Small Holders under Risk

by

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The University of Sydney, NSW, 2006

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Abstract

A generalisation of previous household models is developed to include production, consumption, storage, labour and land allocation decisions under price, yield, storage and investment risks. Implications drawn from this model include, that consumption can be a hedge against price risk and that supply curves are shifted by consumption and by storage decisions. Production possibilities frontiers depend upon the households' aversion to risk and the allocation of land to different crops is affected by every other decision the household makes. Using hypothetical data and focussed on farm households in Vietnam, parameters are estimated for the model and simulations are carried out to examine the consequences of land exchanges and shifts in technology.

For more than a decade, Vietnam has been reforming its agriculture. Following significant changes, Vietnam has attained self-sufficiency in rice production and has become the second largest rice exporter in the world. Rice production has risen from 11.6 million tonnes in 1980 to 36 million tonnes in 2004 (FAOSTAT). However, the share of agriculture in the GDP of Vietnam has fallen from 36 percent in 1986 to 21.8 percent in 2004. Even with reforms, there is extensive rural poverty and a growing disparity between rural and urban incomes (World Bank, 1998).

Vietnam has a range of policies that tend to favour consumers and state-owned enterprises. These include import controls on fertilizers and seeds, output pricing rules and restrictions on rice exports, limited credit provision and collateral for mortgages, and until recently land-use taxes based on rice yields, limitations on land transfers, ceilings land holdings and a system of land-use rights (World Bank, 1998). Land-use rights are assigned to households, often in numerous small and dispersed parcels. As Vietnam opens to world markets, there will be pressure to rationalize land holdings and deregulate agriculture. The benefits may be substantial, but the consequences could also be serious (IFPRI, 1996). At the worst, many households could be displaced from agriculture and live in perpetual poverty.

Of course, Vietnam is one of many countries to undergo structural adjustment and there many studies of development policies. Many of these studies use computable general equilibrium models (*eg*. de Janvry, *et al*., 1992). Other studies use household models built upon microeconomic foundations (*eg*. Ellis, 1998). The purpose of this study is to build on previous models to formulate a dynamic model of household decisions under risk. The model will include production, consumption, storage, labour and land allocation decisions under price, yield, storage and investment risks. It accounts for the major decisions made by households and the main risks they face. Although, the model can be used to analyze import and export restrictions, credit provision, taxes and similar policies, the illustration will emphasize the important role of risk in a household's decisions and the prospects for rationalizing land holdings without displacing households from agriculture.

Basic Household Model

There is an extensive literature on the economics of households. The standard development is given by Nakajima (1970 and 1986) and is based on the earlier work of Chayanov (1966). Ellis (1993) provides a general overview and Singh, Squire and Straus (1986) contrast a wide variety of household models. The model developed by Barnum and Squire (1979) and discussed by Ellis (1993) summarises the essential features. This model includes the choice of whether crop production should be consumed by the household or sold to earn income for the purchase of other goods. The model includes Z-goods that are consumed within the household but are neither produced on the farm nor purchased. They only require the use of labour. Examples are cooking and leisure. It is assumed that there is a labour market and that the household can either hire in or hire out labour. However, the storage of crops for later consumption or sale is not included. Land available for farming is fixed. Decisions are static and there is no carry over of wealth from one year to the next. Risks are not included.

Four types of commodities will be included in this study, as summarized in Table 1. Commodity 1 is produced and sold but is not stored or consumed by the household. Commodity 2 is produced and may be sold, stored and consumed, or a combination of these. Commodity 3 is purchased and consumed but is not produced by the household. Commodity 4 is a Z-good that is consumed by the household but is neither produced on the farm nor purchased. The basic household model includes commodities 2, 3 and 4, except that storage of commodity 2 is not considered.

Table 1. Commodities in the Model.

	Commodity			
	1	2	3	4
Produced	✓	✓		
Sold	✓	✓		
Stored		✓		
Consumed		✓	✓	✓
Purchased			✓	

The household is assumed to maximize its utility from consumption subject to a budget constraint.

$$(1)\qquad J = \max U(Q_2, Q_3, Q_4)$$
subject to:
$$y_2 Y_2 (L_2, X_2, A) - x_2 X_2 A + \ell[T - L_2 A - Q_4] - y_2 Q_2 - q_3 Q_3 = 0.$$

Upper case letters denote quantities and lower case letters denote prices. Subscripts refer to the commodities in Table 1. Indirect utility J results from maximizing the direct utility of consumption U, subject to the budget constraint. In the budget constraint, commodity 2 has yield, Y_2, as a function of labour per unit of area, L_2, variable inputs per unit of area, X_2, and total land area of the farm, A. Commodity 2 sells for price y_2, giving total revenue y_2Y_2. Variable inputs cost x_2 per unit, giving total variable costs x_2X_2A. Labour available to the household is T units of time. Production requires L_2A units and the Z-good, commodity 4, requires Q_4 units. This leaves a surplus of $T - L_2A - Q_4$ units that can be hired out at the wage rate ℓ. If there is a deficit, labour must be hired in at the same wage rate. Consumption of commodity 2 reduces revenue by y_2Q_2 and purchases of commodity 3 require expenditure q_3Q_3.

The household chooses consumption, labour and variable inputs.

Consumption, Q_2, Q_3 and Q_4

$$(2)\qquad \frac{\partial U/\partial Q_2}{\lambda} - y_2 = 0;$$

$$(3)\qquad \frac{\partial U/\partial Q_3}{\lambda} - q_3 = 0;$$

$$(4)\qquad \frac{\partial U/\partial Q_4}{\lambda} - \ell = 0.$$

The shadow price on the budget constraint, λ, is the marginal utility of income. For all three commodities, the marginal utility of consumption divided by the marginal utility of income equals the price of the commodity. Taking ratios shows that marginal rates of substitution equal the negative of the price ratios. For example, $-\partial Q_3/\partial Q_2 = -y_2/q_3$. As in consumer theory, a household maximizes utility by choosing commodities at the point where an isoutility curve is tangent to the budget constraint. Successively changing a price and finding the new points of tangency gives the demand curve for a commodity.

