Endogenous Switching Systems: Issues, Options, and Application to the U.S. Dairy Sector

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Abstract. *This research explores the theoretical and applied issues associated with endogenous switching systems where market prices are bounded by policy instruments such as price supports. Options for estimation of model parameters and their associated standard errors are identified and explored. Application to the U.S. dairy sector illustrates the research tradeoffs between conceptual rigor and empirical tractability that characterize these models. Results suggest that failure to explicitly address the endogenous switching context compromises the estimation results.*

Keywords. *Endogenous switching, simultaneous equations, bounded prices, censored variables.*

The econometric analysis of markets where prices are bounded by governmental policies, such as price ceilings, presents certain complications that do not arise when prices are determined by competitive markets. In the simple case of simultaneous supply and demand equations with no market intervention, the endogeneity of prices in the right-hand side of quantity-dependent equations can be accounted for using either two- or three-stage least squares. These methods, however, are not appropriate with bounded prices, and their use yields biased estimators of the parameters in the structural equations.

With bounded prices, more complicated methods are called for. First, the prices that are controlled cannot be estimated using OLS, but require the use of techniques appropriate for limited dependent variables. Second, the conventionally computed second-stage standard errors on the structural parameters are biased (Maddala, 1983). The objectives of this paper are:

To evaluate and compare different estimation methods for systems of simultaneous equations with censored dependent variables, to explore the generalization of methods that are appropriate for models with one censored variable to models with multiple censored variables, and to evaluate the feasibility of using resampling techniques to compute standard errors for second stage coefficients.

We classified estimation methods in two major groups: maximum likelihood methods, in which the parameters of the structural equations are estimated in one step, and two-stage estimation methods which are similar to two- or three-stage least squares. The first stage consists of estimating instruments for the endogenous variables in the right-hand side of the (structural) equations, and in the second stage the instruments are substituted into the structural equations, which are in turn estimated using standard linear or nonlinear regression techniques.

Conventional second-stage standard errors are biased when two-stage estimation methods are used for models with limited dependent variables. The asymptotic theory for a number of such models may be used to compute correct standard errors for the second stage coefficients (Amemiya, 1977, 1978; Lee, 1990; Lee and others, 1980).[1] Such theory is both complicated and not very general (that is, the asymptotic covariance matrices have to be derived again for each permutation of a model that the analyst wishes to investigate). Hence, simulation methods, which are simple and easily generalized, are an attractive alternative. Moreover, the computationally intense nature of these resampling methods can be easily handled in a standard microcomputer context.

The empirical implications of these methodological issues are demonstrated with an application to endogenous switching models of the U.S. dairy industry. We will revisit the work done in endogenous switching (Liu and others, 1980, 1991). Extant empirical work on endogenous switching in U.S. dairy addresses only fluid and a highly aggregated manufactured product sector. The possibility of extending the models to multiple censored variables has not been explored so far. Disaggregate modeling is particularly relevant in the analysis of U.S. dairy policy, as three different dairy products (American cheese, butter, and

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[1] Sources are listed in the references section at the end of this article.

nonfat dry milk) have bounded prices via purchase prices set by the U S Government While previous work has relied on two-stage estimation techniques, only the (biased) second-stage standard errors were reported Recent efforts by the U S Department of Agriculture and the Department of Agricultural Economics at the University of Wisconsin-Madison have provided us with a new set of data that is both more recent (up to 1990, instead of 1987), and in some cases, more reliable than previously used data In this empirical context two issues arise

1 Comparison of "bias corrected" versus "not bias corrected" two-stage estimation procedures (see the section on endogenous switching systems) In particular, we discuss the feasibility of generalizing the estimation procedures and examining models with multiple censored variables If bias correction could be ignored, this generalization would be quite easy

2 Comparison of conventional second-stage standard errors with bootstrapped standard errors for the parameters of the structural equations The complexity of the asymptotic theory has led researchers to report conventional second-stage standard errors Our empirical results indicate that, as expected from theory, the downward bias in the conventional standard errors is not negligible The good news is that the bootstrap provides a straightforward method for the computation of the second-stage standard errors Moreover, and in market contrast with asymptotic theory, the bootstrap procedure is very easy to generalize, although occasionally it will only be feasible to bootstrap the second stage of the estimation procedure In this sense, the bootstrapped standard errors will be conditional on the empirical distribution of the data and the first stage instruments

