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## **Modeling Policy Options for Irrigation Investment, Buffer Stock Levels and Foodgrain Imports in India**

by

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## MODELING POLICY OPTIONS FOR IRRIGATION INVESTMENT, BUFFER STOCK LEVELS AND FOODGRAIN IMPORTS IN INDIA

### Abstract

With a population near 800 million and an annual growth rate of 2.2%, India continues to face problems in meeting its goals of food security and stability. In the past, irrigation investments and a buffer stock program have been important government of India (GOI) policy instruments for stabilizing foodgrain supplies. This paper develops a food grain sector model that links an optimization model and a simulation model through a stochastic rainfall model. The foodgrain model is used to estimate alternative paths of future irrigation investment across six regions for different buffers stock levels. Optimal operating rules are developed for both irrigation investment and the buffer stock program.

The foodgrain model suggests that India's goal of self-sufficiency needs to be redefined. The complementarities among irrigation investment (new irrigation and rehabilitation), the buffer stock program, and the level of foodgrain imports show that these policy instruments need to be considered together. Each cannot be optimized in isolation. For example, as the relative cost of irrigation continues to climb, foodgrain imports and the buffer stock program may well have to play a larger role in stabilizing foodgrain supplies. If the irrigation budget is reduced by 50% because of higher costs, expected annual foodgrain imports of 8 million tons will be needed to meet future foodgrain targets. The regional distribution of investments may also need to be more targeted. Certain regions such as southern India and Uttar Pradesh may require special emphasis because of their

unique contribution to stability in foodgrain production and their potential for increased irrigation.

The simulation and optimization models together with the stochastic rainfall model provide a powerful tool for analyzing irrigation investment strategies over time and among regions as well as the trade-offs with the buffer stock program and foodgrain imports for meeting GOI's foodgrain objectives. In the final analyses, the effectiveness of GOI's irrigation investment and buffer stock program will depend on a well specified foodgrain import policy. In fact, a more liberal foodgrain import policy is probably in India's own best interests as it tries to increase and bring stability to future foodgrain supplies.

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**MODELING POLICY OPTIONS FOR IRRIGATION INVESTMENT,  
BUFFER STOCK LEVELS AND FOODGRAIN IMPORTS IN INDIA**

Jorge Ramirez and K. William Easter\*

**Introduction**

India has a population of nearly 800 million, growing at an annual rate of about 2.2%. Although since 1960 average real income has increased, about 25% of the population continues to live in conditions of absolute poverty. Most of the poor are in rural areas and depend on agriculture for their living. Agriculture continues to be the dominant sector of the Indian economy, contributing about 40% of the national income and engaging almost 70% of the total labor force of the country.

Agricultural growth will be critical to achieving India's goal of eliminating poverty, creating full employment, and generating self-sustaining economic growth by the year 2000. Success will depend not only on growth in agricultural production, but also on the particular paths chosen to meet growth targets.

In this difficult environment Indian policy-makers, deciding on how irrigation development should proceed, have inherited an additional challenge. They must face the reality that irrigation has grown relatively more expensive and will get even more expensive

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in the future, since the lower cost sites were developed first. The unit cost of establishing and maintaining irrigation capacity is rising at an accelerating rate. Thus, returns to irrigation investment will diminish if output prices increase at a lower rate than costs. In addition, by the year 2010, the Seventh Plan's projections indicate that India will have reached her ultimate irrigation potential, estimated at 113.5 million ha.

Irrigation is one important means of stabilizing annual crop production by mitigating the impact of irregular and inadequate rainfall. Another stabilization option is to carry a buffer stock. Because of the high variability of foodgrain production, the Government of India (GOI) has long used the irrigation option and, in 1969, decided to also build a buffer stock of foodgrains. By 1985, India had 24 million tons of foodgrains in its buffer stock.

This paper presents a model for determining optimal paths of future irrigation investment across different regions for different buffer stock levels in order to meet the goals set by the Planning Commission (self-sufficiency and output stability). A stochastic approach is used to analyze the Indian foodgrain sector and to link the optimization and simulation models which serve as a means for establishing optimal operating rules for irrigation investment and the buffer stock program.

The stochasticity of rainfall is introduced into the Indian foodgrain model, through a multivariate autoregressive model which generates monthly rainfall at the state (geographic) level (figure 1). This model provides us with the flexibility to explore the behavior of the system under statistically real historical rainfall patterns.

### Agricultural Sector Model

The sector is made up of three distinct submodels as shown in figure 1. The first is an optimization model which consists of a dynamic programming model where the objective function involves optimizing the expected value of government expenditures on the irrigation and buffer stock programs. Optimal irrigation investment is found from the programming model which is directly fed into the simulation model, where it is tested under a stochastic environment (rainfall). To perform this test, a buffer stocking policy and/or a foodgrain import policy have to be defined ex-ante. This test is performed with a number of simulations, each for a particular rainfall sequence generated using the multivariate AR(1) model explained below. The simulation process tests the reliability of the system under the stochastic environment, allowing the estimation of optimal operating rules. Finally, sensitivity analysis is performed.

#### The Objective Function

An agricultural sector model consists of, at least, a two-level optimization problem. First, is the problem of optimally allocating public resources under set goals and constraints, and second is the problem of predicting farmers' response. In most cases, the farmer's maximization problem is simplified by estimating 'reaction' or 'behavioral' functions.

In terms of the first problem, it would be ideal if economic policy could be chosen based on a single, well-established objective. But there are often conflicting goals and objectives, and public policy is often formulated on the basis of a qualitative integration of numerous economic, political, social, and technological objectives. Therefore, it could be



argued that the objective function for the agricultural sectoral model should be a weighted set of policy objectives, both economic and non-economic. In India, most major irrigation investment are made by government or quasi-government units, and their activities are guided by national and state agency, and constituency objectives.

Given the various objectives of GOI and provided that they try to satisfy their constituents while operating under budget constraints, the objective function for the Indian model is defined as a government expenditure function. This function can be considered as an approximation of the consumer and producer surplus function, since most of the model's components represent government subsidies.<sup>1</sup> Even a deviation from the "ideal" objective function, from the economic point of view, can be permitted because the optimization model provides only initial optimal values which are later tested under a stochastic environment using a simulation model.

### The Optimization Model

The model disaggregates irrigation into two types of irrigation investment; (1) irrigation by minor schemes which involves a large private investment component ( $j = 1$ ) and (2) irrigation by major and medium schemes which are based primarily on government investments ( $j = 2$ ). In India, this classification corresponds fairly closely to the classification

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<sup>1</sup>When society's objective function includes only economic efficiency goals, Samuelson (1952) has shown that there exists a maximization model that will simulate the market outcome under conditions with downward-sloping demand functions. The optimum is achieved when the sum of producer and consumer surplus is maximized. Yet the optimum level found with optimization models represents a partial equilibrium rather than a general equilibrium. Using a general equilibrium model requires numerous simplifying assumptions and other complications (Norton and Scandizzo, 1981).

by source; ground water and surface water irrigation. The only major exception is tank (small reservoir) irrigation, which is included in minor irrigation and is important in South India. The model also allows for investment to rehabilitate existing major and medium irrigation systems.

