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USERS' GUIDE TO THE MINNESOTA TWO-REGION INPUT-OUTPUT MODEL

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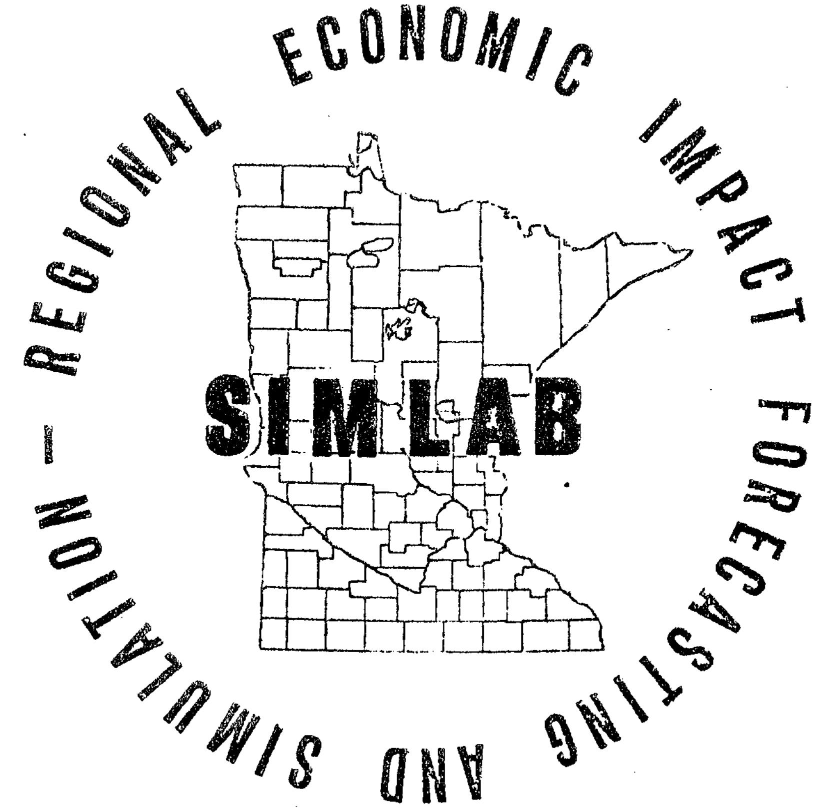


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CONTENTS

## List of Figures



## List of Tables

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# Preface

The Minnesota Two-Region Input-Output Model was initially developed in 1966 by Donald Beard and his colleagues at Northern Natural Gas Company, Omaha, Nebraska, in collaboration with one of the co-authors. The initial two-region program was written in ALGOL. Subsequently, the IBM Service Bureau was employed to translate the program into FORTRAN language. The present version of the FORTRAN source program incorporates, in part, the algorithm of the translated version of the initial program.

The two-region input-output model is currently used in the preparation of the initial series of inter-industry and inter-regional transactions tables for a given region. Two-region tables have been prepared as an initial step in the estimation and forecasting of state and substate effects of agricultural, mineral and industrial resources development in Minnesota, West Minnesota, Northeast Minnesota and the Minneapolis-St. Paul Metropolitan Area. These tables were then reviewed and adjusted to various central totals before being used in the Minnesota Regional Development Simulation Laboratory--SIMLAB for short. The two-region model thus became an integral part of a dynamic computer model of a given regional economy. Use of the two-region program makes possible the repeated reconciliation of a Study Region input-output transactions with the two-region input structure of the U.S. economy.

Use of the Minnesota two-region input-output model as an integral part of SIMLAB provides for a low-cost alternative to the building of dynamic models of regional economic systems. Various internal data balancing procedures in both the two-region program and the SIMLAB program assure the user that the output results are reasonable and valid, provided the appropriate procedures were followed. It is for this purpose of extending the use of both the two-region and the SIMLAB computer programs that this Users' Guide was prepared.

Because of the intricacies of the logical formulation of the two-region input-output program, we have included the initial mathematical representation of the two-region model. The precise and concise statement of relationships among variables is especially helpful to the user who is faced with a need for a rigorous understanding of the exact content of each one of the 25 tables produced by the two-region program. Discussion of the specific tables of data output for a 35-industry breakdown of the 1970 Minnesota economy is cross-referenced to related equations in the mathematical model of the two-region input-output system for such a reader.

For most readers of the Users' Guide, the numerical results of the two-region program and their interpretation in forecasting and planning are adequately covered in the chapter on using the two-region tables. The four tables in this chapter summarize the 25 tables in the complete computer print-out. The 35-industry breakdown is used, also, to reduce the sheer volume of data provided by each computer run. This Users' Guide is, therefore, an introduction to the Minnesota TRIO. Refinements in both data and procedure are available in the various Minnesota Regional Economic Impact Forecasting and Simulation (REIFS) System Reports. Most reports are available from this Department under its Staff Paper Series.

INTRODUCTION

Since the pioneer work of Wassily Leontief in the 1930's, numerous studies of the inter-industry input-output relationship have been completed.[1/] Most of the studies were national in scope or involved large areas. They required considerable time and effort to assemble and process the needed data. Once completed, they were not directly applicable to a region such as a state or multi-county area.

In recent years, there has been an increasing interest by both public and private agencies in the study of energy allocation, environmental monitoring, resource planning, and industrial development of areas. Also, efforts are being made in many states to develop statewide and substate input-output models for the purpose of providing reliable information for policy and management decisions.

As an analytical and forecasting tool, the input-output model is used to measure the impact of changes in volume of economic activity of an industrial sector; to provide a basis for forecasting the probable level of economic activity resulting from private or public investment decisions; and to assess the possible effects of alternative government policies and programs upon an area's economy. The input-output tables may be constructed from primary or secondary data or a combination of primary and secondary data. In the primary data approach, all, or a sample, of the industries are

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[1/] See list of selected references starting on page 79 of this report. References cited are indicated by number in parentheses.

surveyed to obtain the data of annual production, annual sales to the other industries, and annual purchases of inputs from the other industries. In the secondary data approach the industrial outputs are compiled from various census and other secondary sources, from which the sales and purchases among the industries are determined by multiplying the actual (or modified) national input-output coefficients by the estimated total sectoral outputs. In the combination approach, some sectors are based on primary data, especially those which are unique to the region, and others are based on secondary data.

Many regional input-output studies in the past, regardless of which approaches being used, adopted the conceptual framework of the one-region model rather than the more data-demanding interregional model. The main problem with using a one-region model for the study of the inter-industiral relationship at the sub-national level is the loss of the general equilibrium conditions of economic analysis (35).

Leontief's input-output model was first introduced as an analytical tool for dealing with the general equilibrium analysis of the productive system of a national economy as a whole. If this model is to be used for portraying the inter-industrial relationship of a sub-national economy without well-defined interregional linkages of commodity transactions, however, the general equilibrium features are lost. Generally, the one-region-model study treats the interregional transactions of commodities as the "exogenous" sectors of imports and exports. The result of such treatments is that the input-output coefficients no longer express the technical relationship of actual inputs needed for producing a unit of product by an industrial sector, but in fact they represent the inputs procured from the defined study region for producing a unit of product.

The use of the one-region-model-input-output coefficients is not a serious problem if the coefficients are used only for understanding and describing the input requirements of different industries in a region. But if these coefficients are to be used as a basis for computing impact multipliers, or forecasting future outputs, the results will be inaccurate and misleading. This is due largely to the unaccounted technical coefficients which should be considered but are omitted in a one-region model (the coefficients described as Quadrant II and III in the next chapter).[2/]

The two-region input-output model, unlike the one-region model, treats similar industries in the other regions as distinctive individual industries, and the usual input-output analysis applies to this expanded table (Isard, 1951). Explicit in the model are the transactions between the different industries in the two regions which are legitimatized as endogenous components of each region's economic structure. Thus the model preserves the general equilibrium feature of economic analysis when both (or all, if more than two) regions are included in the model. The two-region (or multi-region) input-output coefficients will then approximate the technical coefficients of production which are divided into their intra-regional and interregional components. Thus, the impact multipliers are more reliable and accurate in this model than in the one-region model.

Although the two-region, or multi-region, input-output model is considered to be far more plausible than the one-region in its conceptual framework, the implementation of the model has been hampered by the

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[2/] Greytack (13), Miller (30), and Riefler and Tiebout (36) evaluated the errors of omitting these unaccounted components, namely the interregional "feedback effects," and found the errors were quite significant.

availability of the interregional commodity statistics, time and monetary requirements in gathering the inter-industry transaction data. It is very difficult to develop such a model with the primary data approach. The only recourse available at the present time in implementing such a model is use of the secondary data approach through the disaggregation of national input-output tables. This approach was used to develop the Minnesota two-region input-output model presented in this Guide.

The method used in our study is based on the regional commodity-balance technique outlined by Isard in 1953 (21). This technique is applied to a two-region breakdown of the U.S. input-output tables. One is the Study Region, the other is the Nation Region. The original U.S. input-output table is disaggregated into four principal quadrants with each quadrant containing three groups of sectors--production, final demand and primary input. The four quadrants are as follows:

- First quadrant of intra-regional intersectoral transactions for Study Region;
- Second quadrant of interregional intersectoral transactions from Study Region to the Rest of Nation Region;
- Third quadrant of interregional intersectoral transactions from the Rest-of-Nation Region; and
- Fourth quadrant of the intra-regional intersectoral transactions for Rest-of-Nation Region.

After the compilation of the four quadrants of the transaction tables, a completely balanced regional input-output table for each of the regions can be derived by collapsing the second and third quadrants into the "import" and "export" sectors in each intra-region intersectoral transaction quadrant. This completely balanced regional input-output table corresponds to the traditional one-region input-output model, with the producing sector containing only the inputs purchased from local sources and the interregional flows of commodities included in the import and export sectors.

In the model development, several restrictive assumptions are adopted. One assumption is that the U.S. input structure prevails in the two regions, which means that the U.S. technical coefficients of production define the upper limits of input requirements for each region. The other assumption is that the cross-hauling of a similar product between the regions is canceled and only new interregional flows are shown. This implies that the dollar-volume of transactions in the input-output tables also is expressed in terms of "net" rather than "gross" values.

The main objective of this study is to develop a computationally simple and efficient, but theoretically defensible, two-region input-output model. Two basic data sets are needed for producing the two-region input-output tables: a complete national input-output table, and regional estimates of gross outputs and final demands for a given base year. It is possible to produce a series of regional input-output tables as soon as the national tables are published, if the regional data are available, also, to implement the two-region computer program.

This report is organized into three chapters, following this introduction. In the next chapter, the method of developing the two-region model by disaggregating the national input-output table is presented. Also presented is computational formulation for deriving related coefficients. A case study of the 1970 Minnesota two-region input-output model is presented next. The purpose of this presentation is to help readers understand the model and at the same time examine the outputs produced by the computer program. In the final chapter, the technical aspects of the computer model are described. An Appendix is included also, to provide the reader with a summary listing of the two-region input-output tables (which are available, by request, from the authors).

## BUILDING THE TWO-REGION INPUT-OUTPUT MODEL

The building of a two-region input-output model by a method of disaggregating the national model is presented in this chapter. A great portion of the presentation is given in mathematical notations and formulations of relationships, without which the methodological development of the model is very difficult to describe accurately and concisely.

Before describing the national and two-region models, we refer to the three basic assumptions of the original Leontief input-output model, namely, that:

- A given product is supplied by only one sector;
- The quantity of each input used in any sector is determined by the level of output of the sector; and
- The total effects of carrying on several types of production are the sum of the separate effects.

In addition to Leontief's three assumptions, three more assumptions are needed for the two-region disaggregation, namely, that:

- The input structure at the national level prevails at the regional level;
- An output surplus in the Study Region becomes an import to the Rest-of-Nation Region; and
- The cross-hauling of a similar product between the two regions is indicated only as a net flow.

| | | Consuming Sectors 1 2 . . . . . . N | Final Demand Sectors 1 2 . . . . . K | Total Output |
|---|---|---|---|---|
| Producing Sectors | 1 2 . . . . . . N | $x_{ij}$ | $y_{ik}$ | $X_{i.}$ |
| Primary Input Sectors | 1 . . M | $w_{mj}$ | | $w_m$ |
| Total Inputs | | $X_{.j}$ | $Y_{.k}$ | |

Figure 2.1 Structure of the Intersectoral Transaction Table for the Nation

## 2.1 National Model

### 2.1.1 Inter-Sectoral Transactions

Figure 2.1 shows the compositions of intersectoral transactions at the national level. Three major sectors of this table are: producing sector (X), primary input (W) and final demand (Y). The numerical entries are represented as variables, vectors, and matrices in constant dollar terms.

Notationally, N indicates the number of producing sectors; K indicates the number of final demand sectors; and M indicates the number of primary input sectors.

$x_{ij}$ = the purchase of the output of the ith producing sector by the jth consuming sector for the purpose of producing the output of jth sector (where i = 1,2, . . . , N and j = 1,2, . . . , N)

$y_{ik}$ = the final demand of the sector k for the output from the ith producing sector or primary input class (where i = 1,2, . . . , N + M and k = 1,2, . . . , K)

$w_{mj}$ = the purchase of the mth sector of primary input by the jth consuming sector for the purpose of producing the output of the jth sector (where m = 1,2, . . . , M and j = 1,2, . . . , N)

$X_{i.}$ = the total output of the ith sector (i = 1,2, . . . , N)

$X_{.j}$ = the total input of the jth sector (j - 1,2, . . . , N)

$Y_{i.}$ = total final demand for the output of the ith sector (i = 1,2, . . . , N + M)

$Y_{.k}$ = total final demand of the sector k (k = 1,2, . . . , K)

$W_{.j}$ = the purchase of total primary inputs by the jth consuming sector for the purpose of producing the output of the corresponding jth producing sector (j = 1,2, . . . , N)

The following relationships are held to be true by the above definitions:

$$Y_{i.} = \sum_{k=1}^{K} y_{ik} \qquad \text{where } i = 1,2, \ldots, N + M$$

$$W_{.j} = \sum_{m=1}^{M} w_{mj} \qquad \text{where } j = 1,2, \ldots, N$$

Since we assume that the total output of a sector is equal to the total input of the corresponding sector, we have the relationship of

$$X_{i.} = X_{.j} \quad \text{when } i = j, \qquad \text{where } X_{i.} = \sum_{j=1}^{N} x_{ij} + Y_{i.}$$

and $X_{.j} = \sum_{i=1}^{N} x_{ij} + W_{.j}$

2.1.2 Technical Coefficients

Intersector flows, as shown in Figure 1, can be used as basic data for computing the technical coefficients of production. The technical coefficient shows the value of output from the ith producing sector required to produce a dollar's worth of product in the jth consuming sector. It is computed as follows:

For producing sector,

$$a_{ij} = \frac{x_{ij}}{X_{.j}}, \quad \text{for } i = 1,2, \ldots, N \text{ and } J = 1,2, \ldots, N$$

For primary input sector,

$$a_{ij} = \frac{w_{mj}}{X_{.j}}, \quad \text{for } j = 1,2, \ldots, N,\ m = 1,2, \ldots, M \text{ and } i = N + m$$

For final demand sector,

$$a_{ij} = \frac{y_{ik}}{Y_{.k}}, \quad \text{for } i = 1,2, \ldots, N + M,\ k = 1,2, \ldots, K \text{ and } j = N + k$$

Notationally, we can express the final demand as a function of total output as:

$$\begin{bmatrix} Y_{1.} \\ Y_{2.} \\ . \\ . \\ . \\ Y_{N.} \end{bmatrix} = \begin{bmatrix} X_{1.} \\ X_{2.} \\ . \\ . \\ . \\ X_{N.} \end{bmatrix} - \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1N} \\ a_{21} & a_{22} & \cdots & a_{2N} \\ \cdots & & & \\ \cdots & & & \\ \cdots & & & \\ a_{N1} & a_{N2} & \cdots & a_{NN} \end{bmatrix} \begin{bmatrix} X_{1.} \\ X_{2.} \\ . \\ . \\ . \\ X_{N.} \end{bmatrix} \quad (1)$$

or,

$$Y = X - A \cdot X = (I-A)X$$

where

$Y$ = a column vector of the total final demand of length N

$X$ = a column vector of the total output of length N

$A$ = a matrix of the technical coefficient of size N by N

$I$ = an identity matrix of size N by N

2.1.3 Interdependence Coefficients

One objective of the input-output analysis is to relate output of a sector to the final demand of the output of all of the sectors. By manipulating the data of equation (2), the output of the producing sectors can be expressed as a function of final demands:

$$X = (I - A)^{-1} Y . \quad (3)$$

The elements of the inverse matrix (I - A) in equation (3) are the interdependence coefficients which denote the change in specified output associated with a one-unit change in final demand.