Simultaneous Structural Equation Models with Censored Prices

Consider the following set of demand and supply equations where all variables are expressed as natural logarithms

$$q_{it}^{d} = x_{it}^{d}\ \beta_i^{d} + \alpha_i^{d}\, p_{it} + u_{it}^{d}$$
$$q_{it}^{s} = x_{it}^{s}\ \beta_i^{s} + \alpha_i^{s}\, p_{it} + u_{it}^{s} \qquad (1)$$

$$(i = 1,2, \ \ m),$$

where t indicates time period, q_{it}^{d} is the ith demand equation, q_{it}^{s} is the ith supply equation, the x's are row vectors of exogenous variables, p_{it} is the equilibrium price in the ith market, the β's, are column vectors of parameters, α's are scalar parameters, and u's are stochastic disturbances with mean zero In equilibrium, $q_{it}^{s} \equiv q_{it}^{d} \equiv q_{it}$, and the endogenous variables in the system are the p_{it}'s and the q_{it}'s

Now introduce support prices in some or all markets in the model Without loss of generality, assume that the first 1 to k_t markets are in competitive equilibrium at time t, and that support prices are binding for the remaining $k_t + 1$ to m markets In this case, the equation system in 1 is replaced by

$$q_{it}^{d} = x_{it}^{d}\ \beta_i^{d} + \alpha_i^{d}\, p_{it} + u_{it}^{d}$$
$$q_{gt}^{d} = x_{gt}^{d}\ \beta_g^{d} + \alpha^{d}{}_{g}\ p_{gt} + u_{gt}^{d}$$
$$q_{it}^{s} = x_{it}^{s}\ \beta_i^{s} + \alpha_i^{s}\, p_{it} + u_{it}^{s}$$
$$q_{gt}^{s} = x_{gt}^{s}\ \beta_g^{s} + \alpha_g^{s}\, p_{gt} + u_{gt}^{s} \qquad (2)$$

$$(i = 1, \ \ ,k_t)$$
$$(g = k_t + 1, \ \ ,m),$$

where p_{gt} is the support price in the gth market and all other variables are defined the same as equation system 1 At each period t, the first k_t markets are in competitive equilibrium, and the endogenous variables are the equilibrium prices and quantities in each market The remaining markets are in a government intervention equilibrium, and in those markets, the endogenous variables are the d_{gt}'s and s_{gt}'s the quantities supplied and demanded in each market Because private supply and demand are not equal, this type of model is often referred to as a disequilibrium model Note, however, that private supply and total (private plus government) demand are equal because the operation of price supports requires that the government purchase the quantities supplied in excess of private demand In this sense, both regimes are equilibrium regimes Equations 1 are a special case of equations 2, with $k_t = m$ (that is, no government intervention)

Endogenous Switching Systems: Maximum Likelihood Estimation

Consider the simplest case of equation system 2 where price supports are set for only one market For periods in which the price support is not binding let k = m For periods in which the price support is binding, let k = m-1 Assuming that the disturbances in system 2 are distributed multivariate normal, with mean and variance $(0, \Sigma)$, the joint distribution of the endogenous variables in the system can be found using standard "change of variable" techniques Note that the set of endogenous variables is different in the competitive market equilibrium (where all prices and quantities are endogenous) and in the government intervention equilibrium (in which the price in the intervened market is set exogenously, while quantity demanded and quantity supplied are endogenous) When k = m, the log-likelihood, after dropping the constant term, is

$$\Sigma_{k=m}\left[-\frac{1}{2}\ln|\Sigma| - \frac{1}{2}(u'\Sigma^{-1}u) + \ln|J_{k=m}|\right], \tag{3}$$

where the summation is over the observations with k = m, u is the stacked vector of disturbances for those observations and $J_{k=m}$ is the Jacobian of the transformation Similarly, when k = m-1, the log-likelihood is given by

$$\Sigma_{k=m-1}\left[-\frac{1}{2}\ln|\Sigma| - \frac{1}{2}(u'\Sigma^{-1}u^{*}) + \ln|J_{k=m-1}|\right], \tag{4}$$

where the summation now is over the observations with k = m-1, u^{k} is the stacked vector of disturbances for those observations, and the $J_{k=m-1}$ is the corresponding Jacobian of the transformation Combining equations 3 and 4 we obtain the log-liklihood function for the system

$$L = \Sigma_{all}\left[-\frac{1}{2}\ln|\Sigma|\right] + \Sigma_{k=m}\left[-\frac{1}{2}(u'\Sigma^{-1}u) + \ln|J_{k=m}|\right] + \Sigma_{k=m-1}\left[-\frac{1}{2}(u^{*\prime}\Sigma^{-1}u^{*}) + \ln|J_{k=m-1}|\right] \tag{5}$$