Because of the wide diversity that exists in a country as large as India, the model includes the seventeen major agricultural states and divides them into six regions ( $i = 1, \dots, 6$ ).<sup>2</sup> The regional classification is based on contiguity, net irrigated area and foodgrain production, and is the same as the one used by Sarma and Gandhi, 1989, in their study of foodgrains in India. The diversity among regions is exemplified by the Southern region which has a rainfall pattern that is distinctly different from the pattern found in the other five regions, since it benefits from two annual monsoons.

The *dynamic programming recursive equation* (Bellman, 1962) of the Indian foodgrain model is written as:

$$F_t(\underline{W}_t) = \min_{\underline{W}_{t+1}} [\delta^t f_t(\underline{W}_t, \underline{W}_{t+1}, \underline{G}_t) + F_{t+1}(\underline{W}_{t+1})] \quad (1)$$

$$t = 1, \dots, T$$

and  $\underline{W}_t = \{\underline{IR}_t, \underline{BS}_t\}$ ;

$\underline{G}_t = \{\underline{U}_t, \underline{R}_t\}$ .

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<sup>2</sup>SOUTHERN: Kerala, Tamil Nadu, Andhra Pradesh; NORTHERN: Jammu and Kashmir, Himachal Pradesh, Punjab, Haryana; EASTERN: Orissa, Bihar, West Bengal, Assam; WESTERN: Gujarat, Maharashtra, Karnataka; CENTRAL: Rajasthan, Madhya Pradesh; UTTAR PRADESH: Uttar Pradesh.

The optimal value function  $F_t(\cdot)$  is the minimum present value of the cost function  $f_t(\underline{W}_t, \underline{W}_{t+1}, \underline{G}_t)$  required to meet the foodgrain demand targets, based on population projection and the Planning Commission's objectives of sustained economic growth and poverty alleviation.

In this recursive equation  $\underline{W}_t$  and  $\underline{G}_t$  are state and decision (control) variables respectively, and they represent matrices of  $2I \times J$  entries. The total area irrigated by source  $j$  in region  $i$  at time  $t$  is  $IR_{ijt}$ , while  $BS_t$  is the quantity of foodgrains in the buffer stock program at time  $t$ . The area brought in under new irrigation schemes by source  $j$  in region  $i$  at time  $t$  is  $U_{ijt}$ , while  $R_{i2t}$  is the increased irrigation due to the rehabilitation of medium and major schemes in region  $i$  entering at time  $t$ . Rehabilitation in minor irrigation is not included in the model.

The cost function  $f_t(\underline{W}_t, \underline{W}_{t+1}, \underline{G}_t)$  comprises the following terms: (1) the public cost of the new irrigation entering at time  $t$ ; (2) the public subsidy of operation and maintenance (expenses-receipts) of existing irrigation systems; (3) the public cost of rehabilitation of major and medium schemes (this is also a decision variable); (4) the public subsidy of providing electricity for minor irrigation; (5) the public cost of the foodgrain stocks needed to meet foodgrain demand requirements; and (6) the public subsidy of distributing foodgrains through the foodgrain distribution system. If foodgrain imports are allowed into the country, then two additional components are needed in the objective function: (7) the public cost of purchasing foreign foodgrains and, (8) the public cost of distributing them within the country. The social discount factor is  $\delta$ .

The details of cost function are as follows:

$$f_t(\underline{W}_t, \underline{W}_{t+1}, \underline{G}_t) = \sum \sum C_{ijt} U_{ijt} + \sum \sum O_{ijt} IR_{ijt} + \sum M_{i2t} R_{i2t} + \sum N_{i1t} IR_{i1t} \\ + D_t (Y_t - \sum Y_{it}) + S_t BS_t + (PW_t + D_t) IMP_t - PW_t EXP_t;$$

where:

$C_{ijt}$  is the public cost of developing a new irrigation hectare by source  $j$  in region  $i$  at time  $t$ . These costs are a function of the remaining irrigation potential at time  $t$ ,  $C_{ijt} = \Phi(PIR_{ij} - IR_{ijt})$ , where  $PIR_{ij}$  is the ultimate potential of irrigated area by source  $j$  in region  $i$  and  $IR_{ijt}$  is the total area irrigated by source  $j$  in region  $i$  at time  $t$ ;

$O_{ijt}$  is the per ha operation and maintenance (O&M) cost of existing irrigation systems by source  $j$  in region  $i$  at time  $t$ . In the case of major and medium projects, this parameter represents the actual subsidy (working expenses - government recipients) per ha.;

$M_{i2t}$  is the per ha cost of rehabilitating major and medium schemes in region  $i$  at time  $t$ ;

$R_{i2t}$  and  $BS_t$  were defined above as the new irrigation area due to rehabilitation and the buffer stock;

$N_{i1t}$  is the public subsidy of providing electricity for irrigating one more hectare by minor schemes;

$Y_{it}$  is the foodgrain production of region  $i$  at time  $t$ ;

$Y_t$  is the predicted demand for foodgrains for the whole country at time  $t$  which is exogenously determined. Price induced changes in the quantity demanded can also be considered;

$S_t$  is the unit public capital cost of the buffer stocks at time  $t$ ;

$IMP_t$  and  $EXP_t$  are the foodgrain imports and exports at time  $t$  (these are also decision variables);

$PW_t$  is the foodgrain price in the world market; and,

$D_t$  is the public subsidy for distributing one ton of foodgrain within the country when  $Y_t > \sum Y_{it}$ ; or the procurement cost when  $Y_t < \sum Y_{it}$ .

**Subject to:**

(i) The following state equations for irrigated area:

$$IR_{i2t} = IR_{i2t-1} (1-\alpha_2) + \beta_{i2} U_{i2t} + R_{i2t} \quad i=1,...,I;$$

$$IR_{i1t} = IR_{i1t-1} (1-\alpha_1) + \beta_{i1} U_{i1t} \quad i=1,...,I;$$

in which  $IR_{ijt}$  is the total irrigated area,  $\alpha_j$  is the depreciation factor of existing irrigation systems, and  $\beta_{ij}$  is the percentage of utilization of irrigation potential created in region  $i$  by source  $j$ .

(ii) The state equation for the buffer stock program written as:

$$BS_t = \mu BS_{t-1} + \sum Y_{it} + IMP_t - Y_t - EXP_t$$

$$\text{and } BSMIN_t \leq BS_t \leq BSMAX_t;$$

where  $\mu$  is the depreciation factor for the buffer stock,  $BS_t$ , and  $BSMAX_t$  is a limit on the size of the buffer stock at time  $t$ , which is a function of: (1) the frequency and size of fluctuation in foodgrain production; (2) the limits set by the budget; (3) the ability to procure and store foodgrains; and (4) the extent of the open market price fluctuations permitted. The lower limit of the size of the stock  $BSMIN_t$ , is determined by the financial feasibility of the buffer stock program and GOI's pre-established level for the buffer stock.

(iii) The production functions or "behavioral" functions for the central planning unit as follows:

$$Y_{it} = \Phi_{it}(IR_{it}, FERT_{it}, HYV_{it}, CROP_{it}, MRAIN_{it}) \quad i = 1, \dots, I;$$

where FERT, HYV, CROP and MRAIN are fertilizer, high yielding varieties, cropping land, and average rainfall of region  $i$  for the agricultural season in year  $t$ . Rainfall is the stochastic part of the model and its treatment is explained below. These production functions were estimated using the fixed-effects approach (Judge, 1985) which combines time series and cross-sectional data.

