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**ETS and Technological  
Innovation: A Random  
Matching Model**

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### Summary

The present paper investigates the functioning of an Emission Trading System (ETS) and its impact on the diffusion of environmental-friendly technological innovation in the presence of firms' strategic behaviours and sanctions to non-compliant firms. For this purpose, we study an evolutionary game model with random matching, namely, a context in which a population of firms interact through pairwise random matchings. We assume that each firm has to decide whether to adopt a new clean technology or keep on using the old technology that requires pollution permits to operate and that the strategy whose expected payoff is greater than the average payoff spreads within the population at the expense of the alternative strategy (the so-called replicator dynamics). We investigate the technological dynamics and the stationary states that emerge from the model. From the analysis of the model, we show that by properly modifying the penalty on non-compliant firms, it is possible to shift from one dynamic regime to another and that an increase in permits trade can promote the diffusion of innovative pollution-free technologies.

**Keywords:** Emissions Trading, Technological Innovation, Random Matching, Evolutionary Game, Penalty System, Strategic Behaviour

**JEL Classification:** C62, C63, C73, C78, O33, Q55, Q58

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# ETS and technological innovation: a random matching model

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## Abstract

The present paper investigates the functioning of an Emission Trading System (ETS) and its impact on the diffusion of environmental-friendly technological innovation in the presence of firms' strategic behaviours and sanctions to non-compliant firms. For this purpose, we study an evolutionary game model with random matching, namely, a context in which a population of firms interact through pairwise random matchings. We assume that each firm has to decide whether to adopt a new clean technology or keep on using the old technology that requires pollution permits to operate and that the strategy whose expected payoff is greater than the average payoff spreads within the population at the expense of the alternative strategy (the so-called replicator dynamics).

We investigate the technological dynamics and the stationary states that emerge from the model. From the analysis of the model, we show that by properly modifying the penalty on non-compliant firms, it is possible to shift from one dynamic regime to another and that an increase in permits trade can promote the diffusion of innovative pollution-free technologies.

## 1 Introduction

Emission trading has gained increasing importance in the last years as policy instrument to reduce several different environmental problems.

While the theoretical foundations of the instrument are due to the seminal contributions by several authors in the '60s (e.g. Coase 1960, Dales 1968, Montgomery 1972), the first examples of applications of Emission Trading Systems (henceforth ETS) date back to 1995 when they were successfully implemented in the context of the US Acid Rain Programme to cut NO<sub>x</sub> and SO<sub>2</sub> emissions (Coniff, 2009). More recent applications include water tradable permits to lower pollution and consumption of hydric resources, with different results in different countries and hydrological basins (see Borghesi, 2008).

Among recent applications of ETS, a particularly important role is played by the European Emission Trading Scheme (EU-ETS) for the reduction of carbon dioxide emissions. As Ellerman (2009) has argued, this scheme, that is the first

world's multinational cap-and-trade system for greenhouse gases (GHG) and has created the largest emissions trading market, represents a benchmark for the global GHG emissions trading system that is currently proposed as the main policy instrument to combat climate change in the future (Aldy and Stavins, 2008).

Given the crucial role that the ETS is likely to play in the future international policy agenda, several works have recently investigated its functioning and implications from different perspectives (e.g. Grull and Taschini, 2011; Convery, 2009; Clò, 2008; Carraro et al., 2010; Ellerman and Buchner, 2007; Ellerman and Joskov, 2008; Ellerman, 2009; Ellerman et al., 2010; Costantini et al., 2011). In particular, among these studies a few contributions (e.g. Rogge et al. 2011; Moreno-Bromberg and Taschini, 2011; Brauneis et al., 2011; Borghesi et al., 2012; Cabel and Dechezlepretre, 2012) have examined whether and to what extent the ETS contributes to induce technological innovation and diffusion in the regulated sectors. Several authors have analysed the possible existence of strategic behaviours in the emission trading market (e.g. Hahn, 1984, Hagem and Westkog, 1998, Smith and Swierzbinski, 2007, Wirl, 2009), while others have pointed out the possible emerging of moral hazard behaviours generated from the sanction system in the EU-ETS context (see e.g. Borghesi, 2011) or the optimal environmental policy when firms are not compliant (see e.g. Ino, 2011).

The present paper aims at contributing to the increasing literature on this issue by investigating the functioning of an ETS and its impact on the diffusion of environmental-friendly technological innovations in the presence of strategic behaviours of firms, bounded rationality and sanctions to non-compliant firms.

Differently from all previous contributions in the EU-ETS literature, the present paper adopts a random matching model to analyse the issue described before. The random matching framework is increasingly adopted in game theory to model markets in which frictions and bounded rationality prevent instantaneous adjustment of the level of economic activity. In particular, following the seminal contribution by Maynard Smith (1982) (see Hofbauer and Sigmund 1988, Weibull 1995, Samuelson 1997 for an introduction to evolutionary game theory), several papers have adopted evolutionary game models in which individuals interact with each other during pair wise random matchings. Such a framework seems to fit and has therefore been applied to many different economic contexts and fields, such as: labor economics (to describe the matching of unemployed workers and firms' vacancies), social economics (e.g. to examine the formation of marriages from unmatched individuals), monetary economics (e.g. to analyse the allocation of loans from banks to entrepreneurs, or the role of money in facilitating sales when sellers and buyers meet), and so on. In the present context, the random matching structure of the game will be employed to describe the potential emission trading between heterogeneous firms that can decide whether to adopt a clean technology or keep on using an old polluting technology. The former firms can sell their own permits to the latter, who need them to keep on producing to meet the requirements of the ETS and thus avoid the penalty to non-compliant firms.

Differently from other studies in the ETS literature, moreover, we show that the presence of bounded rationality and imitative behaviors underlying the random matching model may generate path-dependency in the economy. When this is the case, the present model allows to perform comparative statics analyses that show how the basins of attraction of the existing equilibria vary with changes in key parameters of the model, such as the penalty level to non-compliant firms. To the best of our knowledge, the possible existence of path dependency in ETS and the analysis of its dynamic features has been mainly ignored by the existing literature, therefore it represents a further value added of the present work as compared to previous studies.

To investigate this issue, the structure of the paper will be as follows. Section 2 describes the model, distinguishing two possible payoff matrices according to the kind of firms that interact in random pairwise matchings. Section 3 investigates the dynamics emerging under each of the two possible cases and analyses the corresponding Pareto ranking among the equilibria of the model. Section 4 contains some concluding remarks on the main results that descend from the model.

## 2 The model

Let us consider a large population of firms that interact among themselves through pairwise random matchings. Each firm has to choose *ex ante* between two possible strategies: (i) keep on using an old, polluting technology (with production cost  $C_P$ ) and buy the corresponding pollution permits (at price  $p$ ) or (ii) shift to a new, environmental-friendly technology that implies higher production costs ( $C_{NP} > C_P$ ) but requires no pollution permits to operate.

To fix ideas, let us suppose that each firm initially has one permit at disposal and that the firms that use the polluting technology (henceforth firms  $P$ ) need two permits to operate, while the firms that adopt the clean technology (firms  $NP$ ) need no permit for the activity. If so, firms  $P$  need to buy one more permit to keep on producing, whereas firms  $NP$  can sell its permit, so that the conditions for their exchange are obviously satisfied.

