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## Staff Papers Series

A PRODUCER'S WILLINGNESS TO PAY FOR INFORMATION UNDER PRICE UNCERTATNTY: THEORY AND APPLICATION

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## A PRODUCER'S WILLINGNESS TO PAY FOR INFORMATION

UNDER PRICE UNCERTAINTY: THEORY AND APPLICATION

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# A PRODUCER'S WILLINGNESS TO PAY FOR INFORMATION UNDER PRICE UNCERTAINTY: THEORY AND APPLICATION 

ABSTRACT

The theory of the competitive firm under price uncertainty is used to develop a money metric of a producer's willingness to pay for additional information. For a restricted class of utility functions, empirical estimates of the money metric using secondary data can be derived from the firm's risk averse supply or factor demand function. The procedure is illustrated by an application to an agricultural market.

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## I. INTRODUCTION

The central focus of this paper is the development of an easily computable money metric of an agent's willingness to pay for information under risk. Empirical estimates of the value of information are important for obtaining insights into issues such as the informational efficiency of alternative market structures, the effects of the quality of agent's conditional forecasts of market prices on the efficiency of resource use and the social profitability of information supplied by private enterprise and public agencies.

The paper draws on previous contributions to the theory of competitive firm under price uncertainty, namely Rothschild and Stiglitz (1971), Sandmo (1971), and more recently Pope (1978, 1980) and Pope, Chavas and Just (1983). The latter contribution provides insight into the econometric application of the theory and into the validity of producer surplus measures of firm welfare under risk. In these models, the production decision is made given the producer's subjective distribution of output price.

The value of information in this context can be formulated using the Bayesian approach which amounts to a comparison of expected utility levels from choices based on prior information with choices based on additional information. Contributions in this area are numerous and include those of Lindley (1971), Winkler (1972) and more recently Gould (1974) and Hess (1982) who focused on the effects of risk preferences and the nature of the distribution of random events on the value of information.

However, the literature has given little attention to the question of deriving a money metric of an agent's willingness to pay for information under risk that can be applied to an econometric analysis of observed behavior. In part, the problem lies with the empirical application of Bayes' theorem to
explain the observed behavior of risk averse agents. While estimates of the agent's prior distribution of uncertain events may be obtained from observed choices, or in some cases elicited directly, the content of additional information, the process by which additional information becomes avallable and whether the agent behaves as though the prior is updated according to Bayes' theorem raises major difficulties in applying the theorem to estimating the value of information from observed behavior.

The approach presented in this paper is easier to use in empirical application even though it bears a strong resemblance to the Bayesian procedure. For a restricted class of utility functions, it's shown that the money metric of an agent's willingness to pay for additional information can be computed from the firm's risk averse supply or factor demand function.

The problem is specified in section II of the paper and the measures of willingness to pay for additional information are presented in section III. To illustrate the approach, an analytical model is specified in section IV and the results from fitting it to time series data from the U.S. fed cattle industry are reported in section $V$. The empirical results suggest that producers are risk averse and that the bimonthly mean value of information to a typical producer varies from a deflated 12 cents per cwt to 46 cents per cwt over the 1970-80 period depending on the amount of additional information.

## II. THE PROBLEM

The competitive firm under price uncertainty is described in a Bernoullian framework where the agent's expected utility function is a strictly concave, continuous and differentiable function of profits. In this case the primal-dual function can be expressed as

$$
\begin{equation*}
L^{*}=\operatorname{EU}\left[P q^{*}-C\left(q^{*}\right)\right]-\operatorname{EU}[P q-C(q)] \tag{1}
\end{equation*}
$$

where the first and second bracketed terms are the indirect and direct expected utility function respectively, $P$ is stochastic output price, $C(q)$ is the cost function and $E$ is the expectation operator. The first order equation for a minimum is the familiar condition

$$
\begin{equation*}
\frac{\partial L^{*}}{\partial q}=-E\left[U^{\prime}(\pi)\left(P-C^{\prime}(q)\right)\right]=0 \tag{2}
\end{equation*}
$$

where $U^{\prime}(\pi)=d U / d \pi$ and $C^{\prime}(q)$ is positive and continuous.
To describe the different output choices that occur when the agent's distribution of output price is based on different sets of information and to facilitate the derivation of various measures of the value of information, two states of information are defined: the subjective and the more informed state.