## 2.2 Two-Region Model

### 2.2.1 Disaggregation of National Intersectoral Transaction Table

For the two-region model, the outputs of producing sectors and the final demands of a particular region have to be compiled from various census sources. In estimating the regional consumption, the input structure of a region is assumed to be the same as that of the national level, thus the technical coefficients obtained at the aggregate national level can be applied to allocate the inputs of regional industries.

For all of the following notations, a single superscript indicates a particular region, and a double superscript indicates the source of flow by the first letter and the destination of flow by the second letter.

$$x^p_{ij} = a_{ij}\, X^p_j \quad \text{for } i = 1,2, \ldots, N \text{ and } j = 1,2, \ldots, N$$

$$w^p_{mj} = a_{ij}\, X^p_j \quad \text{for } j = 1,2, \ldots, N \quad m = 1,2, \ldots, M, \text{ and } i = N + m$$

$$y^p_{ik} = a_{ij}\, Y^p_{.k} \quad \text{for } i = 1,2, \ldots, N + M,\ k = 1, 2, \ldots, K \text{ and}$$

$$j = N + k$$

where

$X^p_{.j}$ = total output of the jth sector of Region p (where we assume $X^p_{.j} = X^p_{i.}$ for $i = j$)

$Y_{.k}$ = total final demand of the kth class in Region p.

$a_{ij}$ = national technical coefficient relating the ith producing sector and jth consuming sector.

$x^p_{ij}$, $w^p_{mj}$, and $y^p_{ik}$ denote the values of intersectoral transactions of producing sectors, primary input sectors and final demand classes in Region p respectively.

After estimating the intersectoral transactions of a particular region of interest, the amounts of intersectoral transactions of the rest of the nation (denoted by a superscript q) are obtained by a simple subtraction of the former from that of corresponding sector of the national level.

$$x^q_{ij} = x_{ij} - x^p_{ij} \quad \text{where } i = 1,2, \ldots, N \text{ and } j = 1,2, \ldots, N$$

$$w^q_{mj} = w_{mj} - w^p_{mj} \quad \text{where } m = 1,2, \ldots, M \text{ and } j = 1,2, \ldots, N$$

$$y^q_{ik} = y_{ik} - y^p_{ik} \quad \text{where } i = 1,2, \ldots, N + M \text{ and } k = 1,2, \ldots, K$$

In the two-region model, as in the national model, the assumption of equality of total input-output relationship of a sector still holds.

$$X^p_{i.} = X^p_{.j}, \quad \text{when } i = j .$$

$$X^q_{i.} = X^q_{.j}, \quad \text{when } i = j .$$

With the two-region model, the transaction table, as shown in Figure 2.2 now consists of four quadrants each for producing sector, primary input, and

| | | | REGION "p" | | REGION "q" | | |
|---|---|---|---|---|---|---|---|
| | | | Consuming Sectors 1 2 . . . . . . . N | Final Demands 1 2 . . . . . . K | Consuming Sectors 1 2 . . . . . . . N | Final Demands 1 2 . . . . . . K | Total Output |
| REGION "p" | Producing Sectors | 1 2 . . . . . . N | $x^{pp}_{ij}$ (1st quadrant) | $y^{pp}_{ik}$ (1st quadrant) | $x^{pq}_{ij}$ (2nd quadrant) | $y^{pq}_{ik}$ (2nd quadrant) | $X^{p}_{i.}$ |
| | Primary Input Sectors | 1 . . M | $w^{pp}_{mj}$ (1st quadrant) | | $w^{pq}_{mj}$ (2nd quadrant) | | |
| REGION "q" | Producing Sectors | 1 2 . . . . . . N | $x^{qp}_{ij}$ (3rd quadrant) | $y^{qp}_{ik}$ (3rd quadrant) | $x^{qq}_{ij}$ (4th quadrant) | $y^{qq}_{ik}$ (4th quadrant) | $X^{q}_{i.}$ |
| | Primary Input Sectors | 1 . . M | $w^{qp}_{mj}$ (3rd quadrant) | | $w^{qq}_{mj}$ (4th quadrant) | | |
| Total Input | | | $X^{p}_{.j}$ | $Y^{p}_{.k}$ | $X^{q}_{.j}$ | $Y^{q}_{.k}$ | |

Figure 2.2 Structure of the Interregional Intersectoral Transaction Table of the Two-Region Input-Output Model

final demand, instead of a quadrant each at the national level. The inter-regional flows of output have been introduced into the model. The surplus of a producing sector in a region now becomes an export to the other region and the deficit of a consuming sector in a region is assumed to be met by an import from the other region.

Total regional output of a sector becomes:

$$X_{i.}^{p} = \left( \sum_{j=1}^{N} x_{ij}^{pp} + \sum_{k=1}^{K} y_{ik}^{pp} \right) + \left( \sum_{j=1}^{N} x_{ij}^{pq} + \sum_{k=1}^{K} y_{ik}^{pq} \right)$$

$$X_{i.}^{q} = \left( \sum_{j=1}^{N} x_{ij}^{qp} + \sum_{k=1}^{K} y_{ik}^{qp} \right) + \left( \sum_{j=1}^{N} x_{ij}^{qq} + \sum_{k=1}^{K} y_{ik}^{qq} \right)$$

where $i = 1,2, \ldots, N$

Total regional input of a sector becomes:

$$X_{.j}^{p} = \left( \sum_{i=1}^{N} x_{ij}^{pp} + \sum_{m=1}^{M} w_{mj}^{pp} \right) + \left( \sum_{i=1}^{N} x_{ij}^{qp} + \sum_{m=1}^{M} w_{mj}^{qp} \right)$$

$$X_{.j}^{q} = \left( \sum_{i=1}^{N} x_{ij}^{pq} + \sum_{m=1}^{M} w_{mj}^{pq} \right) + \left( \sum_{i=1}^{N} x_{ij}^{qq} + \sum_{m=1}^{M} w_{mj}^{qq} \right)$$

where $j = 1,2, \ldots, N$

The relationships between the national model and the two-region models are:

$$X_{i.} = X_{i.}^{p} + X_{i.}^{q} \qquad \text{where } 1 = 1,2, \ldots, N$$

$$X_{.j} = X_{.j}^{p} + X_{.j}^{q} \qquad \text{where } j = 1,2, \ldots, N$$

$$X_{i.} = X_{.j} \qquad \text{when } i = j$$

2.2.2 Inter-Regional and Intra-Regional Intersectoral Transactions.

In compilation of figures in the two-region transaction tables, the first phase of the procedure is to estimate the total indicated use (or consumption) of a sector's output by all the consuming sectors and final demand sectors in the region. This is derived by:

$$\overset{*}{X}{}^{p}_{i.} = \sum_{j=1}^{N} a_{ij} X^{p}_{.j} + \sum_{k=1}^{K} a_{ik} Y^{p}_{.k}$$

$$\overset{*}{X}{}^{q}_{i.} = \sum_{j=1}^{N} a_{ij} X^{q}_{.j} + \sum_{k=1}^{K} a_{ik} Y^{q}_{.k} \quad \text{where } i = 1,2, \ldots, N + M$$

The next step is to compute the quantity of the output surplus or deficit of a producing sector of the region. The output surplus or deficit is defined as the amount of the difference between the total output and the total indicated use of the same producing sector. If there is a surplus for a sector in the Region p, then this amount will be allocated proportionally to the subsectors of the corresponding consuming sector of Region q. If the difference between the total output and the total indicated use is equal to zero for a sector, there is no inter-regional flow of the output for that particular sector.

We denote the differences in the following forms (where i indicates a row sector number):

$$\left.\begin{aligned} DX^{p}_{i} &= X^{p}_{i.} - \overset{*}{X}{}^{p}_{i.} \\ DX^{q}_{i} &= X^{q}_{i.} - \overset{*}{X}{}^{q}_{i.} \end{aligned}\right\} \quad \text{for } i = 1,2, \ldots, N$$

$$\left.\begin{aligned} DX^{p}_{N+1} &= Y^{p}_{.1} - \overset{*}{X}{}^{p}_{N+1.} \\ DX^{q}_{N+1} &= Y^{q}_{.1} - \overset{*}{X}{}^{q}_{N+1.} \end{aligned}\right\}$$

$$DX^{p}_{N+2} = \sum_{k=2}^{K} Y^{p}_{.k} - \sum_{m=2}^{M} \overset{*}{X}{}^{p}_{N+m.}$$

$$DX^{q}_{N+2} = \sum_{k=2}^{K} Y^{q}_{.k} - \sum_{m=2}^{M} \overset{*}{X}{}^{q}_{N+m.} \qquad \text{for } N+1 < i \leq N+M$$

Each cell of the producing sectors, the primary input sectors, and final demand classes of the four quadrants of the two-region model falls into one and only one of the following conditions: (i specifically denotes a row sector number)

Condition 1. If $DX^{p}_{i} = 0$ and $i \leq N$, then

$$x^{pq}_{ij} = x^{qp}_{ij} = y^{pq}_{ij} = y^{qp}_{ij} = 0$$

$$x^{pp}_{ij} = x^{p}_{ij}$$

$$x^{qq}_{ij} = x^{q}_{ij} \qquad \text{for } j = 1,2,\ .\ .\ .\ , N \text{ for a given } i$$

$$y^{pp}_{ik} = y^{p}_{ik}$$

$$y^{qq}_{ik} = y^{q}_{ik} \qquad \text{for } k = 1,2,\ .\ .\ .\ , K \text{ for a given } i$$

Condition 2. If $DX^{p}_{i} > 0$ and $i \leq N$, then

$$x^{pq}_{ij} = DX^{p}_{i} \left( x^{q}_{ij} \,/\, \overset{*}{X}{}^{q}_{i.} \right)$$

$$x^{qp}_{ij} = 0$$

$$x^{qq}_{ij} = x^{q}_{ij} - DX^{p}_{i} \left( x^{q}_{ij} \,/\, \overset{*}{X}{}^{q}_{i.} \right)$$

$$x^{pp}_{ij} = x^{p}_{ij} \qquad \text{for } j = 1,2,\ .\ .\ .\ , N \text{ for a given } i$$

$$y_{ik}^{pq} = DX_i^p \left( y_{ik}^q \;/\; \overset{*}{X}{}_{i.}^q \right)$$

$$y_{ik}^{qp} = 0$$

$$y_{ik}^{qq} = y_{ik}^q - DX_i^p \left( y_{ik}^q \;/\; \overset{*}{X}{}_{i}^q \right)$$

$$y_{ik}^{pp} = y_{ik}^p \qquad \text{for } k = 1,2, \ldots, K \text{ for a given } i$$

Condition 3. If $DX_i^p < 0$ and $i \leq N$, then

$$x_{ij}^{pq} = 0$$

$$x_{ij}^{qp} = DX_i^q \left( x_{ij}^p \;/\; \overset{*}{X}{}_{i.}^p \right)$$

$$x_{ij}^{qq} = x_{ij}^q$$

$$x_{ij}^{pp} = x_{ij}^p - DX_i^q \left( x_{ij}^p \;/\; \overset{*}{X}{}_{i.}^p \right)$$

for $j = 1,2, \ldots, N$ for a given $i$

$$y_{ik}^{pq} = 0$$

$$y_{ik}^{qp} = DX_i^q \left( y_{ik}^p \;/\; \overset{*}{X}{}_{i.}^p \right)$$

$$y_{ik}^{qq} = y_{ik}^q$$

$$y_{ik}^{pp} = y_{ik}^p - DX_i^q \left( y_{ik}^p \;/\; \overset{*}{X}{}_{i.}^p \right)$$

for $k = 1,2, \ldots, K$ for a given $i$

Condition 4. If $DX^{p}_{N+1} = 0$ and i = N + 1, then

$$y^{pq}_{N+1k} = y^{qp}_{N+1k} = 0$$

$$y^{pp}_{N+1k} = y^{p}_{N+1k} \qquad \text{for } k = 1,2, \ldots, K$$

$$w^{pq}_{1j} = w^{qp}_{1j} = 0$$

$$w^{pp}_{1j} = w^{p}_{1j}$$

$$w^{qq}_{1j} = w^{q}_{1j} \qquad \text{for } j = 1,2, \ldots, N$$

Condition 5. If $D^{p}_{N+1} > 0$ and i = N + 1, then

$$y^{pq}_{N+1k} = D^{p}_{N+1}\left(y^{q}_{N+1k} \;/\; \overset{*}{X}{}^{q}_{N+1.}\right)$$

$$y^{qp}_{N+1k} = 0$$

$$y^{qq}_{N+1k} = y^{q}_{N+1k} - D^{p}_{N+1}\left(y^{q}_{N+1k} \;/\; \overset{*}{X}{}^{q}_{N+1.}\right)$$

$$y^{pp}_{N+1k} = y^{p}_{N+1k} \qquad \text{for } k = 1,2, \ldots, K$$

$$w^{pq}_{1j} = D^{p}_{N+1}\left(w^{q}_{1j} \;/\; \overset{*}{X}{}^{q}_{N+1.}\right)$$

