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Technology and Agricultural Diversification

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Introduction

In recent years there has been a growing interest in the subject of diversification of agricultural production in the developing countries. Unfortunately, very little research on the economics of crop diversification has been done for developing countries. Dalrymple [5] points out that " . . . diversification is more a subject of vague references than actual knowledge. . . . Much more research is needed on diversification at the conceptual and applied levels. Development of a theoretical economic framework could be of significant value in organizing future analysis. One possibly useful starting point is the theory of comparative advantage. The theory should be applied to both production and marketing. . . ."

Interest in agricultural diversification in the developing countries has been heightened by the production increases of the green revolution which, although limited in area covered and number of farmer participants,

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are real and have caused further "revolutions" [30]. One is a higher degree of confidence among researchers in the developing countries that, with well funded and organized research programs, they can create new technologies. Another is the relatively new and generally accepted position of policymakers that peasant farmers, under the right conditions, are capable of rapid adoption of new technology and rapid increases in output. A third is in world grain markets and the price relationships between food grains and feed grains. An increasing number of persons are calling for diversification as a means for both capitalizing on the green revolution and avoiding some of its adverse consequences [3]. All of the above combine to put pressure on those who allocate funds and administer research in the developing countries to concern themselves with a broader range of agricultural commodities. Yet economics as a discipline has contributed little in the way of decision aids to help these people decide on the allocation of research resources among various commodities.

The purpose of this paper is to present and discuss a theoretical model which represents a start toward the development of a conceptual framework designed to analyze questions about the allocation of research resources among alternative crops. First, diversification is defined and discussed. Then a basic model, adapted from the two-factor, two-product model used in international trade theory, is presented. Next, technical change is introduced into the model and some numerical examples tracing the effects of concentrating research on one crop or another are presented. The paper closes with some implications of the results and further considerations of issues in diversification.

Definition of and Interest in Diversification

Crop diversification is defined as an increase in the number of different crops that can profitably be grown on a given piece of land during a given crop season. It concerns the number of crops which simultaneously compete for the same set of production resources [6].

One can think of diversification at either the national or the farm level [14], but it can be of a different nature at each level. Farm level diversification deals with the number of separate enterprises which the individual farmer engages in during any given production period. Conceptually, the total number of enterprises from which an individual farmer might choose would include all of the individual crop and livestock enterprises which are technically possible to produce in the agroclimatic environment in which that farmer is operating. Promoting diversification thus means increasing the number of enterprises which are economically feasible. Technical change then may include both adding to the number of enterprises as well as improving the productivity of enterprises already available to the individual farmer. Although this paper deals only with crop enterprises, the reader is cautioned that, as DeBoer has found, in some areas an integrated crop-livestock production system may be in operation and crop and livestock production cannot be treated as separate and independent enterprises [7].

At the national level, diversification of production can occur either as a result of diversification at the farm level or through the addition of new crops which do not compete for resources with other crops. For example, the latter situation characterizes the pattern of diversification in Thailand [3, 29], while in Taiwan there has been both farm

level diversification and increases in the number of products comprising total agricultural output.

At the national level the chief interest in diversification is to counteract some of the technical, economic and political disadvantages of monoculture [5]. Among these disadvantages are cyclical swings in primary product prices, increased competition from synthetic substitutes, variations in foreign exchange earnings, seasonal fluctuations in employment, and inequalities in the distribution of income among regions.

At the farm level the interests in diversification include those dealing chiefly with the level and stability of farmers' incomes. Interest in crop diversification as a means of reducing the variability of farmers' incomes primarily due to weather has early roots in farm management and production economics research in the United States, particularly in the Great Plains region [24]. Diversification was of interest in increasing farm income through fuller utilization of farm resources. In most cases, however, the main obstacle to diversification was found to be technical constraints on the range of crops which were economically feasible in any given area ^{1/} [12]. Interest of some researchers shifted to firm survival and hence to firm growth [25], while others focused on studies of managerial behavior and decision making [14]. In spite of the attention devoted to the topic of diversification by economists, very little progress was made in developing a theoretical framework for dealing with the issue.

Diversification at the farm level in developing countries has likewise been conceptually and theoretically neglected. During the early 1950's the chief concern was with the obstinacy and traditionalism of peasant farmers. After those myths had been laid to rest, and the

technological developments of the 1960's were realized, attention shifted to the risk aversion of peasant farmers [31]. There is a considerable amount of literature which purports that risk and uncertainty in agricultural production does influence the mix of crops which farmers grow as well as the intensity of factor use and, therefore, crop yields [22, 23, 27]. What is less clear, at this stage, is the degree to which different groups of farmers are risk averters [8].

Diversification at the farm level may not always be beneficial to either the farmer or the economy. Trying to encourage farmers to grow several different crops may result in lower total agricultural output [1]. More generally, Berry and Hymer [2] have shown that under some conditions facilitating the substitution of resources among outputs can lead to lower levels of income and community welfare.

Nevertheless, the current interest in crop diversification at the farm level in developing countries seems to have two bases. First, the variance of returns may differ from one crop to another due to different degrees of variation over time in (a) yield per unit area or (b) demand (price), both domestic and foreign. Concern is with selecting crop combinations which result in farmers achieving the highest level of expected utility, the latter being a function of both the level and variance of income. Some research is under way on this aspect [20]. Second, there may be interest in crop diversification growing out of differential rates of growth of demand (both domestic and foreign^{2/}) among crops. Concern may be with shifts in relative resource allocations over time among crops.

In a technologically dynamic world there is also the issue of allocation of research resources among commodities. What role should production

technology play in agricultural diversification? Hayami and Ruttan [11] have persuasively argued that agricultural innovations are generally induced by factor scarcities, and the nature of particular innovations will be influenced by the nature of factor scarcities and relative factor prices. In a country with an abundant labor supply relative to land, such as Japan, innovations will be of the type which economizes on or makes more productive the land supply (land saving). In a country such as the United States where land has historically been abundant relative to labor, innovations were of a labor saving type.

But even if it is relatively clear within a developing country which factor is relatively scarce and should be made more productive, there remains the question as to the allocation of research resources among several crops, where it is possible to generate improved technology consistent with factor endowments for each of them.

The research resource allocation question is at the heart of diversification of agricultural production. This is a topic which has assumed increasing importance in recent years as part of the green revolution. As country after country has met with success in achieving rapid increases in cereal grain production, notably, wheat and rice, some have been faced with demand constraints for these products.

We turn next to the development of a theoretical framework which will help us to analyze questions concerning the allocation of production research resources among crops. We employ a modification of the standard two-product, two-factor international trade model in our analysis. The basic question which we wish to explore is: In a two-commodity production regime, does it make much difference which commodity experiences technological change in production?

The Basic Model

To analyze certain questions concerning the benefits to be derived from diversification of agricultural production, we need a theoretical model which will enable us to trace through changes in production functions, factor endowments, and relative product prices on output, income and factor rewards. In this section we present such a model, which is adapted from the basic two-factor, two-product model used in international trade theory.^{3/} We will deal with a Cobb-Douglas production world.