In the case of the subjective state, let $f^{\circ}(p)$ denote the agent's prior distribution of output price based on the information available at the time the output decision is made. The optimal quantity of output can be determined by using equation (2) where the expectation, denoted $E^{\circ}$, is taken with respect to $f^{\circ}(p)$. The agent's optimal output choice is represented by $q^{0}$. However, prior to the realization of output price, profit remains a stochastic variable:

$$
\pi^{0}=P q^{0}-C\left(q^{0}\right)
$$

The utility that the agent expects to obtain from producing $q^{0}$ is $E^{0} U\left(\pi^{0}\right)$.
The more informed state is a situation where the agent's beliefs are based on more information than in the subjective state. Let $f^{m}(p)$ denote this more informed distribution on the stochastic variable $P$ which, while not fundamental to our approach, can be viewed as having the properties of a Bayesian posterior distribution obtained from updating $f^{\circ}(p)$ with additional
data such as an independently supplied price forecast. The optimal output choice in the more informed state, denoted by $q^{m}$, can again be determined by equation (2) with expectations, $E^{m}$, taken with respect to the more informed distribution. Prior to realization of the output price, profit is a stochastic variable represented by

$$
\pi^{m}=P q^{m}-C\left(q^{m}\right)
$$

The expected utility in the more informed state is $E^{m} U\left(\pi^{m}\right)$.
The problem is to derive an easily computable money metric of the agent's willingness to pay for the additional information embodied in the more informed distribution $f^{m}(p)$.

## III. VALUE OF INFORMATION

Three different measures of the value of information are presented. The first two are ex-ante measures. In this case, decisions made based on information embodied in the prior $f^{\circ}(p)$ are compared with those made in the more informed state with information embodied in $f^{m}(p)$. The third measure is a special case of the first two measures; it is a measure of the value of perfect information, determined by comparing realized profits from the choice $\mathrm{q}^{0}$ with profits obtained when price is known with certainty.

## Ex-ante Measures

Since the more informed distribution is defined as more descriptive of the stochastic variable $P$ than the prior distribution, a willingness to pay measure of the value of information can be derived by determining what it would be worth to the agent to know the more informed distribution rather than the subjective distribution.

For the first ex-ante measure, denoted by $\mathrm{VI}_{1}$, the maximization of $\mathrm{E}^{\mathrm{o}} \mathrm{U}(\pi)$ yields the optimal quantity, $\mathrm{q}^{\circ}$, produced in the subjective state with corresponding expected utility $E^{0} U\left(\pi^{0}\right)$. However, the expected utility of the choice $q^{\circ}$ in the more informed state is $E^{m} U\left(\pi^{0}\right)=E^{m}\left[U\left(P q^{0}-C\left(q^{0}\right)\right]\right.$. Hence, the value of information can be defined to be the difference in the more informed state between the expected utility of producing $q^{m}$ and the expected utility of producing $q^{\circ}$ :

$$
\begin{equation*}
V I_{1}=E^{m_{U}\left(\pi^{m}\right)-E^{m_{U}}\left(\pi^{0}\right) .} \tag{3}
\end{equation*}
$$

It can be shown that $\mathrm{VI}_{1}$ will always be non-negative. By derivation of quantity $q^{m}$, it is clear that $q^{m}=q^{*}$ in equation ( 1 ) when expectations are taken with respect to $f^{m}(p)$. By definition of the primal-dual problem, $\operatorname{EU}\left(\pi^{*}\right)$ is the maximum value of expected utility that can be attained over all possible values of profit. Thus,

$$
L^{*}=\operatorname{EU}\left(\pi^{*}\right)-E U(\pi) \geq 0
$$

When expectations are taken with respect to $f^{m}(p)$, the expression for $L^{*}$ becomes

$$
E^{m_{U}\left(\pi^{m}\right)-E^{m} U(\pi) \geq 0}
$$

and hence, $\mathrm{VI}_{1}$ is non-negative.
This measure of the value of information, however, is not very useful because utility has only ordinal properties. To avoid this problem a money metric similar to equivalent variation in the certainty case can be derived.