Labour, L_2

$$(5)\qquad y_2 \frac{\partial Y_2}{\partial L_2} - \ell = 0.$$

The marginal value product of labour used in agricultural production equals the wage rate.

Variable Inputs, X_2

(6) $$y_2 \frac{\partial Y_2}{\partial X_2} - x_2 = 0.$$

The marginal value product of variable inputs equals the input price. The condition for variable inputs can be divided by the condition for labour to show that the marginal rate of substitution equals the negative of the price ratio, $-\partial L_2 / \partial X_2 = -x_2 / \ell$. As in producer theory, a household minimizes the costs of producing a given yield by choosing inputs at the point where the slope of an isoquant equals the price ratio. The minimum cost points for all possible yields give the expansion path and the cost function $c(Y)$. The derivative of the cost function is marginal cost, or the supply curve. The household maximizes profits by choosing yield to equate the output price to marginal cost, or $y_2 = \partial c / \partial Y$.

In the basic household model, consumption and production decisions are separable. Two separate models would give the same results. Several authors have investigated situations in which decisions may not be separable. Fabella (1989) included yield risk in a static model. Roe and Graham-Tomasi (1986) included yield risk in a dynamic model. Saha (1994) included price and yield risk in a static model. Fafchamps (1993) modelled labour decisions under risk. And Saha and Stroud (1994) modelled on-farm storage under price risk. There are other situations that have not been investigated. For example, stored commodities are under risk of attack by pests and commodities must be stored to be consumed by the household. Investing in land is, perhaps, the biggest risk of all with major consequences for future consumption. Nor have all these risks been modelled simultaneously.

Dynamic Household Model under Risk

Hertzler (1991) developed a general model for decisions under risk by agricultural households. This model will be specialised for small farms that are part of a village and may be isolated from commodity markets. Households are assumed to behave as if they maximise their expected utility subject to a budget constraint for the change in wealth.

(7) $$J(t_0, W_0) = \max E \int_{t_0}^{t_T} e^{-\rho(t-t_0)} U(Q) dt + e^{-\rho(t_T - t_0)} V(W_{t_T})$$

$$\text{subject to:}$$

$$dW = \delta(W, Q, D)dt + \sigma(Q, D)dz; \qquad W_0 \text{ is known.}$$

A household's satisfaction is summarized by expected utility, J, at initial time t_0 and with initial wealth, W_0. Satisfaction is derived from the utility of consumption, U, discounted at the rate of time preference, ρ, and integrated over all years until the end of the household's time horizon, t_T.

Satisfaction is also derived from the utility of wealth at the end of the planning horizon, V. At the current time t, wealth increases with changes in wealth, dW. A change in wealth has an expected change δdt, where δ is the instantaneous mean, and it has an error term σdz, where σ is a vector of instantaneous standard deviations and dz is a vector of Weiner increments (Dixit and Pindyck 1994, p. 63). The mean and standard deviations are functions of wealth, W, a consumption vector, Q, and a decision vector, D, chosen at time t to apply over a farming season of length dt.

Maximising expected utility over a household's time horizon is equivalent to maximising the Hamilton-Jacobi-Bellman equation in each decision interval. The Hamilton-Jacobi-Bellman equation (8) is a partial differential equation in time and wealth, subject to a boundary condition.

$$(8) \qquad \frac{\partial J}{\partial t} + \max\left[e^{-\rho(t-t_0)}U + \frac{\partial J}{\partial W}\delta + \tfrac{1}{2}\frac{\partial^2 J}{\partial W^2}\sigma\Omega\sigma' \right] = 0;$$
$$\text{with:}$$
$$J(T, W_T) = e^{-\rho(t_T - t_0)}V(W_T)$$

The expression to be maximised includes discounted utility of consumption plus the marginal utility of wealth, $\partial J/\partial W$, multiplied by the instantaneous mean, δ, plus one-half the derivative of the marginal utility of wealth, $\tfrac{1}{2}\partial^2 J/\partial W^2$, multiplied by the instantaneous covariance, $\sigma\Omega\sigma'$. Optimality conditions are the derivatives set equal to zero.

$$(9) \qquad e^{-\rho(t-t_0)}\frac{\partial U/\partial Q}{\lambda} + \frac{\partial \delta}{\partial Q} - \tfrac{1}{2}R\frac{\partial(\sigma\Omega\sigma')}{\partial Q} = 0;$$
$$\frac{\partial \delta}{\partial D} - \tfrac{1}{2}R\frac{\partial(\sigma\Omega\sigma')}{\partial D} = 0;$$
$$\text{where:}$$
$$\lambda(t, W) = \frac{\partial J}{\partial W}; \text{ and } R(t, W) = -\frac{\partial^2 J/\partial W^2}{\partial J/\partial W}.$$

The first optimality condition is for consumption and the second is for production and investment decisions. Terms containing R are marginal risk premiums. To simplify notation, the expected marginal utility of wealth is denoted by λ and the coefficient of absolute risk aversion is denoted by R. The coefficient of risk aversion measures the curvature of expected utility with respect to current wealth. It is distinguished from an Arrow-Pratt coefficient that measures the curvature of utility with respect to terminal wealth. The coefficient of risk aversion encapsulates all of the household's preferences about the future and information about risks.

The instantaneous mean and covariance are defined by the stochastic differential equation for wealth in (7). Hertzler (1991) provided a method for deriving this equation. First the household's wealth is determined.

$$W = bB + aA + y_2 S_2.$$

Upper case letters denote quantities, lower case letters denote prices and numbers as subscripts denote the commodities in Table 1. Wealth is the sum of risk-free investments, *B*, valued at price *b*, land area, *A*, valued at price *a*, and storage, *S*, valued at price *y*.