In these production functions,

$$IR_{it} = \sum IR_{ijt}$$

$$P_{ut} = f(\eta_t, \sum Y_{it}, Y_t)$$

$$FERT_{it} = f(IR_{it}, P_{ut}, t) \quad i = 1, \dots, I$$

$$HYV_{it} = f(IR_{it}, t) \quad i = 1, \dots, I$$

where  $P_{ut}$  is the price of foodgrains at time  $t$ . The price elasticity of the foodgrains demand is  $\eta_t$ . These functions first make the new technology, fertilizer and HYV adoption, endogenous in the optimization and simulation exercises, and second introduce the domestic price level of foodgrains in an exogenous variable which is needed to explain the adoption of fertilizers.

(iv) The following irrigation investment budget and area constraints:

$$\sum C_{ijt} U_{ijt} + \sum M_{i2t} R_{i2t} \leq \text{BUDG}_{it} \quad i=1,\dots,I;$$

$$\sum U_{ijt} \geq \text{MINIR}_{it} \quad i=1,\dots,I;$$

$$\sum \sum C_{ijt} U_{ijt} + \sum M_{i2t} R_{i2t} \leq \text{TBUDG}_t$$

where  $\text{BUDG}_{it}$  is the available budget that can be used for irrigation investment in region  $i$  at time  $t$ , while  $\text{MINIR}_{it}$  is a lower bound of new irrigated area entering region  $i$  at time  $t$ , and  $\text{TBUDG}_t$  is the irrigation investment budget for the whole country at time  $t$ .

(v) The irrigation potential constraints:

$$\text{IR}_{ijt} \leq \text{PIR}_{ij} \quad i = 1,\dots,I \quad j = 1,\dots,J$$

in which  $\text{PIR}_{ij}$  is the ultimate potential of irrigated area by source  $j$  in region  $i$ .

(vii) The electric power generation constraints:

$$\text{IR}_{i1t} \leq \text{EIR}_{i1t} \quad i = 1,\dots,I$$

in which  $\text{EIR}_{i1t}$  is an upper constraint on area irrigated by minor schemes due to the limits of future expansion in rural electrification.

The profitability of groundwater development is strongly affected by the costs of energy. Since subsidized electricity is by far the farmers' cheapest source of energy for pumping, rural electrification is a fundamental requirement for further expansion of minor irrigation. Thus, future development in minor irrigation will be conditional on the rate of expansion in rural electrification. The electric power constraints account for the impacts on irrigation of the limits to expansion of rural electrification.

### The Simulation Model

The simulated model consists of almost the same equations as the optimization model described above, except that the inequality signs are changed to equality signs (See Ramirez, 1990, for details). The simulation model represents the behavior of the system under some predetermined decision variables (new irrigated areas,  $\underline{U}$ , rehabilitated irrigation,  $\underline{R}$ ) and some preestablished operating rules of the system regarding the buffer stock program and foodgrain imports.

### A Stochastic Model for Rainfall in India

Instability of Indian foodgrain production has been extensively studied by a number of researchers who found rainfall to be the main factor contributing to annual foodgrain instability (Ray, 1987; Hazell, 1982; Mehra, 1981). They also found that foodgrain instability has increased with the increase in foodgrain production.

To study this foodgrain instability in India, three statistical descriptors of rainfall are considered of paramount importance (Ray, 1981, Hazell, 1982). These are: (1) the mean values of precipitation at a given geographical points; (2) its variability over time and, (3) the variability of rainfall across different geographical states.

The strong connection between rainfall and the instability of foodgrain output in India suggests that a study of the foodgrain variability in India should include a complete modeling of this particular stochastic process. Moreover, the importance of the cross-correlation structure of foodgrain production at the state level on foodgrain variability (Hazell, 1982) reinforces the need for such an approach. As an input into the foodgrain



model, seasonal rainfall is simulated using a multivariate lag-one Markov model AR(1) with constant parameters (Matalas, 1967).

### The Multivariate AR(1) Model

The multivariate model generates a cross-sectional time series of seasonal rainfall and preserves the following statistics from the historic time series of rainfall: (1) means and standard deviations of seasonal rainfall in every state; (2) lag-zero and lag-one cross-correlation in space; and (3) lag-zero and lag-one serial correlation in time.<sup>3</sup>

For explanatory purposes, the stationary multivariate monthly rainfall series is given

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<sup>3</sup>A variable such as rainfall is a mutually dependent random variable. In other words, the stochastic components represent a set of  $n$  time series dependent among themselves, where  $n$  is the number of variables. When the objective is to generate a new sample of time series at a set of geographical points, the basic requirement is not only to preserve the statistical characteristics at each of the  $n$  series, but also to preserve the mutual dependence among them.

The dependent structure among  $n$  time series can be determined by computing the lag- $k$  cross-correlation between the series. For instance, considering the series  $x_t^{(i)}$  and  $x_t^{(j)}$ , the lag- $k$  cross-correlation coefficient  $r_k^{ij}$  is given by:

$$r_k^{ij} = \frac{\sum_{t=1}^{N-k} (x_t^{(i)} - \bar{x}_t^{(i)}) (x_{t+k}^{(j)} - \bar{x}_{t+k}^{(j)})}{\left[ \sum_{t=1}^{N-k} (x_t^{(i)} - \bar{x}_t^{(i)})^2 \sum_{t=1}^{N-k} (x_{t+k}^{(j)} - \bar{x}_{t+k}^{(j)})^2 \right]^{1/2}}$$

where  $\bar{x}_t^{(i)}$  is the mean of the first  $N-k$  values of series  $i$ , and  $\bar{x}_{t+k}^{(j)}$  is the mean of the last  $N-k$  values of series  $j$ . For  $n$  time series, it is common to represent the correlation structure by the matrix:

$$M_k = \begin{vmatrix} r_k^{11} & r_k^{12} & \dots & r_k^{1n} \\ r_k^{21} & r_k^{22} & \dots & r_k^{2n} \\ \vdots & \vdots & \ddots & \vdots \\ r_k^{n1} & r_k^{n2} & \dots & r_k^{nn} \end{vmatrix}$$

by  $x_t^i$ ,  $i=1,...,n$ , where  $n$  is the number of time series (i.e. number of geographic states in India. Suppose that by using a matrix transformation, we can transform these sequences into normal variables  $y_t^i$ , or, at the least, variables with skewness close to zero. The transformation function is  $g$  (the logarithmic function, for example) such that:

$$Y_t = g(X_t). \quad (2)$$

Then, the general model for the transformed (normalized) sequences  $Y_t$  is:

$$Y_t = \mu + \sigma Z_t \quad (3)$$

where  $\mu$  is an  $(n \times 1)$  matrix of the means and  $\sigma$  is an  $(n \times n)$  diagonal matrix of the standard deviations of the elements of the  $(n \times 1)$  matrix  $Y_t$ . This transformation (Equation 3), removes the periodicity of the monthly rainfall series.

