0-24.9	21	8	1	6	9
25.0-29.9	46	13	5	12	33
30.0-34.9	4	14	15	26	22
35.0-39.9		24	23	34	5
40.0-44.9		16	24		
45.0-49.9	ALTERNATION OF THE PARTY OF THE	3	7		
50.0-54.9			1		

¹ When a narrower class interval is used, the United States series resembles more closely the State distributions.

Corn yields appear to be highly skewed. Extremely low values occur fairly frequently, but high yields are generally not much above the class interval that contains the highest frequency. This indicates that, in these areas, weather conditions

⁴ By Richard J. Foote.

generally are either close to optimum or that some favorable factors are practically always prestat. Occasionally, however, conditions are such that yields are cut very materially.

Because the yield distribution appears to be nonnormal, all of the tests used were non-parametric; that is, they make no assumption about the parameter values of the frequency distribution from which the sample is drawn.

Tests Dealing with Departures from Randomness

A. APPLICATION OF THE THEORY OF RUNS.—The "theory of runs" provides a method for testing the hypothesis that a set of time-ordered observations constitutes a random sequence. The test discussed here has been taken from Hoel.⁵ In this test, the positive deviations from a trend line or from the mean are designated by "a" and the negative deviations by "b," and the number of runs or consecutive sequences of a's and b's are counted. As the length of the runs increases, the total number of runs decreases. By computing the probability of obtaining a given number of runs or less, we obtain an index of the randomness of the sequence. The computations are greatly simplified if a table giving logarithms of N! is used. See Grant⁶ for such a table).

Two tests were applied to corn yields in the United States. The first was based on deviations from a 9-year moving average, using data for the original series for 1866-1949. The second was applied to actual data for the period 1866-1940. Use of actual data after 1940 appeared to be inadvisable because of the sharp upward trend in yields. Use of deviations from trend can be justified since we are not attempting to learn whether a trend exists but rather whether the deviations as such are random. The yield series, together with the moving average, is shown in the upper part of figure 1. These tests gave probabilities of 0.075 and 0.091, respectively. The results indicate that if repeated samples of this size were drawn from a random series, we would expect to get as few runs as shown by the corn series 7.5 and 9.1 percent of the time, respectively. If the 5-percent

point is taken as the level of significance, corn yields do not differ significantly from a random series based on these tests. However, the results are close enough to the 5-percent point to warrant additional investigations.

B. APPLICATION OF THE THEORY OF PHASE LENGTH.

—Wallis and Moore⁷ developed a test based on the number of phases of various lengths expected in a random series. A phase is defined as the interval between consecutive turning points. This test is not sensitive to trend; that is, it may classify a series as random even though there is a persistent trend or long-term oscillations. For this reason, the test is not conclusive in indicating that a series does not differ significantly from a random one; but if non-significant results are obtained, it does indicate that non-random factors are of secondary importance.

The only restriction placed by this test on the nature of the probability distribution is that the probability of two consecutive observations being identical be infinitesimal. This will not necessarily hold for crop yields as published. A method of adjusting for "ties," (that is, consecutive items having the same value) is shown in their paper. This test was applied to corn yields in the United States for the period 1866-1949. The following results were obtained:

Phase dura- tion—Years	Observed Frequency	Expected Frequency	(Observed- expected) ²
			0.09
1	35	33.3	.88
2	18	14.4	.08
3 or more	5	5.2	
			$\chi_{p}^{2} = 1.05$

This yields a probability of 0.61. Thus, if we drew repeated samples from a random series, we would expect to get, 61 percent of the time, a divergence as large as that shown for yields of corn. Therefore, based on this test, we have no reason for assuming that corn yields are non-random.

Timoshenko⁸ has suggested that small ripples be eliminated before this test is applied. He suggests eliminating all "cycles" for which the maximum

⁵ Hoel, Paul G. introduction to mathematical statistics. New York. 1947. pp. 177-183.

⁶ Grant, Eugene L. Statistical quality control. New York. 1946. pp. 547-551.