$$w^{qp}_{1j} = 0$$

$$w^{qq}_{j} = w^{q}_{j} - D^{p}_{N+1}\left(w^{q}_{1j} \;/\; \overset{*}{X}{}^{q}_{N+1.}\right)$$

$$w^{pp}_{1j} = w^{p}_{1j} \qquad \text{for } j = 1,2, \ldots, N$$

Condition 6. If $D^{p}_{N+1} < 0$ and $i = N + 1$, then

$$y^{pq}_{N+1k} = 0$$

$$y^{qp}_{N+1k} = D^{q}_{N+1} \left( y^{p}_{N+1k} \,/\, \overset{*}{X}{}^{p}_{N+1.} \right)$$

$$y^{qq}_{N+1k} = y^{q}_{N+1k}$$

$$y^{pp}_{N+1k} = y^{p}_{N+1k} - D^{q}_{N+1} \left( y^{p}_{N+1k} \,/\, \overset{*}{X}{}^{p}_{N+1.} \right)$$

for $k = 1,2, \ldots, K$

$$w^{pq}_{1j} = 0$$

$$w^{qp}_{1j} = D^{q}_{N+1} \left( w^{p}_{1j} \,/\, \overset{*}{X}{}^{p}_{N+1.} \right)$$

$$w^{qq}_{1j} = w^{q}_{1j}$$

$$w^{pp}_{1j} = w^{p}_{1j} - D^{q}_{N+1} \left( w^{p}_{1k} \,/\, \overset{*}{X}{}^{p}_{N+1.} \right)$$

for $j = 1,2, \ldots, N$

Condition 7. If $D^{p}_{N+2} = 0$ and $N + 1 < i \leq N + M$, then

$$y^{pq}_{N+mk} = y^{qp}_{N+mk} = 0$$

$$y^{pp}_{N+mk} = y^{p}_{N+mk}$$

$$y^{qq}_{N+mk} = y^{q}_{N+mk} \qquad \text{for } m = 2,3, \ldots, M \text{ and } k = 1,2, \ldots, K$$

$$w^{pq}_{mj} = w^{qp}_{mj} = 0$$

$$w_{mj}^{pp} = w_{mj}^{p}$$

$$w_{mj}^{qq} = w_{mj}^{q} \qquad \text{for } m = 2,3, \ldots, M \text{ and } j = 1,2, \ldots, N$$

Condition 8. If $D_{N+2}^{p} > 0$ and $N + 1 < i \leq N + M$, also let $\overset{*}{X}{}_{NS}^{q} = \sum_{m=2}^{M} \overset{*}{X}{}_{N+m.}^{q}$ then

$$y_{N+mk}^{pq} = D_{N+2}^{p} \left( y_{N+mk}^{q} \Big/ \overset{*}{X}{}_{NS}^{q} \right)$$

$$y_{N+mk}^{qp} = 0$$

$$y_{N+mk}^{qq} = y_{N+mk}^{q} - D_{N+2}^{p} \left( y_{N+mk}^{q} \Big/ \overset{*}{X}{}_{NS}^{q} \right)$$

$$y_{N+mk}^{pp} = y_{N+mk}^{p} \qquad \text{for } m = 2,3, \ldots, M \text{ and } k = 1,2, \ldots, K$$

$$w_{mj}^{pq} = D_{N+2}^{p} \left( w_{mj}^{q} \Big/ \overset{*}{X}{}_{NS}^{q} \right)$$

$$w_{mj}^{qp} = 0$$

$$w_{mj}^{qq} = w_{mj}^{q} - D_{N+2}^{p} \left( w^{p} \Big/ \overset{*}{X}{}_{NS}^{q} \right)$$

$$w_{mj}^{pp} = w_{mj}^{p} \qquad \text{for } m = 2,3, \ldots, M \text{ and } j = 1,2, \ldots, N$$

Condition 9. If $D_{N+2}^{p} < 0$ and $N + 1 < i \leq N + M$, also let $\overset{*}{X}{}_{NS}^{p} = \sum_{m=2}^{M} \overset{*}{X}{}_{N+m.}^{p}$ then

$$y_{N+mk}^{pq} = 0$$

$$y_{N+mk}^{qp} = D_{N+2}^{q} \left( y_{N+mk}^{p} \Big/ \overset{*}{X}{}_{NS}^{p} \right)$$

$$y_{N+mk}^{qq} = y_{N+mk}^{q}$$

$$y^{pp}_{N+mk} = y^{p}_{N+mk} - D^{q}_{N+2}\left(y^{p}_{N+mk} \,/\, \overset{*}{X}{}^{p}_{NS}\right)$$

for m = 2,3, . . . , M and k = 1,2, . . . , K

$$w^{pq}_{mj} = 0$$

$$w^{qp}_{mj} = D^{q}_{N+2}\left(w^{p}_{mj} \,/\, \overset{*}{X}{}^{p}_{NS}\right)$$

$$w^{qq}_{mj} = w^{q}_{mj}$$

$$w^{pp}_{mj} = w^{p}_{mj} - D^{q}_{N+2}\left(w^{p}_{mj} \,/\, \overset{*}{X}{}^{p}_{NS}\right)$$

for m = 2,3, . . . , M and j = 1,2, . . . , N

Based on the above nine conditions, we could then carry out the computation to fill each cell of the two-region input-output table portrayed in Figure 2.

### 2.2.3 Balanced Regional Intersectoral Transaction Table

One objective of our two-region input-output model is to produce a balanced regional intersectoral transaction table. In order to construct the Region p's balanced transaction table, we add a total sectoral output of the second quadrant (i.e. interregional output flows from Region p to Region q industries) to the corresponding export sector of the first quadrant, and a total sectoral input of the third quadrant to the corresponding import sector of the first quadrant. The resultant first quadrant is the Region p's balanced intersectoral transaction table.

For Region q's balanced transaction table, we add a total sectoral input of the second quadrant to the corresponding import sector, and a total sectoral output of the third quadrant to the corresponding export sector.

Algebraic expression of the consolidation of the second and third quadrant into the import and export sectors of the first and fourth quadrant are as follows:

Let e designate sector code for both import and export.

For Region p:

$$y^{pp}_{ie} = y^{pp}_{ie} + X^{pq}_{i.} \quad \text{for } i = 1,2, \ldots, N$$

$$y^{pp}_{ie} = y^{pp}_{ie} + W^{pq}_{m.} \quad \text{for } i = N + m \text{ and } m = 1,2, \ldots, M$$

$$w^{pp}_{ej} = w^{pp}_{ej} + X^{qp}_{.j} \quad \text{for } j = 1,2, \ldots, N$$

$$w^{pp}_{ej} = w^{pp}_{ej} + Y^{qp}_{.k} \quad \text{for } j = N + k \text{ and } k = 1,2, \ldots, K$$

where

$$X^{pq}_{i.} = \sum_{j=1}^{N} x^{pq}_{ij} + \sum_{k=1}^{K} y^{pq}_{ik} \quad \text{for } i = 1,2, \ldots, N$$

$$X^{qp}_{.j} = \sum_{i=1}^{N} x^{qp}_{ij} + \sum_{m=1}^{M} w^{qp}_{mj} \quad \text{for } j = 1,2, \ldots, N$$

$$W^{pq}_{m.} = \sum_{j=1}^{N} w^{pq}_{mj} + \sum_{k=1}^{K} y^{pq}_{N+mk} \quad \text{for } m = 1,2, \ldots, M$$

$$Y^{qp}_{.k} = \sum_{i=1}^{N} y^{qp}_{ik} + \sum_{m=1}^{M} w^{qp}_{mk} \quad \text{for } k = 1,2, \ldots, K$$

For Region q:

$$y^{qq}_{ie} = y^{qq}_{ie} + X^{qp}_{i.} \quad \text{for } i = 1,2, \ldots, N$$

$$y^{qq}_{ie} = y^{qq}_{ie} + W^{qp}_{m.} \quad \text{for } i = N + m \text{ and } m + 1,2, \ldots, M$$

$$w^{qq}_{ej} = w^{qq}_{ej} + X^{pq}_{.j} \quad \text{for } j = 1,2, \ldots, N$$

$$w^{qq}_{ej} = w^{qq}_{ej} + Y^{pq}_{.k} \quad \text{for } j = N + k \text{ and } k + 1,2, \ldots, K$$

where

$$X^{qp}_{i.} = \sum_{j=1}^{N} x^{qp}_{ij} + \sum_{k=1}^{K} y^{qp}_{ik} \quad \text{for } 1 = 1,2, \ldots, N$$

$$X^{pq}_{.j} = \sum_{i=1}^{N} x^{pq}_{ij} + \sum_{m=1}^{M} w^{pq}_{mj} \quad \text{for } j = 1,2, \ldots, N$$

$$W^{qp}_{m.} = \sum_{j=1}^{N} w^{qp}_{mj} + \sum_{k=1}^{K} y^{qp}_{N+mk} \quad \text{for } m = 1,2, \ldots, M$$

$$Y^{pq}_{.k} = \sum_{i=1}^{N} y^{pq}_{ik} + \sum_{m=1}^{M} w^{pq}_{mk} \quad \text{for } k = 1,2, \ldots, K$$

Figure 3 portrays the balanced regional intersectoral transaction table of two-region imput-output model.

### 2.2.4 Technical Coefficients

Technical coefficients of four quadrants of two-region model are defined as follows:

$$a^{pp}_{ij} = \frac{x^{pp}_{ij}}{X^{p}_{.j}} \quad \text{(first quadrant)}$$

$$a^{pq}_{ij} = \frac{x^{pq}_{ij}}{X^{q}_{.j}} \quad \text{(second quadrant)}$$

$$a^{qp}_{ij} = \frac{x^{qp}_{ij}}{X^{p}_{.j}} \quad \text{(third quadrant)}$$

$$a^{qq}_{ij} = \frac{x^{qq}}{X^{q}_{.j}} \quad \text{(fourth quadrant)}$$

Under the two region model, the technical relationship among the regional, inter-regional, and intra-regional technical coefficients are:

| | REGION p | | | REGION q | | |
|---|---|---|---|---|---|---|
| | Consuming Sectors 1 2 . . . . . . . N | Final Demands 1 2 . . . e .k | Total Output | Consuming Sectors 1 2 . . . . . . . . N | Final Demands 1 2 . . . e .k | Total Output |
| Producing Sectors 1 2 . . . . N | $x^{pp}_{ij}$ | $y^{pp}_{ij}$ ; $y^{pp}_{ie} + X^{pq}_{i.}$ | $X^{p}_{i.}$ | $x^{qq}_{ij}$ | $y^{qq}_{ij}$ ; $y^{qq}_{ie} + X^{qp}_{i.}$ | $X^{q}_{i.}$ |
| Primary Inputs 1 2 . . | $w^{pp}_{ij}$ | | | $w^{pp}_{ij}$ | | |
| e | $w^{pp}_{ej} + X^{qp}_{.j}$ | | | $w^{qq}_{ej} + X^{pq}_{.j}$ | | |
| . M | | | | | | |
| Total Input | $X^{p}_{.j}$ | $Y^{p}_{.j}$ | | $X^{q}_{.j}$ | $Y^{q}_{.j}$ | |

Figure 2.3 Structure of the Balanced Regional Intersectoral Transaction Table of Two-Region Input-Output Model

$$a^{p}_{ij} = a^{pp}_{ij} + a^{qp}_{ij}$$

$$a^{q}_{ij} = a^{qq}_{ij} + a^{pq}_{ij} \qquad (4)$$

The relationships between the national and regional technical coefficients are:

$$a_{ij} = a^{p}_{ij} = a^{q}_{ij} \qquad (5)$$

For the balanced regional input-output table, the technical coefficients of Region p and Region q are identical to that of the first quadrant and the fourth quadrant of the two-region model respectively, except the import sector. Let $\hat{a}$ denote the technical coefficients of the balanced regional input-output table and e the import sector code.

For Region p:

$\hat{a}^{p}_{ij} = a^{pp}_{ij}$ for all i, except i = e, for each j; and for i = e we have

$$\hat{a}^{p}_{ej} = a^{pp}_{ej} + \sum_{i=1}^{N+M} a^{qp}_{ij}$$

For Region q:

$\hat{a}^{q}_{ij} = a^{qq}_{ij}$ for all i, except i = e, for each j; and for i = e we have

$$\hat{a}^{q}_{ej} = a^{qq}_{ej} + \sum_{i=1}^{N+M} a^{pq}_{ij}$$

We have an identity relationship of:

$\sum_{i=1}^{N+M} \hat{a}^{p}_{ij} = \sum_{i=1}^{N+M} \hat{a}^{q}_{ij} = 1$ , for each j, but the relationship (5) does not hold true any longer, i.e. $a_{ij} \neq \hat{a}^{p}_{ij} \neq \hat{a}^{q}_{ij}$ .

What we have achieved so far is the isolation from the intersectoral transaction of output a component which is attributable to the inter-regional flow. Thus, the technical coefficients expressed by the balanced regional transaction table are quite unique to each region and to a larger extent, we believe, reflect an accurate picture of inter-industry relationship existent in the regional industries.

2.3.5 Interdependence Coefficients[3/]

Notationally, we express the two-region model in the following form:

$$\begin{bmatrix} Y^p \\ Y^q \end{bmatrix} = \begin{bmatrix} X^p \\ X^q \end{bmatrix} - \begin{bmatrix} A^{pp} & A^{pq} \\ A^{qp} & A^{qq} \end{bmatrix} \begin{bmatrix} X^p \\ X^q \end{bmatrix} = \left[ \begin{pmatrix} I & O \\ O & I \end{pmatrix} - \begin{pmatrix} A^{pp} & A^{pq} \\ A^{qp} & A^{qq} \end{pmatrix} \right] \begin{bmatrix} X^p \\ X^q \end{bmatrix} = \begin{bmatrix} I-A^{pp} & -A^{pq} \\ -A^{qp} & I-A^{qq} \end{bmatrix} \begin{bmatrix} X^p \\ X^q \end{bmatrix} \qquad (6)$$

or alternatively as:

$$\left.\begin{aligned} Y^p &= (I-A^{pp})\ X^p - A^{pq}\ X^q \\ Y^q &= -A^{qp}\ X^p + (I-A^{qq})\ X^q \end{aligned}\right\} \qquad (7)$$

Where $Y^p$ and $Y^q$ denote a column vector of total final demands of length N for Region p and q.

$X^p$ and $X^q$ denote a column vector of total outputs of length N for Region p and q.