Let us indicate with  $T$  the sanction that a non-compliant firm  $P$  has to pay if is discovered by the regulatory authority, namely, if it produces with the old technology without purchasing the additional permit that is needed for this purpose. We will denote with  $\theta \in (0, 1)$  the probability of being discovered by the regulatory authority, therefore  $\theta T$  indicates the expected fee for the non-compliant firms  $P$ .

Given the random matching structure of the game, we can obviously distinguish three possible cases depending on the kind of firms that meet up in pairwise matchings.

a) If two firms  $P$  meet, then in principle they both have to pay the fee, since none of them has enough permits to operate. However, they can decide to exchange their permits (i.e. one firm  $P$  sells its permit to the other that has thus the two permits that it needs to operate) and share the expected penalty.

In this case, the exchange price is  $p = \theta T/2$  and the payoffs  $\pi_P$  of the two firms will be:

$$\pi_P = -C_P - \theta T/2$$

so that both firms are better-off with respect to the no exchange case (in which they both have the "full" expected penalty  $\theta T$ ).

b) If two firms  $NP$  meet, the permits are useless for both of them so no permit trade will occur. In this case, the payoffs  $\pi_{NP}$  of the two firms will be:

$$\pi_{NP} = -C_{NP} + \delta$$

where  $\delta \geq 0$  denotes the possible positive spillover deriving to each firm  $NP$  from the diffusion of the new technology (e.g. the positive externality in terms of reduction cost for the new technology that is allowed by the network effects emphasized by much of the empirical literature on this issue).<sup>1</sup>

c) If a firm  $P$  meets a firm  $NP$ , the former can buy from the latter the permit that it needs to avoid the penalty, however the permit exchange may not take place for different reasons. For instance, firm  $P$  might decide not to buy the permit and run the risk to be sanctioned by the regulatory authority since it regards the expected penalty to be sufficiently low. Alternatively, firm  $NP$  could decide not to sell the permit to damage and/or eliminate firm  $P$  as it may represent a potential competitor on the market.<sup>2</sup>

We can, therefore, distinguish two possible subcases within case c):

c.1) No permit exchange occurs between firm  $P$  and  $NP$  (because  $P$  does not buy the permit and/or  $NP$  does not sell it). In this case, if firm  $P$  is not discovered (which occurs with probability  $1 - \theta$ ), the payoffs of the two firms are simply represented by the costs of their respective technologies ( $C_P$  and  $C_{NP}$ ). If, on the contrary, firm  $P$  is discovered (which occurs with probability  $\theta$ ) it will also have to pay the penalty  $T$ , while firm  $NP$  may possibly derive a competitive gain  $\gamma$  from the "punishment" suffered from its competitor  $P$ .<sup>3</sup> In this case, therefore, the expected payoff of firms  $P$  and  $NP$  are given by the probability that  $P$  is actually discovered/not discovered times the corresponding payoffs for each firm as described above, that is, respectively:

$$\begin{aligned}\pi_P &= \theta(-C_P - T) + (1 - \theta)(-C_P) = -C_P - \theta T \\ \pi_{NP} &= \theta(-C_P + \gamma) + (1 - \theta)(-C_{NP}) = -C_{NP} + \theta\gamma\end{aligned}$$

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<sup>1</sup>See, for instance, Borghesi et al. (2012) and the literature cited therein.

<sup>2</sup>A similar use of emission permits for strategic purposes has actually occurred in some applications of ETS. For instance, when a system of water pollution permits was implemented on the Fox River in Wisconsin, the largest firms that possessed most of the permits refused to sell them to the smaller firms to hinder the growth in the production activity of the latter (O'Neill et al., 1983).

<sup>3</sup>One can interpret  $\gamma$ , for instance, as the increase in the revenues and/or in the market share accruing to firm  $NP$  from the closing of the non-compliant firm  $P$  or from the acquisition by  $NP$  of some green labelling that increases the number of its consumers who are concerned with the environmental consequences of the dirty production process used by firm  $P$ .

where  $\gamma \geq 0$  is the competitive gain for  $NP$  from "punishing" firm  $P$ .

c.2) The permit exchange does take place and firms  $P$  buys the permit from firm  $NP$ . In this case, the payoffs of the two firms will simply be, respectively:

$$\begin{aligned}\pi_P &= -C_P - p \\ \pi_{NP} &= -C_{NP} + p\end{aligned}$$

where  $p$  is the price of the tradable permit.

Notice that firm  $P$  will obviously be willing to buy the permit only if its corresponding payoff is higher than the expected payoff from not buying the permit, namely if:

$$-C_P - p > -C_P - \theta T$$

or, equivalently, if  $p < \theta T$ .

Similarly, firm  $NP$  will be willing to sell its permit only if the payoff that it derives from the exchange is higher or at least equal to the expected payoff from not selling the permit, namely if:

$$-C_{NP} + p > -C_{NP} + \theta \gamma$$

that is, if  $p > \theta \gamma$ .

For the permit exchange to actually take place, therefore, the equilibrium price must range between the minimum willingness to accept of firm  $NP$  and the maximum willingness to pay of firm  $P$ , that is  $\theta \gamma < p < \theta T$ .

We can, therefore, distinguish two possible cases that encompass all the possible situations described above:

**Case 1:** If  $\gamma \theta \geq \theta T$ , i.e.  $\gamma \geq T$ , there cannot exist any equilibrium price that satisfies the conditions above so that no trade will take place between firms  $P$  and  $NP$ . In this case (that encompasses cases a), b) and c.1) discussed above), the payoff matrix is as follows:

$$A : \begin{array}{cc} & \begin{array}{cc} P & NP \end{array} \\ \begin{array}{c} P \\ NP \end{array} & \left( \begin{array}{cc} -C_P - \frac{\theta T}{2} & -C_P - \theta T \\ -C_{NP} + \gamma \theta & -C_{NP} + \delta \end{array} \right)\end{array}$$

**Case 2:** If  $\theta \gamma < \theta T$ , i.e.  $\gamma < T$ , the permit exchange is mutually convenient for any  $p \in (\theta \gamma, \theta T)$ . In this case, therefore, summarising the cases a), b) and c.2) above, the payoff matrix is given by:

$$B : \begin{array}{cc} & \begin{array}{cc} P & NP \end{array} \\ \begin{array}{c} P \\ NP \end{array} & \left( \begin{array}{cc} -C_P - \frac{\theta T}{2} & -C_P - p \\ -C_{NP} + p & -C_{NP} + \delta \end{array} \right)\end{array}$$

For the sake of simplicity, we will assume that the equilibrium price sets half way between the minimum willingness to accept of firm  $NP$  and the maximum willingness to pay of firm  $P$ , that is:<sup>4</sup>

$$p = \frac{\theta\gamma + \theta T}{2} = \theta \frac{\gamma + T}{2}$$

If so, the payoff matrix  $B$  becomes:

$$C : \begin{array}{cc} & \begin{array}{cc} P & NP \end{array} \\ \begin{array}{c} P \\ NP \end{array} & \left( \begin{array}{cc} -C_P - \frac{\theta T}{2} & -C_P - \theta \frac{\gamma + T}{2} \\ -C_{NP} + \theta \frac{\gamma + T}{2} & -C_{NP} + \delta \end{array} \right) \end{array}$$

In what follows we will examine the dynamics and the equilibria that emerge under each payoff matrix and the possible shifts in the dynamic regimes from one case to the other (i.e. from matrix  $A$  to matrix  $C$ ) that may derive from changes in the parameter values of  $T$  and  $\gamma$ .