Let us start by assuming a region (thought of as an area within a country or a country which trades in a larger world market) produces two goods, q_1 and q_2 , with two homogeneous factors of production, L and K , where L is the labor input and K is the land (capital) input. Total factor supplies are assumed to be fixed.

Production of our two goods is given by the production functions

$$(1a) \quad q_1 = \alpha_0 L_1^\alpha K_1^{1-\alpha} = \alpha_0 L_1 \left(\frac{K_1}{L_1} \right)^{1-\alpha}$$

$$(1b) \quad q_2 = \beta_0 L_2^\beta K_2^{1-\beta} = \beta_0 L_2 \left(\frac{K_2}{L_2} \right)^{1-\beta}$$

which reflect constant returns to scale. In addition, the fixed supplies of labor and land (capital) are represented by

$$(2a) \quad L_1 + L_2 = \bar{L}$$

$$(2b) \quad L_1 \left(\frac{K_1}{L_1} \right) + L_2 \left(\frac{K_2}{L_2} \right) = \bar{K}$$

Furthermore, we assume that the factors of production are fully employed.^{4/}

The marginal productivities of the factors are:

$$(3a) \quad \frac{\partial q_1}{\partial L_1} = \alpha_0 \alpha \left(\frac{K_1}{L_1} \right)^{1-\alpha} \quad ; \quad \frac{\partial q_1}{\partial K_1} = \alpha_0 (1-\alpha) \left(\frac{K_1}{L_1} \right)^{-\alpha}$$

$$(3b) \quad \frac{\partial q_2}{\partial L_2} = \beta_0 \beta \left(\frac{K_2}{L_2} \right)^{1-\beta} \quad ; \quad \frac{\partial q_2}{\partial K_2} = \beta_0 (1-\beta) \left(\frac{K_2}{L_2} \right)^{-\beta}$$

Let w be the wage rate and r the rental value of land (capital), and p_1 and p_2 be the prices of q_1 and q_2 , respectively. We assume that the wage rates and rental value of land are the same in the production of q_1 and q_2 ; i.e., there is perfect mobility of factors between the production of our two products. Furthermore, since total factor supplies are fixed, w and r are determined implicitly in our model. On the other hand, product prices are assumed to be given to our region; i.e., variations in output of q_1 or q_2 do not affect their prices.

In equilibrium the unit factor rewards are equal to the value marginal products of the factors:

$$(4a) \quad w = p_1 \left(\frac{\partial q_1}{\partial L_1} \right) = p_2 \left(\frac{\partial q_2}{\partial L_2} \right)$$

and

$$(4b) \quad r = p_1 \left(\frac{\partial q_1}{\partial K_1} \right) = p_2 \left(\frac{\partial q_2}{\partial K_2} \right)$$

Equations (4a) and (4b) can be rewritten as either

$$(5a) \quad \frac{p_2}{p_1} = \frac{\frac{\partial q_1}{\partial L_1}}{\frac{\partial q_2}{\partial L_2}} = \frac{\frac{\partial q_1}{\partial K_1}}{\frac{\partial q_2}{\partial K_2}}$$

or

$$(5b) \quad \frac{\frac{\partial q_1}{\partial L_1}}{\frac{\partial q_1}{\partial K_1}} = \frac{\frac{\partial q_2}{\partial L_2}}{\frac{\partial q_2}{\partial K_2}} .$$

Substituting the marginal productivity expressions for our factors given in (3a) and (3b) into (5b) we get,

$$(6) \quad \frac{\alpha}{1-\alpha} \left(\frac{K_1}{L_1} \right) = \frac{\beta}{1-\beta} \left(\frac{K_2}{L_2} \right) .$$

If we let

$$\frac{\alpha}{1-\alpha} = a \quad \text{and} \quad \frac{\beta}{1-\beta} = b ,$$

by using the factor endowment constraint given in (2a) and (2b), we obtain:^{5/}

$$(7) \quad \frac{K_1}{L_1} = \frac{b \left[\left(\frac{L_1}{\bar{L}} \right) \left(\frac{K_1}{L_1} \right) + \left(1 - \frac{L_1}{\bar{L}} \right) \left(\frac{K_2}{L_2} \right) \right]}{a + (b-a) \left(\frac{L_1}{\bar{L}} \right)}$$

and

$$(8) \quad \frac{K_2}{L_2} = \frac{a \left[\left(\frac{L_1}{\bar{L}} \right) \left(\frac{K_1}{L_1} \right) + \left(1 - \frac{L_1}{\bar{L}} \right) \left(\frac{K_2}{L_2} \right) \right]}{a + (b-a) \left(\frac{L_1}{\bar{L}} \right)}$$

Let

$$(9) \quad \ell = \frac{L_1}{\bar{L}}$$

and

$$(10) \quad R = \left[\ell \left(\frac{K_1}{L_1} \right) + (1-\ell) \left(\frac{K_2}{L_2} \right) \right] = \frac{\bar{K}}{\bar{L}}$$

We can then write our output of each product per worker as

$$(11) \quad \frac{q_1}{\bar{L}} = \alpha_0 \ell \left(\frac{bR}{a + (b-a)\ell} \right)^{1-\alpha}$$

and

$$(12) \quad \frac{q_2}{\bar{L}} = \beta_0 (1-\ell) \left(\frac{aR}{a + (b-a)\ell} \right)^{1-\beta}$$

Then, differentiating (11) and (12) with respect to ℓ , and ignoring the constants, we get^{6/}

$$(13) \quad \frac{d \left(\frac{q_1}{\bar{L}} \right)}{d\ell} = \left(\frac{bR}{a + (b-a)\ell} \right)^{1-\alpha} \left(\frac{a + \alpha\ell(b-a)}{a + (b-a)\ell} \right)$$

and

$$(14) \quad \frac{d \left(\frac{q_2}{\bar{L}} \right)}{d\ell} = - \left(\frac{aR}{a + (b-a)\ell} \right)^{1-\beta} \left(\frac{b - (1-\ell)\beta(b-a)}{a + (b-a)\ell} \right) .$$

Therefore, we can write

$$(15) \quad - \frac{d \left(\frac{q_1}{\bar{L}} \right)}{d \left(\frac{q_2}{\bar{L}} \right)} = (bR)^{1-\alpha} (aR)^{\beta-1} [a + (b-a)\ell]^{\alpha-\beta} \left(\frac{a + \alpha\ell(b-a)}{a + (b-a)(1-\beta + \beta\ell)} \right) ,$$

which is the slope of the production possibility frontier.

We can determine the curvature of the transformation curve by obtaining

$$\begin{aligned}
 (16) \quad \frac{d}{d\ell} \left[- \frac{d \left(\frac{q_1}{\bar{L}} \right)}{d \left(\frac{q_2}{\bar{L}} \right)} \right] &= \left[(bR)^{1-\alpha} (aR)^{\beta-1} \frac{[a + (b-a)\ell]^{\alpha-\beta}}{a + (b-a)(1-\beta + \beta\ell)} \right] \times \\
 &\quad (b-a) \left[\frac{(\alpha-\beta)[a + \alpha\ell(b-a)]}{a + (b-a)\ell} + \right. \\
 &\quad \left. \frac{\alpha[a + (b-a)(1-\beta + \beta\ell)] - \beta[a + \alpha\ell(b-a)]}{a + (b-a)(1-\beta + \beta\ell)} \right]
 \end{aligned}$$

The transformation curve will be concave to the origin if this derivative is negative. Conversely, the transformation curve will be concave, upward (away from the origin), if the derivative is positive.