The AR(1) model of the series  $z^i$ , where  $i=1,...,n$ , may be represented in matrix form by:

$$Z_t = A_1 Z_{t-1} + B e_t, \quad (4)$$

where  $Z_t$  is an  $(n \times 1)$  vector of elements  $z^i$ ,  $A_1$  and  $B$  are  $(n \times n)$  matrix parameters, and  $e_t$  is an  $(n \times 1)$  vector of independent, normally distributed random variables with a mean of zero and a variance of one. The random vector  $e_t$  is assumed to be uncorrelated in time and

space. The correlation structure of  $Z_t$  indicates a lag-zero and a lag-one cross-correlation in space, and a lag-zero and lag-one serial correlation in time.

The multivariate correlation function for the AR(1) model has the following properties:

$$M_k = A_1 M_{k-1}, \quad k > 0 \quad \text{or} \quad (5)$$

$$M_k = A_1^k M_0, \quad k \geq 0. \quad (6)$$

The moment estimates of the parameters  $A_1$  and  $B$  may be obtained from:

$$A_1 = M_1 M_0^{-1} \quad (7)$$

$$BB^T = M_0 - A_1 M_1^T \quad (8)$$

where  $M_0$  and  $M_1$  are the lag-zero and lag-one correlation matrices of  $Z_t$  (Salas, et al., 1980).

The estimation of the parameters of the multivariate AR(1) model requires the solution of Equations (7) and (8). The most common procedure is to assume  $B$  has a lower triangular matrix and to use the square root method (Young and Pisano, 1968) when  $D$  is a positive definite matrix. Alternatively, a method proposed by Lane (1979) can be used

when  $D$  is, at least, a positive semidefinite matrix. The first procedure was used to estimate the parameters for the AR(1) model.

### The Stochastic Model for the Indian Case

Monthly rainfall data for the period 1949-87 was obtained from different sources in order to estimate the parameters of the stochastic model described above. An algorithm developed by Mejia (1988) was used to run the AR(1) stochastic model for rainfall. Mejia's algorithm uses a pivotal approach to simplify the dimensionality of the problem. Conceptually, this approach selects some geographical states as pivotal stations, and assigns the remaining stations (satellites) to one of these pivotal stations. The algorithm is then designed to preserve the spatial statistics described in the previous section both among the pivotal stations, and, simultaneously, between each pivot and its associated satellite stations. For the particular case of India, one pivotal state was chosen from each of the regions, and the remaining geographical states in each region were defined as the satellites for that particular pivotal state.

### Solution Algorithm for the Indian Sector Model

The solution algorithm for the Indian Model is as follows:

- (i) Two states of rainfall, high and low, are defined for each geographical region  $i$ ,  $i = 1, \dots, 6$ . This resulted in  $k = 1, \dots, 64$  possible combinations of states of rainfall when the six regions are considered together.<sup>4</sup>

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<sup>4</sup>Ideally, one would like to have several discrete values for the realization of rainfall. Unfortunately, adding an extra discrete value, let us say 3, complicates the problem computationally. In this case, there would be 729 possible combinations.

(ii) The discrete probabilities of all these combinations are estimated based on historic information, and the probability  $p_k$  of having a particular combination of states of rainfall is estimated in the following manner:

$$p_k = n_k/N \quad k=1, \dots, 64. \quad (9)$$

These probabilities are estimated based on 1,000 (N) years of generated rainfall using the multivariate AR(1) model.

(iii) The dynamic programming recursive equation (1) is run for the expected value of the objective function. Let  $p_k$  be the probability of a particular combination of rainfall states, then the expected value is taken over the possible rainfall combinations so that:

$$E[f_{kt}(\underline{W}_t, \underline{W}_{t+1}, \underline{G}_{t,k})] = \sum p_k f_{kt}(\underline{W}_t, \underline{W}_{t+1}, \underline{G}_{t,k}).$$

This optimization exercise yields optimal levels of irrigation investment for all regions,  $\underline{G}_t = \{\underline{U}_t, \underline{R}_t\}$ ,  $t=1, \dots, T$ , and consequently the operating rules for the system.

(iv) The simulation approach is then used to test the optimal operating rules. This is done for two reasons. First, the operating rules found in the solution of the dynamic recursive equation (1) are for expected values of the stochastic component of the model rainfall. Therefore, a test of the system under different stochastic realizations of rainfall was necessary. Second, in past modeling exercises (Kuhner and Harrington, 1975) it was found that the objective function in the neighborhood of the optimal solution is often rather flat.

This means that a suboptimal or inferior solution might not only be adequate, but, moreover, may actually be more desirable than the theoretical optimum. An equivalent situation occurs when many solutions have values near the optimal solution. For some optimization problems, linear combinations of solutions are also solutions, and hence the number of nearly optimal solutions becomes infinite (Harrington, 1978).

(v) Finally, sensitivity analysis is applied to steps (iii) and (iv) to identify those decisions which remain intact or nearly intact under a variety of changing goals and conditions. The property of remaining intact has been called *system resilience* and has been the objective of some study in the past (Rogers, 1988). We use resilience as a means to test alternative policies for meeting GOI's foodgrain security and stability objectives.

### Optimization and Simulation Results

The optimization and simulation models are written in GAMS (General Algebraic Modeling System)<sup>5</sup>, and are linked to the stochastic rainfall program written in Fortran 77. Additional computer programs and DOS batch files<sup>6</sup> are also used for this purpose.

The optimization model is executed in a sequential and recursive manner. Each individual optimization program consists of approximately 600 variables and 600 equations, with close to 1800 non-zero elements. The problem is linear in all its equations with the exception of those representing the rehabilitation of irrigation systems. The time-horizon of fifteen years (1986-2000) is divided into three sub-periods of five years each, which

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<sup>6</sup>The stochastic program for rainfall, the simulation and optimization models, and the additional programs which link these models are available from the first author.

correspond to the planning periods of India, 1986-90, 1991-95, 1996-2000. This division makes replication of the GOI planning process possible and, at the same time, simplifies the solution of the model.

In contrast, the simulation model is run for the complete planning period (1986-2000). This is performed several times, once for each realization of rainfall, in order to explore the behavior of the foodgrain sector under this stochastic environment.

### Validation

The validation for the Indian model was accomplished in a simple two-stage check. First, the stochastic model for rainfall was validated by comparing the statistics (means, variances, lag-zero and lag-one correlation matrices) of the actual and generated rainfall. The result of this validation was that the model works as expected, generating sequences which preserved the desired set of statistics.

The second stage was the validation of the simulation and optimization models. To validate these two models, production during the period 1975-84 was simulated. The percentage absolute deviations (PAD) were calculated to measure model performance. The estimated PADs were in general less than 10%, showing a good fit between actual and estimated production.

### Optimization Results

Over the fifteen year planning horizon, there is substantial differences in irrigation investment among regions. Uttar Pradesh had the largest irrigation development, both in

major and medium irrigation projects, and in minor irrigation. The Western region had the largest investment in irrigation rehabilitation. Sensitivity analysis indicates that minor irrigation is less sensitive to parameter changes than major and medium irrigation projects.