⁷ Wallis, W. Allen, and Moore, Geoffrey H. A significance test for time series. Natl. Bur. Econ. Research. Tech. Paper 1, 1941.

⁸ TIMOSHENKO, V. P. VARIABILITY IN WHEAT YIELDS: PART I. CYCLES OR RANDOM FLUCTUATIONS? Food Research Inst. Wheat Studies. 18:291-377. 1942.

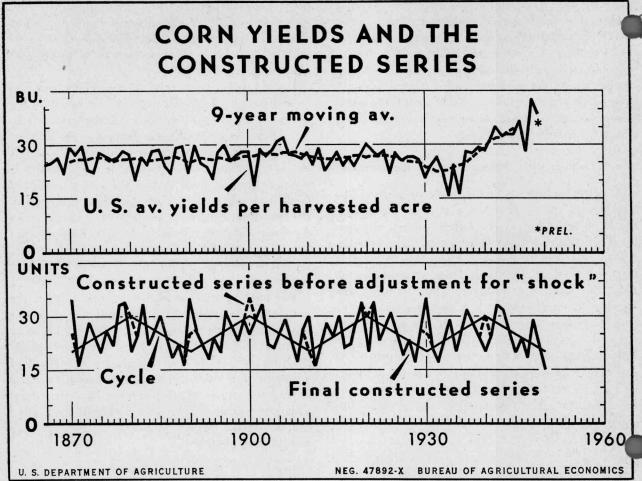


FIGURE 1.

between two successive minima is less than $\frac{1}{2}$ standard deviation above both minima. When this was done for the corn series; $P\left(\chi_p^2\right)$ was reduced to 0.36. But this is not valid since some of the expected phases are only of "ripple" magnitude.

These two analyses yielded average phase lengths of 1.48 and 1.65 years, respectively. The expected average duration of phases in a random series is 1.49 years.

Timoshenko also computes the average amplitude of cycles, but this appears to be mainly of value in comparing various series. He gives no tests of significance. Kendall⁹ indicates that methods based on the study of oscillations in the original series are not satisfactory in ascertaining the nature of an

oscillatory system. The use of a correlogram, discussed below, gives more reliable results, at least in those cases in which a sufficiently long series is available.

Attributes of a Random Series

For a random series, the serial correlation coefficient \mathbf{r}_1 for first differences should equal -0.5 and the correlation between $\mathbf{x}_{i+1} - \mathbf{x}_i$ and \mathbf{x}_i should equal -0.71. Scatter diagrams indicate that these are approximately true for yields of corn.

Autoregressive Series and the Correlogram¹⁰

Two types of oscillations in time series have

⁹ Kendall, M. G. contributions to the study of oscillatory time-series. Natl. Inst. Econ. and Social Research, England. Occasional Papers IX, 76 pp. Cambridge. 1946.

¹⁰ For a more detailed discussion of this subject see FOOTE, RICHARD J. THE STATISTICAL ANALYSIS OF CYCLES OR OSCILLATIONS IN TIME SERIES. Bur. Agr. Econ. November, 1950.

received wide attention in the literature: (1) haronic or periodic and (2) autoregressive.

Harmonic cycles are characterized by regularity in time; that is, their peaks and troughs recur at regular intervals. A series made up of a number of harmonics of varying lengths may give a jagged appearance, particularly if large superposed errors are present, but the separate cycles can be readily unscrambled by use of a periodogram, provided sufficient data are available.

Autoregressive series are not "cycles" in the strict sense of the word, since their peaks and troughs do not recur at regular intervals. They are characterized by frequent variations in both period and amplitude. In many cases, however, fluctuations around the up-and-down movement for any given oscillation are fairly smooth. Autoregressive series can be represented by nonperiodic mathematical models and can be analyzed by means of a correlogram.