To facilitate the development of this measure, consider the simple case when $f^{\mathbb{m}}(p)$ has only two parameters: a mean and variance. In Figure 1 , the meanvariance ( $E, V$ ) space has been given for the more informed state where $O A B$ is the mean-variance frontier of response possibilities, and the $U^{m}$ curves represent isoutility where $\bar{U}_{1}^{m}>\bar{U}_{2}^{m}>\bar{U}_{3}^{m}$. Let point $A$ correspond to the

Figure 1.

optimal output level in the more informed state, $q^{m}$, and the random variable profits, $\pi^{m}$. Let decisions made in the subjective state lead to production $q^{\circ}$. Then the point corresponding to production level $q^{0}$ must lie on or below the mean-variance frontier $O A B$ because this curve represents the set of all efficient output levels in the more informed state. Suppose that output level $q^{0}$ can be represented by point $C$ which by necessity, lies on a lower isoutility curve $\mathrm{U}_{2}^{\mathrm{m}}$. Let $\mathrm{VI}_{2}$ be the amount of monetary payment that must be given to an agent who produces $q^{0}$ so that his expected utility in the more informed state would have been the same as if he had produced $q$. $\mathrm{VI}_{2}$ is a monetary (as opposed to utility) measure of the value of information. It is illustrated by the distance on the vertical axis between points $C$ and $D$.

Stated in general terms, define a nonstochastic variable $\mathrm{VI}_{2}$ such that

$$
\begin{equation*}
E^{m_{U}}\left(\pi^{m}\right)=E^{m_{U( }}\left(\pi^{o}+V I_{2}\right) . \underline{1 /} \tag{4}
\end{equation*}
$$

To show that $\mathrm{VI}_{2}$ is non-negative, recall that $U^{\prime}(\pi)>0$ implies $U\left(\pi_{1}\right)>$ $U\left(\pi_{2}\right)$, if $\pi_{1}>\pi_{2}$. Since it has already been shown from the primal-dual problem that $E^{m} U\left(\pi^{m}\right) \geq E^{m} U\left(\pi^{0}\right)$, then by equation (4), $E^{m} U\left(\pi^{0}+V I_{2}\right)>E^{m} U\left(\pi^{0}\right)$. By definition of expectations,

$$
\begin{equation*}
\int U\left(\pi^{o}+V I_{2}\right) f^{m}(p) d p>\int U\left(\pi^{o}\right) f^{m}(p) d p \tag{5}
\end{equation*}
$$

But by the properties of integrals, expression (5) implies $U\left(\pi^{\circ}+V I_{2}\right) \geq U\left(\pi^{0}\right)$ for all p. Since $U^{\prime}(\pi)>0, \pi^{0}+V I_{2} \geq \pi^{0}$. And hence, $V I_{2}$ is non-negative.

The empirical advantage of this approach lies in the ease of obtaining a money metric of the value of having the additional information embodied in $f^{m}(p)$. In general, knowledge of the agent's utility function and $f^{m}(p)$ are required to compute the value of information. However, knowlege of the initial beliefs $f^{\circ}(p)$ are not required. Estimates of $f^{m}(p)$ may come
about through public or private price forecasts or research that yields insights into factors determining the distribution of P. Given knowledge of the agent's utility function, our measure of the value of information becomes a key input into determining the social or private profitability of efforts to supply agents with the knowledge embodied in $f^{m}(p)$.

## Ex Ante Measures for a Class of Utility Functions

The usefulness of this approach becomes more apparent if the expected utility function is restricted to a member of the following class:

$$
\begin{equation*}
\mathrm{EU}=\mathrm{E} \mathrm{\pi}+\mathrm{g}(\mathrm{q}, \sigma) ; \stackrel{\rightarrow}{\sigma}=\left(\sigma_{2}, \sigma_{3}, \ldots, \sigma_{\mathrm{k}}\right) \tag{6}
\end{equation*}
$$

where $\sigma_{k}$ represents the $k t h$ central moment of price. It has been shown by Pope and others that the indirect expected utility function corresponding to (6) is related to the risk averse supply function as follows

$$
\begin{equation*}
\frac{\partial E U\left(\pi^{*}\right)}{\partial E P}=q^{*} \tag{7}
\end{equation*}
$$

Pope, Chavas, and Just show that if equation (7) holds, producer surplus, given by the area behind the risk averse supply curve, is a money metric of utility.