Next the household's wealth is differentiated using stochastic calculus to give the change in wealth under risk. Then assumptions must be made about a household's expectations for the future. For example, the commodity price at the beginning of a season may be *y*, the actual price at the end of the season will be $y + dy$, but the price that the household expects is $y + E\{dy\}$. Both actual and expected prices can be modelled by stochastic differential equations. Although these equations can be arbitrarily complex, it is typical to assume log-normal distributions. In other words, the percentage change in prices is normally distributed and prices themselves are always positive.

$$db = b\delta_W dt;$$
$$da = a\delta_a dt + a\sigma_a dz_a;$$
$$dy_1 = y_1\delta_{y_1} dt + y_1\sigma_{y_1} dz_{y_1};$$
$$dy_2 = y_2\delta_{y_2} dt + y_2\sigma_{y_2} dz_{y_2};$$
$$dp_2 = \sigma_{p_2} dz_{p_2};$$

Expected rates of change are terms like δdt and errors are terms like σdz. The price of a risk-free investment, *b*, has a known rate of return and no error term. If the risk-free investment is simply to hoard money, then the rate of return is zero. The land price, *a*, and commodity prices, *y*, have both expected rates of change and error terms. Commodities stored and sold between harvests may attract a premium, *p*, which has no expected change, only an error term. In addition, the storage premium can be positive or negative and, hence, is assumed to be normally distributed.

Assumptions are also required for the household's expectations about yields and storage.

$$dY_1 = Y_1\sigma_{Y_1} dz_{Y_1};$$
$$dY_2 = Y_2\sigma_{Y_2} dz_{Y_2};$$
$$dS_2 = S_2\delta_{S_2} dt + S_2\sigma_{S_2} dz_{S_2};$$

Yields, *Y*, have no expected change, only error terms but are log-normally distributed. Storage, *S*, has both an expected rate of physical degradation and an error term.

Finally, the original equation for wealth can be solved for risk-free investments and substituted into the differential equation for wealth. As a result, the risk-free rate of return becomes the opportunity cost of investment. With these assumptions, wealth changes with capital gains and depreciation on investments, with revenues and costs of production, with revenue from labour and with expenditures on consumption. The instantaneous mean is:

$$(10)\qquad \delta(W,Q,D) = \delta_W W + a[\delta_a - \delta_W]A + \left[y_2\left[\delta_{y_2} - \delta_W - \delta_{S_2}\right] + p_2 - s_2\right]S_2$$
$$+y_1\left[1+\delta_{y_1}\right]Y_1(L_1,X_1,K_1,A) - x_1X_1K_1A + y_2\left[1+\delta_{y_2}\right]Y_2(L_2,X_2,K_1,A) - x_2X_2(1-K_1)A$$
$$+\ell\left[T - L_1K_1A - L_2(1-K_1)A - Q_4\right] - y_2\left[1+\delta_{y_2}\right]Q_2 - q_3Q_3.$$

On the left hand side, the instantaneous mean is a function of wealth, W, consumption vector, Q, and decision vector, D. On the right-hand side, wealth, W, increases at the risk-free rate δ_W. Land area, A, is valued at price a and has an expected rate of capital gains above the risk free rate of $\delta_a - \delta_W$. Commodities go into storage at a level, S, at the commodity price, y, and have a rate of return on investment above the risk-free rate of $\delta_y - \delta_W$. Stored commodities will physically degrade at the rate δ_S, with a degradation cost per unit of $y\delta_S$. Commodities sold between harvests will attract a price premium, p, and storage will cost s dollars per unit. Production of a commodity gives yield, Y, which is expected to sell for price $y[1 + \delta_y]$. Yield is a function of labour per unit of area, L, variable inputs per unit of area, X, the proportion of the farm in that commodity, K, and the total land area, A. Because the proportions sum to one, the proportion of the farm in commodity 1 is K_1 and the proportion in commodity 2 is $(1 - K_1)$. Therefore, yield of commodity 2 is actually a function of K_1. Surplus labour is the total time available to the household, T, less the time devoted to producing commodities, L_1K_1A and $L_2(1 - K_1)A$, less the time consumed as the Z-good, Q_4. Consumption of the commodity that is also produced and stored, Q_2, reduces revenue by the expected sale price multiplied by the quantity consumed. Consumption of the commodity that is only purchased, Q_3, costs q_3 per unit.

Corresponding to the instantaneous mean is the instantaneous error term:

$$\sigma(Q,D)dz = aA\sigma_a dz_a + S_2\sigma_{p_2}dz_{p_2} - y_2S_2\sigma_{S_2}dz_{S_2}$$
$$+ y_1Y_1(L_1,X_1,K_1,A)\sigma_{y_1}dz_{y_1} + y_2\left[Y_2(L_2,X_2,K_1,A) + S_2 - Q_2\right]\sigma_{y_2}dz_{y_2}$$
$$+ y_1Y_1(L_1,X_1,K_1,A)\sigma_{Y_1}dz_{Y_1} + y_2Y_2(L_2,X_2,K_1,A)\sigma_{Y_2}dz_{Y_2}.$$

There are seven sources of risk for land prices, the price premium for storage, any degradation during storage, commodity prices and yields. If the household chooses larger quantities, the error term will increase, except for consumption of commodity 2. Consumption reduces the amount that is exposed to price risk, $Y_2 + S_2 - Q_2$.

The instantaneous covariance is found by squaring the error term.

$$(11) \quad \sigma(Q,D)\Omega\sigma'(Q,D) = a^2A^2\sigma_a^2 + S_2^2\sigma_{p_2}^2 + y_2^2S_2^2\sigma_{S_2}^2 + y_1^2Y_1^2(L_1,X_1,K_1,A)\sigma_{y_1}^2 + y_2^2[Y_2(L_2,X_2,K_1,A)+S_2-Q_2]^2\sigma_{y_2}^2 + y_1^2Y_1^2(L_1,X_1,K_1,A)\sigma_{Y_1}^2 + y_2^2Y_2^2(L_2,X_2,K_1,A)\sigma_{Y_2}^2.$$

Only variances are included and all covariances are set to zero. Covariances between prices and yields are surely not zero, particularly for a small village. They are set to zero because the notation becomes too cumbersome otherwise.

Optimal Decisions

More specific versions of the optimality conditions in equation (9) are derived using the instantaneous mean in equation (10) and the instantaneous covariance in equation (11). The optimality conditions below are like those of the basic household model except for marginal risk premiums. These are terms beginning with *R*.