### 3 Dynamics of the game

Let us indicate with  $x(t) \in [0, 1]$  the share of firms that adopt strategy  $P$  at time  $t \in [0, +\infty)$ . As a consequence,  $1 - x(t)$  denotes the share of firms adopting the alternative strategy  $NP$ . Variable  $x$  represents, therefore, the distribution of the two strategies in the population of firms; if  $x = 1$  (respectively,  $x = 0$ ) then all firms adopt strategy  $P$ , that is, they all keep on using the polluting technology (respectively, all firms adopt strategy  $NP$ , i.e. they all shift to the clean technology).

At any time  $t$  a large number of pairwise matchings occur between firms that randomly interact.

Given the random matching structure of the game,  $x$  (respectively,  $1 - x$ ) measures the probability of "meeting" a firm that has adopted strategy  $P$  (respectively,  $NP$ ).

Let us assume, for the sake of simplicity, that the adoption process of the two strategies can be described by the well-known *replicator dynamics* (see Weibull 1995):

$$\dot{x} = x(1 - x) [\Pi_P(x) - \Pi_{NP}(x)] \quad (1)$$

where  $\Pi_P(x)$  and  $\Pi_{NP}(x)$  indicate the expected payoffs of strategies  $P$  and  $NP$ , while  $\dot{x}$  denotes the time derivative of  $x(t)$ , namely,  $\dot{x} = dx(t)/dt$ . According to replicator dynamics, the strategy whose expected payoff is greater

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<sup>4</sup>This is equivalent to assuming that firms  $P$  and  $NP$  have the same bargaining power. If this is not the case, the equilibrium price will obviously tend towards one extreme or the other of the range of values  $(\theta\gamma, \theta T)$  according to the respective importance and bargaining power of the two firms.



than the average payoff spreads within the populations at the expense of the alternative strategy, namely:

$$\dot{x} \gtrless 0 \text{ iff } \Pi_P(x) - \Pi_{NP}(x) \gtrless 0, \forall x \in (0, 1) \quad (2)$$

### 3.1 Dynamics of the game when the payoff matrix $A$ applies

If  $\gamma \geq T$ , the payoff matrix  $A$  applies (i.e. tradable permits are exchanged only between firms  $P$  but not between heterogeneous firms,  $P$  and  $NP$ ). In this case, the expected payoffs for the two strategies are as follows:

$$\Pi_P(x) = (-C_P - \frac{\theta T}{2})x + (-C_P - \theta T)(1-x) = -C_P - \theta T + \frac{\theta T}{2}x$$

$$\Pi_{NP}(x) = (-C_{NP} + \gamma\theta)x + (-C_{NP} + \delta)(1-x) = -C_{NP} + \delta + (\gamma\theta - \delta)x$$

so that the replicator dynamics become:

$$\dot{x} = x(1-x) [\Pi_P(x) - \Pi_{NP}(x)] = x(1-x) \left[ C_{NP} - C_P - \theta T - \delta + \left[ \left( \frac{T}{2} - \gamma \right) \theta + \delta \right] x \right] \quad (3)$$

where  $\frac{T}{2} - \gamma < 0$ , since  $\gamma \geq T$ .

Notice that the payoff  $\Pi_P(x)$  is a strictly increasing function of the share of polluting firms  $x$ : in fact, the higher is the share of polluting firms, the higher the probability for a firm  $P$  to meet a similar firm and thus share the expected penalty ( $\frac{\theta T}{2}$ ) rather than having to pay it all ( $\theta T$ ) as it occurs when it meets a firm  $NP$ .

Also observe that the payoff  $\Pi_{NP}(x)$  is a strictly increasing function of  $x$  if  $\gamma\theta - \delta > 0$ , while it is strictly decreasing if  $\gamma\theta - \delta < 0$ . This is also consistent with what one would reasonably expect: if for a firm  $NP$  the expected gain from meeting a firm  $P$  ( $\gamma\theta$ ) is higher than the gain from meeting another firm  $NP$  ( $\delta$ ), then its payoff will increase with the number of firms  $P$ . The opposite obviously occurs if the sign of the relationship between  $\gamma\theta$  and  $\delta$  is reversed. Notice that if  $\delta = 0$ , that is, the meeting of two firms  $NP$  does not generate any positive spillover for each of them, then only the former condition can apply and the payoff  $\Pi_{NP}(x)$  is always strictly increasing in the share of polluting firms  $x$ .

As one easily observe from equation (3), the payoff differential  $\Pi_P(x) - \Pi_{NP}(x)$  is strictly increasing (decreasing) in  $x$  if  $\left( \frac{T}{2} - \gamma \right) \theta + \delta$  is positive (negative). As shown below, we will therefore distinguish two possible cases in the description of the dynamics of the model according to the sign of the previous expression.

The following Proposition illustrates the taxonomy of the dynamic regimes that may occur in the context  $\gamma \geq T$ .

**Proposition 1** When  $\gamma \geq T$ , dynamics (3) can lead to the following possible dynamic regimes:

1) If  $C_{NP} - C_P \geq \max \{ \theta T + \delta, \theta (\frac{T}{2} + \gamma) \}$ , then whatever the initial distribution of strategies  $x(0) \in (0, 1)$ ,  $x$  will always converge to the steady state  $x = 1$  (see Figure 1).

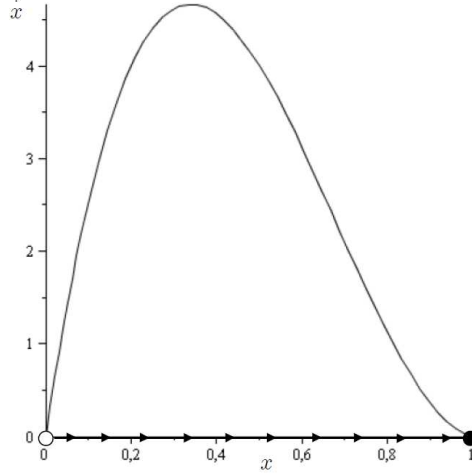
2) If  $C_{NP} - C_P \leq \min \{ \theta T + \delta, \theta (\frac{T}{2} + \gamma) \}$ , then whatever the initial distribution of strategies  $x(0) \in (0, 1)$ ,  $x$  will always converge to the steady state  $x = 0$  (see Figure 2).

3) If  $\theta (\frac{T}{2} + \gamma) < C_{NP} - C_P < \theta T + \delta$  and  $(\frac{T}{2} - \gamma) \theta + \delta > 0$ , then there exists a repulsive inner steady state:

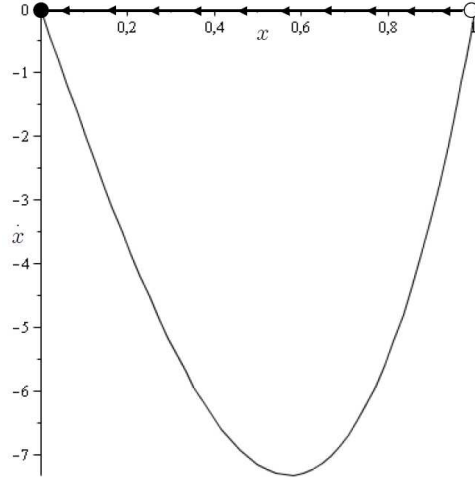
$$\bar{x} := (C_P - C_{NP} + \theta T + \delta) / \left[ \left( \frac{T}{2} - \gamma \right) \theta + \delta \right] \in (0, 1)$$

If  $x(0) \in [0, \bar{x})$ , then  $x$  converges to the steady state  $x = 0$ , while if  $x(0) \in (\bar{x}, 1]$ , then it converges to the steady state  $x = 1$  (see Figure 3).