To derive an explicit expression for $\mathrm{VI}_{2}$, the supply function in the more informed state can be stated as $q^{m}=q\left(p, \sigma^{m}\right)$. Then, $V I_{2}$ is given by

$$
\begin{equation*}
\mathrm{VI}_{2}=\int_{\mathrm{p}_{\mathrm{q}} \mathrm{o}}^{\overline{\mathrm{p}}^{\mathrm{m}}} \mathrm{q}\left(\mathrm{p}, \mathrm{o}^{\mathrm{m}}\right) \mathrm{dp}-\mathrm{q}^{0} \cdot\left(\overline{\mathrm{p}}^{\mathrm{m}}-\mathrm{p}_{\mathrm{q}^{\circ}}\right) \tag{8}
\end{equation*}
$$

where the lower limit of integration is the value of $p_{q}$ o satisfying the expression $q^{0}=q\left(p_{q}, \vec{\sigma}^{m}\right)$. To show that this condition is the money metric $\mathrm{VI}_{2}$, it follows from (6) that expanding (8) yields

$$
\begin{align*}
V I_{2}= & \bar{p}^{-m} q^{m}-C\left(q^{m}\right)-g\left(q^{m}, \sigma^{\frac{m}{m}}\right) \\
& -\bar{p}^{-q^{0}}+C\left(q^{0}\right)+g\left(q^{0}, \sigma^{m}\right) \tag{9}
\end{align*}
$$

which is precisely condition (4) when expected utility is of the form (6).2/
This result is depicted in Figure 2. If the agent's prior beliefs are such that $\mathrm{q}^{0}$ is observed, then (9) is given by the triangular area a. Area $b$ depicts the value of (9) when the agent's beliefs are such that $q^{\prime}$ is observed. Empirical estimates of these values for the fed cattle industry appear in a later section of this paper.

Measures of the Value of Perfect Information
A special case of the measure developed above is the measure of the value of perfect information. Note that the choice $q^{\circ}$ which maximizes expected utility based on the prior $f^{\circ}(p)$, yields, ex-post, the realized profits given by $\pi^{r}=p^{r} q^{\circ}-C\left(q^{0}\right)$ where $p^{r}$ is the realized price. If the price $p^{r}$ were known before the choice $q^{\circ}$ is made, i.e., if the agent had perfect information, then utility is maximized when the choice $q^{*}$ maxmizes profits, i.e., $\pi^{*}=$ $p^{r} q^{*}-C\left(q^{*}\right)$. Corresponding to (5), the value of perfect information is given by

$$
\begin{equation*}
V I_{3}=\pi^{*}-\pi^{r} \tag{10}
\end{equation*}
$$

where it follows from the primal-dual problem, that (10) cannot be negative. The graphical analysis of this measure is similar to Figure 2 except the risk averse supply (factor demand) function is replaced by the traditional supply (factor demand) function.

The usefulness of this approach now becomes apparent. Even though $U(\pi)$ is generally not known, $q^{0}$ and $\pi^{r}$ are observable; and $\pi^{*}$ can be estimated.

$$
-10-
$$

Figure 2


In this case, if the establishment of a forward market is being contemplated or if consideration is being given to a policy of announcing the price of output at the time production decisions are made, our procedure gives insights into output response and changes in profits in a rather straightforward manner.
III. EMPIRICAL FRAMEWORK

Equation (6) does not provide much insights into the functional form of the indirect utility function because $\pi$ depends on, among other factors, the underlying production function. For notational convenience, let $V$ denote the form of the indirect utility function. The procedure employed here is to approximate $V$ by a second order Taylor series expansion.

Let the parameters of $V$ be represented as the vector $\vec{W}=\left(\mathrm{p}_{1}, \overline{\mathrm{p}}, \vec{\sigma}\right)$. $\rightarrow$ where $p_{1}$ is a vector of variable input prices. When all the parameters have been normalized around their mean values, expanding $V$ around $\vec{W}=\overrightarrow{0}$ yields:
. $V=\vec{V}(0)+\sum_{i=1}^{\ell} \frac{\partial \overrightarrow{V(0)}}{\partial W_{i}} W_{i}+1 / 2 \sum_{i=1}^{\ell} \sum_{j=1}^{\ell} \frac{\partial^{2} V(0)}{\partial W_{i} \partial W_{j}} W_{i} W_{j}+$ higher order terms.
Truncate the expression at the second order and substitute the following terms:
$\stackrel{\rightarrow}{V(0)}=\alpha_{0} ; \frac{\partial \stackrel{+}{V(0)}}{\partial W_{i}}=\alpha_{i} ; \frac{\partial^{2} V(0)}{\partial W_{i} \partial W_{j}}=\beta_{i j}$.
Hence,
$\overrightarrow{V(W)} \simeq \vec{V}(W)=\alpha_{0}+\sum_{i=1}^{\ell} \alpha_{i} W_{i}+1 / 2 \sum_{i=1}^{\ell} \sum_{j=1}^{\ell} \beta_{i j} W_{i} W_{j}$.
By Young's theorem, there is symmetry between cross partial derivatives.
Thus, $\beta_{i j}=\beta_{j 1}$. Let $W_{i}=p_{1 i}$ where $1=1,2 \ldots, n ; W_{n+1}=\bar{p} ;$ and $W_{s+n}=\sigma$ where $s=2,3, \ldots, k$. From the partial derivatives of (12) with respect to $W$