The analysis suggests that development of the unexploited potential for minor irrigation should have high priority. In addition, increased investments in crop production should be given high priority in the Southern region because of their compensating effect on the variability of foodgrain production in the other five regions. Crop losses due to monsoon failure do not occur in the Southern region at the same time as they do in the rest of India because the south benefits from two annual monsoons.

Besides estimating the optimal irrigation investment allocation across Indian states the model can also be used to look at the trade-off between irrigation investment and foodgrain imports (Ramirez, 1990). Figure 2, for example, presents the trade-off that exists between these two expenditures. The expected annual level of imports is estimated for different percentage reductions in the budget for irrigation. This is performed for three initial levels of the buffer stock program: 10, 15 and 25 million tons respectively. A reduction in the budget for irrigation means that foodgrain imports will need to be increased to fill the foodgrain deficit that would arise due to the decline in new irrigated area. For example, if the irrigation budget is reduced by 50% expected annual foodgrain imports of 8 million tons would be required to meet future foodgrain targets.

Likewise, Figure 3 shows the effect that a reduction in the irrigation budget has on the variability of foodgrain imports. The larger the reduction in the budget for irrigation development, the larger the standard deviation of imports during the planning horizon



(1986-2000). For a budget reduction of 50%, the variability of foodgrain imports increases to 4 million tons.

### Simulation Results

The simulation exercise is dependent on some pre-specified operating rules for the buffer program. Operating rules refer to GOI's decision on when and what quantity of foodgrains to import for the buffer stock program. Since the rule that GOI has followed in the buffer stock program was not available, it was estimated based on GOI's past experience (1971-85). The equation which best represented GOI's behavior in terms of importing foodgrains in any given year, is as follows:

$$IMP_t = F(BS_{t-1}, Y_{t-1}) , \quad (10)$$

where  $IMP_t$  is the foodgrain imports in year  $t$ ;  $BS_{t-1}$  is the buffer stock level in the year  $t-1$ , and  $Y_{t-1}$  is the foodgrain production in year  $t-1$ .

Figure 4 shows the exceedence probability of import levels if GOI decides to implement this operating rule. For example, there exists a 20% probability that import levels of 10 million tons are exceeded at any given year. Figure 5 presents the E-V frontier for this operating rule. Smaller expected levels of government expenditure are accompanied by larger levels of expenditure variability. Each point on this frontier corresponds to a particular maximum buffer stock level. Based on this information, GOI can choose the buffer stock level that would be consistent with its attitude toward risk. Risk averse behavior from GOI, in this particular case, refers to the willingness to accept higher expected levels of government expenditure for less expenditure variability. Consequently,

the higher GOI's risk aversion, the larger the maximum level selected for the buffer stock, which corresponds to a low-variance in government expenditures for foodgrain stocks.

In choosing a desired level of buffer stock, GOI also has to consider the probability of failure of the operating rule. Failure of the operating rule, in this particular case, is defined as the event where the buffer stock level falls below the minimum permissible stock level (3 million tons). As Figure 6 shows, if GOI chooses a low buffer stock level, the probability of having a failure would be high. But, if GOI chooses buffer stock levels experienced in the recent past, 25 million tons for example, then this probability would be lower, approximately 13%.

### Conclusions

The Indian foodgrain model highlights the complementarities among irrigation investment (new irrigated area and the rehabilitation of the existing systems), buffer stock operations and the level of foreign trade, in achieving the Planning Commission goals of foodgrain security and stability. The analysis shows that there is considerable room for combining these three policy instruments in order to achieve foodgrain security in the country.

India's goal of self sufficiency has to be redefined. Although the agricultural sector can meet the expected target levels of foodgrains up to year 2000, it cannot provide enough foodgrain to deal with the high variability of foodgrain production. Foodgrain imports will be necessary for the buffer stock program and to reduce instability in food supplies. The level of the foodgrain imports clearly depends on the operating policy for the buffer stock program, and on the irrigation investment policy which suggests that irrigation investment policy should be linked to buffer stock and foreign trade policies. The definition of two of

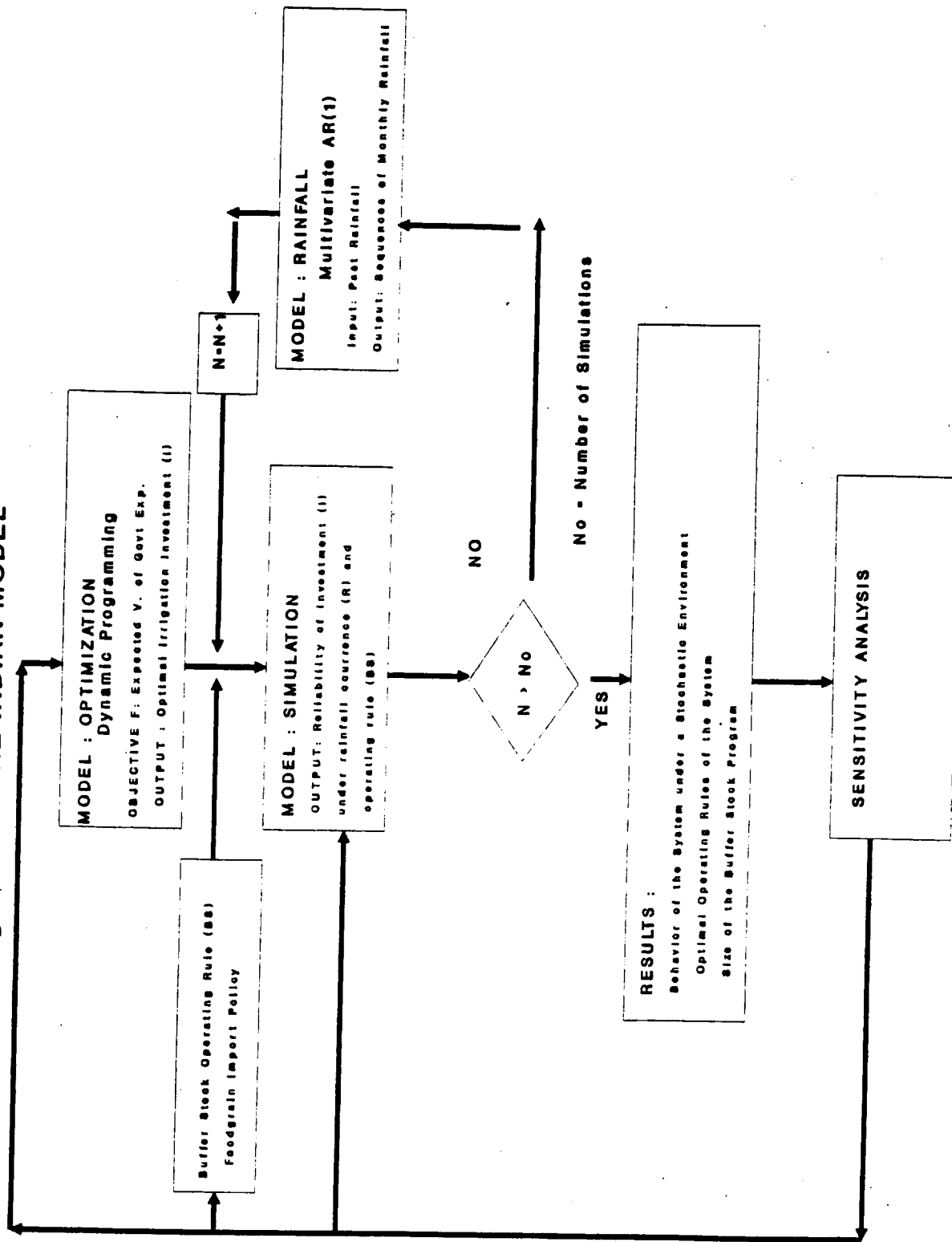
these policies determines the success and demands placed on the remaining policy. Future imports can be determined by the level of irrigation investment and the operating rule of the buffer stock program. In turn, the buffer stock operations are a function of decisions on how much foodgrain to import and the amount of irrigation investment.