We are now in a position to solve our model for the equilibrium values of the outputs and inputs. We need to know the factor endowments, \bar{K} and \bar{L} ; the coefficients of our production functions, α and β ; and the product prices, p_1 and p_2 . Given these values we (a) set (15) equal to $-\frac{p_2}{p_1}$ and solve for ℓ ; ^{7/} (b) using (9) we then solve for L_1 , and together with (2a) we solve for L_2 ; (c) we can then solve for K_1 and K_2 using equation (10); and (d) having solved for the above values of the L_i 's and K_i 's, we compute q_1 and q_2 from (1a) and (1b).

In our model factor prices are internally determined and we may wish to determine their values. Total factor returns are determined by

$$(17a) \quad w \cdot \bar{L} = p_1 \frac{\partial q_1}{\partial L_1} \cdot L_1 + p_2 \frac{\partial q_2}{\partial L_2} \cdot L_2$$

and

$$(17b) \quad r \cdot \bar{K} = p_1 \frac{\partial q_1}{\partial K_1} \cdot K_1 + p_2 \frac{\partial q_2}{\partial K_2} \cdot K_2.$$

Consequently, the implicit factor prices are

$$(18a) \quad w = \frac{1}{\bar{L}} \left(p_1 \frac{\partial q_1}{\partial L_1} \cdot L_1 + p_2 \frac{\partial q_2}{\partial L_2} \cdot L_2 \right)$$

and

$$(18b) \quad r = \frac{1}{\bar{K}} \left(p_1 \frac{\partial q_1}{\partial K_1} \cdot K_1 + p_2 \frac{\partial q_2}{\partial K_2} \cdot K_2 \right).$$

We can then see how factor prices change as a result of either changes in (a) the coefficients of the production functions, (b) the level of total factor endowments, or (c) product prices.

Finally, we wish to modify the basic competitive trade model so that our region can be an exporter of both q_1 and q_2 as well as an exporter of one and an importer of the other. We wish to avoid the exchange rate problem which arises in the classical trade model, and which would be particularly troublesome if our country or region exported some of both q_1 and q_2 .

The total income of our country or region will be

$$(19) \quad Y = p_1 q_1 + p_2 q_2$$

We assume that the output of our region is a small part of total production and trade, and therefore, prices of the outputs can be treated as given. Further, we are interested in the case where our region can be an exporter of both q_1 and q_2 .

Let q_1^d be the amount of total output of q_1 consumed at home. Then

our expenditure equation would be

$$(20) \quad E = p_1 q_1^d + p_2 q_2^d + S$$

where S represents expenditures on things which are not produced in the region and do not enter into the production processes for the q_i 's. In our formulation, $E = Y$. In this way we avoid the problem of handling exchange-rate adjustments without moving too far away from reality; i.e., it is quite realistic to think of a region buying goods which are not produced in the region and do not compete for local resources in their production.

There are a number of features of our model which represent some possibly serious oversimplifications of the real world. It is important to highlight these in order to place the usefulness of the theoretical model in a proper perspective. At its present stage of development, we feel the model is a useful conceptual tool for examining certain questions concerning diversification of agricultural production, but that certain refinements and extensions of it are required to test its validity and usefulness in specific problem situations.

First, the model assumes that all resources are fully employed, a standard assumption underlying production possibility curves. Yet the literature on agricultural development is replete with discussions of the seasonal or secular unemployment or underemployment of some resources, particularly labor. To the extent that one or more factors of production is under- or unemployed, the "observed" or "measured" production possibility frontier will lie inside or to the left of the frontier which would obtain with full employment. One way for this to come about is for there

to exist an institutionally defined minimum wage which is above the marginal value product of labor at full employment levels (implicitly solved for in our model).

Another reason why our observed production possibility frontier might lie inside the full employment frontier is that some labor is devoted to the production of what Hymer and Resnick [13] have defined as Z-goods, which are consumed, but not traded.

It would be possible but by no means easy to test for either the extent of under-or unemployment or the amount of labor devoted to the production of Z-goods, and the degree to which an estimated production possibility frontier deviated from one obtained with full employment of resources. Concentrating just on the question of under- or unemployment, one would first have to define what is meant by full employment of labor (hours or days of employment per worker in a given period). Further, one would have to verify whether or not there were wage rate rigidities which resulted in a wage rate being above the marginal value product of labor. Considering variations in the nature of agricultural production among regions of a country, variations in the seasonal demands for labor, and variations in the opportunity costs of labor both seasonally and among regions, it will be a difficult task to determine the extent of under-or unemployment and its impact on production. But if, in a particular situation, labor is greatly underutilized, approximations of the extent of such underutilization would be useful in many respects.

There is one other aspect of the underutilization of resources which is highly relevant to the next section of the paper dealing with technological change. Let us assume that there is some technological advance

which raises the demand for labor and capital and increases output. If we start from a situation of underutilization of resources, but this is not acknowledged, the resultant increase in output could consist of two components: (1) increased factor productivity and (2) fuller utilization of resources. If (2) is not recognized the estimated gains in production attributed "purely" to technological advance will be biased upward.

A second assumption of our model is that we are dealing with a Cobb-Douglas production world of the type already discussed, and this world holds throughout the full range of production -- from complete specialization in q_1 , to complete specialization in q_2 . We would like to make two points about this assumption. There is no need to assume that the agricultural production world is Cobb-Douglas. Other forms of production functions such as quadratic or CES production functions may be more appropriate in some (many) circumstances. We encourage others to derive the explicit form of the production possibility frontier using other forms of the production functions. We found it easiest to work with the Cobb-Douglas form, but hold no special brief for its universal applicability.

Furthermore, there is no reason to expect a particular form of the production functions to hold over the full range of possible factor substitution. At best, any given form may be a good approximation over a given (and sometimes small) range of resource substitution among the two production functions. Two possibilities are illustrated in Figures 1a and 1b. In Figure 1a, q_1 and q_2 are complementary in production in the range of Oa for q_1 and Ob for q_2 , but substitutes in the range of $a\hat{q}_1$ for q_1 and $b\hat{q}_2$ for q_2 . If the range of complementarity can be determined, the axis can be appropriately transformed to 0 and the analysis carried