$$
\begin{equation*}
\frac{\partial V^{\sim}}{\partial W_{n+1}}=\alpha_{n+1}+\sum_{j=1}^{\ell} \beta_{n+1, j} W_{j} \tag{13}
\end{equation*}
$$

By condition (7) equation (13) can be expressed as
$q^{*}=\alpha_{n+1}+\sum_{j=1}^{\ell} \beta_{n+1, j} W_{j}+R$
where $R$ is a residual due to the truncation of the Taylor series at the second order.

A final difficulty remains before (14) can be fit to data. The subjective variables $\bar{p}$ and $\sigma$ are not observable. Hence, an auxiliary model must be formulated as an analogue of producers' forecasts to obtain instruments for $\bar{p}$ and $\sigma$. Our procedure, which is briefly discussed in the next section, is to use the $\bar{p}$ and $\sigma$ forecasts given by an ARIMA ( $2,1,0$ ) model.
IV. EMPIRICAL RESULTS

Aggregate bimonthly data on cattle slaughter for the period from the second bimonth of 1970 to the fifth bimonth of 1980 were used to estimate the supply equation (14). The input prices Included feeder cattle, corn and soybean meal. The mean and variance of bimonthly fed cattle price were used as the relevant moments of the aggregate subjective distribution of fed cattle price. Bessler examined the problem of obtaining estimates of the moments of aggregate subjective distributions and found that the ARIMA model gave the best estimates. Hence, estimates of the agents' conditional forecasts three to four bimonths in the future were obtained by using an ARIMA model. However, it is recognized that conditional forecasts obtained by using other models may have provided a better fit of the total supply equation to the data. Because there was first order autocorrelation in the disturbance terms, it was necessary to transform the data. A modified Cochrane-Orcutt procedure was used to obtain a maximum likelihood estimate of the autocorrelation coefficient.

The empirical estimates of the parameters of (14) appear in Table 1. Overall, the model fits the data remarkably well. Coefficient estimates on the price of corn and feeder cattle are significant and of the expected sign. The soybean meal price coefficient is not significantly different from zero indicating perhaps that soybean meal is not an extensively used Input for cattle feeding in the United States.

Important for our purposes here is the significance and expected signs of the coefficients on the ARIMA forecast of mean and variance of cattle price. These results suggest that the supply function is upward sloping and that fed cattle producers are risk averse.

Using equation (9) along with the parameter estimates reported in Table 1, estimates of the value of information are obtained from simulations based on two more informed distributions of fed cattle prices. These distributions are hypothetical because they are not based on additional analysis or composite forecasts of the fed cattle price series. They are a more accurate description of fed cattle prices in the sense that for each bimonth the mean price and variance values selected are closer to the realized price than is the ARIMA forecast. It must also be emphasized that since the demand for fed beef is not infinitely elastic, the value of information estimates obtained must be interpreted as the value to a single or small group of producers whose output response to new information has a negligible impact on market price.

Bimonthly estimates of the value of information for the two hypothetical distributions mentioned above are reported in Table 2 for the years 1978 to 1980. Descriptive statistics of the value of information estimates for the entire period 1970-1980 are reported at the bottom of the table. The fed cattle price and the corresponding ARIMA forecast and variance

$$
-14-
$$

Table 1. Parameter Estimates of the Market Risk Averse Supply Function of Fed Cattle Production, 1970-1980.

| Independent Variables | Coefficient Estimates |
| :---: | :---: |
| Constant | 93,668,000.** |
| Corn Price | -7,773,499.* |
| Soybean Meal Price | 13,849. |
| Feeder Cattle Price | -1,548,600.** |
| Mean Fed Cattle Price | 1,095,800.* |
| Variance of Fed Cattle Price | -546,440.* |
| $\mathrm{R}^{2}$ on Transformed Data . 57 |  |

All input prices were divided by USDA's index of price paid by farmers. THe ARIMA forecasts are of the deflated mean fed cattle price received by farmers for fed cattle in the U.S. The dependent variable is U.S.D.A. estimates of bimonthly commercial cattle slaughter.