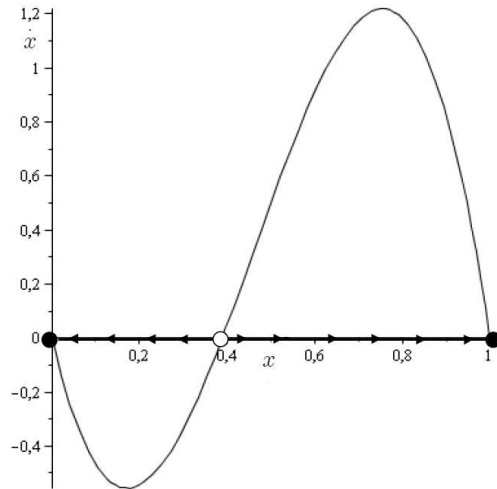
4) If  $\theta T + \delta < C_{NP} - C_P < \theta (\frac{T}{2} + \gamma)$  and  $(\frac{T}{2} - \gamma) \theta + \delta < 0$ , then whatever the initial distribution of strategies  $x(0) \in (0, 1)$ ,  $x$  will always converge to the inner steady state  $\bar{x} \in (0, 1)$  in which the alternative strategies  $P$  and  $NP$  coexist (see Figure 4).



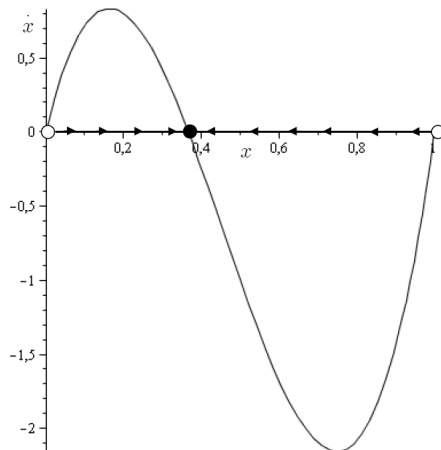
**Figure 1:** Whatever the initial distribution of strategies  $x(0) \in (0, 1)$ ,  $x$  converges to the steady state  $x = 1$ . Parameter values  $\delta = 2$ ,  $\gamma = 130$ ,  $\theta = .4$ ,  $C_{NP} = 115$ ,  $C_P = 42$ ,  $T = 100$ .



**Figure 2:** Whatever the initial distribution of strategies  $x(0) \in (0, 1)$ ,  $x$  converges to the steady state  $x = 0$ . Parameter values  $\delta = 2$ ,  $\gamma = 130$ ,  $\theta = .4$ ,  $C_{NP} = 45$ ,  $C_P = 42$ ,  $T = 100$ .



**Figure 3:** Bi-stable dynamic regime (path-dependence): if  $x(0) \in [0, \bar{x})$ , then  $x$  converges to the steady state  $x = 0$ , while if  $x(0) \in (\bar{x}, 1]$ , then it converges to the steady state  $x = 1$ . Parameter values  $\delta = 50$ ,  $\gamma = 130$ ,  $\theta = .4$ ,  $C_{NP} = 125$ ,  $C_P = 42$ ,  $T = 100$ .



**Figure 4:** Coexistence dynamic regime: whatever the initial distribution of strategies  $x(0) \in (0, 1)$ ,  $x$  always converges to the inner steady state  $\bar{x} \in (0, 1)$  in which the alternative strategies  $P$  and  $NP$  coexist. Parameter values  $\delta = 2$ ,  $\gamma = 130$ ,  $\theta = .4$ ,  $C_{NP} = 95$ ,  $C_P = 42$ ,  $T = 100$ .

The Proposition above suggests that there can be multiple equilibria of the game: two extreme equilibria ( $x = 0$  and  $x = 1$ ) in which all firms adopt the same (clean and dirty, respectively) technology and an inner equilibrium that can be either repulsive or attracting. In the former case, the system is path-dependent: the trajectories of the economy may lead to one extreme or the other depending on the initial distribution of polluting firms in the population so that the technological adoption strategy is self-enforcing. In the latter case the trajectories will lead to an attracting equilibrium in which both technological adoption strategies coexist.

Notice that the bi-stable dynamics characterizing case 3 above can occur if and only if  $(\frac{T}{2} - \gamma)\theta + \delta > 0$ , namely, only if the payoff differential  $\Pi_P(x) - \Pi_{NP}(x)$  is strictly increasing in  $x$ , which generates a self-enforcing mechanism leading to extreme equilibria.<sup>5</sup> On the contrary, the coexistence regime characterizing case 4 above can occur if and only if  $(\frac{T}{2} - \gamma)\theta + \delta < 0$ , namely, the payoff differential  $\Pi_P(x) - \Pi_{NP}(x)$  is downward sloping in  $x$ .

The following Proposition describes how a change in  $\theta$  and/or  $T$  modifies the inner equilibrium value  $\bar{x}$ .

**Proposition 2** *In the context  $\gamma \geq T$ , if  $\theta(\frac{T}{2} + \gamma) < C_{NP} - C_P < \theta T + \delta$  and  $(\frac{T}{2} - \gamma)\theta + \delta > 0$  (bi-stable regime), then  $\frac{\partial \bar{x}}{\partial T} > 0$  and  $\frac{\partial \bar{x}}{\partial \theta} > 0$ . If*

<sup>5</sup>Since  $\gamma \geq T$ , observe that for the condition above to apply it must be  $\delta > 0$ , that is, a positive spillover must derive from diffusion of the new technology.

$\theta T + \delta < C_{NP} - C_P < \theta \left(\frac{T}{2} + \gamma\right)$  and  $\left(\frac{T}{2} - \gamma\right) \theta + \delta < 0$  (*coexistence regime*), then  $\frac{\partial \bar{x}}{\partial T} < 0$  and  $\frac{\partial \bar{x}}{\partial \theta} < 0$ .

The above Proposition suggests that an increase in  $\theta$  and/or  $T$  shifts the repulsive inner equilibrium  $\bar{x}$  to the right, thus enlarging the attraction basin of the "virtuous" equilibrium  $x = 0$  in which no firm pollutes any longer. The opposite applies when the inner equilibrium  $\bar{x}$  is an attractor: in this case an increase in the expected penalty (due to higher penalty level  $T$  and/or higher probability of being discovered  $\theta$  by the regulatory authority for non-compliant firms) shifts  $\bar{x}$  to the left, thus increasing the share of non-polluting firms at the equilibrium.