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3/ The derivation of interdependence coefficients from the balanced regional transaction table is similar to that of the national model. We formulate as follows:

$$\begin{bmatrix} X^p \\ X^q \end{bmatrix} = \begin{bmatrix} I - A^{pp} & 0 \\ 0 & I - A^{qq} \end{bmatrix}^{-1} \begin{bmatrix} Y^p \\ Y^q \end{bmatrix} = \begin{bmatrix} (I - A^{pp})^{-1} & 0 \\ 0 & (I - A^{qq})^{-1} \end{bmatrix} \begin{bmatrix} Y^p \\ Y^q \end{bmatrix}$$

or alternatively,

$$X^p = (I - A^{pp})^{-1}\ Y^p \quad \text{for Region p, and}$$
$$X^q = (I - A^{qq})^{-1}\ Y^q \quad \text{for Region q.}$$

$A^{pp}$, $A^{pq}$, $A^{qp}$, and $A^{qq}$ denote a matrix of the technical coefficients of size N by N for quadrant first, second, third, and fourth respectively.

$A^p$ and $A^q$ denote a matrix of technical coefficients of size N by N for Region p and q. I denotes an identity matrix of size N by N. 0 denotes a zero matrix of size N by N.

The interdependence coefficients for the two-region model are derived as follows:

From equation (6),

$$\begin{pmatrix} X^p \\ X^q \end{pmatrix} = \begin{pmatrix} I - A^{pp} & - A^{pq} \\ -A^{qp} & I - A^{qq} \end{pmatrix}^{-1} \begin{pmatrix} Y^p \\ Y^q \end{pmatrix}$$

$$= \begin{pmatrix} \{(I-A^{pp}) - A^{pq} (I-A^{qq})^{-1} A^{qp}\}^{-1} & \{(I-A^{pp}) - A^{pq} (I-A^{qq})^{-1} A^{qp}\}^{-1} A^{pq} (I-A^{qq})^{-1} \\ \{(I-A^{qq}) - A^{qp} (I-A^{pp})^{-1} A^{pq}\}^{-1} A^{qp} (I-A^{pp})^{-1} & \{(I-A^{qq}) - A^{qp} (I-A^{pp})^{-1} A^{pq}\}^{-1} \end{pmatrix} \begin{pmatrix} Y^p \\ Y^q \end{pmatrix}$$

$$= \begin{pmatrix} H_{pp} & H_{pp} Z_{pq} \\ H_{qq} Z_{qp} & H_{qq} \end{pmatrix} \begin{pmatrix} Y^p \\ Y^q \end{pmatrix} = \begin{pmatrix} H_{pp} & H_{pq} \\ H_{qp} & H_{qq} \end{pmatrix} \begin{pmatrix} Y^p \\ Y^q \end{pmatrix} \qquad (8)$$

Or alternatively as

$$\left.\begin{aligned} X^p &= H_{pp} (Y^p + Z_{pq} Y^q) = H_{pp} Y^p + H_{pq} Y^q \\ X^q &= H_{qq} (Y^q + Z_{qp} Y^p) = H_{qp} Y^p + H_{qq} Y^q \end{aligned}\right\} \qquad (9)$$

Where

$$Z_{pq} = A^{pq} (I - A^{qq})^{-1}$$

$$Z_{qp} = A^{qp} (I - A^{pp})^{-1}$$

$$H_{pp} = \{(I - A^{pp}) - Z_{pq} A^{qp}\}^{-1}$$

$$H_{pq} = H_{pp} Z_{pq}$$

$$H_{qq} = \{(I - A^{qq}) - Z_{qp} A^{pq}\}^{-1}$$

$$H_{qp} = H_{qq} Z_{qp}$$

We consider $Z_{pq}$ as the feed-back-effect matrix multiplier of $1 increase in final demand in Region q to the export demand of Region p's outputs. $Z_{qp}$ as the feed-back-effect matrix multiplier of $1 increase in final demand in Region p to the export demand of Region q's outputs.

$H_{pp}$ and $H_{qq}$ is the intra-regional impact matrix multiplier of $1 increase in final demand in Region p and Region q to outputs required to expand in their respective regions, or the interdependence coefficients matrix of the first and fourth quadrant

$H_{pq}$ is the inter-regional impact matrix multiplier of $1 increase in final demand in Region q to the outputs required to expand in Region p, or the interdependence coefficients matrix of the second quadrant.

$H_{qp}$ is the inter-regional impact matrix multiplier of $1 increase in final demand in Region p to the outputs required to expand in Region q, or the interdependence coefficient matrix of the third quadrant.

With the given interdependence coefficients of the four quadrants, $H_{pp}$, $H_{pq}$, $H_{qp}$, and $H_{qq}$, we would be able to assess the impact of final demand change to the output requirements of Region p and q under three possible occurances:

1) final demand change occurs in Region p but does not occur in Region q;

2) final demand change occurs in Region q but does not occur in Region p; and

3) simultaneous final demand change occurs in both regions.

The outputs required to expand in both regions to meet the final demand changes can be assessed by studying the following combinations of interdependence coefficient matrices. For the above occurances:

1) Region p by $H_{pp}$, and Region q by $H_{qp}$;

2) Region p by $H_{pq}$, and Region q by $H_{qq}$; and

3) Region p by $H_{pp} + H_{pq}$, and Region q by $H_{qq} + H_{qp}$.

The question of what will be the overall effects of output required to expand in the nation as a whole with respect to occurance 1) and 2) can be studied by:

a) $H_{pp} + H_{qp}$ for the overall effect of $1 increase in Region p's final demand, and

b) $H_{qq} + H_{pq}$ for the overall effect of $1 increase in Region q's final demand.

2.2.6. Short-Run Output Forecasting Model

Equation (9) can be converted for the short-run forecasting of output requirements, with the given base year technical coefficient matrices of $A^{pp}$, $A^{qq}$, $A^{pq}$, $A^{qp}$ and the estimates of the future year final demand vectors, $Y^p_f$ and $Y^q_f$. The forecasting models are:

for the Region p's future year outputs $(X^p_f)$,

$$X^p_f = [(I-A^{pp}) - A^{pq}(I-A^{qq})^{-1} A^{qp}]^{-1} [Y^p_f + A^{pq}(I-A^{qq})^{-1} Y^q_f];$$

and for the Region q's future year outputs $(X^q_f)$,

$$X^q_f = [(I-A^{qq}) - A^{qp}(I-A^{pp})^{-1} A^{pq}]^{-1} [Y^q_f + A^{qp}(I-A^{pp})^{-1} Y^p_f].$$

# USING TWO-REGION INPUT-OUTPUT TABLES

The preceeding chapter, the computer model in the general case, was presented. This chapter, a specific application of the model, is demonstrated for the state of Minnesota. A 35-industry breakdown of the state economy is used.

## 3.1 Industry Classification and Data Sources

Table 1 of computer print-outs presents the industry sectoral classification used for the study (for summary listing of tables, see Appendix). The productive and processing activities of the Minnesota economy are grouped into five major areas: (1) Agriculture, Forestry and Fisheries; (2) Mining and Construction; (3) Manufacturing; (4) Transportation, Communication, and Utilities; and (5) Trade, Finance and Services. The Standard Industrial Classification System was used to further classify the major industry groups in a 35-industry breakdown of the Minnesota economy.

In addition to producing sectors, the 1970 Minnesota two-region input-output model contains final demand and primary input sectors. The final demand sectors are the ultimate users of the products and services produced by the producing sectors. The final demand sectors are (1) personal consumption expenditures, (2) gross fixed capital formation, (3) net inventory change, (4) net export, (5) federal government purchases, and (6) state and local government purchases. Primary inputs are the portion of production cost paid by the producing sectors as employee salaries, wages, proprietor's income, rents, or payments to producers outside the region for goods and services purchased, and value added by the time of sales, which can be ragarded as the residual or difference between the sale value and the cost of material and services. These items are considered in the national product accounting system as non-producing inputs and are classified into: (1) employee compensation (or personal income), (2) imports, and (3) value added.

There are two data sets which are essential in the preparation of the two-region input-output model.

- U.S. Base

The year 1970 was chosen for this study. The computer tape containing the national input-output transactions was acquired from the Office of Business Economics, U.S. Department of Commerce. This tape contains dollar transactions of 87 industrial sectors classified by the OBE. Classification of individual industries into sectors is reproduced in Table 1 of the computer print-out for the 87-sector model. The data for the 87 industrial sectoral transactions were then aggregated to 35 sectoral levels (listed now in Table 1 for the 35-industry breakdown). With the aggregated 35-sector transaction, the gross output and total final demands at the national level were then obtained by summing up the column entries of the producing sectors and final demands of this transaction table. Table 2 shows the gross outputs of the 35 producing sectors of the nation in 1970.

- Minnesota Data Base

The data at the regional level required by the two-region input-output computer model is the sector total output estimates and the total final demand estimates. The individual sector total outputs were obtained from secondary sources including publications of the U.S. Bureau of Census and publications of Minnesota state agencies. The data sources for each of the major economic categories are listed in the next chapter. The figures of total final demand estimates are obtained from the Minnesota State Energy Agency.

## 3.2 Direct Requirements at the National Level

After the compilation of the 35-sector national transaction table, the direct requirements (or technical coefficients) table of the nation is calculated from this transaction table by dividing each column entry by the column total. The results are reproduced in Table 3. Each column entry in Table 3 shows the input that the sector numbered at the top of the column required from the sector named on the left side of the table, to produce a dollar of output of the column sector. For example, to produce one dollar of output, the livestock and its products sector (sector 1) used \$.20 of its own products, purchased \$.25 of products from the other agricultural products sector (2), \$.10 of products from the food and kindred products sector (8), purchased a total of \$.71 of products from the 35 procuding sectors (36), paid \$.06 per \$1 output for wages and salaries to farm operators or hired laborers, and paid or received rents, interests, profits and taxes of \$.23 (39) as value added. The other column sector of this table can be explained in a similar manner.

The coefficients of this direct requirements table are regarded as the technical aspect of input mixes for producing a \$1 unit of output. When a sector increases outputs, the required total inputs are obtained from input sectors according to the percentages shown in the column of this table. In developing the two-region input-output tables, we assume that the input structure such as presented in this direct requirements table at the national level would prevail at the sub-region level, so that the disaggregation of the national transaction table can be carried out. With the given estimates of gross outputs and final demands of one of the regions (Region p) the first step in disaggregating the national transaction table is done by applying the national direct requirements coefficients to the given estimates of sectoral gross output and final demand for one of the regions. The resulting regional transactions[4/] table indicates the inputs required from other industries to support the estimated level of output of each regional industry. Having estimated the regional transaction table for Region p, the transaction table for Rest-of-Nation can be easily calculated by subtracting Region p's requirements from the corresponding cell of the national transaction table.

3.3 Surplus or Deficit of Regional Production

The regional transaction table derived by applying the national direct requirements coefficients to the estimated regional gross outputs as mentioned in the previous section, is not balanced in itself. That is to say that the total indicated use (demand) of a sector, which is the sum of the column entries in a row sector in the regional transaction table might not be equal to the gross output supply of the sector represented by that row. We define the output surplus as the amount by which the estimated gross output exceeds the total indicated use, and define the output deficit as for the case in reverse.

Table 4 presents the results calculated for Minnesota in the year 1970, and lists the sectors from the largest surplus producing sector, to the largest deficit producing sector. The sectors listed in Table 4 include 35 producing sectors and two primary input sectors. Two primary input sectors head the top of the list with a surplus of $3,007 million for employee compensation, and $1,162 million for the value added sector. Following these two are the 35 producing sectors. Among the 35 producing sectors, only eleven sectors produced a surplus. The largest surplus was produced in the manufacturing sector with $988.5 million, followed by the food, kindred

[4/] This table is not reproduced as computer print-out in our program, but it was computed and stored internally within the computer.

products sector, the livestock and products sector, and the iron and ferrous ores sector, with $552.7 million, $547.3 million, and $532.0 million respectively. The largest deficit producing sector was the other manufacturing sector with $1,496.3 million, followed by the finance, insurance and real estate (F.I.R.E.) sector, and the construction sector, both having more than a billion dollar deficit ($1,281 and $1,183.5 respectively).

In the two-region input-output model, the assumption is made that the surplus output of the given region is being exported to the Rest-of-Nation, and the deficit output of the given region is being made up by the imports from the Rest-of-Nation. Thus, the figures in Table 4 apply not only for Minnesota but also for Rest-of-Nation.

Table 5 presents the figures of Table 4 in the form of the ratio of the sectoral output surplus or deficit to the total indicated use of the corresponding sector of the regions.

The iron and ferrous ores sector tops the list of all the surplus producing sectors in Minnesota with a positive ratio of 12.2 which indicates that the surplus produced by this sector in Minnesota in 1970 was 12.2 times more than the total amount used in Minnesota in the same year. From the standpoint of Rest-of-Nation, Minnesota's surplus output was equivalent to 29.3 percent of the Rest-of-Nation's total requirements for this product.

The state and local government enterprises, which include public utilities and transportation, produced twice more than used in the Minnesota industries and final consumers, and the amount was equivalent to 3 percent of the Rest-of-Nation's 1970 total consumption.

The machinery manufacturing sector and the railroad transportation sector in Minnesota also produced considerable surplus as compared to the total use in Minnesota in 1970, with the output exceeding the total use by 84.3 and 69.7 percent respectively, which is equivalent to 1.6 and 1.3 percent of the Rest-of-Nation's total use of the sectoral outputs in 1970.

On the other hand, Minnesota industries imported more than three-fourths of the total use of outputs required from the other mining sector, the other utilities sector, and the non-ferrous ores sector, from Rest-of-Nation to meet production requirements in 1970.

### 3.4 Inter- and Intra-Regional Transactions

Based on the output surpluses or deficits of Minnesota's sectoral production as listed in Table 4, the four quadrants of the two-region input-output table can now be compiled (Figure 3.2). The four quadrants are: (1) the first quadrant of intra-regional intersectoral transactions of Minnesota's produced outputs to Minnesota's purchasing and final demand sectors; (2) the second quadrant of inter-regional intersectoral transactions of Minnesota's produced outputs to Rest-of-Nation's purchasing and final demand sectors; (3) the third quadrant of inter-regional intersectoral transactions of Rest-of-Nations produced outputs to Minnesota's purchasing and final demand sectors; and (4) the fourth quadrant of intra-regional intersectoral transactions of Rest-of-Nation's produced outputs to Rest-of-Nation's purchasing and final demand sectors.

The mathematical solutions for allocating the surpluses or deficits are presented in Section 2.3 of the previous chapter. The allocation of the surplus outputs produced in Minnesota to Rest-of-Nation's purchasing and final demand sector resulted in the transaction table of Quadrant 2, which is represented in Table 8. (Total Intersectoral Transfers in the Two-Region Input-Output System: Rest-of-Nation Purchases of Specified Region A, (Minnesota) Gross Output by Purchasing Sector, 1970.) The allocation of surpluses produced in Rest-of-Nation resulted in the transaction table of quadrant 3, which is represented in Table 7. (Total Intersectoral Transfers in the Two-Region Input-Output System: Region A (Minnesota) Purchases of Specified Rest-of-Nation Gross Output, by Purchasing Sector, 1970.)