The parameter estimates of the structural model can now be obtained, along with an estimate of the covariance matrix of the system, through maximization of equation 5 using numerical methods

Generalization of this method to multiple censored variables is straightforward The main difference is that with multiple censored variables the number of equilibria depends on the number of markets in which the price supports are binding at any one time The log-likelihood function equation 4 would be replaced by a set of functions, each one corresponding to an observed set of combinations of markets in competitive equilibrium and markets in government intervention equilibrium The log-likelihood function equation 3 would continue to be appropriate for observations in which all markets are in competitive equilibrium With multiple censored variables, the first two terms on the right-hand side of equation 5 remain unchanged, but the third term is replaced by a set of terms, one for each equilibrium in which at least one price bound is binding

While this approach is quite simple conceptually (and the convenience of obtaining unbiased estimates of the standard errors is not to be overlooked), empirical implementation is difficult, because unless the covariance matrix is constrained to be diagonal, with each additional market included in the system, the number of parameters to be estimated increases exponentially At the same time, as the number of censored variables increases, the number of observations corresponding to each equilibrium will diminish (provided that the additional price supports are binding for at least one observation) The net result is an increasingly difficult numerical optimization problem (See Quandt for a comprehensive discussion of maximum likelihood estimation methods for what he calls "market disequilibrium models ")

The main advantages of the maximum likelihood approach for the estimation of the parameters in an endogenous switching system of simultaneous equations are its conceptual simplicity and the straightforward computation of unbiased standard errors These advantages should be weighted against some drawbacks Numerical optimization of equation 5, for example, is far from trivial To keep the number of structural parameters in the model manageable, imposing restrictive assumptions in terms of the functional forms used may be necessary These restrictions may be inappropriate in some contexts

A more important shortcoming is that the maximum likelihood approach does not lend itself easily for the estimation of models with complex error structures Multivariate normality was the crucial assumption in our previous derivation If one wishes to entertain serial correlation in addition to cross-equation correlation between the residuals in the model, the simplicity of the maximum likelihood approach is greatly reduced [2] This is particularly important if the researcher has reason to believe that serial correlation is present in the model, but has no strong priors about the order of the corresponding ARMA process One approach that could be helpful here would be to use two-stage procedures to obtain an initial estimate of the model, including the moving average and autoregressive coefficients in each

[2]See the discussion of maximum likelihood estimation of univariate and multivariate ARMA models in Brockwell and Davis

equation If, for example, a low-order AR representation seems appropriate for all equations, the "rho transformed" model could be estimated by using maximum likelihood using the "rho transformed" variables

Endogenous Switching Systems: Two-Stage Estimation Procedures

Again, the discussion begins with single-censored variable models For this special case, two-stage estimation procedures offer the advantage of considerable computational simplicity Furthermore, this approach allows the analyst to use complex functional forms and error structures in the estimation of the structural equations Assuming serially uncorrelated errors in the reduced form equations while at the same time allowing more general error structures in the structural equations, however, may pose questions about the logical consistency of the model The advantages of two-stage procedures have to be measured against some drawbacks, the most important of which has already been mentioned the standard errors on the parameter estimates of the structural equations are biased, if computed by conventional methods The reduced form equations for the observations where k = m are

$$p_{it} = x_t\pi_i + u_i$$
$$p_{mt} = x_t\pi_m + u_m, \quad (6)$$

where i=1,2 m-1 p_m is the censored price, the p_i are all the other endogenous variables in the model, the x's are row vectors of exogenous variables, and the π's are column vectors of reduced form parameters The reduced form equations when k = m-1 are

$$p_{it} = x_{it}^{k}\pi^*_i + u^*_i, \quad (7)$$

where $x^* = (x, p_{gt})$ and $p_{mt} \equiv p_{gt}$ Denote the probability of an observation belonging to the competitive regime by F(c), where F(c) is the standard normal distribution, and where $c = (p_{g,t} - x_{t}\pi)/\sigma$ The probability of an observation being in the government intervention equilibrium is (1-F(c)) The expected value of the censored variable can then be written as

$$E(p_m) = F(c)^*E\left[p_m \mid p_m > p_g\right] + (1 - F(c))^*p_g, \quad (8)$$

and the conditional expectation as well as F(c) are obtained using a Tobit model [3] The expected value of the other instruments is given by

$$E(p_i) = F(c)^*(x\pi_i + E(u_i|p_m>p_g)) + (1-F(c))^*(x^*\pi_i + E(u^*_i|p_m \le p_g))$$
$$= F(c)^*x\pi_i + (1-F(c))^*x^*\pi^*_i + F(c)^*E(u_i|p_m>p_g) + (1-F(c))^*E(u^*_i|p_m \le p_g) \quad (9)$$