In what follows, we intend to point out the possible Pareto dominance relationships between the stationary states of the dynamic system analysed above. For this purpose, let us consider that the average payoff of the population of firms is given by:

$$\bar{\Pi}(x) = x \cdot \Pi_P(x) + (1 - x) \cdot \Pi_{NP}(x)$$

Therefore  $\bar{\Pi}(1) = \Pi_P(1)$  and  $\bar{\Pi}(0) = \Pi_{NP}(0)$  hold. When the two extreme equilibria  $x = 0$  and  $x = 1$  are both attractors, it seems important to emphasize under which conditions the firms' payoffs are higher in  $x = 0$  than in  $x = 1$ . This occurs when:

$$\Pi_{NP}(0) > \Pi_P(1)$$

i.e.:

$$-C_{NP} + \delta > -C_P - \frac{\theta T}{2}$$

which can be rewritten as follows:

$$\frac{\theta T}{2} + \delta > C_{NP} - C_P$$

Therefore if the cost differential  $C_{NP} - C_P$  between the clean and the dirty technologies is sufficiently low, then both the firms and the citizens are better-off in  $x = 0$  than in  $x = 1$ : the former because they get a higher payoff, while the latter because they live in a non-polluted environment. Notice that the condition above requires that the expected penalty  $\theta T$  and/or the spillover effect  $\delta$  are sufficiently high so that all firms are highly motivated to shift to the clean technology.

If, on the contrary, the condition above does not apply, then the firms' payoffs are higher in  $x = 1$  than in  $x = 0$ , while the opposite applies for the citizens, at least in terms of their benefits from a clean environment.<sup>6</sup>

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<sup>6</sup>Notice that in the present model we focus attention on the firms' profit rather than on the welfare of the whole collectivity. However, given the many and well-documented health damages provoked by environmental degradation (cf. WHO, 2005), it seems reasonable to

### 3.2 Dynamics of the game when the payoff matrix $C$ applies

If  $\gamma < T$ , the payoff matrix  $C$  applies (i.e. tradable permits are exchanged not only between homogeneous firms  $P$ , but also between heterogeneous firms,  $P$  and  $NP$ ). In this case the expected payoffs are:

$$\begin{aligned}\Pi_P(x) &= (-C_P - \frac{\theta T}{2})x + (-C_P - \bar{p})(1-x) = -C_P - E\bar{p} + \left(E\bar{p} - \frac{\theta T}{2}\right)x = \\ &= -C_P - \theta \frac{\gamma + T}{2} + \theta \frac{\gamma}{2}x\end{aligned}$$

$$\begin{aligned}\Pi_{NP}(x) &= (-C_{NP} + \bar{p})x + (-C_{NP} + \delta)(1-x) = -C_{NP} + \delta + (E\bar{p} - \delta)x = \\ &= -C_{NP} + \delta + \left(\theta \frac{\gamma + T}{2} - \delta\right)x\end{aligned}$$

and the replicator dynamics become:

$$\begin{aligned}\dot{x} &= x(1-x) [\Pi_P(x) - \Pi_{NP}(x)] = x(1-x) \left[ C_{NP} - C_P - \delta - E\bar{p} + \left(\delta - \frac{\theta T}{2}\right)x \right] = \\ &= x(1-x) \left[ C_{NP} - C_P - \delta - \theta \frac{\gamma + T}{2} + \left(\delta - \frac{\theta T}{2}\right)x \right]\end{aligned}\quad (4)$$

Notice that the payoff function  $\Pi_P(x)$  is always strictly increasing in  $x$  so that the polluting strategy is self-enforcing.<sup>7</sup> The payoff of the non-polluting technology  $\Pi_{NP}(x)$  is, instead, strictly increasing in  $x$  if  $\theta \frac{\gamma + T}{2} - \delta > 0$ , namely, if the price of the tradable permits ( $\theta \frac{\gamma + T}{2}$ ) sold to firm  $P$  is higher than the benefit gained from meeting a firm  $NP$  ( $\delta$ ). Stated differently, in this case the payoff of firm  $NP$  increases with  $x$  since the firm  $NP$  is more likely to meet a firm  $P$  which makes it better off. The opposite obviously applies if the price of the tradable permits sold to firm  $P$  is lower than the spillover effect from meeting a firm  $NP$ .

The payoff differential  $\Pi_P(x) - \Pi_{NP}(x)$  is strictly increasing in  $x$  if  $\delta - \frac{\theta T}{2} > 0$ , strictly decreasing if  $\delta - \frac{\theta T}{2} < 0$ . This is consistent with our apriori intuition: if the benefit obtained by the matching of two firms  $NP$  ( $\delta$ ) are higher than that

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argue that citizens would be better-off in a perfectly clean world ( $x = 0$ ) than in an extremely polluted one ( $x = 1$ ). The opposite result will obviously emerge when the firms' profits are higher in  $x = 1$  than in  $x = 0$  (i.e.  $\Pi_{NP}(0) < \Pi_P(1)$ ). In that case, the firms' interests are likely to conflict with the welfare of society as a whole. The welfare analysis of the whole collectivity, however, goes beyond the scope of the present paper. We therefore leave it for future extensions of the present work.

<sup>7</sup>This occurs because, if  $\gamma > 0$ , the price that a firm  $P$  pays when it buys the pollution permit from another firm  $P$  ( $\frac{\theta T}{2}$ ) is higher than what it pays when it buys it from a firm  $NP$  ( $\theta \frac{\gamma + T}{2}$ ).

from the meeting of two firms  $P$  ( $\frac{\theta T}{2}$ ), then the payoff of the former firms grow faster than that of the latter as firms  $NP$  spread through the population. The opposite obviously applies if  $\delta$  is lower than  $\frac{\theta T}{2}$ .

The following Proposition illustrates the taxonomy of the dynamic regimes that may occur in the context  $\gamma < T$ .

**Proposition 3** *When  $\gamma < T$ , dynamics (4) can lead to the following possible dynamic regimes:*

1) *If  $C_{NP} - C_P \geq \max\left\{\theta\frac{\gamma+T}{2} + \delta, \theta\left(T + \frac{\gamma}{2}\right)\right\}$ , then whatever the initial distribution of strategies  $x(0) \in (0, 1)$ ,  $x$  will always converge to the steady state  $x = 1$  (see<sup>8</sup> Figure 1).*

2) *If  $C_{NP} - C_P \leq \min\left\{\theta\frac{\gamma+T}{2} + \delta, \theta\left(T + \frac{\gamma}{2}\right)\right\}$ , then whatever the initial distribution of strategies  $x(0) \in (0, 1)$ ,  $x$  will always converge to the steady state  $x = 0$  (see<sup>8</sup> Figure 2).*

3) *If  $\theta\left(T + \frac{\gamma}{2}\right) < C_{NP} - C_P < \theta\frac{\gamma+T}{2} + \delta$  and  $\delta - \frac{\theta T}{2} > 0$ , then there exists a repulsive inner steady state:*

$$\bar{x} = \left(C_P - C_{NP} + \delta + \theta\frac{\gamma+T}{2}\right) / \left(\delta - \frac{\theta T}{2}\right) \in (0, 1)$$

*If  $x(0) \in [0, \bar{x}]$ , then  $x$  converges to the steady state  $x = 0$ , while if  $x(0) \in (\bar{x}, 1]$ , then it converges to the steady state  $x = 1$  (see<sup>8</sup> Figure 3).*

4) *If  $\theta\frac{\gamma+T}{2} + \delta < C_{NP} - C_P < \theta\left(T + \frac{\gamma}{2}\right)$  and  $\delta - \frac{\theta T}{2} < 0$ , then whatever the initial distribution of strategies  $x(0) \in (0, 1)$ ,  $x$  will always converge to the inner steady state  $\bar{x} \in (0, 1)$  in which the alternative strategies  $P$  and  $NP$  coexist (see<sup>8</sup> Figure 4).*

Notice that, as occurred under matrix  $A$ , even in the present context a bi-stable (path-dependent) dynamic regime takes place only if the spillover parameter  $\delta$  is sufficiently high (more precisely,  $\delta > \frac{\theta T}{2}$ , see case 3 above).