Consumption, Q_2, Q_3 and Q_4

$$(12) \quad e^{-\rho(t-t_0)}\frac{\partial U/\partial Q_2}{\lambda} - y_2[1+\delta_{y_2}] + Ry_2^2[Y_2+S_2-Q_2]\sigma_{y_2}^2 = 0;$$

$$(13) \quad e^{-\rho(t-t_0)}\frac{\partial U/\partial Q_3}{\lambda} - q_3 = 0;$$

$$(14) \quad e^{-\rho(t-t_0)}\frac{\partial U/\partial Q_4}{\lambda} - \ell = 0.$$

Because commodities 3 and 4 are not subject to risk, optimality conditions (13) and (14) are the same as equations (3) and (4) for the basic household model. Commodity 2 is subject to risk, however. Compared with equation (2), optimality condition (12) now includes a marginal risk premium.

Labour, L_1 and L_2

$$(15) \quad y_1[1+\delta_{y_1}]\frac{\partial Y_1}{\partial L_1} - \ell K_1 A - Ry_1^2Y_1\sigma_{y_1}^2\frac{\partial Y_1}{\partial L_1} - Ry_1^2Y_1\sigma_{Y_1}^2\frac{\partial Y_1}{\partial L_1} = 0;$$

$$(16) \quad y_2[1+\delta_{y_2}]\frac{\partial Y_2}{\partial L_2} - \ell(1-K_1)A - Ry_2^2[Y_2+S_2-Q_2]\sigma_{y_2}^2\frac{\partial Y_2}{\partial L_2} - Ry_2^2Y_2\sigma_{Y_2}^2\frac{\partial Y_2}{\partial L_2} = 0.$$

Compared with equation (5) of the basic household model, optimality conditions (15) and (16) include marginal risk premiums for both price and yield risks.

Variable Inputs, X_1 and X_2

$$(17)\quad y_1\left[1+\delta_{y_1}\right]\frac{\partial Y_1}{\partial X_1} - x_1K_1A - Ry_1^2Y_1\sigma_{y_1}^2\frac{\partial Y_1}{\partial X_1} - Ry_1^2Y_1\sigma_{Y_1}^2\frac{\partial Y_1}{\partial X_1} = 0;$$

$$(18)\quad y_2\left[1+\delta_{y_2}\right]\frac{\partial Y_2}{\partial X_2} - x_2(1-K_1)A - Ry_2^2[Y_2+S_2-Q_2]\sigma_{y_2}^2\frac{\partial Y_2}{\partial X_2} - Ry_2^2Y_2\sigma_{Y_2}^2\frac{\partial Y_2}{\partial X_2} = 0.$$

Compared with equation (6), optimality conditions (17) and (18) also include marginal risk premiums for price and yield risks. The conditions for labour and variable inputs generalize the results of Roe and Graham-Tomasi (1986).

Storage, S_2

$$(19)\quad y_2\left[\delta_{y_2} - \delta_W - \delta_{S_2}\right] + p_2 - s_2 - RS_2\sigma_{p_2}^2 - Ry_2^2S_2\sigma_{S_2}^2 - Ry_2^2[Y_2+S_2-Q_2]\sigma_{y_2}^2 = 0.$$

The basic household model does not include storage decisions. Optimality condition (19) generalizes the results of Saha and Stroud (1994) to include not only price risks but also degradation risks.

Land, K_1 and A

$$(20)\quad y_1\left[1+\delta_{y_1}\right]\frac{\partial Y_1}{\partial K_1} - x_1X_1A - \ell L_1A - Ry_1^2Y_1\sigma_{y_1}^2\frac{\partial Y_1}{\partial K_1} - Ry_1^2Y_1\sigma_{Y_1}^2\frac{\partial Y_1}{\partial K_1}$$
$$+y_2\left[1+\delta_{y_2}\right]\frac{\partial Y_2}{\partial K_1} + x_2X_2A + \ell L_2A - Ry_2^2[Y_2+S_2-Q_2]\sigma_{y_2}^2\frac{\partial Y_2}{\partial K_1} - Ry_2^2Y_2\sigma_{Y_2}^2\frac{\partial Y_2}{\partial K_1} = 0;$$

$$(21)\quad a[\delta_a - \delta_W] - Ra^2A\sigma_a^2$$
$$+y_1\left[1+\delta_{y_1}\right]\frac{\partial Y_1}{\partial A} - x_1X_1K_1 - \ell L_1K_1 - Ry_1^2Y_1\sigma_{y_1}^2\frac{\partial Y_1}{\partial A} - Ry_1^2Y_1\sigma_{Y_1}^2\frac{\partial Y_1}{\partial A}$$
$$+y_2\left[1+\delta_{y_2}\right]\frac{\partial Y_2}{\partial A} - x_2X_2(1-K_1) - \ell L_2(1-K_1) - Ry_2^2[Y_2+S_2-Q_2]\sigma_{y_2}^2\frac{\partial Y_2}{\partial A} - Ry_2^2Y_2\sigma_{Y_2}^2\frac{\partial Y_2}{\partial A} = 0.$$

Neither the basic household model nor other models include decisions about land allocation. In optimality condition (20), the household chooses the proportion of land allocated to growing commodities 1 and 2. These crops will be rotated for biological reasons and diversified to manage risks. In optimality condition (21), the household either chooses the area of the farm or, if the area is fixed, calculates the shadow price of land.

Illustration of the Household Model

The optimality conditions are a nonlinear system of equations that must be solved simultaneously. For this, utility and yield functions are required.

Utility of Consumption, U

$$(22) \quad U(Q) = (Q_2 - \beta_2)^{\alpha_2} (Q_3 - \beta_3)^{\alpha_3} (Q_4 - \beta_4)^{\alpha_4}.$$

This functional form of utility leads to a linear expenditure system.