Next, examine the two conditional expectations in equation 9 Starting with

$$F(c)^*E(u_i|p_m>p_g) = F(c)*\frac{\text{cov}(u_m,u_i)}{\sigma} * \frac{f(c)}{F(c)} = \frac{cov(u_m,u_i)}{\sigma}*f(c) \quad (10)$$

where sigma is the estimated standard deviation from the Tobit model, and f(c) is the density function corresponding to F(c) For the second conditional expectation we use [4]

$$(1-F(c))^*E(u^*_i| p_m \le p_g) = -(1-F(c)) * \frac{cov(u^*_i,u_g)}{\sigma} * \frac{f(c)}{(1-F(c))} = -\frac{cov(u^*_i,u_g)}{\sigma}*f(c) \quad (11)$$

Use equations 10 and 11 to rewrite equation 9 as

$$E(p_i) = F(c)^*x\pi_i + (1-F(c))^*x^*\pi^*_i + cov^{**}\frac{f(c)}{\sigma}$$
$$= x\pi_i + (1-F(c))p_g\pi_g + cov^{**}\frac{f(c)}{\sigma} \quad (12)$$

where cov^* is simply the sum of the covariance terms in equations 10 and 11 and is a parameter to be estimated The last term in equation 12 is similar to Heckman's bias correction term, and it fulfills the same function In what follows, we will use the expression "bias correction term" to refer to the last term in equation 12

In contrast with maximum likelihood methods, the generalization of two-stage estimation methods from the single to the multiple censored variables

[3]Note that in equation 8 and in the remaining equations for price expectations the time subscripts are omitted for notational convenience

[4]Standard results from multivariate normal theory are being used for these derivations See Johnson and Kotz, 1972, Chapter 36

case is not straightforward Computationally it may be more difficult than maximum likelihood estimation To see this note that with multiple censored variables one could estimate each instrument estimated separately or estimate all instruments simultaneously With k censored variables, there are 2^k possible equilibrium solutions the competitive equilibrium solution in all markets, plus all the possible combinations of competitive equilibrium in some markets and government intervention equilibrium in some other markets Denote these equilibria as E_k, with k=1,2, 2^k The unconditional expected price in market i is the weighted average of the conditional expected prices corresponding to each of the 2^k possible equilibria

Without loss of generality, let the first c equilibria be such that the price in market i is the competitive equilibrium price, denoted by p_i In equilibria c+1, c+2, to 2^k the price support is binding in market i, denoted by $p_{g(i)}$, where the subscript indicates that the price support is binding, and that the price refers to market i Finally, let $F(E_k)$ denote the probability of the kth equilibria being observed The unconditional expected price in market i is given by

$$E(p_i) = \sum_{k=1}^{c} F(E_i) * E(p_i | E_k) + p_{g(i)} \sum_{k=c+1}^{2k} F(E_k) \quad (13)$$

While equation 13 is an expanded version of equation 8, its evaluation is much harder The reason is that evaluation of the F( ) functions in equation 13 requires integration of the multivariate normal density function over as many variables as price supports are binding in that particular equilibrium With as little as three such variables, reliable results may be very difficult and time consuming to obtain More than three binding price supports could make evaluation of equation 13 a practical impossibility, although Monte Carlo integration can always be tried In addition, the conditional price expectations cannot be obtained with a single equation Tobit model because the conditional distribution for each of the different equilibria will be different This requires the use of a multiple-equation simultaneous Tobit model

Similar arguments make the evaluation of the counterpart to equation 9 quite difficult Note that in evaluating the conditional expectations of the disturbances, the conditioning terms are now the particular equilibria to which the observation belongs The manipulations in equations 10 and 11 allowed us to derive concise expressions that could then be included in equation 12, but no similar manipulations are available for the more complex conditional expectations in the case of multiple-censored variables This discussion leads one to question the feasibility of estimating the instrumental variables one by one if there are observations for which more than one price support is binding Available analytical results and software may allow for this approach for cases with up to three censored variables

An alternative is to estimate all instruments simultaneously, using a ML approach Computing the expected prices would require the evaluation of the conditional expectations discussed above (equation 13) Analytical results are currently available for a few special cases of the bivariate and trivariate normal distributions