Still remaining is the estimation of the intra-regional transaction table for each region. Transactions among sectors within Minnesota are estimated by subtracting each entry of Table 7 (quadrant 3) from the corresponding entry in the regional transaction table of Minnesota mentioned in Section .2. The result is Table 6 (Total Intersectoral Transfers in the Two-Region Input-Output System: Region A (Minnesota) Purchases of Specified Region A Gross Output by Sector, 1970). For the intra-regional intersectoral transactions within Rest-of-Nation, the estimation was made by subtracting each entry of Table 8 (quadrant 2) from the corresponding entry in the Rest-of-Nation's regional transaction table. Table 9 is the resultant table for intra-regional transactions of Rest-of-Nation.

This completes a brief description of the disaggregation of the national transaction table to the four quadrants of the transaction tables of the two-region model.

### 3.5 Balanced Regional Intersectoral Transaction

After compilation of the four quadrants of the transaction tables for the two-region model, the balanced regional intersectoral transaction table for Minnesota can be accomplished by merging the total inputs (row 40) of quadrant 3 (Table 7) into the import sector (row 38) of quadrant 1 (Table 6) and the total outputs (column 43) of quadrant 2 (Table 8) into the export sector (column 40) of quadrant 1 (Table 6). After the merger of quadrants 2 and 3 into quadrant 1, recomputing the total inputs (row 40) and the total outputs (column 43) of quadrant 1 (Table 6) would result in the balanced Minnesota intersectoral transaction table of Table 10. The term "balanced" here is used to imply that the total output (column 43) and total input (row 40) of the corresponding sector in the row and the column of this table are being equated.

Similar methods of merging quadrants 2 and 3 into quadrant 4 (Table 9) would yield a balanced regional intersectoral transaction table for Rest-of-Nation. However, the merging of quadrants 2 and 3 into quadrant 4 differs from that of Minnesota. We now merge total outputs (column 43) of quadrant 3 (Table 7) into the export sector (column 40) of quadrant 4 (Table 10) and the total inputs (row 40) of quadrant 2 (Table 8) into the import sector (row 38) of quadrant 4.

What we have achieved so far by the compilation of the balanced regional intersectoral transaction table under the framework of the two-region model are: (1) that the intersectoral transfers of output between the producing sectors and consuming sectors (or purchasing sectors) within a region account for only the goods and services produced by the local industries, and (2) that the inter-regional flow of goods and services is being identified and included in the import or export sectors.

### 3.5.1 Direct Requirements

Table 11 shows the direct requirements of outputs derived from the balanced Minnesota intersectoral transaction table (Table 10). In order to present the difference of the direct requirements of outputs between Minnesota and the nation as a whole (Table 3), we reproduced the row sector 36, which is an aggregation of the direct requirements coefficients of the 35 producing sectors and the import sector (row 38) from both Table 3 and Table 11 on Table Columns 1 and 2 of Table .1 list the aggregated direct requirements for the nation and Minnesota respectively. Column 3 indicates the foreign imports for the nation and column 4 indicates the imports from foreign and domestic origins for Minnesota. The difference of the aggregated direct requirements between Minnesota and the nation is due largely to the amount of imports.

Among the 35 producing sectors in Minnesota, the petroleum industry (sector 13) depended upon imports for about fifty percent of its total inputs, as compared to only five percent for the nation as a whole. This ratio of import to total input is the highest among Minnesota's industrial sectors. In spite of a large surplus of output produced by the iron and ferrous ores sector in Minnesota, this sector relied upon imports for about forty percent of the total inputs, largely due to the very high portion of total inputs constituted by outputs from other industries located outside Minnesota. The third largest importing sector was the non-ferrous metal ores sector which imported about 31 percent of the total inputs for its production, followed by the primary metals sector and the gas utilities sector, with 28 and 27 percents respectively. For the rest of the sectors, imports constituted less than one fourth of their total inputs.

To what extent does the resultant balanced regional transaction table as an outcome of disaggregating the national transaction table reflect the particular economic conditions existing in Minnesota? In order to give an answer to this question, we chose the petroleum industrial sector for an illustration. Reproduced in Table 3.2 are the direct requirements coefficients of column 13 from the national table (Table 3) and the balanced Minnesota regional table (Table 11). In order to produce a $1 output in the petroleum industrial sector in the nation, the petroleum industrial sector imported 4.7 cents of products from foreign countries, purchased 44.4 cents of products from the other mining sector, 7.1 cents of products from its own

Table 3.1 Comparison of Direct Requirements Coefficients Between the National Table (Table 3) and the Balanced Minnesota Table (Table 10)

| | Industry | Aggregated Coefficient of 35 Producing Sectors | | Import Sector | |
|---|---|---|---|---|---|
| | | Nation | Minnesota | Nation 1/ | Minnesota 2/ |
| | | (1) | (2) | (3) | (4) |
| 1 | Livestock and Prod. | .70 | .68 | .01 | .03 |
| 2 | Other Agric. Prod. | .51 | .40 | .01 | .12 |
| 3 | Agri., For., Fish. | .35 | .31 | .14 | .17 |
| 4 | Iron, Ferro Ores | .43 | .33 | .29 | .40 |
| 5 | Nonferro. Metal Ores | .41 | .23 | .13 | .31 |
| 6 | Other Mining | .37 | .26 | .06 | .18 |
| 7 | Construction | .55 | .42 | * | .13 |
| 8 | Food, Kindred Prod. | .71 | .67 | .03 | .06 |
| 9 | Lumber, Furniture | .55 | .40 | .04 | .19 |
| 10 | Pulp, Paper Prod. | .58 | .50 | .06 | .15 |
| 11 | Printing, Publishing | .50 | .44 | * | .06 |
| 12 | Chemicals Products | .65 | .45 | .03 | .22 |
| 13 | Petroleum Ind. | .75 | .30 | .05 | .50 |
| 14 | Stone, Clay, Glass | .49 | .36 | .02 | .16 |
| 15 | Primary Metals | .60 | .38 | .07 | .28 |
| 16 | Fabricated Metals | .57 | .37 | .02 | .22 |
| 17 | Machinery Exc. Elect. | .56 | .44 | .03 | .15 |
| 18 | Electrical Mfg. | .54 | .41 | .05 | .18 |
| 19 | Other Manufacturing | .60 | .41 | .03 | .23 |
| 20 | Railroad Trans. | .34 | .25 | .01 | .10 |
| 21 | Truck. Warehousing | .33 | .26 | 0 | .07 |
| 22 | Other Transportation | .45 | .38 | .09 | .16 |
| 23 | Communication | .22 | .17 | .01 | .06 |
| 24 | Electric Utilities | .56 | .41 | * | .15 |
| 25 | Gas Utilities | .41 | .16 | .02 | .27 |
| 26 | Other Utilities | .78 | .72 | 0 | .05 |
| 27 | Wholesale | .33 | .27 | * | .06 |
| 28 | Retail | .24 | .19 | * | .05 |
| 29 | F.I.R.E. | .32 | .25 | * | .07 |
| 30 | Hotel, Repair Serv. | .43 | .32 | 0 | .11 |
| 31 | Business Services | .47 | .40 | * | .07 |
| 32 | Medical, Educational | .28 | .21 | * | .07 |
| 33 | Other Services | .63 | .55 | .03 | .11 |
| 34 | Federal Gov. Entp. | .25 | .21 | .04 | .08 |
| 35 | State and Local Gov. | .48 | .33 | 0 | .15 |

* Less than .01

1/ Includes only foerign imports

2/ Includes foreign imports and inter-regional flows.

sector, 4.0 cents from the finance, insurance and real estates sector, 3.1 cents from the business service sector, and 2.7 cents from the chemical products sector. For a comparison, the petroleum industrial sector in Minnesota imported 49.6 cents of products from both domestic and foreign sources, purchased 7.0 cents of products from the other mining sector in Minnesota, and 3.8 cents from its own sector in Minnesota. Other mining in Minnesota includes quarrying, but not petroleum (which thus overstates slightly its direct requirements from this industry in Minnesota and, thus, understates total imports).

Deviation of Minnesota's direct requirements coefficients from the national direct requirements coefficients was especially pronounced for the other mining sector (i.e. 7.0 cents for Minnesota vs. 44.4 cents for the nation), except for slight understatement of imports, the balanced regional transaction table compiled under the two-region model reflects fairly well the fact that Minnesota lagged behind the nation in oil mining activities due to its lack of natural resources endowment. Based on the sector-by-sector comparison of direct requirements coefficients between Minnesota and the nation for other producing sectors, however, we believe that the balanced regional intersectoral transaction table represents reasonably well the inter-industry relationship existing in the region of interest, so as to give useful information for analytical purposes.

### 3.5.2 Direct and Indirect Requirements

Table 12 shows the direct and indirect requirements coefficients derived from the balanced Minnesota intersectoral transaction table. The method of derivation of these coefficients is a straight forward inversion of the so-called Leontief matrix,notationally expressed as (I-A) which is composed of the direct requirements coefficients (quadrant 1's A matrix) of the 35

TABLE 3.2 Comparison of Technical Coefficients of Petroleum Industry Sector Between the Nation and Minnesota, 1970

| | Industry | Nation | Minnesota |
|---|---|---|---|
| 1 | Livestock and Prod. | 0 | 0 |
| 2 | Other Agric. Prod. | 0 | 0 |
| 3 | Agri., For., Fish. | 0 | 0 |
| 4 | Iron, Ferro Ores | 0 | 0 |
| 5 | Nonferro. Metal Ores | 0 | 0 |
| 6 | Other Mining | .444206 | .070195 |
| 7 | Construction | .016837 | .011295 |
| 8 | Food, Kindred Prod. | .001482 | .001482 |
| 9 | Lumber, Furniture | .000032 | .000021 |
| 10 | Pulp, Paper Prod. | .006149 | .006149 |
| 11 | Printing, Publishing | .000095 | .000095 |
| 12 | Chemicals Products | .026864 | .015032 |
| 13 | Petroleum Ind. | .071134 | .037930 |
| 14 | Stone, Clay, Glass | .002428 | .001947 |
| 15 | Primary Metals | .001955 | .000950 |
| 16 | Fabricated Metals | .004603 | .003650 |
| 17 | Machinery Exc. Elect. | .003374 | .003374 |
| 18 | Electrical Mfg. | .000410 | .000331 |
| 19 | Other Manufacturing | .003374 | .001895 |
| 20 | Railroad Trans. | .001703 | .001703 |
| 21 | Truck. Warehousing | .005549 | .005549 |
| 22 | Other Transportation | .048443 | .047155 |
| 23 | Communication | .001072 | .000770 |
| 24 | Electric Utilities | .005959 | .003403 |
| 25 | Gas Utilities | .011288 | .011288 |
| 26 | Other Utilities | .001356 | .000285 |
| 27 | Wholesale | .012329 | .011935 |
| 28 | Retail | .000441 | .000395 |
| 29 | F.I.R.E. | .040107 | .028790 |
| 30 | Hotel, Repair Serv. | .000504 | .000428 |
| 31 | Business Services | .031405 | .027938 |
| 32 | Medical, Educational | .000221 | .000198 |
| 33 | Other SErvices | .002396 | .001995 |
| 34 | Federal Gov. Entp. | .000662 | .000501 |
| 35 | State and Local Gov. | 0 | 0 |
| 36 | Subtotals | .746398 | .296675 |
| 37 | Employee Comp. | .088358 | .088358 |
| 38 | Import | .046981 | .496703 |
| 39 | Value Added | .118264 | .118264 |
| 40 | Totals | 1.000000 | 1.000000 |

producing sectors and an identity matrix (I) of the similar size.

It is not our intention to analyze separately the direct and indirect requirements coefficients derived from a balanced regional intersectoral transaction table, from that of the information calculated by utilizing the four quadrants of the two-region intersectoral transaction tables. The reader is reminded that the analysis of direct and indirect requirements coefficients herein is also applicable to the analysis of similar coefficients of the first quadrant of the two-region input-output model as put forth in section 2.7.[5/]

The objective of computing the direct and indirect requirements table is to apply these coefficients in assessing the impact of change in final demand of products upon the productive activities in an economy. Each coefficient of the table indicates in dollar terms how much the outputs of the sector in the row in which the coefficient is located will change resulting from a one dollar change in final demand for products and services of the column in which the coefficient is located. For example, according to the coefficients of Table 12, the economy-wide impact of a one dollar increase in the final demand of products produced by the livestock and it products sector (sector 1) would require: (1) the output of its own sector to increase by $1.32, which includes one dollar of final delivery to the ultimate consumers; (2) the output of the other agricultural products sector (sector 2) to expand by $.36; (3) the output of the food and kindred products sector (sector 8) to expand by $.16; (4) the output of the finance, insurance and real estates sector (sector 29) to expand by $.07; and (5) the outputs of all of the 35 producing sectors to expand by a total of $1.76. All other

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5/ Use of the direct and indirect requirements coefficients derived from the single-region model is similar to their use when derived from the balanced regional transaction table which does not take into account the inter-regional interdependent relationships.

producing sectors of this table can be explained in a similar manner. Diagonal entries of this table are greater than one, due to a one dollar delivery to the final demand by each sector which is included in the entries. The column totals (row 36) show the total outputs from all the producing sectors to expand in order to deliver one dollar of final products to the final demands. There are varying degrees of output expansion effect brought about in the sectors of the Minnesota economy. We list five of the most expansive and the least expansive sectors below:

| | |
|---|---|
| The most expansive sectors in Minnesota | |
| Food and kindred products sector (8) | $2.36 |
| Livestock and its products sector (1) | 2.30 |
| Other utilities sector (26) | 2.10 |
| Other services sector (33) | 1.95 |
| Pulp, paper products sector (10) | 1.86 |
| The least expansive sectors in Minnesota | |
| Gas utilities sector (25) | $1.23 |
| Retail sector (28) | 1.28 |
| Communication sector (23) | 1.29 |
| Federal government enterprises sector (34) | 1.31 |
| Medical, education sector (32) | 1.34 |

The rest of the sectors fall between these two groups. Each entry of Table 12 is called the "direct and indirect requirements coefficient," for the reason that there are two kinds of effects embodied in each coefficient. That is, the direct requirement and the indirect requirement. With the information of the direct requirements coefficient table (Table 11) we are able to isolate the indirect requirement of output expansion from the direct and indirect requirement table (Table 12). For example, in the food and kindred products sector (column 8), a one dollar increase in final demand of the output of this sector would require its own sectoral output to expand by \$1.25, and the outputs from the livestock and its products sector (row 1) and the other agricultural products sector (row 2) to expand by \$.36 and \$.19, respectively. Looking into column 8 of Table 11, which indicates that in order to produce \$1.00 of the food and kindred products sectoral output, the direct requirements of outputs from the livestock and its product sector, and the other agricultural sector are \$.22 and \$.07, respectively. However, in order for these two sectors to increase their outputs, they in turn require outputs from all other sectors and themselves. The amounts of requirements from these two sectors are indicated on columns 1 and 2 of Table 11. Because of this interrelationship of output requirements among the producing sectors, a one dollar increase in final demand of output would bring about a chain of effects throughout the economy. Thus, the indirect requirements are represented by the effects of this chain of output expansion expressed in dollar volume. In the above example, a one dollar increase in final demand of the products from the food and kindred products sector is associated with total product requirements of \$1.25, which includes \$.16 for direct requirements (column 8, row 8, Table 11), and \$1.09 for indirect requirements (\$1.25 - \$.16 = \$1.09). The direct and indirect requirements of the livestock and products sector, as mentioned before, totals \$.36, which includes \$.22 of direct requirements (column 8, row 1, Table 11) and \$.14 of indirect requirements (\$.36 - \$.22 = \$.14).