Figure 1a

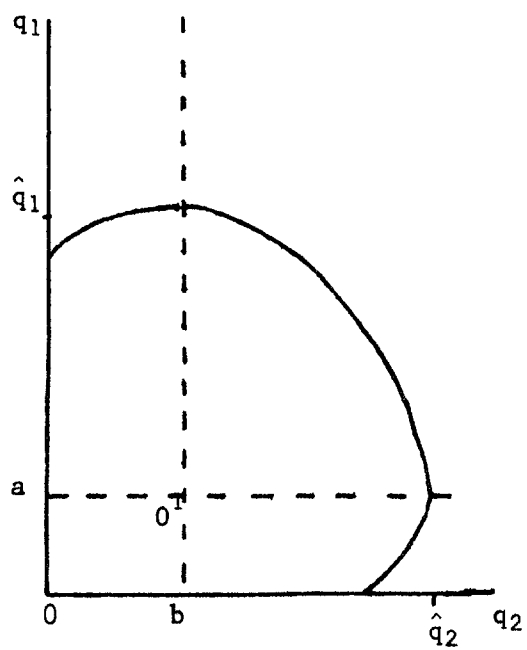
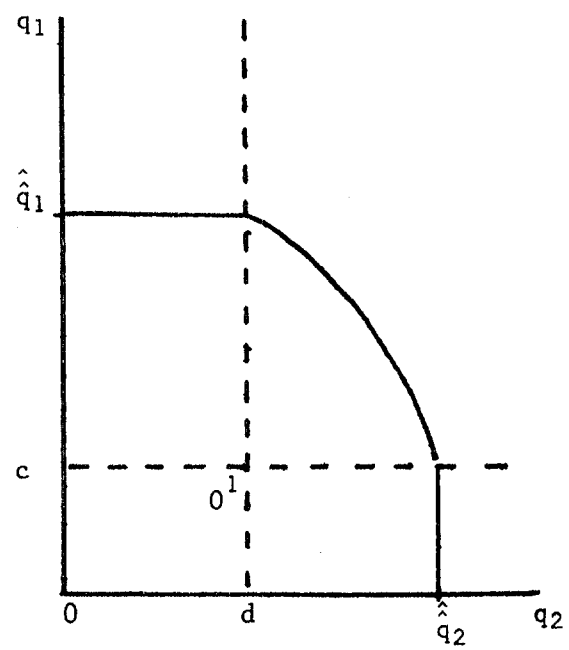


Figure 1b



out with the range of production being within $\hat{a}q_1$ and $\hat{b}q_2$ for q_1 and q_2 , respectively. Figure 1b illustrates a situation where it is possible to increase the production of q_1 up to O_d or q_1 up to O_c without reducing the outputs of q_1 or q_2 , respectively. In these ranges, q_1 and q_2 are supplementary in production. Beyond these levels, q_1 and q_2 compete for the same resources. One can think of other examples, but the ones presented above illustrate our point about any particular form of the production functions holding for only a part of the full range of possible production.

So far, our discussion has been in terms of a competitive world in which product prices are assumed given to our country or region. But, we are also interested in the general case of maximizing total returns subject to the production possibility curve where changes in output of our country or region can influence product prices.

Consider the following demand equations for our two outputs, q_1 and q_2 .

$$(21a) \quad p_1 = a_{10} + a_{11}q_1 + a_{12}q_2$$

$$(21b) \quad p_2 = a_{20} + a_{21}q_1 + a_{22}q_2$$

where

$$a_{10}, a_{20} > 0$$

$$a_{11}, a_{22} \leq 0$$

$$a_{12}, a_{21} < 0 \quad \text{if } q_1 \text{ and } q_2 \text{ are substitutes}$$

$$a_{12}, a_{21} > 0 \quad \text{if } q_1 \text{ and } q_2 \text{ are complements}$$

$$a_{12}, a_{21} = 0 \quad \text{if } q_1 \text{ and } q_2 \text{ are independent}$$

We can think of a_{10}, a_{20} as containing all the other variables which would

enter our demand equations, such as income, but these are held constant at specified levels. The expression for total revenue is

$$\begin{aligned}
 (22) \quad TR &= P_1 q_1 + P_2 q_2 \\
 &= a_{10} q_1 + a_{11} q_1^2 + a_{12} q_2 q_1 + a_{20} q_2 + a_{21} q_1 q_2 + a_{22} q_2^2 \\
 &= a_{10} q_1 + a_{20} q_2 + a_{11} q_1^2 + a_{22} q_2^2 + (a_{12} + a_{21}) q_1 q_2
 \end{aligned}$$

And, the expressions for marginal revenues with respect to q_1 and q_2 are

$$(23a) \quad \frac{\partial TR}{\partial q_1} = a_{10} + 2a_{11} q_1 + (a_{12} + a_{21}) q_2$$

$$(23b) \quad \frac{\partial TR}{\partial q_2} = a_{20} + 2a_{22} q_2 + (a_{12} + a_{21}) q_1$$

Total revenue will be at a maximum when

$$(24) \quad \frac{\partial TR}{\partial q_1} = \frac{\partial TR}{\partial q_2} = 0$$

or, when

$$(25a) \quad q_1 = \frac{-(a_{12} + a_{21}) q_2 - a_{10}}{2a_{11}}$$

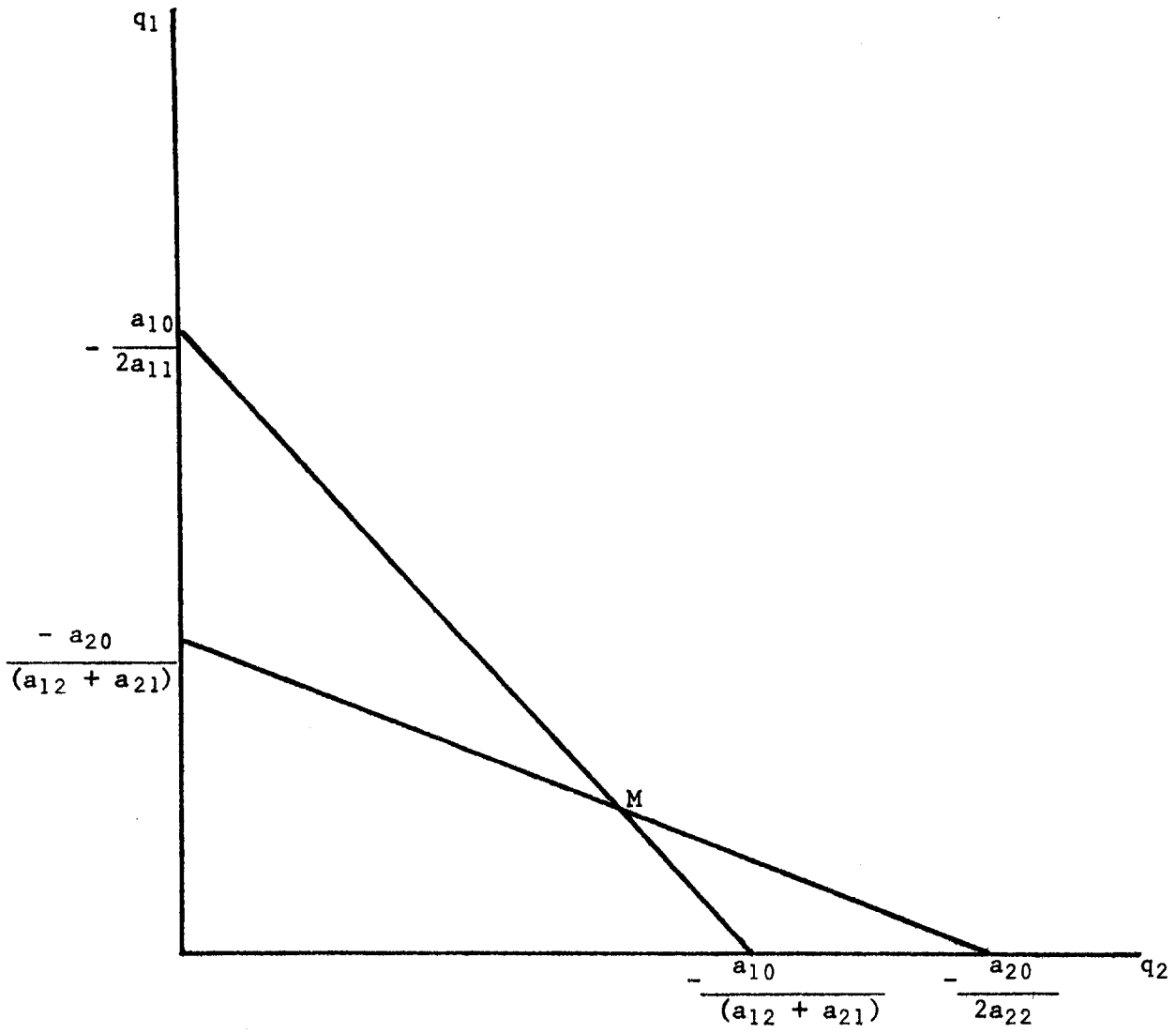
$$(25b) \quad q_1 = \frac{-2a_{22} q_2 - a_{20}}{(a_{12} + a_{21})}$$