As it clearly emerges from the Proposition above, the dynamic regimes that may occur with matrix  $C$  (when permits are traded between heterogeneous firms) are similar to those that result from matrix  $A$  (when permits are traded only between polluting firms), although under different parameter values. In both cases (in particular under cases 3 of Propositions 1 and 3), we can have a bi-stable dynamics so that hysteresis takes place in the model. This implies that two economies that take part to the same ETS and undergo the same legislation in terms of sanctions to non-compliant firms may lead to two opposite outcomes ( $x = 0$  where none pollutes or  $x = 1$  where everyone pollutes) depending on the share of firms  $x(0)$  that initially adopt the new technology  $NP$ . On the contrary, when cases 4 of Propositions 1 and 3 apply, the dynamics emerging from the payoff matrices  $A$  and  $C$  are independent of the initial conditions and

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<sup>8</sup>Please note that although the referred figures in the statement are related to the matrix  $A$ , from a qualitative point of view they can fit also for the cases under scrutiny.

there always exists a unique steady-state that is globally attractive ( $x = 0$ ,  $x = 1$ , or  $x = \bar{x}$ ).

It is important to emphasize that -ceteris paribus- a rise in the penalty level  $T$  shifts the economy from the regime analyzed in Proposition 1 (case  $\gamma \geq T$ ) to that of Proposition 3 (case  $\gamma < T$ ), thus increasing the overall number of transactions in the ETS as it induces also firms  $P$  and  $NP$  to exchange permits.

The following Proposition describes how the inner equilibrium identified in Proposition 3 is modified by a change in the penalty level and/or in the monitoring capacity of the regulatory authority that affects the probability to discover non-compliant firms.

**Proposition 4** *In the context  $\gamma < T$ , if  $\theta(T + \frac{\gamma}{2}) < C_{NP} - C_P < \theta\frac{\gamma+T}{2} + \delta$  and  $\delta - \frac{\theta T}{2} > 0$  (bi-stable regime), then  $\frac{\partial \bar{x}}{\partial T} > 0$  and  $\frac{\partial \bar{x}}{\partial \theta} > 0$ . If  $\theta\frac{\gamma+T}{2} + \delta < C_{NP} - C_P < \theta(T + \frac{\gamma}{2})$  and  $\delta - \frac{\theta T}{2} < 0$  (coexistence regime), then  $\frac{\partial \bar{x}}{\partial T} < 0$  and  $\frac{\partial \bar{x}}{\partial \theta} < 0$ .*

When a bi-stable dynamics regime applies (case 3 of Proposition 3 above), an increase of  $T$  raises the value of the repulsive inner steady-state  $\bar{x}$ , therefore it increases the basin of attraction of  $x = 0$  with respect to that of  $x = 1$ . Stated differently, when the system is path-dependent an increase of  $T$  raises the likelihood that the system may converge to the steady state  $x = 0$  (where all firms adopt the non-polluting technology  $NP$ ).

When the inner steady-state  $\bar{x}$  is globally attracting (case 4 of Proposition 3 above), an increase in  $T$  reduces the value of  $\bar{x}$ . In other words, in this case a rise in the penalty level (that shifts the system from matrix  $A$  to matrix  $C$ ) increases the share of non-polluting firms  $NP$  at the equilibrium.

In both cases, therefore, a rise in the penalty level implemented by the regulatory authority that increases permits trade tends to promote the diffusion of the new non-polluting technology, as it increase either the attraction basin of the totally clean outcome ( $x = 0$ ) or the share of clean firms at the equilibrium.<sup>9</sup>

The same applies to an increase in the monitoring effort/ability of the regulator that raises the value of  $\theta$ , thus making more difficult for non-compliant firms to escape the sanction.

Finally, it is important to underline that the dynamics of the economy may lock the system into a "poverty-trap". As a matter of fact, in some cases the dynamic regime may lead the system towards the "dirty" steady-state  $x = 1$ , although the firms' profits would be higher in the "clean" steady-state  $x = 0$ , in which also the overall collectivity would most likely be better-off. To show that this may be the case, consider Proposition 3. In this context, we have:

$$\Pi_{NP}(0) > \Pi_P(1)$$

for:

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<sup>9</sup>Notice that, when  $\gamma = T$ , the two matrices  $A$  and  $C$  coincide so that they have the same inner equilibrium  $\bar{x}$ . As a consequence, the comparative statics results concerning  $\bar{x}$  described in the previous Propositions hold true even when an increase in  $T$  shifts the regime from matrix  $A$  to matrix  $C$ .



$$-C_{NP} + \delta > -C_P - \theta \frac{T}{2} + \theta \gamma$$

or equivalently when:

$$\theta \left( \frac{T}{2} - \gamma \right) + \delta > C_{NP} - C_P \quad (5)$$

Recalling that the condition for a bi-stable dynamics under Proposition 3 is:

$$\theta \left( T + \frac{\gamma}{2} \right) < C_{NP} - C_P < \theta \frac{\gamma + T}{2} + \delta \quad (6)$$

it turns out that the two conditions (5) and (6) can simultaneously apply if:

$$\theta \left( T + \frac{\gamma}{2} \right) < \theta \left( \frac{T}{2} - \gamma \right) + \delta$$

that is, if:

$$\theta \left( \frac{T}{2} + \frac{3}{2} \gamma \right) < \delta$$

This condition suggests that if the positive spillover effect  $\delta$  that firms  $NP$  enjoy when they meet on the market is sufficiently large, then all firms would be better-off by adopting the new technology but the bi-stable dynamics may still lead the economy in the opposite direction if many firms are initially reluctant to change technology and keep on using the old one (i.e. if  $x(0)$  is initially above the repulsive inner equilibrium  $\bar{x}$ ).<sup>10</sup> In other words, in this case the economy may end up in a situation that is Pareto-dominated for the firms and most likely also for the society as whole.

### 3.3 Dynamics of the game when $\theta$ is endogenous

So far, we have assumed that the monitoring capacity of the regulatory authority and thus the probability  $\theta$  of non-compliant firms of being discovered is exogenously given. However, this may not be the case. In fact, the monitoring capacity and effort of the regulatory authority in discovering and sanctioning non-compliant firms can actually be endogenously determined by the number of polluting firms that the authority has to control. In this section we intend to analyse how results may change if we account for this possibility by endogenising the probability  $\theta$ . For this purpose, we will focus on the case in which heterogeneous firms can exchange emission permits (matrix  $C$  above).<sup>11</sup>

<sup>10</sup>Notice that a positive technological spillover  $\delta$  is a necessary but not a sufficient condition to satisfy the condition above, since such a spillover has to be sufficiently high for this to occur.

<sup>11</sup>A similar analysis can obviously be performed also in the case of matrix  $A$ . We omit it for space reasons and prefer to focus on matrix  $C$  since in this latter case the permit market is more extended as it includes also the trade between firms  $P$  and  $NP$ .