The above discussion gives some idea as to the usage of the coefficients of Table 11 and Table 12 in analyzing the possible impact of an increase in final demand of goods and services from a producing sector.

## 3.6 Direct Requirements of the Two-Region Model

### 3.6.1 Intra-Regional Direct Requirements

Table 13 shows the intra-regional direct requirements coefficients for Minnesota. Figures in this table are derived from dividing each column entry of Table 6 (intra-regional intersectoral transactions table of Minnesota) by the Minnesota gross output estimate of the sector corresponding to the column.

Table 16 is a similar table for Rest-of-Nation. Figures in this table are computed from Table 9 (intra-regional intersectoral transactions table of Rest-of-Nation) by dividing each column entry of this table by the Rest-of-Nation gross output estimate of the sector corresponding to the column. Each figure shows the direct requirements per dollar of output for a local purchasing sector listed at the top of the table from the local producing sector listed on the left side of the table.

### 3.6.2 Inter-Regional Direct Requirements

Table 14 shows the inter-regional direct requirements of output produced in the Rest-of-Nation by a Minnesota producing sector. Figures in this table are computed by dividing each column entry in Table 7 (the inter-regional intersectoral transfers of Rest-of-Nation's outputs to Minnesota) by the Minnesota's gross output estimate of the sector corresponding to the column. Each figure indicates the direct requirements per dollar of output for a Minnesota purchasing sector listed at the top of the table from the Rest-of-Nation producing sectors listed in the left side fo the table. The sum of a column total (row 40) of this table and the corresponding column total of Table 13 should be equal to 1.0.

Table 15 shows the inter-regional direct requirements of output produced in Minnesota by a Rest-of-Nation producing sector. Figures in this table are computed by dividing each column entry in Table 8 (the inter-regional intersectoral transfers of Minnesota outputs to Rest-of-Nation) by the Rest-of-Nation gross output estimate of the sector corresponding to the column. Each figure shows the direct requirements per dollar of output for a Rest-of-Nation purchasing sector listed at the top of the table from the Minnesota producing sectors listed in the left side of the table. The sum of a column total (row 40) of this table and the corresponding column total of Table 16 should be equal to 1.0.

## 3.7 Direct and Indirect Requirements

In Chapter 2, Section 2.3.5, we present the mathematical solution of the equation system representing the producing sectors, which will lead to the production of the direct and indirect requirements or interdependence coefficients tables. The solution utilizes the four quadrants of direct requirements tables mentioned in the previous section and involves a series of matrix multiplications and inversions. The coefficient of the direct and indirect requriements in the four quadrants are derived simultaneously. These coefficients can be used to obtain estimates of outputs required by each sector in the regions to meet the possible changes of final demands in each region or in both regions.

### 3.7.1 Intra-Regional Direct and Indirect Requirements

Table 21 presents the intra-regional direct and indirect coefficients of Minnesota (Quadrant 1). This table is similar to the one which we discussed in Section 3.5 for the balanced regional transaction table (Table 12). Essentially, a direct and indirect requirements coefficient in this table expresses the output required from a Minnesota's producing sector to meet a one dollar increase in demand for the output from this sector, which takes place in Minnesota.

Table 24 presents the intra-regional direct and indirect requirements coefficients of Rest-of-Nation (Quadrant 4). This table is similar to Table 21 mentioned above, but the coefficients relate the output required from a Rest-of-Nation producing sector to meet a one dollar increase in demand for the output from this sector, which takes place in Rest-of-Nation.

### 3.7.2 Inter-Regional Direct and Indirect Requirements

Table 22 presents the direct and indirect requirements coefficient related to the output required from a producing sector in Rest-of-Nation to a one dollar increase in demand for the output of the sector, which takes place in Minnesota (Quadrant 3).

Table 23 presents the similar direct and indirect requirements coefficients related to the output required from a producing sector in Minnesota by a one dollar increase in demand for the output of the sector, which takes place in Rest-of-Nation (Quadrant 2).

A coefficient in these two tables expresses the output required from a region's producing sector to meet a one dollar increase in demand for the sectoral output of the other region.

## 3,8 Economic Effects of Change in Final Demand

The purpose of this section is to discuss the uses of the intra- and inter-regional direct and indirect requirements tables in analyzing the economic impact of a change in final demand under the two-region input-output model.

For the illustrative purpose, we hypothesize that the demand for food and kindred products (sector 8) has increased by one million dollars in each region and have compiled Table 3.3. The first four columns of Table 3.3 are calculated by multiplying each entry of column 8 of the direct requirements tables (Tables 13, 14, 16, and 15) and the direct and indirect requirements tables (Tables 21, 22, 24, and 23) by $1,000,000. The columns 1 and 2 show the Minnesota food and kindred products sector paid to each of the producing sectors for the purpose of obtaining the materials and services required to produce the one-million dollars of new food and kindred products, with column 1 indicating the payments which are made to the producing sectors in Minnesota and column 2 indicating the payments which are made to the producing sectors in Rest-of-Nation. For example, in Minnesota $223,623 of materials would be purchased directly from the livestock and livestock product sector, $72,066 would be purchased from the other agricultural sector, $3,815 would be purchased from agriculture, forestry and fisheries sector, $163,946 would be purchased from food and kindred products sector, $188,106 would be paid in employee salaries and wages, $29,839 would be paid on foreign imports, and so on for a total of $965,875, which would be spent directly within Minnesota, for producing $1,000,000 of new food and kindred products. These are called intra-regional direct effects. The above total of $965,875 does not include the amounts spent by Minnesota food and kindred products sector in Rest-of-Nation for procuring the materials and services in order to make $1,000,000 products. This amount is equal to $34,125 (column 2, row 40). The entries in column 2 show how this $34,125 was distributed among the sectors in Rest-of-Nation. These are called the inter-regional direct effects of making $1,000,000 of new food and kindred products in Minnesota and represent the additional market for the Rest-of-Nation producing sectors which sell directly to the Minnesota food and kindred products sector.

TABLE 3.3 Output Reguired in the Regions for the Food and Kindred Products Industry to Increase Food and Kindred Products by $1,000,000 in Each Region -- 1970

| | | Direct Output Requirements | | | |
|---|---|---|---|---|---|
| | | Minnesota (Quadrant 1) (1) | Minnesota from Rest-of-Nation (Quadrant 3) (2) | Rest-of-Nation (Quadrant 4) (3) | Rest-of-Nation from Minnesota (Quadrant 2) (4) |
| | | (Dollars) | | | |
| 1 | Livestock and Prod. | 223,623 | 0 | 220,253 | 3,370 |
| 2 | Other Agric. Prod. | 72,066 | 0 | 71,909 | 158 |
| 3 | Agri., For., Fish. | 3,815 | 693 | 4,508 | 0 |
| 4 | Iron, Ferro Ores | 0 | 0 | 0 | 0 |
| 5 | Nonferro, Metal Ores | 0 | 0 | 0 | 0 |
| 6 | Other Mining | 99 | 530 | 629 | 0 |
| 7 | Construction | 2,244 | 1,101 | 3,345 | 0 |
| 8 | Food, Kindred Prod. | 163,946 | 0 | 163,061 | 885 |
| 9 | Lumber, Furniture | 640 | 342 | 982 | 0 |
| 10 | Pulp, Paper Prod. | 27,275 | 0 | 27,161 | 114 |
| 11 | Printing, Publishing | 5,480 | 0 | 5,479 | 1 |
| 12 | Chemicals Products | 4,527 | 3,564 | 8,091 | 0 |
| 13 | Petroleum Ind. | 1,260 | 1,103 | 3,263 | 0 |
| 14 | Stone, Clay, Glass | 10,147 | 2,509 | 12,656 | 0 |
| 15 | Primary Metals | 88 | 93 | 181 | 0 |
| 16 | Fabricated Metals | 23,519 | 6,148 | 29,667 | 0 |
| 17 | Machinery Exc. Elect. | 2,440 | 0 | 2,402 | 38 |
| 18 | Electrical Mfg. | 23 | 6 | 29 | 0 |
| 19 | Other Manufacturing | 5,213 | 4,069 | 9,282 | 0 |
| 20 | Railroad Trans. | 9,892 | 0 | 9,764 | 128 |
| 21 | Truck. Warehousing | 16,211 | 0 | 16,177 | 34 |
| 22 | Other Transportation | 4,442 | 123 | 4,565 | 0 |
| 23 | Communication | 2,545 | 1,000 | 3,545 | 0 |
| 24 | Electric Utilities | 2,324 | 1,746 | 4,069 | 0 |
| 25 | Gas Utilities | 2,430 | 0 | 2,422 | 9 |
| 26 | Other Utilities | 104 | 391 | 496 | 0 |
| 27 | Wholesale | 36,959 | 1,218 | 38,177 | 0 |
| 28 | Retail | 1,798 | 213 | 2,011 | 0 |
| 29 | F.I.R.E. | 9,557 | 3,757 | 13,314 | 0 |
| 30 | Hotel, Repair Serv. | 1,303 | 231 | 1,534 | 0 |
| 31 | Business Services | 31,792 | 3,945 | 35,738 | 0 |
| 32 | Medical, Educational | 418 | 49 | 467 | 0 |
| 33 | Other Services | 5,610 | 1,128 | 6,738 | 0 |
| 34 | Federal Gov. Entp. | 519 | 167 | 686 | 0 |
| 35 | State and Local Gov. | 267 | 0 | 258 | 8 |
| 36 | Subtotals | 672,578 | 34,125 | 701,958 | 4,744 |
| 37 | Employee Compensation | 188,106 | 0 | 187,163 | 943 |
| 38 | Import | 29,839 | 0 | 29,839 | 0 |
| 39 | Value added | 75,353 | 0 | 74,989 | 363 |
| 40 | Totals | 965,875 | 34,125 | 993,949 | 6,051 |

TABLE 3.3 Output Required in the Regions for the Food and Kindred Product Industry to Increase Food and Kindred Products by $1,000,000 in Each Region -- 1970 (continued)

| | | Direct and Indirect Output Requirements | | | |
|---|---|---|---|---|---|
| | | Minnesota | Minnesota from Rest-of-Nation | Rest-of-Nation | Rest-of-Nation from Minnesota |
| | | (Quadrant 1) (5) | (Quadrant 3) (6) | (Quadrant 4) (7) | (Quadrant 2) (8) |
| | | (Dollars) | | | |
| 1 | Livestock and Prod. | 361,307 | 1,114 | 167,455 | 3,038 |
| 2 | Other Agric. Prod. | 190,775 | 1,617 | 15,908 | 569 |
| 3 | Agri., For., Fish. | 17,598 | 3,705 | 27,571 | 673 |
| 4 | Iron, Ferro Ores | 419 | 442 | 4,159 | 63 |
| 5 | Nonferro. Metal Ores | 87 | 1,330 | 3,947 | 58 |
| 6 | Other Mining | 1,543 | 16,221 | 3,983 | 56 |
| 7 | Construction | 10,855 | 9,671 | 5,564 | 75 |
| 8 | Food, Kindred Prod. | 1,245,241 | 1,796 | 1,244,457 | 3,717 |
| 9 | Lumber, Furniture | 4,874 | 5,102 | 7,069 | 116 |
| 10 | Pulp, Paper Prod. | 54,105 | 3,361 | 16,443 | 238 |
| 11 | Printing, Publishing | 21,541 | 3,753 | 7,374 | 131 |
| 12 | Chemicals Products | 21,119 | 26,502 | 30,545 | 296 |
| 13 | Petroleum Ind. | 9,313 | 11,535 | 7,062 | 83 |
| 14 | Stone, Clay, Glass | 15,640 | 5,994 | 5,716 | 84 |
| 15 | Primary Metals | 8,530 | 22,255 | 9,065 | 109 |
| 16 | Fabricated Metals | 36,904 | 14,694 | 6,444 | 88 |
| 17 | Machinery Exc. Elect. | 13,085 | 4,051 | 5,979 | 106 |
| 18 | Electrical Mfg. | 3,871 | 3,496 | 5,945 | 78 |
| 19 | Other Manufacturing | 16,984 | 23,976 | 10,472 | 144 |
| 20 | Railroad Trans. | 19,588 | 1,963 | 3,213 | 49 |
| 21 | Truck Warehousing | 32,781 | 1,889 | 8,419 | 93 |
| 22 | Other Transportation | 12,759 | 3,310 | 14,558 | 145 |
| 23 | Communication | 10,603 | 6,747 | 6,953 | 67 |
| 24 | Electric Utilities | 6,163 | 8,048 | 2,705 | 52 |
| 25 | Gas Utilities | 5,418 | 1,840 | 2,485 | 35 |
| 26 | Other Utilities | 479 | 2,056 | 4,115 | 109 |
| 27 | Wholesale | 72,824 | 7,637 | 22,799 | 210 |
| 28 | Retail | 15,836 | 3,691 | 3,432 | 43 |
| 29 | F.I.R.E. | 49,908 | 32,311 | 5,992 | 118 |
| 30 | Hotel, Repair Serv. | 3,255 | 1,341 | 7,505 | 85 |
| 31 | Business Services | 65,159 | 16,831 | 6,972 | 85 |
| 32 | Medical, Educational | 3,069 | 695 | 17,546 | 163 |
| 33 | Other Services | 20,971 | 9,185 | 106,343 | 946 |
| 34 | Federal Gov. Entp. | 2,457 | 1,800 | 2,954 | 45 |
| 35 | State and Local Gov. | 3,055 | 2,438 | 3,946 | 51 |
| 36 | Subtotals | 2,358,179 | 262,398 | 1,805,095 | 12,020 |
| 37 | Employee Compensation | 0 | 0 | 0 | 0 |
| 38 | Import | 0 | 0 | 0 | 0 |
| 39 | Value added | 0 | 0 | 0 | 0 |
| 40 | Totals | 2,358,179 | 262,398 | 1,805,095 | 12,020 |

A similar analysis can be made for $1,000,000 increase in demand for food and kindred products in Rest-of-Nation, by using the intra-regional (column 3) and inter-regional (column 4) direct requirement coefficients in evaluating the direct effects of $1,000,000 in new food and kindred products.