Yield, Y_1 and Y_2

$$
\begin{aligned}
(23) \quad Y_1(L_1, X_1, K_1, A) &= f_1(L_1, X_1) g_1(K_1) h(A); \\
f_1(L_1, X_1) &= \gamma_1 L_1^{\eta_1} X_1^{\mu_1} e^{\nu_1 X_1}; \\
g_1(K_1) &= K_1^{\phi_1} e^{\theta_1 (1 - K_1)}; \\
h(A) &= \omega A^{\xi} e^{\psi A}; \\
(24) \quad Y_2(L_2, X_2, K_1, A) &= f_2(L_2, X_2) g_2(K_1) h(A); \\
f_2(L_1, X_1) &= \gamma_2 L_2^{\eta_2} X_2^{\mu_2} e^{\nu_2 X_2}; \\
g_2(K_1) &= (1 - K_1)^{\phi_2} e^{\theta_2 K_1}.
\end{aligned}
$$

Yields are assumed to be multiplicatively separable into functions *f*, *g* and *h*. Function *f* is the yield per unit of land area. It is assumed to be a Cobb-Douglas function in labour and a transcendental function in variable inputs. Function *g* incorporates yield interactions from crop rotations among commodities. If there are no yield interactions, ϕ will be 1 and θ will be zero. In this case, g_1 will equal K_1 and g_2 will equal $(1 - K_1)$. Function *h* incorporates economies of farm size and is the same for both commodities. If there are no economies of size, ω and ξ will be 1, ψ will be zero and function *h* will equal *A*. If there are no yield interactions or economies of size, the yields per farm are simply the yields per unit of area multiplied by the areas allocated to each crop, $Y_1 = f_1 K_1 A$ and $Y_2 = f_2(1 - K_1)A$.

Parameters for illustrating the model are listed in Table A1 of the Appendix. The model is solved using the Reduced Gradient and Newton-Raphson algorithms implemented by Solver in Excel. Table 2 contains the baseline solution for consumption, decision variables and functions in the model.

Table 2. Baseline Solution.

		Commodity			
	General	1	2	3	4
Q			1420.92	439.14	2072.85
L		366.48	907.07		
X		5.55	4.91		
S			4652.74		
K		0.68			
a	76724.92				
$U(Q)$	514.97				
$Y(L,X,K,A)$		1080.39	1237.80		
$f(L,X)$		163.99	300.46		
$g(K)$		0.69	0.43		
$h(A)$	9.61				
$Y+S-Q$			4469.62		
$T-L_1K_1A-L_2(1-K_1)A-Q_4$	-460.42				

Demand for Commodity 2

Interestingly, the household can hedge against price risk by consuming commodity 2. The demand curve is shown in Figure 1.

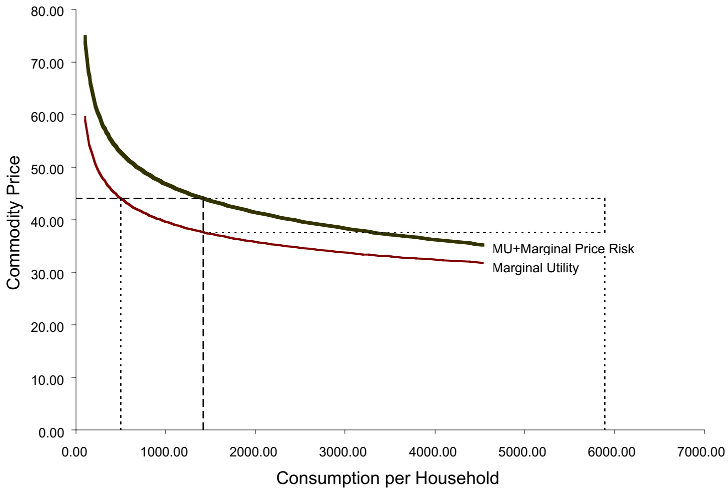

Figure 1: Demand for Commodity 2 Under Price Risk.

The demand curve is marginal utility plus the marginal risk premium. At low consumption, there is less of a hedge against risk and the marginal risk premium is greater. At high consumption, there is more of a hedge and the marginal risk premium is less. If there was no price risk, the marginal risk premium would collapse to zero and the household would consume 504 units where the expected price per unit of 54 intersects marginal utility. Instead the household hedges against price risk and consumes 1,421 units. Yield plus storage is 5,891 units. Subtracting consumption consumed leaves 4,470 units unhedged.

Storage might also be a hedge against price risk (Saha and Stroud, 1994). Unfortunately in optimality condition (19), storage introduces marginal risk premiums for the seasonal price and physical degradation during storage. Therefore, like yield, storage is undertaken because the household expects to make a profit from a risky situation.

Commodity Supply under Risk

Optimality conditions (15) through (18) define the derived demands for labour and variable inputs. These derived demands can be rearranged to show that the expansion path for inputs is unaffected by risk. For example along an isoquant, $dY = (\partial Y/\partial L)dL + (\partial Y/\partial X)dX = 0$, or the marginal rate of substitution is $dL/dX = -(\partial Y/\partial X)/(\partial Y/\partial L)$. Putting wage and input costs on the right-hand sides and dividing condition (17) by condition (15) shows that the marginal rate of substitution equals the negative price ratio.

$$\frac{dL}{dX} = -\frac{\partial Y/\partial X}{\partial Y/\partial L} = -\frac{x}{\ell}.$$

The expansion path is unaffected by risk because labour and variable inputs are purchased at the beginning of each season for known prices. The expansion path is shown in Figure 2

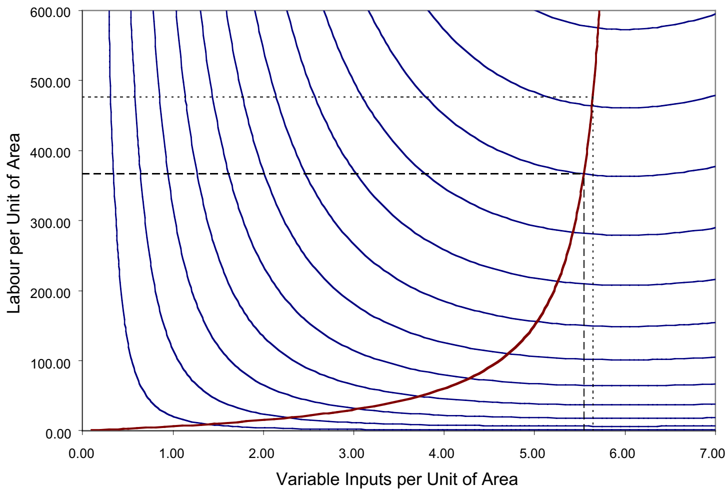

Figure 2. Expansion Path for Commodity 1.