Let us assume, for the sake of simplicity, that the probability that a non-compliant firm  $P$  is actually discovered by the regulatory authority is a linear function of the overall share of polluting firms, that is:

$$\theta(x) := a + bx$$

where:  $a \geq 0$ ,  $b \geq 0$  and  $0 \leq a + bx \leq 1 \forall x$ .

Notice that we intentionally imposed no apriori condition on the sign of the parameter  $b$ . In fact, an increase in the share  $x$  of polluting firms may have conflicting effects on the monitoring capacity of the regulatory authority, so that the sign of  $b$  is apriori ambiguous. On the one hand, the higher is the share of polluting firms, the lower is the probability for each of them of being discovered if it keeps producing without purchasing the additional emission permit that is requested by law ( $b < 0$ ). On the other hand, the higher is the share of polluting firms, the higher is likely to be the monitoring effort of the regulatory authority and thus also the probability for non-compliant firms of being discovered ( $b > 0$ ).

Assuming  $\theta(x) := a + bx$ , the expected payoffs become:

$$\begin{aligned} \Pi_P(x) &= -C_P - \frac{\gamma + T}{2}(a + bx) + \frac{\gamma}{2}x(a + bx) = \\ &= -C_P - a\frac{\gamma + T}{2} + \frac{1}{2}(a\gamma - b(\gamma + T))x + b\frac{\gamma}{2}x^2 \end{aligned}$$

$$\begin{aligned} \Pi_{NP}(x) &= -C_{NP} + \delta + \left( \frac{\gamma + T}{2}(a + bx) - \delta \right)x = \\ &= -C_{NP} + \delta + \left( a\frac{\gamma + T}{2} - \delta \right)x + b\frac{\gamma + T}{2}x^2 \end{aligned}$$

therefore, the following replication dynamics apply:

$$\dot{x} = x(1-x) \left[ C_{NP} - C_P - \delta - a\frac{\gamma + T}{2} + \left( \delta - b\frac{\gamma + T}{2} - a\frac{T}{2} \right)x - b\frac{T}{2}x^2 \right] \quad (7)$$

Observe that the graphs of the payoff functions  $\Pi_P(x)$  and  $\Pi_{NP}(x)$  are given by two convex (U-shaped) parabola if  $b > 0$ , while they can be represented as two concave (bell-shaped) parabola if  $b < 0$ . Both parabola, therefore, have a minimum (respectively, maximum) that may lie or not within the interval  $(0, 1)$ . The payoff differential:

$$f(x) := C_{NP} - C_P - \delta - a\frac{\gamma + T}{2} + \left( \delta - b\frac{\gamma + T}{2} - a\frac{T}{2} \right)x - b\frac{T}{2}x^2$$

is a concave parabola if  $b > 0$ , whereas is a convex parabola if  $b < 0$ .

It follows that we can have two steady-states in  $(0, 1)$ ,  $\bar{x}_1$  and  $\bar{x}_2$ , with  $\bar{x}_1 < \bar{x}_2$ . In such case, if  $b > 0$ , we have four steady-states,  $x = 0$  and  $\bar{x}_2$

being attractive, while  $x = 1$  and  $\bar{x}_1$  are repulsive (see Figure 5); if, on the contrary,  $b < 0$  we still have the same four steady states but with opposite stability properties:  $x = 0$  and  $\bar{x}_2$  repulsive, while  $x = 1$  and  $\bar{x}_1$  are attractive (see Figure 6).

Observe that it is:

$$f(0) = C_{NP} - C_P - \delta - a \frac{\gamma + T}{2} < 0 \quad (8)$$

for  $C_{NP} - C_P < \delta + a \frac{\gamma + T}{2}$  and:

$$f(1) = C_{NP} - C_P - (a + b) \left( \frac{\gamma}{2} + T \right) < 0 \quad (9)$$

for  $C_{NP} - C_P < (a + b) \left( \frac{\gamma}{2} + T \right)$ .

Also notice that the value of  $x$  that maximizes  $f(x)$  (if  $b > 0$ ) or minimizes  $f(x)$  (if  $b < 0$ ) is the solution of the following equation:

$$f'(x) = \delta - b \frac{\gamma + T}{2} - a \frac{T}{2} - bTx = 0$$

namely:

$$x_e = \frac{\delta - b \frac{\gamma + T}{2} - a \frac{T}{2}}{bT}$$

where  $x_e > 0$  when:

$$\delta - b \frac{\gamma + T}{2} - a \frac{T}{2} \geq 0 \quad \text{if } b \geq 0 \quad (10)$$

while  $x_e < 1$  when:

$$\delta - b \frac{\gamma + T}{2} - a \frac{T}{2} \geq bT \quad \text{if } b \geq 0 \quad (11)$$

### 3.3.1 Case $b > 0$

When  $b > 0$ , the necessary and sufficient condition to have 4 steady-states is that conditions (8)-(11) are simultaneously satisfied and that  $f(x_e) > 0$  also holds.

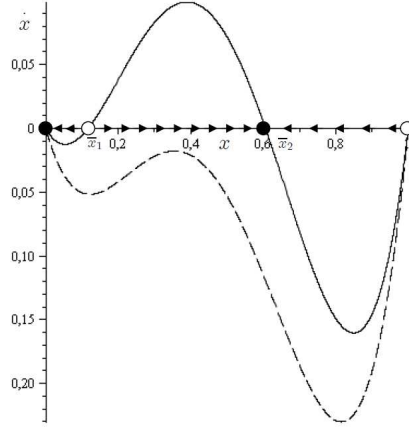
Among this set of conditions, (10)-(11) jointly ensure that the value of  $x$  that maximizes  $f(x)$  lies in the interval  $(0, 1)$ , namely  $x_e \in (0, 1)$  iff:

$$b \frac{\gamma}{2} + (a + b) \frac{T}{2} < \delta < b \frac{\gamma}{2} + (a + 3b) \frac{T}{2}$$

which can also be expressed in terms of the penalty  $T$  as follows:

$$\frac{2\delta - b\gamma}{a + 3b} < T < \frac{2\delta - b\gamma}{a + b} \quad (12)$$

provided  $a + b \neq 0$ .<sup>12</sup>



**Figure 5:** Graphes of  $\dot{x}$  corresponding to two different values of  $C_{NP} - C_P$ , that give rise to two different dynamic regimes; in one regime, the steady states  $\bar{x}_2$  and  $x = 0$  are locally attracting, in the other the steady state  $x = 0$  is globally attracting. The dotted line refers to the cost difference  $C_{NP} - C_P = 31$  the continuous line to the cost difference  $C_{NP} - C_P = 31.5$  The other parameter values are  $a = 0.4$ ,  $b = 0.3$ ,  $T = 47$ ,  $\delta = 22$   $\gamma = 3$ .