Assuming q_1 and q_2 to be substitutes in consumption, a general solution for maximum total revenue is given in Figure 2, where maximum total revenue is at point M, the point which satisfies both equations (25a) and (25b).

From (22) we can generate a revenue surface consisting of a family of iso-revenue lines. Holding TR constant at \overline{TR} , we have

$$(26) \quad a_{11} q_1^2 + a_{22} q_2^2 + (a_{12} + a_{21}) q_1 q_2 + a_{10} q_1 + a_{20} q_2 = \overline{TR}$$

Figure 2



This is an expression for a conic section whose shape depends on the values of the coefficients.

$$(a_{12} + a_{21})^2 - 4a_{11}a_{22} = 0; \text{ parabola, two parallel lines, or imaginary}$$

$$(a_{12} + a_{21})^2 - 4a_{11}a_{22} < 0; \text{ an ellipse, a single point, or imaginary}$$

$$(a_{12} + a_{21})^2 - 4a_{11}a_{22} > 0; \text{ a hyperbola, or two intersecting lines.}$$

Let us first look at a special case where variation of output does not affect price; i.e., the competitive case. In this situation our demand curves are

$$p_1 = k_1 = \bar{p}_1$$

$$p_2 = k_2 = \bar{p}_2$$

The total revenue function is

$$TR = \bar{p}_1 q_1 + \bar{p}_2 q_2 = k_1 q_1 + k_2 q_2$$

Holding TR constant at \overline{TR} , we have

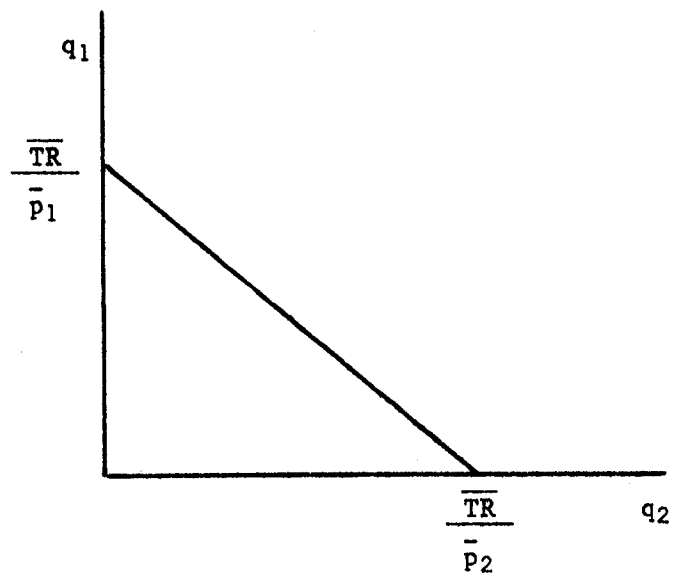
$$\overline{TR} = \bar{p}_1 q_1 + \bar{p}_2 q_2$$

or

$$\bar{p}_1 q_1 = \overline{TR} - \bar{p}_2 q_2$$

which is a straight line as shown in Figure 3.

Figure 3



The slope of our iso-revenue line is

$$-\frac{\frac{\overline{TR}}{\overline{p}_1}}{\frac{\overline{TR}}{\overline{p}_2}} = -\frac{\overline{p}_2}{\overline{p}_1}.$$

To derive the general case, we first differentiate (26) and obtain

$$(27) \quad \frac{dq_1}{dq_2} = -\frac{[2a_{22}q_2 + (a_{12} + a_{21})q_1 + a_{20}]}{[2a_{11}q_1 + (a_{12} + a_{21})q_2 + a_{10}]}$$

which is the expression for the slope of our iso-revenue line.

We are concerned with that portion of the conic section which is negatively sloped and convex to the origin. A necessary condition is that