The above models can serve as a tool to help in the Indian agricultural sector planning. The simulation and optimization models, together with the stochastic simulation of rainfall, provide a very powerful tool for analyzing the reliability of particular irrigation investment strategies, as well as the trade-offs among irrigation investment, the operation of the buffer stock program, and foodgrain imports.

The set of models can also help GOI in designing the operating rules for the buffer stock program and in examining the reliability of the foodgrain sector in terms of the probability of needing imports in any given year. In India there is no clear operating rule on how foodgrain imports should be used to provide buffer stocks. This study shows that the effectiveness of the irrigation investment and the buffer stock program in reducing foodgrain instability and providing food security, cannot be determined without a clear foodgrain import policy.

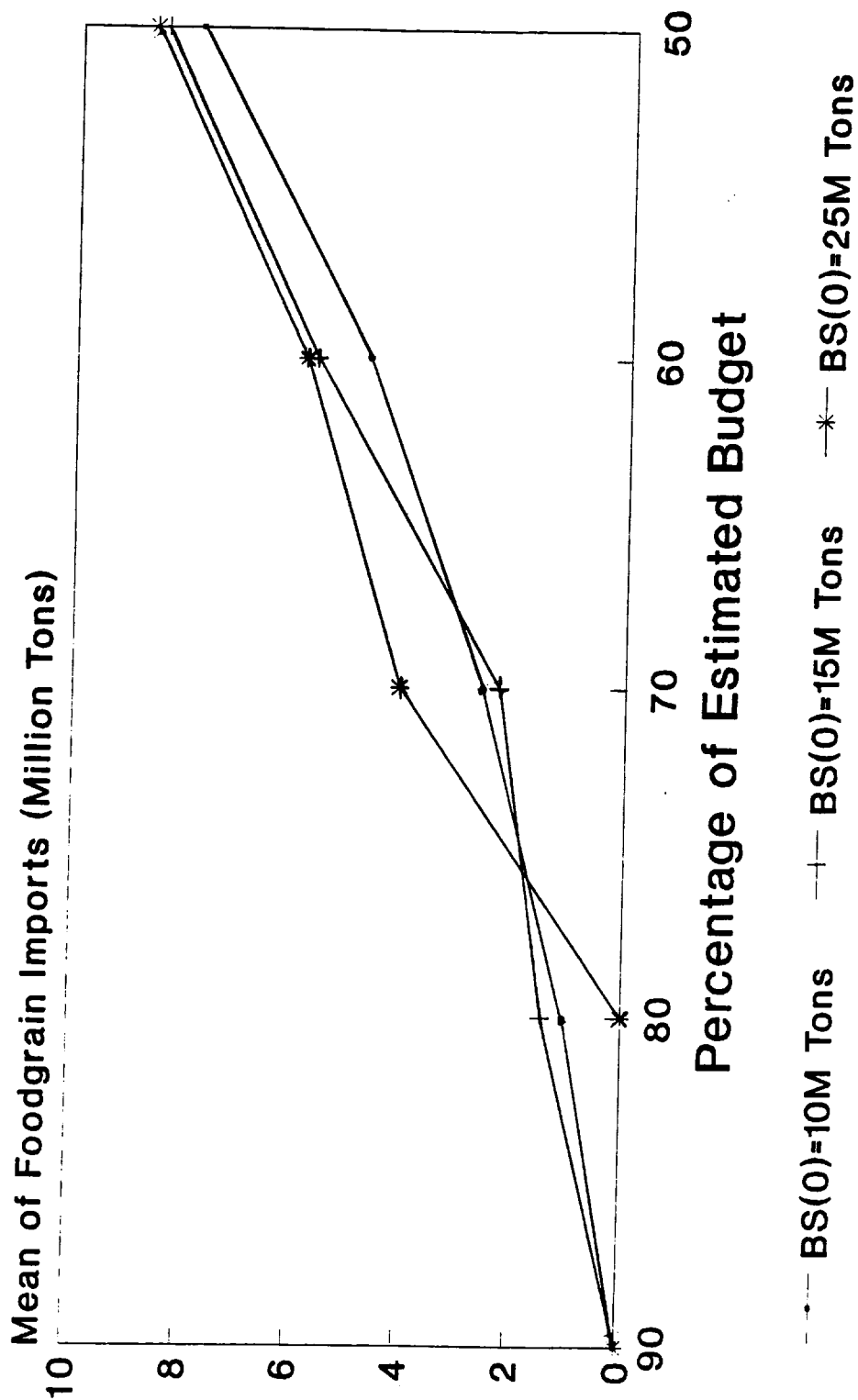
As in any model, the one presented here is subject to limitations and potential extensions. Alternative approaches can be developed to complement and/or modify the existing models. One possible alternative would be to transform the GOI objective function into a utility function that would include a risk-aversion coefficient. Additional improvements that might be tried include the conjunctive use of water, and the consideration of salinity and waterlogging as externalities in irrigation development. Finally, improved estimates of the cost of new irrigation and irrigation rehabilitation would substantially improve the empirical results.