The remaining conditions (8), (9) and  $f(x_e) > 0$ , that are needed to have 4 steady states, are all dependent on the cost difference between the two technologies  $C_{NP} - C_P$ . More precisely, as can be clearly seen from conditions (8)-(9), the cost difference between the clean and the dirty technology must be sufficiently low to have the dynamic regime with four equilibria described in the section above. In fact, an increase in the difference  $C_{NP} - C_P$  shifts upwards the concave parabola  $f(x)$ . A relatively low increase in the cost of the two technologies moves the attracting equilibrium  $\bar{x}_2$  to the right (thus raising the number of polluting firms at the equilibrium) and the repulsive equilibrium  $\bar{x}_1$  to the left (which reduces the attraction basin of the "virtuous" equilibrium  $x = 0$ ), which is consistent with what one would reasonably expect. But if the increase in the difference  $C_{NP} - C_P$  is very high, the parabola may shift above the horizontal axis, so that there is no longer any inner equilibrium. Figure 5 shows two graphes of  $\dot{x}$ , corresponding to different values of  $C_{NP} - C_P$ , that give rise to two different dynamic regimes; in one regime, the steady states  $\bar{x}_2$  and  $x = 0$  are locally attracting, in the other the steady state  $x = 0$  is globally attracting.

Summing up, when  $b > 0$ , if the difference in the technological costs is sufficiently low and the penalty level has intermediate values as described above,

<sup>12</sup>Notice that it is always  $a + b \geq 0$  since we have:  $0 \leq a + bx \leq 1 \forall x$ .

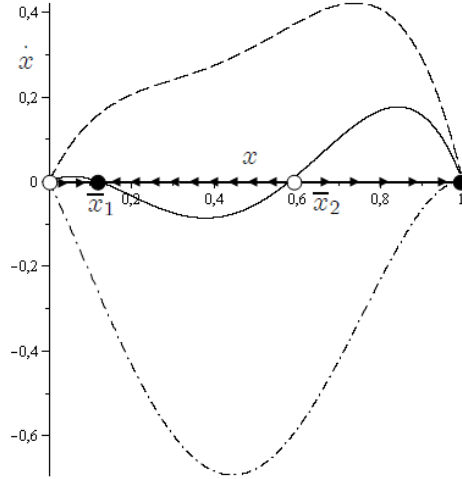
then we can have 4 steady states, that is, a path-dependent economy with one inner equilibrium  $\bar{x}_2$  in which the two strategies  $P$  and  $NP$  coexist.

### 3.3.2 Case $b < 0$

A similar reasoning applies in the case  $b < 0$ . When  $b < 0$ , we have 4 steady-states iff:  $f(0) > 0, f(1) > 0, f(x_e) < 0$  and  $x_e \in (0, 1)$ . The former three conditions crucially depend on the difference  $C_{NP} - C_P$  (that has to be sufficiently high for the vertical intercept of the curve to be positive as well as its value at  $x = 1$ ). As to the latter condition  $x_e \in (0, 1)$ , it is easy to check that it holds iff:

$$a + 3b > 0 \quad \text{and} \quad \frac{2\delta - b\gamma}{a + b} < T < \frac{2\delta - b\gamma}{a + 3b}$$

Thus, when  $b < 0$ , if the difference in the technological costs is sufficiently high and the penalty level has intermediate values as described above, we have 4 steady-states with opposite stability features with respect to the case  $b > 0$ , namely:  $x = 0$  and  $\bar{x}_2$  repulsive, while  $x = 1$  and  $\bar{x}_1$  are attractive.



**Figure 6:** Graphes of  $\dot{x}$  corresponding to two different values of  $C_{NP} - C_P$ , that give rise to two different dynamic regimes; in one regime, the steady states  $\bar{x}_1$  and  $x = 1$  are locally attracting, in the others, the steady state  $x = 1$  or  $x = 0$  are globally attracting. The dotted line refers to the cost difference  $C_{NP} - C_P = 23.5$ , the continuous line to the cost difference  $C_{NP} - C_P = 22$ . The other parameter values are  $a = 0.7, b = -0.3, T = 47, \delta = 4, \gamma = 3$ .

Observe that an increase in the cost difference  $C_{NP} - C_P$  shifts the attracting equilibrium  $\bar{x}_1$  to the right (thus increasing the share of polluting firms  $P$  at the equilibrium) and the repulsive equilibrium  $\bar{x}_2$  to the left (which extends the

attraction basin of  $x = 1$  where pollution is maximum). This seems consistent with our intuition: the higher is the cost of the clean technology with respect to the polluting technology, the lower is the number of firms that decide to invest in the new technology and the more attractive is the "business-as-usual" solution in which firms prefer to keep on using the traditional polluting technology.

Even in this case, however, if the increase in the cost difference  $C_{NP} - C_P$  is remarkably high, the parabola will shift above the horizontal axis, so that the inner coexistence equilibria  $\bar{x}_1$  and  $\bar{x}_2$  cease to exist and there remains only one attracting equilibrium,  $x = 1$ . Figure 6 shows two graphs of  $\dot{x}$ , corresponding to different values of  $C_{NP} - C_P$ , that give rise to two different dynamic regimes; in one regime, the steady states  $\bar{x}_1$  and  $x = 1$  are locally attracting, in the other the steady state  $x = 1$  is globally attracting.

## 4 Conclusions

The present paper has examined how the implementation of an ETS may affect the diffusion of new environmental-friendly technologies, taking into account both the penalty to non-compliant firms established in the ETS and the possible strategic behaviour of single firms. For this purpose, we have set up and analysed an evolutionary game model with random matching. While this framework does not aim to be necessarily realistic (although it fits many contexts, possibly including also the pairwise meetings in local ETS), it allows to explain learning processes and to emphasize specific mechanisms that may derive from strategic interaction among economic agents.

As shown in the paper, we can have two alternative payoff matrixes depending on the relationship between two crucial parameters,  $T$  and  $\gamma$ , that capture the penalty level and the incentive of clean firms to act strategically, respectively. In one case, only polluting firms exchange permits among themselves, whereas in the other case permits can be traded between heterogeneous firms (polluting and non-polluting). We have shown that by properly increasing the penalty level the regulatory authority can shift from one dynamic regime to the other (i.e. we can pass from the former to the latter case) and that an increase in permits trade promotes the diffusion of innovative pollution-free technologies at the equilibrium.

In both cases, moreover, multiple equilibria emerge from the model, with dynamics leading either to extreme equilibria or to inner equilibria. When the dynamics lead to extreme equilibria, all firms imitate the others and select the same (polluting or non-polluting) strategy. When they converge to an inner attracting equilibrium, then there coexist heterogeneous choices in the population of firms, with some firms that adopt the clean technology and others that remain with the old polluting technology. When the inner equilibrium is, instead, a source (i.e. a repulsive steady state), the system is characterized by path-dependency. This suggests that in a context characterized by bounded rationality and imitative behaviours as the one described in this paper, the initial share of innovative firms that adopt the new non-polluting technology may

play a key role in determining the final outcome of the ETS. If the dynamic trajectories are path-dependent, in fact, two economies that take part to the same ETS and undergo the same penalty system (as it occurs, for instance, in the European ETS) might end up in opposite situations as to the diffusion of the new technology depending on the initial share of non-polluting firms.

Finally, the number of possible equilibria can further increase (up to four alternative steady-states) if we assume that the probability of discovering non-compliant firms is not exogenously given, but rather a function of the number of polluting firms. In any case, whatever the number of possible equilibria, it is also possible to rank them and analyse which one Pareto-dominates the others.

Further research will be needed in the future to deepen the present analysis. In particular, it would be desirable to extend the evolutionary game proposed here from pairwise random matchings to the case of  $n$  firms possible meetings, so that each firm can simultaneously match up and exchange permits with any other firm in the market rather than with a single firm. This would strengthen the realism of the model, potentially adding further complexity to the possible dynamics that derive from it.

## 5 References

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