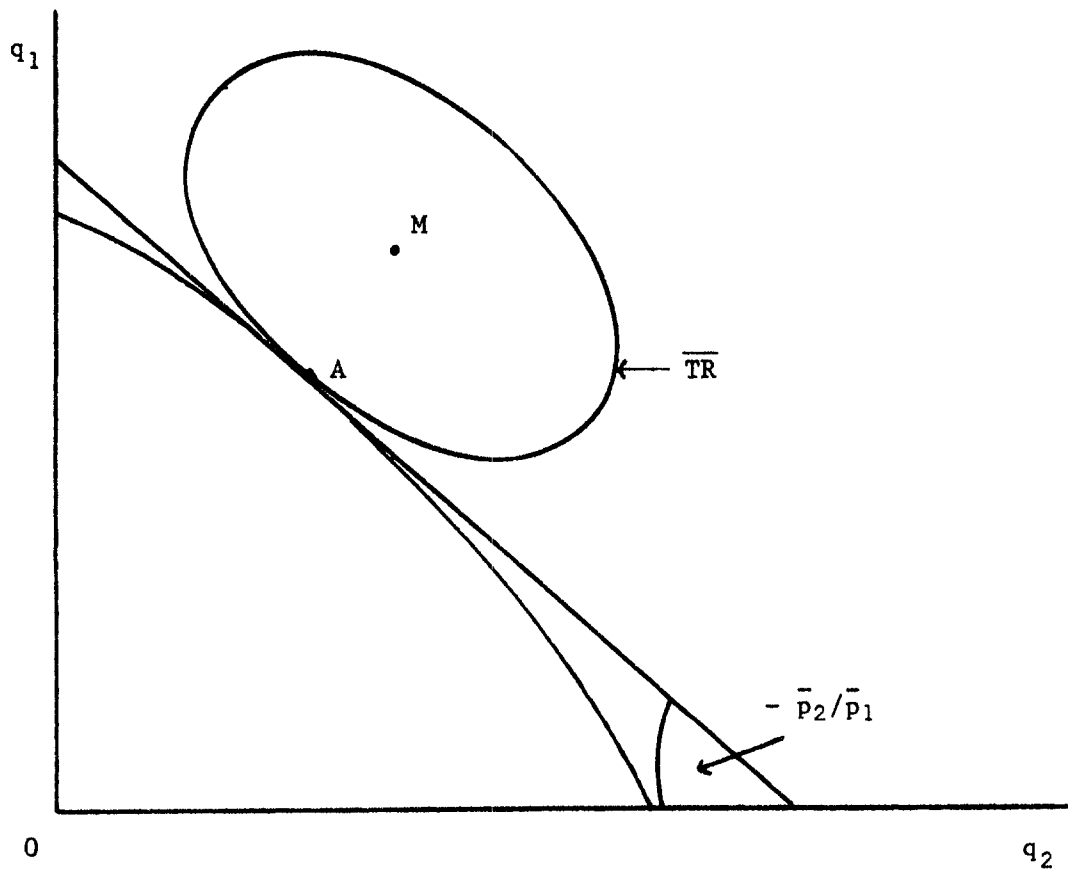
$$\frac{dq_1}{dq_2} < 0.$$

For this to be true, both the numerator and denominator must be negative or both must be positive. A sufficient condition for convexity is that

$$(28) \quad \frac{\frac{d^2 q_1}{dq_2^2}}{2} > 0.$$

Some combinations of our production possibility frontier and our revenue function are illustrated in Figure 4.^{8/} Our equilibrium output is at point A. We have shown this to be the equilibrium output which would satisfy two different demand conditions. The straight price line with slope $-\overline{p}_2/\overline{p}_1$ corresponds to the competitive market situation.

Figure 4



But at point A the slope of the production possibility frontier also equals the slope of the iso-revenue ellipse, \overline{TR} .

As we shall see in the next section, the nature of the demand curves and, therefore, the iso-revenue curves, is very important when one evaluates changes in the production possibility frontier.

Technological Change

We now wish to examine the consequences of certain types of technological change in the context of our two-commodity, two-factor world. National (regional) research leaders are faced with the question of the allocation of research resources among commodities. They could put all their effort into increasing the output of q_1 , or all into q_2 , or allocate some combination of total research resources to both q_1 and q_2 .

We will assume that the relevant research institutions have a given amount of total resources to spend on research. The case where these resources are devoted entirely to improving the output of q_1 or q_2 is considered. Further, we assume that the available resources can generate increases in production of equal absolute amounts in either q_1 or q_2 ; i.e., the increase in production of q_1 with all resources devoted to that product equals the increase in production of q_2 if all resources were devoted to its production.^{9/}

We can rewrite our production functions, equations (1a) and (1b), as either

$$(1a') \quad q_1 = \alpha_0 \tau_1 L_1^\alpha K^{1-\alpha}$$

$$(1b') \quad q_2 = \beta_0 L_2^\beta K^{1-\beta}$$

or

$$(1a'') \quad q_1 = \alpha_0 L_1^\alpha K_1^{1-\alpha}$$

$$(1b'') \quad q_2 = \beta_0 \tau_2 L_2^\beta K_2^{1-\beta}$$

where τ_1 and τ_2 represent fixed levels of technological change of the neutral type. We restrict our analysis to neutral technological change because in the Cobb-Douglas world with constant returns to scale it is not possible to represent non-neutral technological change without changing the production coefficients, α or β [4].

The specified technological changes would appear in equations (11) and (12) and we can derive the new expressions for the slopes of the new production possibility frontiers as either

$$(15a') \quad - \frac{d \left(\frac{q_1}{\bar{L}} \right)}{d \left(\frac{q_2}{\bar{L}} \right)} = \tau_1 (bR)^{1-\alpha} (aR)^{\beta-1} [a + (b-a)\ell]^{\alpha-\beta} \left(\frac{a + \alpha\ell(b-a)}{a + (b-a)(1-\beta + \beta\ell)} \right)$$

or

$$(15b') \quad - \frac{d \left(\frac{q_1}{\bar{L}} \right)}{d \left(\frac{q_2}{\bar{L}} \right)} = \frac{1}{\tau_2} (bR)^{1-\alpha} (aR)^{\beta-1} [a + (b-a)\ell]^{\alpha-\beta} \left(\frac{a + \alpha\ell(b-a)}{a + (b-a)(1-\beta + \beta\ell)} \right)$$

Technological change in the form of τ_1 would increase the absolute value of the slope of our production possibility frontier, while technological change of the τ_2 type would reduce the slope.

Next we derive the production possibility frontier for assumed values of \bar{L} , \bar{K} , α and β , and look at the effect of changes in technology in either the production of q_1 or q_2 . We will assume that our region is endowed with 300 units of \bar{K} and 750 units of \bar{L} , if our region has an

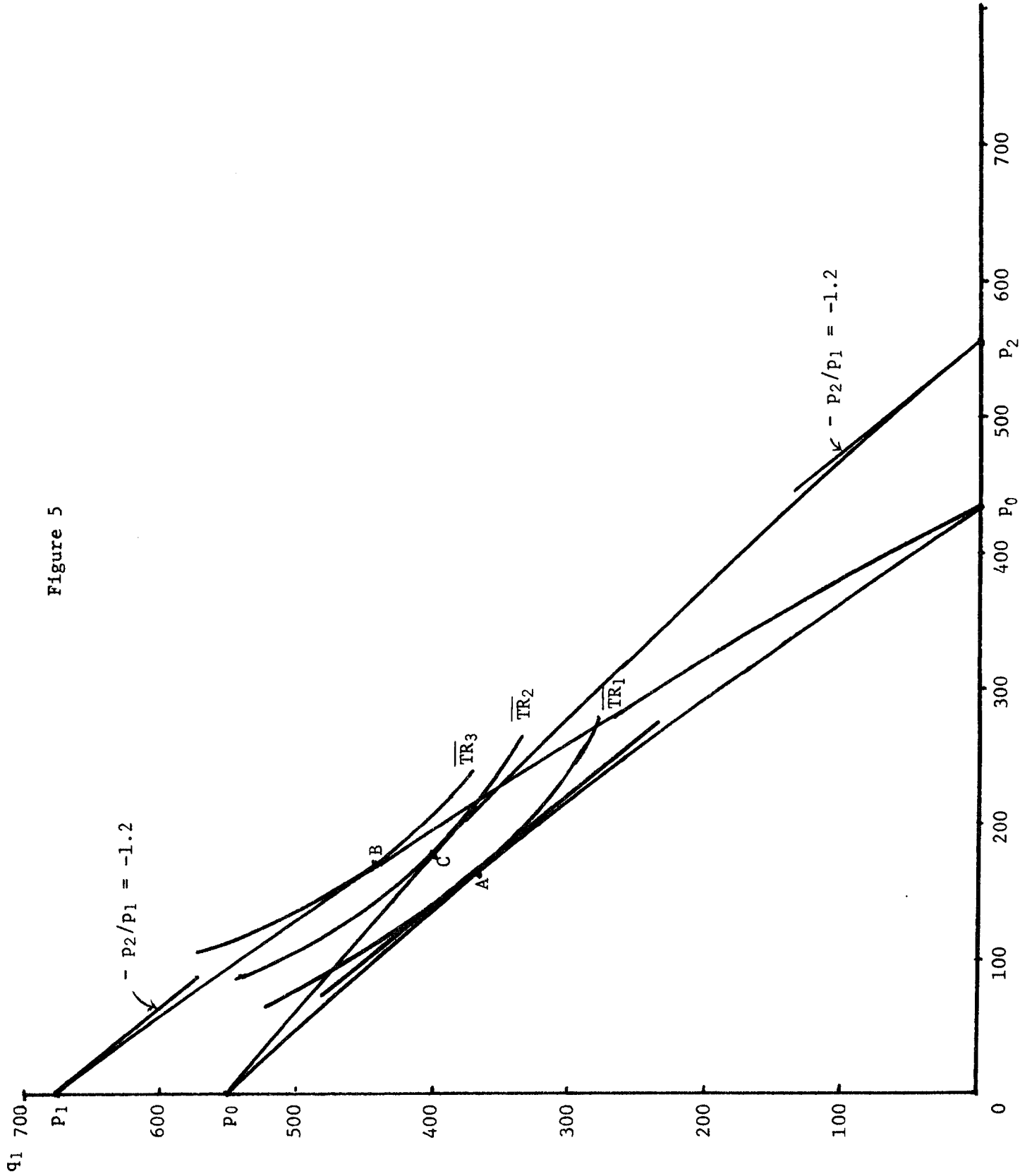
abundant supply of labor relative to capital. Let $\alpha = .67$ and $\beta = .4$; i.e., production of q_1 is relatively more labor using than the production of q_2 .