* Indicates significance of a two-tailed t-test at the . 05 percent level. ** Indicates significance of a two-tailed t-test at the .01 percent level.
Table 3. Bimonthly Estimates of the Value of Information of Two More Accurate, Though Hypothetical, Distributions of Fed Cattle Price and the Value of Perfect Information in Deflated Dollars Per Cwt Live Weight of Fed Cattle Produced. 1978-1980

|  | Fed Cattle <br> Price <br> Realized | ARIMA Forecast |  | Value of Information |  | Value of Perfect Information |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $\begin{aligned} & \text { Mean } \\ & \text { Price } \end{aligned}$ | Variance |  |  |  |
| 78-1 | 20.818 | 18.905 | 11.649 | 0.0276 | 0.2589 | 0.5877 |
| 78-2 | 23.915 | 18.800 | 11.614 | 0.1808 | 0.8350 | 1.3762 |
| 78-3 | 26.145 | 18.725 | 11.587 | 0.1651 | 0.9910 | 1.5710 |
| 78-4 | 25.204 | 19.600 | 11.724 | 0.2447 | 1.0243 | 1.6288 |
| 78-5 | 26.336 | 22.100 | 12.334 | 0.0243 | 0.3818 | 0.7951 |
| 78-6 | 26.301 | 24.805 | 13.396 | 0.0007 | 0.1177 | 0.4158 |
| 79-1 | 30.325 | 25.535 | 13.923 | 0.0390 | 0.5481 | 1.1269 |
| 79-2 | 34.234 | 25.895 | 13.757 | 0.9533 | 2.9656 | 4.2381 |
| 79-3 | 32.211 | 26.575 | 13.846 | 0.4280 | 1.5932 | 2.5502 |
| 79-4 | 29.027 | 28.620 | 14.341 | 0.0436 | 0.3245 | 0.8304 |
| 79-5 | 29.281 | 32.700 | 15.579 | 0.0020 | 0.0027 | 0.2891 |
| 79-6 | 28.050 | 33.300 | 16.064 | 0.0385 | 0.0040 | 0.1052 |
| 80-1 | 27.850 | 30.910 | 15.856 | 0.1066 | 0.0093 | 0.0781 |
| 80-2 | 25.923 | 29.970 | 15.825 | 0.0095 | 0.0083 | 0.2279 |
| 80-3 | 24.731 | 29.260 | 15.793 | 0.0598 | 0.0069 | 0.0822 |
| 80-4 | 25.653 | 28.225 | 16.080 | 0.0017 | 0.0481 | 0.3627 |
| 80-5 | 24.554 | 27.085 | 16.241 | 0.0083 | 0.0010 | 0.1153 |
| Mean (1970-1980) |  |  |  | 0.1156 | 0.3082 | 0.4607 |
| Std. Deviation |  |  |  | 0.1731 | 0.5074 | 0.7176 |
| Minimum |  |  |  | 0.0000 | 0.0000 | 0.0052 |
| Maximum |  |  |  | 0.9533 | 2.9656 | 4.2381 |

[^0]are also reported. For 1978 through the fourth bimonth of 1979 , the ARIMA model generally underestimated price and for the remainder of the period, fed cattle price was overestimated. The variance of the forecast increased over the period.

The results indicate that for a group of producers whose output levels have no noticeable effect on market price, the value of information embodied in distribution ( $D-I$ ) (with the more informed mean 50 percent closer to the realized price than the ARIMA mean and with more informed and ARIMA variances equal) averages about 12 cents per cwt over the entire period and ranges from a low of nearly zero to a high of 95 cents per cwt. The value of information embodied in an even more accurate forecast (D-II) (with the more informed mean equal to the realized price and with more informed variance only one half of 'the ARIMA forecast variance) averages about 31 cents per cwt for the entire period, ranging from approximately zero to a high of $\$ 2.97$ cents. The high occurred in the second bimonth of 1979 which serves to point out that the value of information is larger the greater the difference between $q^{0}$ and $q^{m} \cdot q^{0} w 111$ tend to be smaller than $q^{m}$ when $\vec{p}^{0}$ is smaller than $\vec{p}^{m}$ and when $\sigma^{\circ}$ is larger than $\sigma^{m}$. $q^{0}$ will tend to be larger than $q^{m}$ when the opposite relationships occur between the parameters of the subjective and more informed distributions.