Figure 1. THE INDIAN MODEL



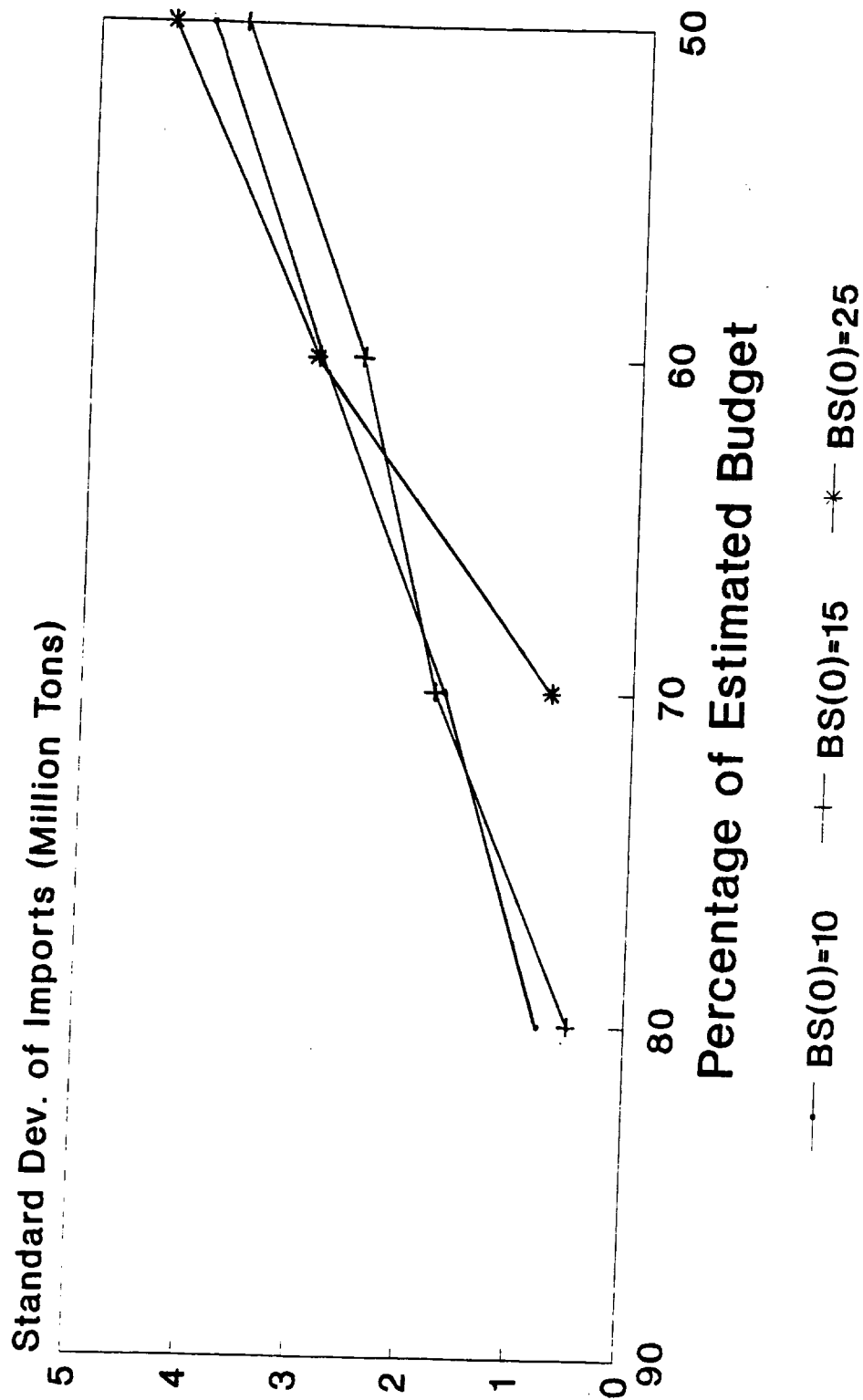
# Figure 2

## Foodgrain Imports vs Irrigation Budget



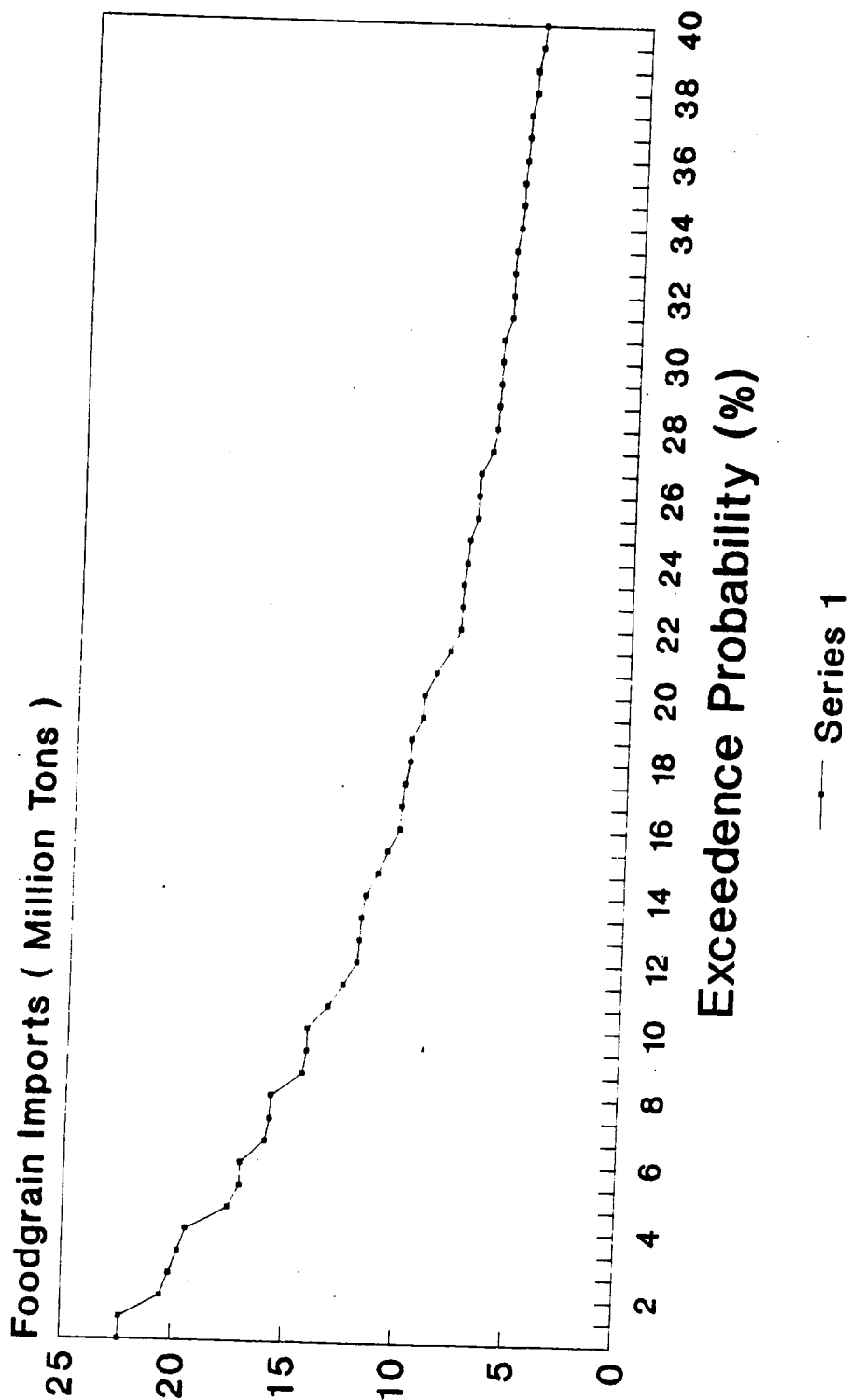
# Figure 3

## Foodgrain Imports vs Irrigation Budget



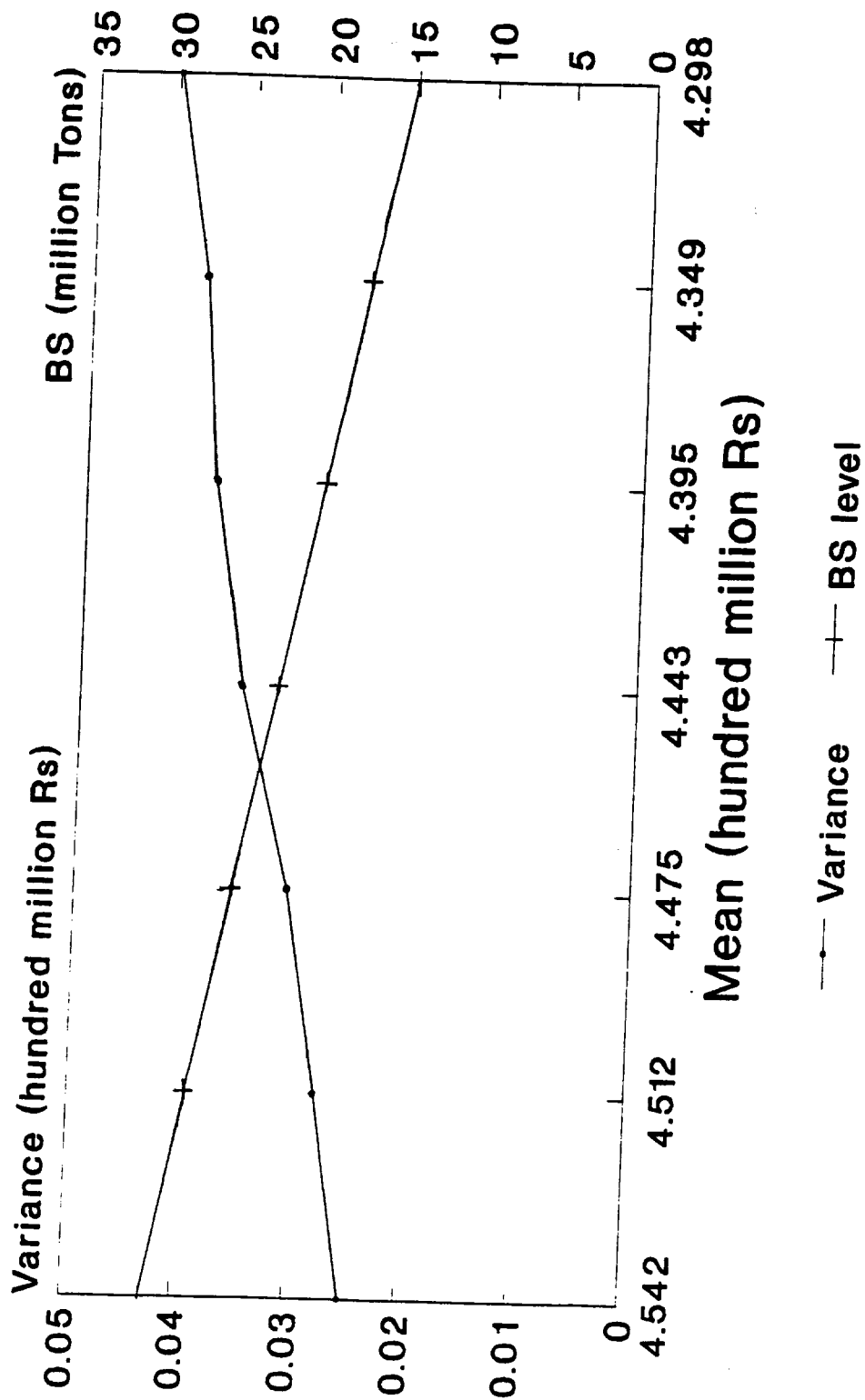