Our initial production possibility curve is P_0P_0 in Figure 5. If all our resources were devoted to the production of q_1 we would get 554.3 units of that commodity; if all the resources went into the production of q_2 , we would get 432.8 units of q_2 . Let us assume the following unit product prices, $p_1 = \$1.00$ and $p_2 = \$1.20$, and that we have a competitive market situation. Then our equilibrium solution would be $q_1 = 364.0$, $q_2 = 163.7$ and total revenue would be \$560.4 (point A).

Next consider a new production possibility curve, P_1P_0 , which results from a given technological change, τ_1 . With the same price ratio as before, $p_2/p_1 = 1.20$, we would get complete specialization in q_1 and an output of 676.2 units. Similarly, if the technological change were τ_2 , our new production possibility curve would be P_0P_2 and we would have complete specialization in q_2 and an output of 554.3 units. The gross revenues would be \$676.2 and \$665.2, respectively. The gross revenue from producing q_1 is only slightly higher than that from q_2 under the new technological situations. The fact that the changes in technology result in complete specialization is based on the lack of curvature in the production possibility curve, a characteristic of the model we are using.

Now suppose that instead of our country or region being a price taker, changes in its outputs affected their prices. In such a situation, we would have a family of iso-revenue curves such as those in Figure 5, labeled \overline{TR}_1 , \overline{TR}_2 and \overline{TR}_3 . As we have seen, the type of conic section which our iso-revenue lines represent depends on the values of the parameters

Figure 5



in our demand equations. In our particular case, technological change in q_1 gives us a new output combination at point B and technological change in q_2 gives us a new output combination at C. Also, in these circumstances total revenue from B would be larger than total revenue from C.^{10/}

Some Implications

There are a number of interesting implications which flow from our model. We will now discuss some of them.

First, if our country or region is a price taker, the extent to which a given technological change results in a higher degree of specialization in production depends upon the concavity of the production possibility curve. If our production possibility curve is very flat then technological change in the production of one commodity would drive our region toward complete specialization in that commodity. This tendency would be blunted the more concave the production possibility curve. This is a point which research administrators, economists and those concerned with a nation's food and fiber policies should be aware of since the allocation of agricultural research resources, ceterus paribus, could bring about drastic shifts in the agricultural output mix. The impact of the new, high yielding varieties of wheat on pulse production in northwestern India is a case in point. The new wheat technology has led to almost complete specialization in wheat production on many farms in this region, resulting in a marked decline in pulse production and a sharp rise in pulse prices. Since pulses are the major source of protein for most of the residents of the region, some feel that while total agricultural production has gone up there has been a decline in the average nutritional quality of consumer diets.

Second, if our region faces downward sloping demand curves for its products, technological change in one product will result in a shift in the terms of trade against that product. The extent of such a shift will depend, among other things, on the relative own- and cross-price elasticities (flexibilities) of demands for the two products. The more responsive the change in the price of the product experiencing technological change to a given change in output relative to the other product, the greater will be the unfavorable shift in terms of trade.^{11/} We can see this in Figure 5. At our initial point, $-p_2/p_1 = 1.20$. At point B, $-p_2/p_1 > 1.20$, and at point C, $-p_2/p_1 < 1.20$. The shift in terms of trade will tend to dampen the push toward specialization as a result of a given change in technology in the production of one of the products.

Third, intervention in the markets for q_1 and q_2 by government (or other groups) in the form of price support measures in a situation with downward sloping market demand curves can yield results similar to the competitive model; i.e., intervention can result in a higher degree of specialization than would result from a market solution. This does not automatically follow because governments can also set the relative support prices in ways which will shift the terms of trade against the commodity experiencing the technological change.

Fourth, the question as to which commodity should receive research resources depends very much on society's developmental objectives and policies. For example, suppose it is the primary concern of policy makers to increase the incomes of producers, and relative prices are unimportant. Then, one rule which could be followed is to increase the production of the commodity with the highest price and income elasticities. In this

way one would tend to minimize the extent to which a shift in the terms of trade tends to counteract the effect of technological change. On the other hand, suppose one of the commodities is a wage good, it has lower price and income elasticities than the non-wage good, and it is the policy makers' desire to keep the price of the wage good as low as possible. In this case, it would make sense to invest research resources in bringing about technological change in the wage good which is the one with the most inelastic demand; i.e., we want to maximize the shift in terms of trade against the wage good. These are but two of many possible situations.

Finally, we should be cognizant of the fact that the price elasticity of demand which a region or country faces depends on both domestic and export demand parameters. It is possible for the domestic demand curve to be quite price inelastic, but the export demand curve facing our country or region to be quite price elastic, e.g., the case of corn in Thailand. In such a situation it would be important for the country or region to follow price policies which did not exclude domestic production from entering export markets, if the policy objectives were to minimize the adverse effect on terms of trade for corn of a change in output. On the other hand, if the name of the game is to keep domestic prices as low as possible, then export barriers might be erected, e.g., the case of the rice premium in Thailand!

Summary and Conclusions

We have attempted to construct a theoretical model which would give us some insights into questions of diversifying agricultural production. We have not dealt with questions of relative risk and uncertainty among

crops. This is important. But there is more to the question of diversification than risk and uncertainty. One other major area which we have focused on is the allocation of research resources and the generation of new technology among commodities.