Because of the unsatisfactory results obtained when periodograms were applied to economic, meteorological, and agricultural data, Yule11 developed a new concept known as an "autoregressive series." In such a series, the assumption is made that, once a disturbance has occurred, certain cyclical processes are set in motion which will ontinue, subject to a large or small degree of damping, until upset by another disturbance. Yule gave as his initial example a freely swinging pendulum at which boys throw peas. It is immaterial to the theory whether the pendulum is initially in motion or at rest. In the economic field, a similar example is perhaps given by the advance in coffee prices late in 1949. A rumor of severe frost damage in Brazil caused these prices to advance. With the price rise, dealers and consumers began to accumulate stocks, which caused a further advance in price. Eventually, stocks appeared to be large enough, buying slowed down or stopped altogether, and prices began to decline. The extent of the initial decline would depend on the degree of actual or estimated damage to the crop, the extent to which dealers and consumers decided to reduce their inventories, and any changes in other price-making factors which had occurred since the

11 Yule, G. Udney. On a method of investigating periodicities in disturbed series, with special reference to wolfer's sunspot numbers. Royal Soc. London Phil. Trans. A226, pp. 267-298. 1927.

date of the original rumor. Several highly damped oscillations might occur before prices reached equilibrium.

A correlogram can be used to test whether systematic oscillations of either the harmonic or the autoregressive type prevail in a time series. If in a time series, \mathbf{r}_1 is defined as the simple correlation between the original data and the same data lagged by 1 time unit, \mathbf{r}_2 is the simple correlation between the original data and the same lagged by 2 time units, and, in general, \mathbf{r}_k is the simple correlation between the original data and the same lagged by k time units, then the correlogram consists of the \mathbf{r}_k plotted against k.

A test for systematic oscillations is not the same as a test for randomness, since a non-random series is not necessarily oscillatory. On the other hand, one of the characteristics of a random series is that the serial correlation coefficients be zero except for sampling variations.

Rohloff¹² computed a correlogram for corn yields in the United States, using data for the period 1866-1947. He used two sets of data: (a) deviations from a third-degree polynomial and (b) deviations from a 9-year moving average. Neither gave any evidence of an oscillatory pattern or of significant deviations of the serial correlation coefficients from zero. Examination of the fit of the polynomial indicated that the sharp rise in yields since 1940 was primarily responsible for the significant reduction in deviations by the use of the third-degree term. The polynomial gave a poor fit during most of the entire period. Because of this, some experimental work was done with the corn-yield data for the period 1866-1940 with no adjustment made for trend. This also yielded unsatisfactory results. Hence, for the computations discussed below, deviations from the 9-year moving average from 1870 through 1945 were used.

The Autoregression Equation

Autoregressive series can be fitted by equations of the following type:

 $X_t = -a_1 X_{t-1} - a_2 X_{t-2} - \ldots - a_p X_{t-p} - a_0 + \epsilon_t$, where ϵ_t represents the disturbance or shock. This is called an autoregression equation, since the

¹² ROHLOFF, A. C. STATIONARY TIME SERIES TECHNIQUES APPLIED TO U. S. CORN YIELDS, 1866-1947. Unpublished manuscript. 1949.

value of X at time t depends mainly on previous values of the same series.

If the analyst has reason to think that he is dealing with an autoregressive series, Kendall¹³ suggests the following method for determining the number of terms needed in the equation. Consider the lagged values of the original series as the variables X1, X2, X3 . . . up to X5, say. Let X0 be the original series. Then X_1 represents X_0 lagged by 1 year, etc. (Xo corresponds to Xt in the above equation, X_1 to X_{t-1} , etc.) Compute the following simple and partial correlation coefficients: ro1, ro2.1, ro3.12, ro4.123, ro5.1234. Also compute the following quantities: $1 - R^2_{0.1} = 1 \mathbf{r}^{2}_{01}$, 1 - $\mathbf{R}^{2}_{0.12}$, 1 - $\mathbf{R}^{2}_{0.123}$, 1 - $\mathbf{R}^{2}_{0.1234}$, 1 -R²₀₋₁₂₃₄₅. At some point the partial correlation coefficients will approach zero except for sampling errors, and the $1 - R^2$ will become relatively constant. If this point is reached at r_{03·12} and 1 $-R^{2}_{0.123}$, for example, then a 2-term equation is enough.