The columns 5 and 6 of Table 3.3 show the Minnesota intra-regional and inter-regional direct and indirect requirements of producing a $1,000,000 of new food and kindred products. When the food and kindred products sector buys materials and services from other sectors, these sectors in turn make purchases from their respective customers, and each of these customers also makes purchases from their respective customers. These added or indirect purchases resulting from the initial direct purchases by the food and kindred product sector are called indirect effects. Within Minnesota, the total direct effects for the 35 producing sectors is $672,578 (column 1, row 36), and the total direct and indirect effects for the 35 producing sectors is $2,358,179 (column 5 total), which includes the total indirect effects of $1,685,601 ($2,358,179 - $672,578 = $1,685,601). This means that from the initial direct purchases of $672,578 by the Minnesota food and kindred product sector to produce the $1,000,000 of products, an additional output totaling $1,685,601 is produced by the 35 producing sectors in Minnesota. This additional amount is sold as inputs to other sectors whose outputs are required in making the final products by the Minnesota food and kindred products sector. Beside the purchases made within Minnesota by the food and kindred products sector, this sector also makes direct purchases of $34,125 input from the Rest-of-Nation producing sectors. For the Rest-of-Nation sectors to supply $34,125 of outputs to Minnesota's food and kindred products sector, they must produce $263,398 of gross output (column 6, row 36) of which $34,125 is sold directly to the Minnesota sector and $229,273 ($263,398 - $34,125 - $229,273) is sold as inputs to other Rest-of-Nation sectors whose outputs are required in making the final products sold to the Minnesota food and kindred product sector. Columns 7 and 8, which list the intra-regional and inter-regional direct and indirect effects of producing the $1,000,000 of the new food and kindred products in Rest-of-Nation, can be interpreted in the same manner as the Minnesota example.

When the direct and indirect requirements (columns 5 to 8 of Table 3.3) are summed the <u>total effects</u> (row 36) are estimated. The estimate of the total effects is based on the assumption that the $1,000,000 increase in demand for food and kindred products occurs in both Minnesota and Rest-of-Nation. The total impact of output expansion due to the change in demand thus can be evaluated with variations in the assumption regarding the regional location of the final demand change as follows:

(1) If the demand change occurs only in Minnesota, the economy-wide effects of outputs required to increase to meet the new demand will be a total of $2,358,179 for Minnesota and $262,398 for Rest-of-Nation, with the distribution of requirements of output from each sector as described in columns 5 and 6 respectively.

(2) If the change occurs only in Rest-of-Nation, the economy-wide effects of outputs required to increase to meet the new demand will be a total of $12,020 for Minnesota and $1,805,095 for Rest-of-Nation, with the distribution of requirements of output from each sector as described in columns 8 and 7 respectively.

(3) If the change occurs in both regions, the economy-wide effects of outputs required to increase to meet the new demand will be a total of $2,370,199 ($2,358,179 + $12,020 = $2,370,199) for Minnesota and $2,068,303 ($1,805,095 + $262,398 = $2,068,303) for Rest-of-Nation. The distribution of requirements from each sector of the regions can be obtained for Minnesota by summing up the corresponding row entries of

columns 5 and 8, and for Rest-of-Nation by summing up in a similar way of entries in columns 6 and 7.

The magnitude of over-all effects in the national economy brought about by the $1,000,000 increase in demand for food and kindred products in each region can be assessed by summing up the corresponding row entries in columns 5 and 6 for Minnesota, and columns 7 and 8 for Rest-of-Nation. The result is the total effects of $2,620,577 ($2,358,579 + $262,398 = $2,620,577) due to the $1,000,000 increase in demand for these products in Minnesota and $1,817,115 ($1,805,095 + $12,020 = $1,817,115) for the same increase in demand for Rest-of-Nation.

In the above discussion, the analysis was focused on the total effect brought about by the increase in demand for food and kindred products. In order to give the over-all view of the total effects of all the 35 producing sectors, we list these effects in Table 3.4 for comparisons among sectors and between regions. The entries of each column are expressed as the total effects per $1.00 change in demand of products of the sector listed in the left. The intra-regional total effects in Minnesota ranges from a high of $2.3045 per $1.00 increase in the livestock and its products sector (sector 1) to a low of $1.2336 per $1.00 increase in the gas utilities sector (sector 25). In Rest-of-Nation, this effect ranges from a high of $4.1226 in the finance, insurance, and real estate sector (sector 29) to a low of $1.0794 in the other utilities sector. A comparison of these effects between the two regions indicates that there are only seven sectors in Minnesota which have a greater total effects than that of Rest-of-Nation. They are: the livestock and livestock products sector (sector 1), the agriculture, forestry, and fisheries sector (sector 3), the iron and ferrous ores sector (sector 4), the food and kindred products sector (sector 8), the other utilities sector (26), the hotel, repair services sector (sector 30), and the medical and education

TABLE 3.4. Comparison of Intra- and Inter-Regional Total Effects of $1 Increase in Final Demands for Specified Industry Output, by Region -- 1970

| Industry No. | Industry Title | Minnesota[1/] (Quadrant 1) | Rest-of-Nation[2/] (Quadrant 4) | Minnesota from Rest-of-Nation[3/] (Quadrant 3) | Rest-of-Nation from Minnesota[4/] (Quadrant 2) |
|---|---|---|---|---|---|
| | | (dollars) | | | |
| 1 | Livestock and Prod. | 2.3045 | 1.9740 | .2405 | .0216 |
| 2 | Other Agric. Prod. | 1.6590 | 1.9642 | .3547 | .0040 |
| 3 | Agri., For., Fish. | 1.5475 | 1.2549 | .1748 | .0020 |
| 4 | Iron, Ferro Ores | 1.5093 | 1.1101 | .3089 | .0469 |
| 5 | Nonferro. Metal Ores | 1.3584 | 1.3525 | .4236 | .0013 |
| 6 | Other Mining | 1.3877 | 2.6962 | .2955 | .0007 |
| 7 | Construction | 1.6549 | 2.3574 | .4422 | .0009 |
| 8 | Food, Kindred Prod. | 2.3582 | 1.8051 | .2624 | .0120 |
| 9 | Lumber, Furniture | 1.6375 | 1.7949 | .4759 | .0008 |
| 10 | Pulp, Paper Prod. | 1.8561 | 2.2352 | .3736 | .0077 |
| 11 | Printing, Publishing | 1.7589 | 1.8660 | .2469 | .0017 |
| 12 | Chemicals Products | 1.7376 | 2.4760 | .6451 | .0012 |
| 13 | Petroleum Ind. | 1.4490 | 1.7961 | .9096 | .0005 |
| 14 | Stone, Clay, Glass | 1.5001 | 1.5059 | .3879 | .0011 |
| 15 | Primary Metals | 1.6064 | 3.3485 | .6227 | .0024 |
| 16 | Fabricated Metals | 1.5943 | 2.1139 | .6141 | .0015 |
| 17 | Machinery Exc. Elect. | 1.7195 | 2.0206 | .4594 | .0197 |
| 18 | Electrical Mfg. | 1.6585 | 1.8003 | .4506 | .0011 |
| 19 | Other Manufacturing | 1.6685 | 3.0172 | .6512 | .0012 |
| 20 | Railroad Trans. | 1.3782 | 1.5076 | .2849 | .0073 |
| 21 | Truck, Warehousing | 1.4123 | 1.5829 | .2234 | .0018 |
| 22 | Other Transportation | 1.6029 | 1.9433 | .2645 | .0027 |
| 23 | Communication | 1.2880 | 1.6017 | .1460 | .0003 |
| 24 | Electric Utilities | 1.5987 | 2.0272 | .4654 | .0040 |
| 25 | Gas Utilities | 1.2336 | 1.4887 | .4886 | .0030 |
| 26 | Other Utilities | 2.1033 | 1.0794 | .4126 | .0199 |
| 27 | Wholesale | 1.4319 | 2.4189 | .2024 | .0007 |
| 28 | Retail | 1.2830 | 1.4997 | .1454 | .0005 |
| 29 | F.I.R.E. | 1.3829 | 4.1226 | .2035 | .0007 |
| 30 | Hotel, Repair Serv. | 1.4949 | 1.2026 | .3294 | .0005 |
| 31 | Business Services | 1.6526 | 3.0059 | .2309 | .0008 |
| 32 | Medical, Educational | 1.3387 | 1.0944 | .2093 | .0005 |
| 33 | Other Services | 1.9476 | 2.2395 | .3592 | .0023 |
| 34 | Federal Gov. Entp. | 1.3117 | 1.2431 | .1423 | .0006 |
| 35 | State and Local Gov. | 1.5261 | 2.0135 | .4309 | .0335 |

1/ Column totals (row 36) of Table 21
2/ Column totals (row 36) of Table 24
3/ Column totals (row 36) of Table 22
4/ Column totals (row 36) of Table 23

sector (sector 32).

A comparison of the inter-regional total effects (columns 4 and 5) shows, in general, a $1.00 increase in demand for a product in Minnesota brings about a larger total effect of output expansion in Rest-of-Nation than the effects in the opposite direction (i.e., each entry in the column 3 is larger than the corresponding entry in column 4 of Table 3.4). For example, a $1.00 increase in demand for petroleum products in Minnesota would require outputs from Rest-of-Nation industrial sectors to expand by $.9096 (column 3, row 13), but for the opposite case of a $1.00 increase in demand for petroleum products in Rest-of-Nation, there is no effect at all in Minnesota.

Table 25 of computer printout presents a summary table for the purpose of comparing the resultant total effects brought about by a $1.00 increase in demand for output from each of the 35 producing sectors in each region. The total effects for Minnesota in this table is equivalent to the summation of columns 1 and 3 in Table 3.4 and that for Rest-of-Nation is equivalent to the summation of columns 2 and 4.

A brief input-output analysis under the two-region model was presented in this chapter. The intent of the discussion was not only to explain how to read the tables produced by the computer model but also to discuss the ways in which the tables can be used for analytical purposes, such as an economic impact analysis based on the direct and indirect requirements tables of the two-region input-output model. It is not our intent to present a detailed analysis of the computer output of the two-region model nor to cover the whole range of analytical tools available for the input-output analysis (e.g., the analyses based on direct, indirect and induced requirements under so-called "closed" input-output model, the output and employment multiplier analysis, or the economic impact analysis based on change in output). This publication is intended simply to help the reader in understanding of the working of the two-region input-output computer model so that it can be applied to particular studies of regional economic systems. The computerized procedure has demonstrated high applicability in building regional input-output models for impact measurement, forecasting, and policy analysis.[6/]

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6/ The computer specifications and restrictions which apply in this program are as follows:

1. Language Used: Fortran Extended 3.0
   Kronos 2.1
   CDC Cyber 74
2. Computer Installed: CDC 6000/7000/ Cyber Series
3. Memory Requirement: Central Memory 135000 words, 12 random access mass storage files, one disc, and one tape drive.
4. Subroutine Required: MXTRP -- matrix transpose subroutine.
5. Program Operating Capacity: Maximum of 114 sectors consuming and final demand (or primary input) sectors.

# PART IV

## TWO-REGION INPUT-OUTPUT COMPUTER PROGRAM

This chapter describes the computational aspects of the two-region input-output model. It is intended to be used in implementation of the computer program we developed.

### 4.1 Input Requirements

There are two basic data sets, national data set and regional data set which are required in order to use this program.

#### 4.1.1 National Direct Requirements and Final Demand

Before using the two-region input-output computer model, it is necessary to investigate the availability of input-output transactions table at large area or at national level. Our objective in using the two-region input-output computer model is to disaggregate the selected input-output table of the national level to the four quadrants of intra- and inter-regional input-output tables, which include the region of interest. There are several factors that need to be considered before accepting the one to be used as a basis for disaggregation: (1) The similarity of the two region input structures. There is no certain way in evaluating the input-mix of manufacturing a product in different regions, due to different technologies involved in making a similar product by different manufacturers. This is further complicated by the factor of aggregating the diversity of products into a sectoral product as often used in the input-output analysis. The aggregation bias, which presents many problems in usual commodity analysis in economics, has been compounded by the geographical aggregation of inputs used by the industries in the input-output analysis of a regional economy. To minimize the aggregation bias -- in this case the disaggregation bias, we suggest that, if there exists an input-output table at the level of aggregation which covers the region of interest, it is better to use such a table rather than a table derived from this model.

The second factor to be considered in the selection of a national table is the detail of industrial classification covered by the existing input-output tables. If the focus of the study is on a few particular sectors, it is desirable to choose the one which has information regarding these particular sectors.

The third factor is the year in which the existing input-output study was conducted. Since the industrial structure changes over the years, it is desirable to select one which is not quite out-of-date.

In the selection of the national input-output table, these factors should be carefully weighed before accepting it as a basis for disaggregation.

At the national level, input-output tables for the U.S. economy have been published by the Office of Business Economics, U.S. Department of Commerce for the year 1947, 1958, 1963, 1967 and 1970 (25,56,57). Computer tapes are available for distribution at the three levels of industrial classification -- 87,370, and 478 industrial level for the data since 1963.

In recent years, numerous input-output tables at the state level have been published.[7/] The most extensive study of input-output tables for all the states in the nation was made by the Harvard Economic Research Project (HERP) under the sponsorship of the Office of Economic Research, Economic Administration, U.S. Department of Commerce (34). This project developed the multi-regional input-output model (MIRO) to produce the input-output table

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7/ The states include Arizona, Georgia, Iowa, Kansas, Minnesota, Mississippi, Nebraska, New Mexico, North Carolina, Oklahoma, Texas, Washington, and West Virginia, etc. Other recent studies involving several states or a part of a state as region include parts of California, Pennsylvania, Oklahoma and Texas, and the lower Colorado Region.

for fifty states for the year 1963. The model uses the OBE 1963 U.S. input-output tables to provide estimates of state gross outputs and estimates of inter-state commodity (trade) flows and, by using iterative methods, to arrive at the 78 industrial sectoral input-output table for each state in the nation.

The above mentioned input-output tables, especially the one published by the OBE, are worthwhile to be examined for possible adaptation as the national table for the two-region input-output model.

In addition to the need of the national direct requirement coefficients table, this computer program requires the personal income estimates for each producing sectors, and the estimates of final demand class totals at the delineated national level.

### 4.1.2 Regional Gross Outputs and Final Demand

The data required from the region of interest is the gross output estimates for producing sectors and the final demand (class) totals. The sector definition and classification at the regional level should conform to those at the national level. There are several sources of information that can be utilized to compile the regional gross outputs. We list according to the major industry areas for the sources where the user might be able to look for pertinent data.