Based on equation (10), the value of perfect information appears in last column of Table 2. The estimated mean value of perfect information is about 46 cents per cwt although the range in value is from approximately one-half cent per cwt to $\$ 4.24$ per cwt. Again, the largest value of information occurred in the same year as the previous cases, a year when forecast price was low and the variance of forecast price was reasonably high.

The demand for beef is surely downward sloping and thus these estimates pertain to an individual producer or to a small group of producers whose response to information has a negligible effect on output price. Since the estimates in Table 2 are in terms of dollars per cwt of fed cattle produced, multiplying by mean bimonthly production provides an upper bound to the values of information to the industry. For the industry, these estimates are $74.3,198.0$ and 296.0 million dollars for $D-I, D-I I$, and the value of perfect information, respectively. The usefulness of these values depends, of course, on the output response of producers to new information. In the case of the first hypothetical distribution ( $D-I$ ) average bimonthly output response estimates over the $1970-1980$ period are only 16 percent. The corresponding values for $D-I I$ and the value of perfect information are 3.8 percent and 7.5 percent respectively. Thus, while the 74.3 million dollar figure may be a close approximation, the other two are almost surely grossly overestimated; or stated another way, depending on the elasticity of demand, when additional information is disseminated to the entire industry, its value per unit of output produced is likely to be substantially lower than the mean values reported in Table 2.

## V. CONCLUSION

An easily computable money metric of a risk averse agent's willingness to pay for additional information under price uncertainty was developed in this paper. The procedure was empirically demonstrated for a restricted class of utility functions by fitting a risk averse supply function to time series data from the U.S. fed cattle industry. While this paper makes a contribution to methods for estimating the value of information, numerous hurdles remain. Consideration of both price and production risk can complicate the empirical analysis, depending on the specification of the random variables, because condition (7) may not hold. Another problem is the empirical derivation of market welfare measures when demand is downward sloping. It is easily shown that the forecast is invalidated in this case if account is not taken of the proportion of the agents in the industry who choose to adopt or modify the forecast. This issue is addressed in a forthcoming paper by the authors.

## FOOTNOTES

1/ Lindley (1971) describes a similar measure for the value of information, $Z$, given by $E^{\beta} U\left(\pi^{\beta}-Z\right)=E^{\beta} U\left(\pi^{0}\right)$, where expectations are taken with respect to the Bayesian posterior distribution $f^{\beta}(p)$. Although both Lindley's Z and our $\mathrm{VI}_{2}$ are measures of the amount the agent is willing to pay to obtain more information, in general they may not be equal. There is also a subtle difference in interpretation. In the Bayesian approach $Z$ is the amount of money which must be given up by the agent when he produces $q^{\beta}$ so that he has the same amount of utility in the more informed state as producing $q^{\circ}$. In our case, $\mathrm{VI}_{2}$ is the amount of money that must be given to the agent when he produces $q^{0}$ so that his expected utility in the more informed state is the same as if he had produced $q^{m}$. Whether other measures, such as the distance $A-E$, or $F-G$ in figure 2 provide equivalent measures to $V I_{2}$ depends on the form of $E^{m_{U}}\left(\pi^{m}\right)$.

2/ The value $\mathrm{VI}_{2}$ can also be obtained from the risk averse factor demand function, $-\partial \operatorname{EU}\left(\pi^{*}\right) / \partial C=X^{*}$, where $C$ is the price of input $X$, in a manner analogous to (9).

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[^0]:    a/ D-I denotes a hypothetical distribution of fed cattle price where the bimonthly mean is 50 percent closer to the bimonthly realized price than is the ( $2,1,0$ ) ARIMA forecast; but identical variance. D-II denotes a hypothetical distribution of fed cattle price where the bimonthly mean is equal to the realized bimonthly price and the variance is 50 percent of the ( $2,1,0$ ) ARIMA forecast variance.