An alternative that would still allow estimation of all price instruments simultaneously is to use some probability distribution other than the multivariate normal In particular, one would be looking for a distribution that has closed-form expressions for the distribution function, and that does not impose undue restrictions on the covariance matrix of the system The first requirement is nicely fulfilled by several members of the family of multivariate logistic distributions Unfortunately these fail the second requirement severe restrictions are imposed on the structure of the correlation between any pair of variables (Pudney, 1989, p 295) It is an open question whether the data in a particular application support those restrictions or not A more general class of functions, the Generalized Extreme Value Functions (GEV's), could be used Flexible functional forms can be used, so that no unnecessary restrictions are imposed *a priori* on the covariance matrix To our knowledge very little applied work using GEV's exists, but this might be an alternative worth exploring The numerical optimization problem of maximizing the likelihood function would still be difficult with GEV's but perhaps more tractable

Proper estimation of the instruments in a model with multiple censored variables seems to present sufficient difficulties to grant consideration to the following proposal estimate each censored variable separately with a Tobit model, and compute the expected value as in equation 8 That is, estimate the instrumental variable for each censored variable ignoring whether the other censored variables are at or above the censoring point Then, estimate the expected value of all other instruments simply by regressing them on the full set of exogenous variables in the model This implies ignoring the fact that some exogenous variables appear only in some regimes (and ought to be weighted by the probability of observing the regime) as well as ignoring the multivariate equivalent of the bias correction term in equation 12. This proposal is of interest only in the case of multile censored variables. If there is only one censored variable, proper computation of the instruments is sufficiently straightforward to make consideration of the procedure we just have outlined unnecessary. In the case of multiple censored variables, in contrast, the simplifications gained from ignoring cross-equation information in the estimation of the instruments are considerable. The loss of information that this implies, in a statistical sense, may be more than compensated for by the gain of economic information that could result from considering a more disaggregate model with multiple censored variables.

For illustration purposes, the proper (or biased corrected) and improper (not biased corrected) instruments are contrasted in the empirical section of the paper. The model presented includes only one censored variable, but if ignoring bias correction in this context proved to be of little consequence one might be more encouraged to ignore it in more complicated models, where such an approach would entail substantial gains in terms of computational simplicity.

Computing Standard Errors for Endogenous Switching Models

There are two ways to approach the problem of computing appropriate standard errors for simultaneous-equation endogenous switching models. First, there is the statistical high road: derive and compute the asymptotic covariance matrix for the particular model specification in which one is interested. Lee and Maddala discuss both general methods that can be used in such derivations and particular cases for which the covariance matrices have been derived. Once the asymptotic covariance matrices have been derived, programming them is not necessarily difficult, but the process can be cumbersome. Moreover, although some very general expressions have been derived, that is, expressions that are valid for a wide class of models with censored or truncated endogenous variables, the covariance matrices for special cases are all different. This means that slight changes in the model specification may require extensive reprogramming of the covariance matrices.

The second approach uses resampling techniques. In particular, any two-stage procedure could be bootstrapped, which would yield estimates of the variance of the structural parameter estimates that result from the empirical distribution of the data and from the particular estimation procedure selected. If the residuals for each regression cannot be assumed to be white noise, the bootstrap resampling should take place from the endogenous and exogenous variables, including all lagged variables in the model.[5] Using the bootstrap implies re-estimating the model for each bootstrap data set. The number of replications in the literature varies, with 200 to 500 replications common. For our empirical application, a conservative approach is followed, and 1,000 replications are used. The instruments were computed only once, and then the second stage of the estimation procedure was bootstrapped. In this sense the bootstrapped standard errors are conditional on the data and the first-stage instruments. The alternative to bootstrap both the first and second stages took too long to be feasible.

An Application to the U.S. Dairy Sector

Consider a structural model of the U.S. dairy sector consisting of six equations, as follows:

1) retail demand for fluid milk products,
2) retail demand for manufactured dairy products,
3) retail supply of fluid milk products,
4) retail supply of manufactured dairy products,
5) wholesale supply of fluid milk products, and
6) wholesale supply of manufactured dairy products.

Each equation is specified as linear in the logarithms of the endogenous and exogenous variables. The right-hand side of each equation includes exogenous variables as well as endogenous prices. The specific variables included in each equation are detailed in the tables that follow.

The model has two possible solutions: a market equilibrium solution and a government intervention solution. In the latter, the wholesale price of manufactured dairy products is set by the government. The fluid milk market is always in competitive equilibrium. In the manufactured dairy products market, wholesale demand may fall short of wholesale supply if the purchase price is above the market price. The difference is made up by government purchases (CCC).