Our comparative static model would indicate that there is nothing inherently good or bad about diversification of production. Which way a country goes depends heavily on demand conditions, and price policy and developmental objectives. But these objectives provide useful guidelines to research administrators for determining the allocation of research resources among commodities. It would appear from our analysis that the allocation of research resources among commodities should be guided by, among other things, their relative long-run demand and price prospects as well as price objectives. Price policies can and should play an important role not only in the allocation of traditional resources among commodities in a region^{12/} but also in helping to determine the allocation of research resources.

FOOTNOTES

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^{1/} One could argue that in most cases the main obstacles to diversification are economic rather than technical; i.e., with sufficiently high product prices almost any crop can be grown under artificially controlled ("greenhouse") environmental conditions almost anywhere in the world. But one can also view constraints to diversification as technical in the sense that technological advances which lower the unit costs of production or distribution improve the relative profitability and economic feasibility of growing a crop. It is in this latter context that we view technical constraints to diversification.

^{2/} For an excellent discussion of the problems which some countries face in entering export trade, see J. Norman Efferson [9].

^{3/} The theoretical model presented in this section is adapted from Johnson [15]. For a more general discussion see Kemp [17].

^{4/}The limitations of this assumption are discussed more fully later in this section.

^{5/}The explicit derivation is as follows:

$$\begin{aligned}
 a \left(\frac{K_1}{L_1} \right) &= b \left(\frac{K_2}{L_2} \right) \\
 &= b \left(\frac{\frac{\bar{K}}{\bar{L}} - \frac{L_1}{\bar{L}} \cdot \frac{K_1}{L_1}}{1 - \frac{L_1}{\bar{L}}} \right) \\
 &= \frac{b \left(\frac{\bar{K}}{\bar{L}} \right)}{1 - \frac{L_1}{\bar{L}}} - \frac{b \left(\frac{L_1}{\bar{L}} \right) \left(\frac{K_1}{L_1} \right)}{1 - \frac{L_1}{\bar{L}}} .
 \end{aligned}$$

or,

$$a \left(\frac{K_1}{L_1} \right) + \frac{b \left(\frac{L_1}{\bar{L}} \right) \left(\frac{K_1}{L_1} \right)}{1 - \frac{L_1}{\bar{L}}} = \frac{b \left(\frac{\bar{K}}{\bar{L}} \right)}{1 - \frac{L_1}{\bar{L}}} .$$

Solving for $\frac{K_1}{L_1}$ we get

$$\frac{K_1}{L_1} = \frac{b \left(\frac{\bar{K}}{\bar{L}} \right)}{\left[1 - \frac{L_1}{\bar{L}} \right]} \cdot \frac{\left[1 - \frac{L_1}{\bar{L}} \right]}{\left[a \left[1 - \frac{L_1}{\bar{L}} \right] + b \left(\frac{L_1}{\bar{L}} \right) \right]}$$

(continued)

$$\begin{aligned}
&= \frac{b \left(\frac{\bar{K}}{\bar{L}} \right)}{a + (b-a) \left(\frac{L_1}{\bar{L}} \right)} \\
&= \frac{b \left[\left(\frac{L_1}{\bar{L}} \right) \left(\frac{K_1}{L_1} \right) - \left(1 - \frac{L_1}{\bar{L}} \right) \left(\frac{K_2}{L_2} \right) \right]}{a + (b-a) \left(\frac{L_1}{\bar{L}} \right)} .
\end{aligned}$$

The expression for $\frac{K_2}{L_2}$ can be derived in a similar way. The denominator in the right hand side of (8) obtains when $\left(1 - \frac{L_1}{\bar{L}} \right)$ is substituted for $\frac{L_2}{\bar{L}}$.

6/ The detailed derivation of equation (13) is as follows:

$$\begin{aligned}
\frac{d \left(\frac{q_1}{\bar{L}} \right)}{d \ell} &= \left(\frac{bR}{a + (b-a)\ell} \right)^{1-\alpha} + \ell(1-\alpha) \left(\frac{bR}{a + (b-a)\ell} \right)^{-\alpha} \left(- \frac{bR(b-a)}{[a + (b-a)\ell]^2} \right) \\
&= \left(\frac{bR}{a + (b-a)\ell} \right)^{1-\alpha} + \ell(1-\alpha) \left(\frac{bR}{a + (b-a)\ell} \right)^{1-\alpha} \left(- \frac{(b-a)}{a + (b-a)\ell} \right) \\
&= \left(\frac{bR}{a + (b-a)\ell} \right)^{1-\alpha} \left(1 - \frac{\ell(1-\alpha)(b-a)}{a + (b-a)\ell} \right) \\
&= \left(\frac{bR}{a + (b-a)\ell} \right)^{1-\alpha} \left(\frac{a + \alpha\ell(b-a)}{a + (b-a)\ell} \right)
\end{aligned}$$

7/ The equation we wish to solve will be of the general form

$$\frac{(r + sl)^Y (r + ul)}{(v + wl)} = k$$

This expression can be solved for using an iterative procedure. One has to guard against the slope of the price line being such as to give a corner solution (complete specialization in one product). In such a case the equality in the above expression will not hold.

8/ In our model we derive an expression for the slope of the production possibility curve, but its level can be determined only in conjunction with the explicit forms of the production functions and the levels of resource endowments. A general solution for profit maximization is possible, but quite complicated because of the large number of constraints which must be satisfied. A general solution would be much simpler if one were to work with an explicit form of the production possibility curve, although in this situation the production functions could not be completely and uniquely specified.

9/ While not considered in this paper, one might want to look at alternative assumptions. First, research efforts may not be equally productive for both q_1 and q_2 . Therefore, equal absolute increases in production under complete specialization would require different levels of research inputs. Second, there is no a priori reason to assume that research resources will be devoted entirely to either increasing q_1 or q_2 . However, to the extent that there might be significant economies of scale in research which can be captured through specialization, concentrating on one commodity or another might improve the productivity of research

institutions. This could be an important consideration for many developing countries which still have relatively small quantities of research resources. Third, research is not costless, as assumed in this paper, and one should investigate the returns to investments in research.

^{10/} It may also be the case that short-run inflexibilities in the marketing system could give rise to downward sloping demand curves in the short run in what would otherwise be a competitive market situation. One observes the problems of markets being unable to handle, in the short run, changes in output mix in both developed and less developed countries. There is some evidence that in developing countries, significant changes in the production of one crop can cause temporary distortions between the relative price structure in a region relative to the prices in a larger marketing area. Lele [21], in her study of sorghum grain marketing in western India found that distortions in intermarket price differentials arose when the volume of grain production and marketings pressed against the supply of transport services.

^{11/} This is most easily seen in the case where q_1 and q_2 are independent in consumption, i.e., $a_{12} = a_{21} = 0$. In this case the shift in terms of trade between the two products depends only on the relative values of their own price elasticities (flexibilities) of demand.

^{12/} The role of price in the allocation of resources among crops in developing countries was highlighted by Raj Krishna [18] and subsequently, by many other analysts.

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