Each point on the expansion path is a least-cost combination of inputs to produce a given yield. Figure 2 identifies two of the many possible points. The lower point is the baseline solution of 366 units of labour and 5.55 units of variable inputs. The higher point is the solution without price or yield risks of 476 units of labour and 5.64 units of variable inputs.

All least-cost combinations together give the total cost curve as a function of yield, or $c(Y)$. The derivative of total costs is marginal costs. As a result, the optimality conditions for labour and variable inputs are equivalent to a dual condition in which yield is the decision variable.

$$y_1\left[1+\delta_{y_1}\right] = \frac{\partial c}{\partial Y} + R y_1^2 Y_1 \sigma_{y_1}^2 + R y_1^2 Y_1 \sigma_{Y_1}^2 .$$

On the left-hand side is marginal revenue and on the right-hand side is the supply curve under risk, including marginal risk premiums for price and yield risks. A similar dual condition gives the supply curve for commodity 2. Optimal yield of commodity 1 is shown by the supply curve in Figure 3.

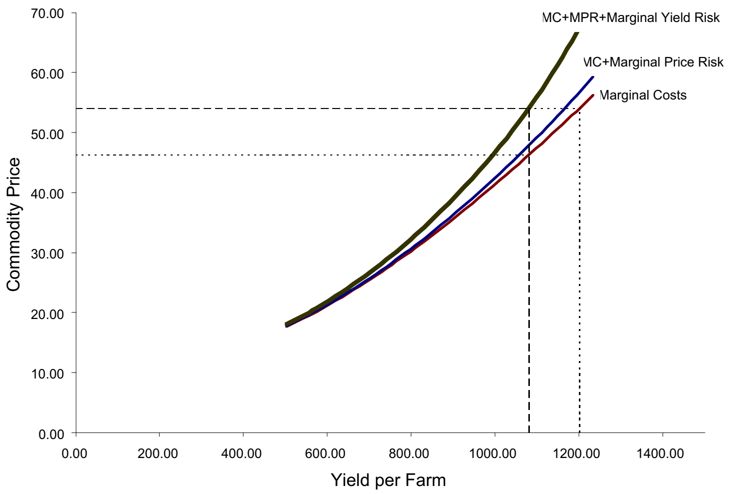

Figure 3. Supply of Commodity 1.

The supply curve is the top curve in which marginal risk premiums for price and yield risk are added to the marginal cost curve. The expected marginal revenue of 54 intersects the supply curve at an optimal yield of 1,080 units. The required labour and variable inputs are the lower point on the expansion path in Figure 2. Commodity 1 is grown on 68% of the farm. Suppose this percentage remains fixed and price and yield risks can be completely hedged. The marginal risk premiums would collapse toward zero and the household would produce 1,201 units where expected marginal revenue intersects the marginal cost curve. The required labour and variable inputs are the higher point on the expansion path. However, as risks are hedged, the allocation of land will change and the marginal cost curve will shift. Because decisions are not separable, shifting the supply curve changes all other decisions. Changing other decisions, in turn, shifts the supply curve even more. Indeed, the supply curve in Figure 3 only applies to the baseline solution.

Production Possibilities Frontier

The production possibilities frontier shows the yields of commodities 1 and 2 for all possible land allocations. The optimal allocation is defined by condition (20). To help in interpreting this condition, suppose there are no yield interactions between commodities with $g_1(K_1) = K_1$ and $g_2(K_1) = 1 - K_1$. Also suppose there are no economies of farm size with $h(A) = A$. Then the optimal allocation simply equates the gross margins per unit of land area.

$$y_1\left[1+\delta_{y_1}\right]f_1 - x_1X_1 - \ell L_1 - Ry_1^2Y_1\sigma_{y_1}^2 f_1 + Ry_1^2Y_1\sigma_{Y_1}^2 f_1$$
$$= y_2\left[1+\delta_{y_2}\right]f_2 - x_2X_2 - \ell L_2 - Ry_2^2[Y_2 + S_2 - Q_2]\sigma_{y_2}^2 f_2 + Ry_2^2Y_2\sigma_{Y_2}^2 f_2$$

These are not simple gross margins but have marginal risk premiums for price and yield risks. With yield interactions and economies of farm size, the gross margins are more complicated yet. Because an analytical solution is intractable, the production possibilities frontier and the optimal allocation of land are calculated numerically, as shown in Figure 4.

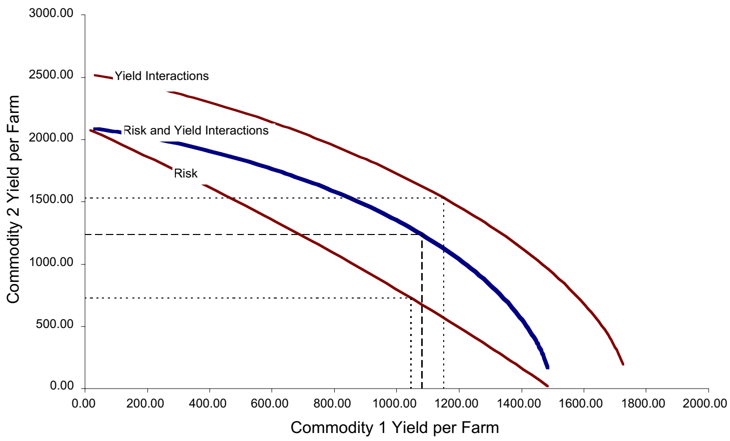

Figure 4. Production Possibilities Frontier.

The production possibilities frontier is the middle curve in the figure. At the optimum, 68% of the land is allocated to commodity 1 and 32% to commodity 2. Yields are 1,080 units and 1,238 units per farm. If there were risks, but no yield interactions, the frontier would be the lower curve with optimal yields of 1,046 and 729 units. Notice that this lower curve is not a straight line. The production possibilities frontier would be straight only if there were no risks and no yield interactions. Then, growing more of one commodity would simply displace the other commodity from the land. With risk, however, commodities interact through the marginal risk premiums. Conversely, if there were yield interactions but no risk, the frontier would be the higher curve in the figure with optimal yields of 1,150 and 1,531 units. Because it includes both risks and yield interactions, the production possibilities frontier is more curved than either of the other curves.