Table 1 defines the variables used in the model. The variables QF, FUSE, DINV, and D are treated as exogenous. There are small governmental purchases even when the market price is above the support price. When this happens, CCC also becomes an exogenous variable. The endogenous

[5] When the residuals can be assumed to be white noise, it is customary to resample from the residuals to generate the bootstrap data sets. See Efron (1982).

Table 1—Description of variables used in model

QFL	retail and wholesale supply and demand of fluid milk products
QM	retail supply and demand of manufactured dairy products, wholesale demand of manufactured dairy
QMWS	wholesale supply of manufactured dairy products
PRF	consumer price index, fluid milk products
PRM	consumer price index, manufactured dairy products
PWF	producer price index, fluid milk products
PWM	producer price index, manufactured dairy products
P1	minimum price for class 1 milk
P2	minimum price for class 2 milk
D	class 1 price differential
QF	farm-level milk production
FUSE	farm-level milk use
CCC	net government removals of manufactured dairy products
DINV	change in manufactured dairy product inventories
C	intercept term
A87	a dummy variable equal to 1 starting in the first quarter of 1988, equal to zero before that
INT	A87*LN(PRF/CPI)
CPI	consumer price index, all items
BEV	consumer price index, non-alcoholic beverages
DFA	deflated expenditures on fluid milk products advertising
T	trend=1,2
PCE	personal consumption expenditures
DMA	deflated expenditures on manufactured dairy products advertising
Q i	dummy variable equal to 1 in quarter i, zero otherwise
PFE	producer price index, fuel, energy, and related products

In addition, the following identities hold

QMWS=QF-QFL-FUSE
QM=QF-QFL-FUSE-CCC-DINV

Last, the following notational conventions are used

AR(i)	ith "rho" coefficient in an autoregressive process
LN	natural logarithm
LAG(x,i)	variable x, lagged i periods

variables in the market equilibrium solution are QFL, PRF, PRM, PWM, PWF, and P2 In the government intervention equilibrium, the endogenous variables are QFL, PRF, PRM, PWF, P2, and CCC Given the identities defined above there is no need for a separate equation for CCC Note that the model cannot include separate equations for wholesale demand for fluid milk products and manufactured dairy products because the wholesale demands are identical to the retail supplies Inclusion of the wholesale demand equations would result in a model with eight equations in six unknowns

The model is estimated using quarterly data from 1975 1 to 1990 4 (Cornick Eisenhauer and Cox 1992) Since quarterly time series data are used, serial correlation between the residuals is expected All structural equations are first estimated using ordinary least squares We compute the residuals for each equation and estimate their correlation and partial correlation coefficients for 12 lags The results from this exercise are used as a basis to reformulate the time series structure imposed on the error terms of the equations The results in tables 2-7 correspond to versions of each equation for which there is little evidence of serial correlation in the residuals (with the exception of equation 6, which exhibits fourth-order serial correlation) No correction for serial correlation is made in the estimation of the instruments used in the structural equations The six-equation model is estimated two different ways first, using the two-stage procedure described in section 4, and second, estimating the instruments for the censored price using a Tobit model, but the instruments for the other prices are estimated on the full set of exogenous variables, which ignores both the bias correction term and the fact that some variables need to be weighted by the probability of observing the regime in which the variable occurs In the following discussion, the first procedure is referred to as "bias corrected" and the second approach as "not bias corrected" The objective of comparing these two procedures is to evaluate the impact of bias correction on the regression results

For the bias corrected model, standard errors for all structural coefficients are also computed in two different ways the conventional standard errors are computed at the second stage, and the model is bootstrapped with computed standard errors after 1,000 replications of the model The objective of this comparison is to evaluate the expected downward bias in the nonbootstrapped standard errors

Table 2 presents the results for the retail fluid demand equation, which is estimated as a function of retail fluid price, retail price of nonalcoholic beverages, and personal consumption expenditures (all deflated by the retail CPI for all items) the deflated advertising expenditures for fluid milk products, lagged demand, plus several dummy variables Parameter estimates are almost identical with and without bias correction, with one important exception the own price coefficient is -0 037 without bias correction and it increases to