Despite the fact that the correlogram gave no evidence of an oscillatory pattern, these values were computed for the corn-yield series, using deviations from the 9-year moving average. The following results were obtained:

Order of partial r	Value of partial r	Value of 1— R ²	
01	- 0.065	0.9958	
02.1	1478	.9741	
03.12	2035	.9338	
04.123	2247	.8866	
05.1234	184	.8566	

These computations indicate that even a 5-term autoregression equation would "explain" less than 15 percent of the variation in the original series. This is in contrast to the sample which Kendall cites—namely, sheep numbers in England—for which a 2-term equation would explain 75 percent of the variation. If repeated samples of this size were drawn from a population for which the true correlation was zero, multiple correlations of the size obtained for the corn-yield series would be expected to occur slightly more than 5 percent of the time. So they are not significant, according to the usual criterion. The partial correlations

also do not differ significantly from zero. These results confirm the evidence given by the correl gram that significant oscillatory patterns do not prevail in the United States corn-yield series. But they do not rule out the possibility of the existence of other types of repeating patterns.

Analysis of Other Types of Patterns in Time Series

Under an earlier research project in the Bureau of Agricultural Economics, some work was done on the statistical analysis of patterns in time series which cannot be represented by any simple type of mathematical equation. Some of the patterns found in yield series are similar to the patterns formed by the combination of a number of harmonic series of regular length, but they do not reoccur with sufficient regularity over the entire series to indicate a compound harmonic type. Sanderson¹⁴ discussed the probability of obtaining such patterns from random series under various assumptions. In the two test cases which he considers in detail. based on first differences and actual data, respectively, of yields of corn in the United States, 1866-1938, the extent to which patterns were repetitive did not differ significantly from what could have been expected if the yield series were random.

Application of These Tests to a Constructed Series

The fact that these tests indicated no significant deviations from randomness for yields of corn is, of course, no definite proof that the corn-yield series is a random one. In general, tests of significance can merely indicate whether a given series does or does not differ significantly from a certain test series, in this case a random one. The analyst can never prove that the given series is random. In view of this fact, it is sometimes useful to apply tests of significance to constructed series to see whether the results differ from what might be expected before the tests were applied. A series was constructed, based on a regular pattern superimposed on a regular cycle, with a "shock effect" added to cover the cycle partially. The pattern used is similar to those discussed by Bean for yields of specified crops. The lower part of figure

¹³ KENDALL, M. G. OSCILLATORY MOVEMENTS IN ENGLISH AGRICULTURE. Royal Statis. Soc. Jour. Pt. II, 106, pp. 92-117. 1943.

¹⁴ SANDERSON, FRED H. RESEARCH PROJECT ON "REPEATED PATTERNS" IN WEATHER AND CROP SERIES. Unpublished manuscript. 1940.

1 shows the final series, together with the cycle. The observations have been labeled as specific years facilitate plotting them along with the corn yields; but such labels are purely arbitrary. As this series would be essentially predictable, given the cycle and the superimposed pattern, it is, by definition, non-random.

Application of the theory of runs to the constructed series gave a probability of 0.089, about the same as for the corn-yield series.

Application of the theory of phase length gave the following results:

Phase dura- tion—Years	Observed Frequency	Expected Frequency	(Observed- expected) ²
1	26	32.5	1.3
2	21	14.1	3.4
3 or more	4	5.0	.2
			$\chi_{\rm p}^2 = 4.9$

As with the corn-yield series, fewer 1-year phases and more 2-year phases than expected were found; the number of phases of 3 or more years were about equal to expectations. A probability of 0.09 was given, materially less than the 0.61 obtained for yields of corn, but still not significant if the 5-percent point is taken as the acceptance level.