How can risk shift the production possibilities frontier? A frontier is supposed to identify the set of all efficient allocations. Any point interior to the frontier is supposed to be inefficient. This is certainly the case for mean-variance and portfolio models. In this model, the frontier shifts for the same reason the supply curves do. Decisions are not separable because risks are log-normally distributed which implies that the marginal risk premiums increase with yields. A household chooses how much land to allocate to each crop and how risky those crops will be. For example, a household will use less labour and variable inputs because a lower yield is less risky. As a result, a risk-averse household will be on a lower production possibilities frontier. A risk-neutral household will choose more labour and variable inputs and be on a higher frontier. Every household chooses a different frontier of efficient allocations.

Demand for Land

Optimality conditions (20) and (21) define the demand for land. For a given price, the optimal quantity of land can be chosen, or for a fixed quantity, the shadow price of land can be calculated. Again to help with the interpretation, suppose there are no yield interactions or economies of farm size. Combining and rearranging the optimality conditions gives the shadow price of land.

$$a = \frac{y_2\left[1+\delta_{y_2}\right]f_2 - x_2 X_2 - \ell L_2 - R y_2^2\left[Y_2 + S_2 - Q_2\right]\sigma_{y_2}^2 f_2 + R y_2^2 Y_2 \sigma_{Y_2}^2 f_2}{\delta_W - \delta_a} - \frac{R a^2 A \sigma_a^2}{\delta_W - \delta_a}.$$

On the right-hand side, the first ratio is the gross margin per unit of land area capitalized at a real rate of return. The real rate of return is the risk-free rate minus the expected rate of capital gains on land. The second ratio adjusts the price of land for the marginal risk premium on capital gains. With yield interactions and economies of farm size, the shadow price of land is more complicated and is solved numerically, as shown in Figure 5.

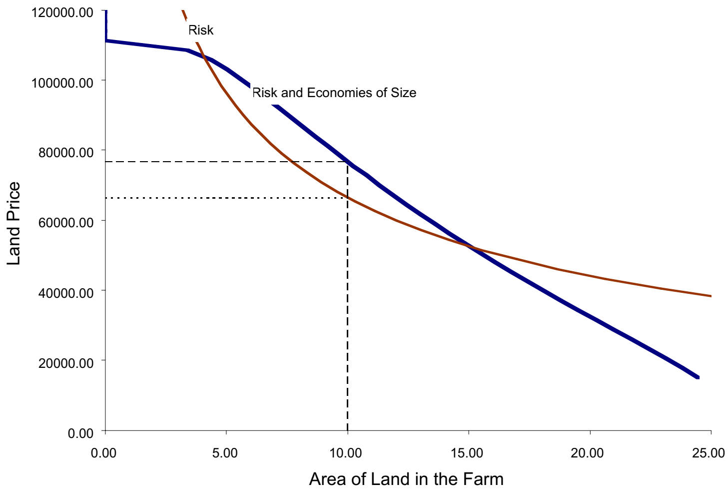

Figure 5. Demand for Land.

The demand curve for land includes risks and economies of size. A farm of less than 3.40 units of land is not viable and the demand curve is kinked. Above 3.40 units of land, the demand curve slopes downward because increasing the size of the farm increases the exposure to risk. A risk-averse household is willing to pay less if it already has a large farm. On a farm with 10 units of land, with risks but no economies of size, the shadow price of land would be 66,387 per unit. With risks and economies of size, the shadow price of land is 76,725 per unit. A farm with less than 15 units of land has economies of size and will produce more efficiently if it becomes larger. Conversely, a farm with more than 15 units of land has diseconomies of size and will produce more efficiently if it becomes smaller.

Surplus Labour

Restructuring farm sizes may be beneficial. A potential problem, however, is displacing households from the land, leaving them unemployed. An indicator of employment is the surplus labour of a household, as shown in Figure 6.

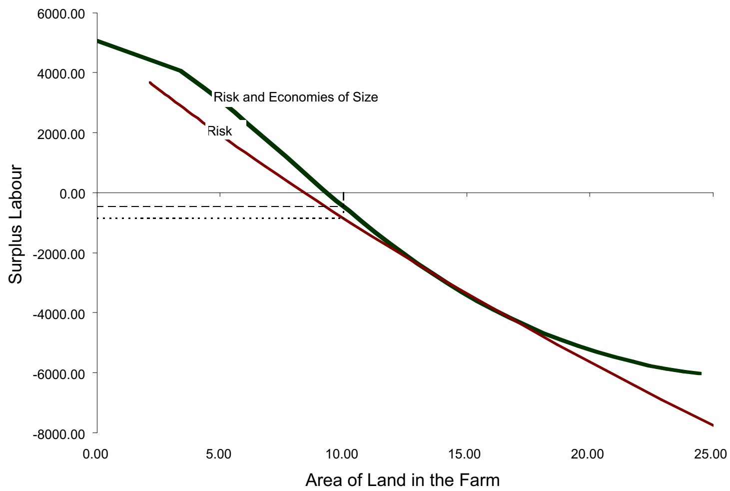

Figure 6. Surplus Labour by Farm Size.

Households on small farms have a positive surplus and hire out their labour. Households on larger farms have a negative surplus and hire in labour from other households. Economies of size cause the household to hire out more of its own labour and hire in less of other household's labour, except at the most efficient farm size of 15 units of land area. For an area of 10 units, the surplus is –460 units of labour. If there were no economies of size the surplus would be –852 units of labour. Increasing farm sizes to 15 units of land will absorb labour into farming rather than displace it. For example, six households each farming 5 units of land will have a surplus of 3,080 units of labour per household or 18,480 units in total for all households. If the first four households sell their land to the fifth and sixth households, four households will have of 0 units of land and 2 households will have 15 units of land. The first four households will have a surplus of 5,087 units of labour each and the fifth and sixth households will have a surplus of –3,407 units of labour each, or 13,534 units in total for all households. The surplus is reduced and the households employ more, not less, of their labour in agriculture. Of course, if the fifth household then sells its land to the sixth household, five households will have no land and a surplus of 25,435 units of labour and the sixth household will have 30 units of land and a surplus of –6,143 units of labour, or 19,294 units in total for all households. As farms increase beyond the most efficient size, surplus labour goes up and households are displaced from agriculture. Further, if there were risks but no economies of size, amalgamating farms would always displace households.