Table 2—Retail fluid demand

	Not bias corrected	Bias corrected		
	Parameters	Parameters	Second Stage t-values	Bootstrap t-values
C	-1 591	-1 563	-4 870	-5 604
A87	0 023	0 029	1 787	1 458
INT	0 245	0 296	2 130	1 637
LN(PRE/CPI)	-0 037	-0 067	-0 944	-0 999
LN(BEV/CPI)	-0 016	-0 016	-1 814	-1 628
LN(PCE/CPI)	0 185	0 185	4 066	4 568
LN(DFA)	0 003	0 003	0 833	0 926
T	-0 003	-0 003	-4 667	-4 538
Q1	-0 027	-0 027	-3 151	-3 515
Q2	-0 070	-0 070	-11 339	-12 017
Q3	-0 043	-0 043	-12 286	-14 052
LAG(RFD,1)	0 501	0 507	5 196	6 001

-0 067 with bias correction All coefficients have the expected signs, but note the use of a dummy variable for observations after 1987, and an interaction term including the dummy and the own-price coefficient Dropping the dummy and interaction term resulted in a change of sign in the own-price coefficient Use of the second-stage t-values or the bootstrapped t-values seems to make little difference The coefficient "INT" loses significance at conventional levels when the bootstrapped t-values are used, but this is a variable of little economic interest

Table 3 presents the results for retail manufactured demand, which is estimated as a function of own price and personal consumption expenditures (both deflated by the retail CPI for all items), deflated advertising expenditures on manufactured dairy products and dummy variables for the second and third quarters The equation is estimated as an AR(2) process Demand seems slightly more inelastic if the bias corrected own-price coefficient is used instead of the not-bias-corrected one Perhaps more significant is the change in the coefficient on advertising expenditures the elasticity of demand with respect to advertising is estimated to be 0 012 without bias correction, and it drops almost 50 percent, to 0 007, with bias correction The t-values are virtually identical with and without using the bootstrap

Table 4 presents results for retail fluid supply, estimated as a function of retail price and the price of fuel and energy, with both deflated by the wholesale price of fluid milk products Quarterly dummies and a time trend were included in the equation, which was estimated as an AR(1) process All coefficients have the expected signs and are virtually identical with and without bias correction Note, however, that statistical inferences change for the energy price and trend coefficients if the bootstrapped t-values are used instead of the conventional second stage t-values In both cases, the coefficients are statistically significant at conventional levels according to the second-stage t-values, and not statistically significant according to the bootstrapped t-values

Table 5 shows the results for retail manufactured supply, estimated as a function of own price and the price of fuel and energy, both deflated by the wholesale price of manufactured dairy products A time trend and quarterly dummies are included in the equation, which is estimated as an AR(2,4) process Note that the supply response is more inelastic according to the bias-corrected parameter estimates, and that the AR(3) coefficient is more

Table 3—Retail manufactured demand

	Not bias corrected	Bias corrected		
	Parameters	Parameters	Second Stage t-values	Bootstrap t-values
C	-3 428	-3 405	-19 769	-15 202
LN(PRM/CPI)	-0 112	-0 094	-0 737	-0 579
LN(PCE/CPI)	0 426	0 430	5 685	5 468
LN(DMA)	0 012	0 007	0 389	0 410
Q2	0 046	0 043	2 260	2 307
Q3	0 010	0 004	0 193	0 204
AR(2)	-0 591	-0 581	-5 376	-4 683

Table 4—Retail fluid supply

	Not bias corrected	Bias corrected		
	Parameters	Parameters	Second Stage t-values	Bootstrap t-values
C	2 656	2 632	107 981	7 545
LN(PRF/PWF)	0 221	0 254	1 654	0 886
LN(PFE/PWF)	–0 038	–0 043	–4 182	–1 229
Q1	–0 045	–0 041	–6 155	–5 223
Q2	–0 084	–0 079	15 370	–11 856
Q3	–0 051	–0 051	–15 781	–17 312
T	0 001	0 001	4 410	0 639
AR(1)	0 581	0 485	4 469	3 609

Table 5—Retail manufactured supply

	Not bias corrected	Bias corrected		
	Parameters	Parameters	Second Stage t-values	Bootstrap t-values
C	2 643	2 582	37 963	13 042
LN(PRM/PWM)	0 205	0 126	1 442	0 933
LN(PFE/PWM)	–0 051	–0 050	–1 550	–1 129
Q1	–0 078	–0 057	–2 253	–1 890
Q2	0 034	0 036	1 286	1 275
Q3	0 022	–0 006	–0 192	–0 160
T	0 005	0 005	9 972	1 798
AR(2)	–0 162	–0 375	–2 792	–2 221
AR(4)	0 276	0 245	1 833	1 470

than twice as large according to the bias-corrected estimates The coefficients on the first quarterly dummy and on the time trend are statistically significant at conventional levels using second-stage t-values, but lose that significance if the bootstrapped t-values are used