The above indicates that, although based on cerin commonly used tests, the yields of corn do not deviate significantly from a random series, they also probably would not deviate significantly from a series of the type used in the constructed example. Thus it appears that these tests are not sensitive in distinguishing between these two types of series—that is, a random one and one based on a composite of a regular cycle plus regular patterns.¹⁵

New Test of Significance Needed

In analyzing crop yields for repeating patterns of the type discussed by Bean, it frequently is necessary to fit rather flexible trends to bring out the patterns. If such trends are fitted in such a

way as to make the repeating patterns, based on deviations from them, as nearly alike as possible, the usual tests of significance for the correlations between patterns in two or more different periods do not apply. The following method would yield a valid test under such circumstances. ¹⁶ The method is outlined in mathematical terms in the next paragraph but is discussed on a less mathematical basis in the paragraph following that.

Proposed Test: Let the yield series be represented by a set of x_{ij} , where i indicates the period and j indicates the year within the period. Let the r_{ik} represent a set of simple correlations of the data in each period with the corresponding data in each of the other periods. If there are m periods, there will be m(m-1) such correlations. We

shall now fit sets of yij such that either the minimum absolute value of the rik or the average absolute value of the rik when based on xij - yij is as great as possible. The yij will represent the trend values for each period and could be fitted with or without restrictions.¹⁷ By determining the expected distribution of the rik when the xij are random and independently distributed, a test of significance for such correlations would be obtained. Allowance might also need to be made for the fact that the number and length of the periods and the starting dates all are chosen in such a way as to make the rik as large as possible. The mathematical derivation of this distribution will be complicated by the fact that yield series, in general, have a skewed rather than a normal distribution and the test should be applicable to such series. If the trends found in the yield series are themselves correlated from period to period, this might form a desirable restriction in fitting the yii, so that a more sensitive test would be obtained.

In non-mathematical terms, this test involves the following: Suppose we have data covering 80 years and we believe that patterns, in terms of deviations from certain trends, repeat at 11-year intervals. If the first pattern starts within the first 3 years of data, we would have 7 such complete repe-

¹⁵ The computation of either a correlogram or a periodogram for the constructed series probably would have revealed the cycle used and possibly the shock effect. But trend factors which are assumed to prevail in the crop-yield series are of a non-oscillatory nature and thus were not revealed by the correlogram. The nature of the trend factors for the yield series is indicated in somewhat more detail in the next section of this paper.

¹⁶ This method was suggested by Glenn Burrows, Bureau of Agricultural Economics.

¹⁷ It should be noted that this problem differs from the usual one of fitting trends or polynomials where the best fit for the data as such is desired.

titions. If we correlated the first 11 years of data, in terms of deviations from trend, with the second 11 years of data, the third, the fourth, etc.; and the second 11 years of data with the third, the fourth, the fifth, etc.; using all possible combinations; we would have 21 such correlations, each based on 11 observations. The trends could be fitted either mathematically or graphically. As major interest is centered on fitting the trends in such a way as to bring out the repeating patterns, it probably would be easier to fit them graphically than mathematically. But if we wish to obtain the expected mathematical distribution for such correlations, the trends would have to be fitted by mathematical methods. The use of orthogonal polynomials might well prove useful. After working out the expected distribution for such correlations, we would have a means of testing the significance of the 21 correlation coefficients after allowing for the characteristics of the trends that were used to obtain the deviations from trend which are actually correlated. It is probable that considerable time would be required to develop this distribution. Only after it had been developed would we know how much error is involved in using the standard tests of significance when dealing with correlations based on deviations from trends fitted in such a way as to maximize the correlations.

Conclusions

Based on several standard tests, yields of corn, after allowing for trend, give no evidence of departing significantly from a random series. However, using the same tests, a constructed series based essentially on a regular pattern superimposed on a regular cycle also gave no evidence of significant departures from a random series. The pattern used for the constructed series resembles those which Bean believes he has found in cropyield series. Thus, the standard tests do not appear to be sensitive in distinguishing between a random series and one made up of repeating patterns of the type used in the constructed series. An improved test is outlined, but to work out the details substantial mathematical work would be required.

The authors and certain other statisticians are convinced that the evidence which Bean has accumulated so far warrants additional work in this field. Even if the chance of success is relatively slight, all possibilities should be followed up as even moderately improved success in forecasting crop yields would bring great economic benefit.

Mimeographed indexes for volumes 1 and 2 are now available upon request.