# Figure 4

## Exceedence Probability of Imports



# Figure 5

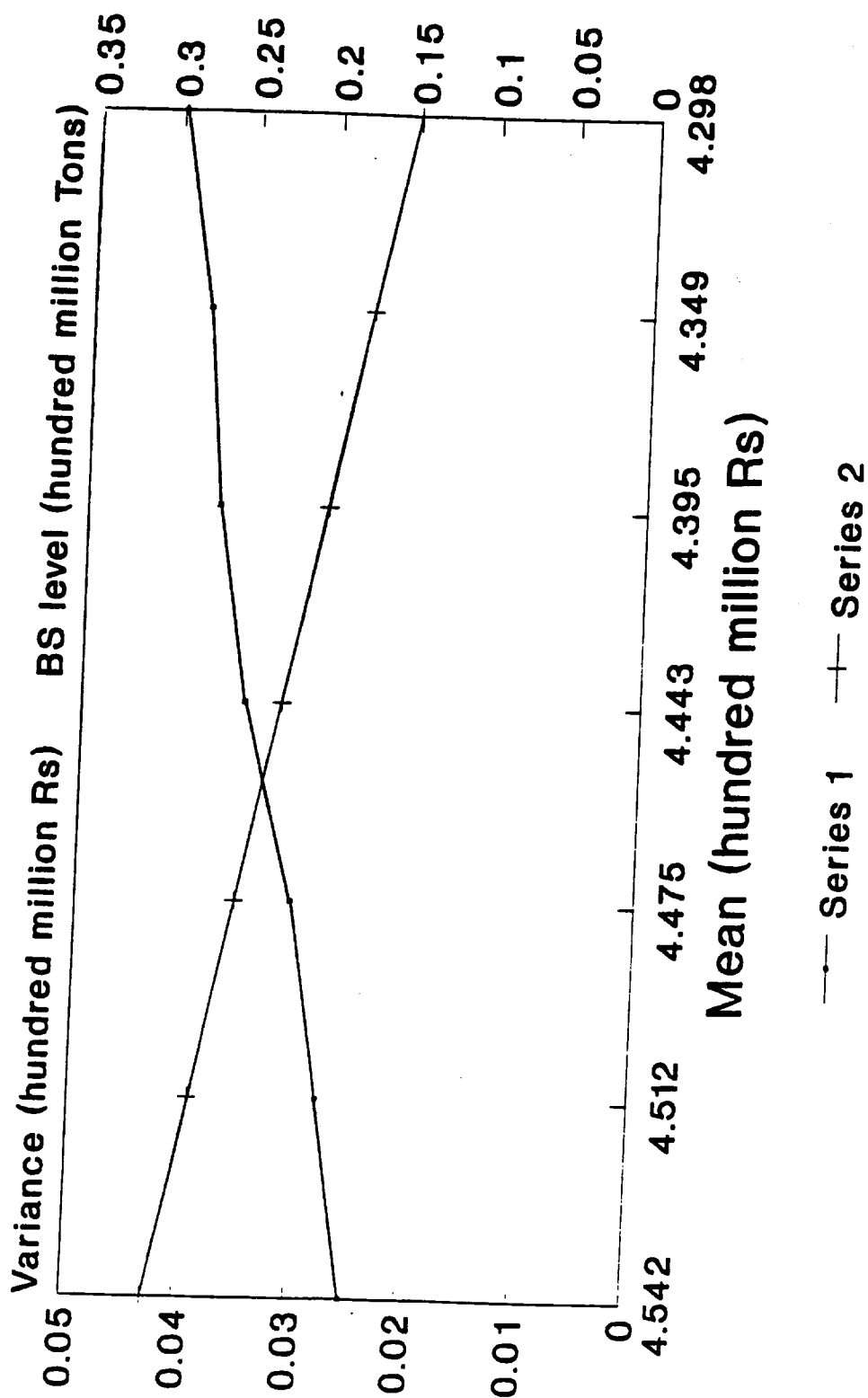
## E-V Frontier for Government Expenditure





# Figure 6

## E-V Frontier for BS levels



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