Wholesale fluid supply results are presented in table 6 Supply is estimated as a function of own price and price of fuel and energy, both deflated by class 1 price Quarterly dummies, a time trend and lagged supply, are also included in the equation All the coefficients have the expected signs, and are almost indistinguishable regardless of the estimation method used The t-values present an anomalous pattern for this equation, in the sense that the bootstrapped t-values are generally larger than the second-stage t-values, contrary to what was expected However, in all cases, inferences concerning significance at conventional levels are the same regardless of the set of t-values used

Wholesale manufactured supply results are presented in table 7 The equation is very similar to the wholesale supply equation, and the regressors are own price and price of fuel and energy, both deflated by class 3 prices Two quarterly dummies, a time trend and lagged supply are included in the equation As in other equations, most parameter estimates are very similar regardless of estimation method Moreover, the own-price coefficient changes by about a third and has the wrong sign

Table 6—Wholesale fluid supply

	Not bias corrected	Bias corrected		
	Parameters	Parameters	Second Stage t-values	Bootstrap t-values
C	1 255	1 239	4 707	5 606
LN(PWF/P1)	0 047	0 053	2 445	6 613
LN(PFE/P1)	–0 022	–0 020	–2 400	–3 138
Q1	–0 044	–0 044	–6 922	–8 406
Q2	–0 081	–0 082	–16 020	–18 075
Q4	–0 052	–0 052	–17 400	–20 551
T	0 000	0 000	1 000	1 319
LAG(WFS,1)	0 529	0 537	5 147	6 149

Table 7—Wholesale manufactured supply

	Not bias corrected	Bias corrected		
	Parameters	Parameters	Second Stage t-values	Bootstrap t-values
C	2 739	2 754	73 150	3 721
LN(PWM/P3)	-0 312	-0 472	-2 389	-12 975
LN(PFE/P3)	0 179	0 181	5 234	3 316
Q2	0 145	0 154	11 835	13 535
Q4	-0 089	-0 095	-8 556	7 688
T	0 006	0 007	13 088	9 849
LAG(WMS,1)	0 345	0 358	5 444	6 245

(implying negatively sloped supply) despite repeated attempts to obtain more reasonable results This, perhaps, reflects the high level of aggregation in this manufactured dairy product category No statistical inferences are changed if the bootstrapped t-values are used instead of the second-stage t-values

Summary and Conclusions

Regardless of whether the analyst chooses a one-step or a two-step procedure, the use of maximum likelihood methods seems to be the only satisfactory alternative A special difficulty associated with maximum likelihood estimation of the instruments in the presence of multiple censored variables was the need for multiple integration of a multivariate probability density function Using a multivariate normal probability density function may render this problem intractable In this context, the use of alternative closed form distribution functions, such as the Generalized Extreme Value Functions, may prove useful Univariate Tobit models and ordinary least squares regressions could be used to generate starting values for the maximum likelihood estimation

The main conclusions that can be derived from that application are quite modest First, when bias correction is ignored we found the resulting bias in the parameter estimates to be quite small with few exceptions Computational simplicity, in the context of these data and model, may be a sufficient argument to recommend use of methods that ignore bias correction However, the generality of these results for other research contexts, particularly those with multiple market endogenous switching, is an open question Second, while we also found the bias associated with conventional second stage standard errors to be rather small either asymptotic theory or resampling techniques should be used to generate correct second stage standard errors The use of the bootstrap was illustrated, and the simplicity and generality of the approach were emphasized

Several areas require further research Our analysis indicates that the dynamic specification of the dairy sector model is particularly important, yet we derived that specification in an *ad hoc* fashion While considerable research on dairy sector dynamics has been carried out at the farm level, it seems necessary to extend that research into the dynamics of the wholesale and retail components of the dairy sector

Our analysis was carried out entirely in terms of an aggregate "manufactured dairy products' category In contrast, the U S dairy price support program operates through purchase prices for three different manufactured dairy products butter American cheese, and nonfat dry milk Therefore, to evaluate more fully the effects of the operation of the price support program, the analysis should be performed at a more disaggregate level To do this, it will be necessary to explore several possibilities estimate the instruments in multiple-censored variable models using maximum likelihood techniques maximum likelihood estimation of the structural equations possibly after using the "not bias corrected" approach to generate both starting values and hypotheses concerning the time series structure of the residuals in the model Neither of these alternatives will be easy or straightforward The potential lack of generality of the research results presented here, particularly for multiple market endogenous switching models, suggests we need to further evaluate these alternatives

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