Land Trading

On way to restructure farm sizes is by trading of land among households. If no trading is allowed, the shadow price of land will vary among households. If trading is allowed, the price of land will reach equilibrium. One possible equilibrium is shown in Figure 6.

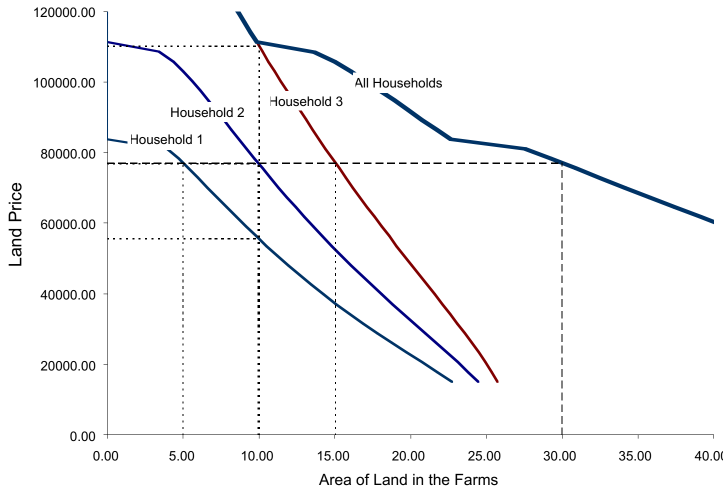

Figure 7. Trading of Land Among Households.

In the figure, three households are the same except for different degrees of absolute risk aversion. Household 2 has the degree of risk aversion listed in Table 2 and the same demand curve as in Figure 5. Household 1 is 1.8 times as risk averse and Household 3 is 0.5 times as risk averse as Household 2. With no trading of land, each household farms 10 units. Household 3 would be willing to pay 110,195 per unit of land and Household 1 would be willing to accept 55,528 per unit. With trading, Household 3 buys 5 units of land from Household 1 at an equilibrium price of 76,998 per unit. Household 3 actually pays less than they were willing to pay and Household 1 actually receives more than they were willing to accept. Although trading is voluntary, all households volunteer. As measured by the maximand in equation (8), trading of land increases everyone's welfare. With no trading, surplus labour for all three farms was –1,384 units. With trading, surplus labour is –1,191 units. The efficiencies of Household 3 are more than offset by the inefficiencies of Household 1. Even with economies of farm size, trading of land may displace the more risk-averse households from agriculture.

Concluding Comments

The behaviour of households under risk is complex. Through the marginal risk premiums, each decision affects every other. Decisions about consumption and investments in land cannot be separated. Consumption reduces wealth and decreases the household's ability to invest. Consumption and production decisions cannot be separated. Consumption of commodities that are produced by the household is a hedge against price risk. Hedging against price risk shifts the marginal risk premium and, hence, shifts the supply curve. Storage is not an effective hedge because storage, itself, is risky. Investment and production decisions cannot be separated because the quantity of land shifts the supply curve and also shifts the production possibilities frontier.

The only decision that can be separated is determining the expansion path of least-cost combinations of inputs. Decisions about inputs are made at the beginning of each season at known prices. However, the optimal choice of inputs is indirectly affected by risk because yields depend upon risks. Supply curves will shift with marginal risk premiums for price and yield risks. The more risk-averse a household, the less they will supply and the fewer inputs they will use.

Unlike the expansion path for inputs, the production possibilities frontier for yields is directly affected by risks. The production possibilities frontier curves out even if there are no yield interactions. Yields of different commodities interact through the marginal risk premiums and a household diversifies their cropping as a hedge against price and production risks. With risk and yield interactions, a household also rotates crops around the farm. There is no simple rule for the optimal allocation of land among crops. The entire model must be solved as a simultaneous system.

The demand curve for land slopes downward. A risk-averse household is willing to pay less for land as their farm size increases. This is because a larger farm increases the household's exposure to risk. If there are economies of farm size, there will be a minimum size that is viable. Above the minimum size and below the most efficient size, economies of size will increase the household's willingness to pay for land. Above the most efficient size, diseconomies of farm size will lower the household's willingness to pay.

Restructuring of land may displace households and leave them unemployed. However, if there are economies of farm size, restructuring can absorb more labour into agriculture, not less. Restructuring may take place with trading of land among diverse households. Trading is always voluntary and the welfare of all households will improve. However, trading need not achieve efficient farm sizes and risk-averse households may be displaced.

In further research, the model will be expanded for more diverse types of households. For example, households headed by men differ from households headed by women (de Janvry, *et al.*, 1992; Sawit and O'Brien, 1995). Men and women may have different objectives but they are linked by a common budget constraint. The household model might also be aggregated into a village model with trading of labour and land among diverse households. For example, the household that sells land may not be displaced from the village. They may start a bank or other business. The village model will allow policies for development to be analysed in detail. It is unlikely that any single policy will both foster development and prevent displacement of households. Finally, wealth and the degree of risk aversion might be made endogenous to the model. Many households have no social safety net and will remain viable only if wealth remains above a subsistence level. Determining the behaviour of subsistence households under risk will require the solution of the model as a stochastic dynamic program.

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Appendix

Table A1. Parameters for Baseline Solution.

	General	Commodity 1	2	3	4
W	100000				
R	0.00001				
λ	0.006				
T	7000				
A	10				
ℓ	8				
y		60	40		
x		120	80		
p			30		
s			5		
q				40	
δ_W	0.05				
δ_a	0.04				
δ_y		-0.1	0.1		
δ_S			0.1		
σ_a	0.1				
σ_y		0.4	0.3		
σ_p			10		
σ_Y		0.2	0.1		
σ_S			0.4		
α			0.6	0.2	0.1
β			50	10	1000
γ		6	4		
η		0.4	0.5		
μ		1.2	1.5		
ν		-0.2	-0.3		
ϕ		0.9	0.8		
θ		-0.1	0.1		
ω	0.36				
ξ	1.6				
ψ	-0.04				