- Agriculture, Forestry and Fisheries.
  - Census of Business -- Retail Trade, U.S. Bureau of Census
  - Census of Agriculture, U.S. Bureau of Census
  - Livestock Slaughter, Annual Summary, USDA
  - Poultry Slaughter, USDA
  - Poultry and Egg Situation, USDA
  - County Business Patterns, U.S. Bureau of Census
  - Agricultural Prices Annual Summary, USDA
  - Farm Income Situation, USDA

  In Minnesota:
  - Minnesota Agricultural Statistics, Minnesota Dept. of Agriculture
  - Minnesota Feeder Pig Industry, Minnesota Dept. of Agriculture

Minnesota Grain-Fed Cattle Marketing by County, Minnesota Dept. of Agriculture
South and West Central Farm Management Report, University of Minnesota
Fact Sheet, Poultry, Minnesota Agricultural Extension Station, University of Minnesota

- Mining and Construction.
  Census of Construction, U.S. Bureau of Census
  Census of Mineral Industries, U.S. Bureau of Census
  Survey of Current Business, U.S. Bureau of Census
  Mineral Yearbook, Bureau of Mines
  Census of Governments, U.S. Bureau of Census
  Construction Review, U.S. Department of Commerce
  Dodge Construction Contract Statistics, F.W. Dodge Company, McGraw-Hill

  In Minnesota:
  U.S. Department of Commerce data tape on the value of Building Permits issued by various local governments for private construction
  State Governments' Nonpublished data sources

- Manufacturing.
  Census of Manufacturers, U.S. Bureau of Census
  Country Business Patterns, U.S. Bureau of Census
  Statistical Yearbook of the Electric Utility Industry, the Edison Electrical Institutes
  Census of Governments -- Local Governments, U.S. Bureau of Census

  In Minnesota:
  Minnesota Department of Employment Service employee estimates by different SIC industries classification
  Production per workers estimate from Federal Reserve Bulletin
  Minnesota Public Commission Reports

- Transportation, Communication, and Utilities
  Statistics of Communication Carriers, Federal Communication Commission
  AM-FM Broadcast Financial Data, Federal Communication Commission
  TV Broadcasting Financial Data, Federal Communication Commission
  Statistical Yearbook of the Electric Utility Industry, The Edison Electrical Institutes
  County Business Pattern, U.S. Bureau of Census
  Census of Transportation, Commodity Transportation Survey, U.S. Bureau of Census

  In Minnesota:
  County Business Pattern tapes supplied by U.S. Bureau of Census

- Trade, Finance and Services.
  County Business Pattern, U.S. Bureau of Census
  Census of Business -- Wholesale and Retail Trade, U.S. Bureau of Census

<u>Census of Business - Selected Services</u>, U.S. Bureau of Census
<u>Statistics of Income, Business Income Tax Return</u>, Internal Revenue Service
<u>Census of Government</u>, U.S. Bureau of Census

In Minnesota:
<u>County Business Pattern</u> tapes supplied by U.S. Bureau of Census
Publications by Minnesota Department of Employment
<u>Comparative Statements of Cash Receipts and Class Disbursements by Classification</u>, Minnesota State Finance Department
<u>Federal Outlay in Minnesota, 1972</u>, Office of Economic Opportunity

## 4.2 Input Data Set-Up

### 4.2.1 Order of Input Data

There are 8 input data sets which need to be arranged in the order as specified below. These data sets are the inputs to the two-region input-output computer main program. Some pre-processing manipulation of the original national transaction data and regional data sets are required. The explanation of pre-processing computation is included in the next section of glossary of parameters and matrices. The reader is reminded to read that section for a cross reference of the name and content of matrices and parameters used in this section.

Input 1 Parameters N. K, M, AEQL, IYEAR, NAME, are read according to a (3I5, I1, 2X, I4, A10) format. This is an integer format right justified.

Input 2 Industrial sector codes and names according to a (5A10) format. Punch sector code first and then the sector name in each card. The order of the cards are the N producing sector codes and names followed by a blank card; then the M-1 primary input sectors followed by a blank card, and finally the K-1 final demand classes.

Input 3 Matrix "A" is read (of dimension (N+M) x (N+K)), one element per card-read according to a (2I3, E16.8, 54X, I4) format.

In the first two integer fields of 3 column each, the row and column of each element are specified (right justified); in the next 16 columns the data is specified in E format. The final 4 columns of each card are either blank or contain a "1111". Only the last card read of the "A" matrix contains a "1111".

Input 4 Vector "UBAR" is read (of dimension N) according to an (I6, E16.8, 54X, I4) format. In the first field of 6 columns, the subscript of each element is specified - right justified. In the next 16 columns, the data is specified in E format. Columns 77 - 80 are left blank except the last card of the "UBAR" vector where "1111" is punched.

Input 5 Elements of vector "YNUM" (dimensioned (K-1)) are read according to an (I6, E16.8, 54X, I4) format. The same instructions apply here as for "Input 3".

Input 6 Elements of the vector "LVEC" (dimensioned N) are read according to same format and conditions as above.

Input 7 Elements of the vector "RVEC" (dimensioned N + K) are read according to same format and conditions as above.

Input 8 Matrix "ASUPA" is read according to same format as "Input 2" above. The entire matrix, a portion of it, or none of it may be read depending on the option AEQL in "Input 1"[8/].

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8/ Any elements of a vector or matrix not specifically read will be set to equal zero.

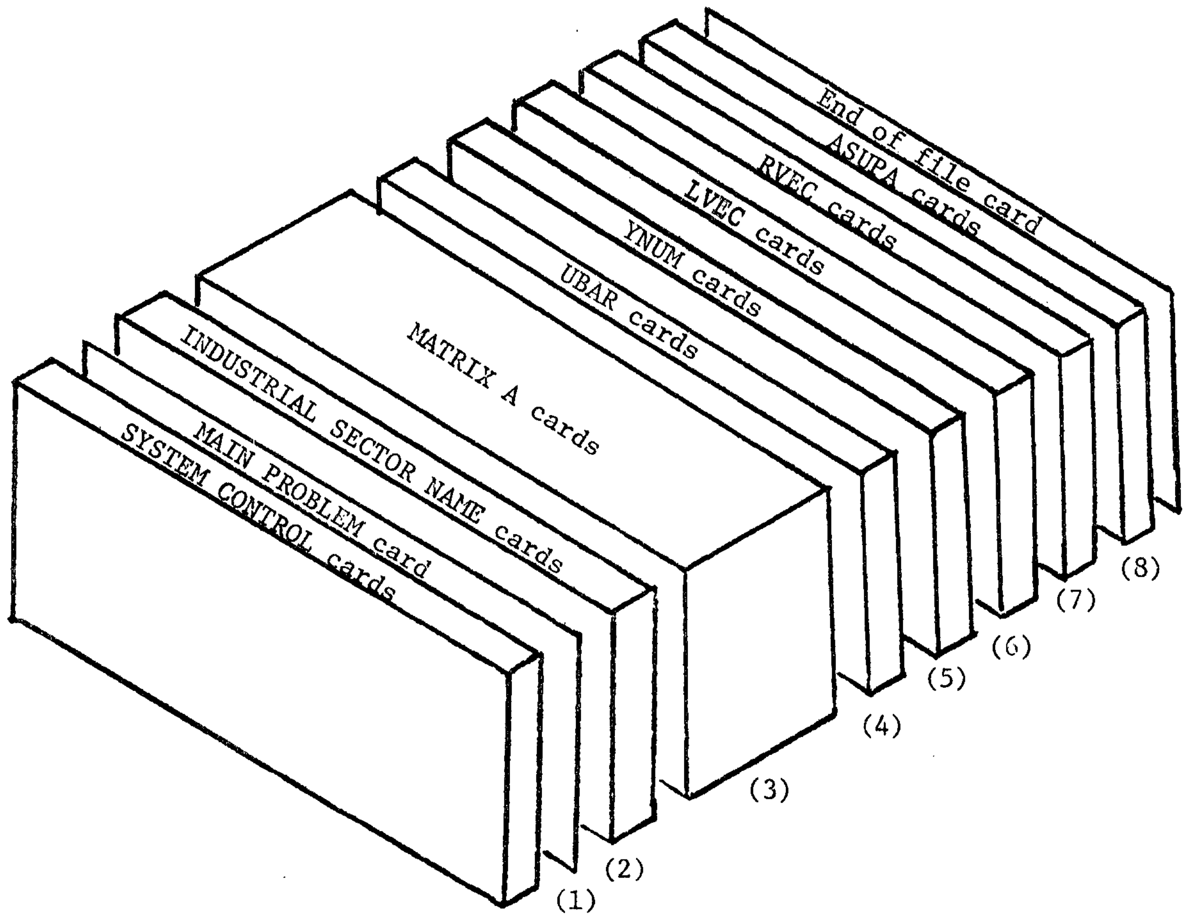



FIGURE 4.1-Typical TRIO Job Setup Using Card Decks

System control cards for using card inputs under Kronos 2.0 Cyber 74 CDC 6000/7000 are:

a. NAME, Time, Control memory card

b. Account card

c. GET, TWOMOD.card (TWOMOD is the binary compiled TRIO program)

d. LOAD, TWOMOD. card

e. EXECUTE, TWOMOD. card

f. End of Record card

### 4.2.2 Glossary of Parameters, Vectors, and Matrices

The glossary of parameters, vectors, and matrices used in the previous section is listed below:

N The number of producing sector.

K The number of final demand classes plus one.

M The number of primary input sectors plus one.

AEQL Reading option switch for "ASUPA" matrix which will be explained later. If this switch is set to equal to "0" none zero element of "ASUPA" matrix will be read. If this switch is set equal to "1", the "ASUPA" matrix will not be read.

A "A" matrix refers to the direct requirements coefficients at the national level. These coefficients include not only the producing sectors but also K-1 final demand classes. The latter can be changed by the AEQL switch to override or modify the reading in K-1 final demands coefficients. Element "$a_{ij}$" of "A" matrix is dimensioned and numbered according to the location of the element in the column and row of "A" matrix. i is equal to or greater than 1 but smaller or equal to N + M. j is equal to or greater than 1 but smaller or equal to N + K, i.e.,

$$1 \leq i \leq N + M \qquad \text{and} \qquad 1 \leq j \leq N + K$$

RVEC A vector with elements "$r_i$" where $1 \leq i \leq N + K$. For $i \leq N$ each element $r_i = \frac{X^p_{i.}}{X_{i.}}$ for $i \leq N$. Where $X^p_{i.}$ is equated to the estimated gross output of sector i of Region p, and $X_{i.}$ is equated to the estimated gross output of sector i at the national level. For $N < i \leq N + K$ $r_i = \frac{Y^p_{.i}}{Y_{.i}}$ where $Y^p_{.i}$ is the estimated total final demand of ith class in Region p, and $Y_{.i}$ is the estimated total final demand of ith class at the national level. RVEC is used for allocating the total dollar inputs at the national level to two regions p and q.

LVEC A vector with elements "$l_i$" where $1 \leq i \leq N$. Each element $l_i = \frac{X_{N + 2i}}{X_{N + 2}}$ denotes the ratio of personal income payment by each industry ($X_{N + 2i}$) to the total personal income ($X_{N + 2}$) at the national level. LVEC is used in adjusting and allocating the total personal income at the national level back to each sector i at the regional level.

YNUM A vector with elements "$y_i$" where $N + 2 \leq i \leq N + K$. Each element $y_i$ denotes the final demand class total at the national level.

UBAR A vector with elements "$u_j$" where $1 \leq j \leq N$. Each element of UBAR is the summation of the row elements in a column sector of producing sector ($1 \leq i \leq N$) and the element pertaining to the import sector ($i = N + 3$), i.e.,

$$u_j = \sum_{i=1}^{N} A_{ij} + A_{N+3j} \quad \text{for } 1 \leq j \leq N.$$

ASUPA A matrix with elements "$a_{ij}$", where $1 \leq i \leq N + M$ and $N + 2 \leq j \leq N + K$. ASUPA is the similar matrix as "A" but only includes the final demand classes. This matrix can be used to modify the "A" matrix's final demand classes.

IYEAR Year designation for outputting.

Name The name of state or region of interest.

## 4.3. Computational Procedures

The purpose of this and the following sections are to explain the steps involved in computation of the two-region computer model. The variables are described as follows:

| Variable | Description |
| --- | --- |
| ZSOL | A solution vector with elements $Z_j$ where $1 \leq j \leq N$. ZSOL may be considered either a row or column vector but its elements are subscripted j for consistency with ZVECA and ZVECB below. Each element, $Z_j$, represents the total transactions (purchases or output) of one of the N sectors in the producing sector at the national level. |
| ZVECA | A row vector with elements $ZVECA_j$ where $1 \leq j \leq N$. Each element, $ZVECA_j$, represents the total transactions (purchases or output) of one of the N sectors in the producing sector of Region p only. |

ZVECB A row vector with elements $ZVECB_j$ where $1 \leq j \leq N$. Each element, $ZVECB_j$, represents the total transactions (purchases or output) of one of the N sectors in the producing sector of Region q only.

YVECA A row vector with elements $YVECA_j$ where $N + 2 \leq j \leq N + K$. Each element, $YVECA_j$, gives the total dollar purchases of one of the K-1 classes in the final demand of Region p only. (See YNUM above.)

YVECB A row vector with elements $YVECB_j$ where $N \leq 2 \leq j \leq N + K$. Each element, $YVECB_j$, gives the total dollar purchases of one of the K-1 classes in the final demand of Region q only. (See YNUM above.)

CVECA A column vector with elements $CVECA_i$ where $1 \leq i \leq N + M$. Each element, $CVECA_i$, gives the amount of the output of one of the N producing sectors or M-1 primary input sectors at the national level consumed by all of the N purchasing sectors and K-1 final demand classes in Region p only.

CVECB A column vector with elements $CVECB_i$ where $1 \leq i \leq N + M$. Each element, $CVECB_i$, gives the amount of the output of one of the N producing sectors or M-1 primary input sectors at the national level consumed by all of the N purchasing sectors and K-1 final demand classes in Region q only.

TVECA A column vector with elements $TVECA_i$ where $1 \leq i \leq N + M$ but i not equal to N + 1 and N + 3 to N + M - 1. Each element, $TVEC_i$, shows the surplus or deficit of productions of a sector's output compared to consumption of that sector's output in Region p. A surplus indicates an export from Region p to q while a deficit indicates an import into p from q.

TVECB A column vector with elements $TVECB_i$ where $1 \leq i \leq N + M$ but i not equal to N + 1 and N + 3 to N + M - 1. Each element, $TVECB_i$, shows the surplus or deficit of production of a sector's output compared to consumption of that sector's output in Region q. A surplus indicates an export from Region q to p while a deficit indicates an import into q from p.

TBARA A column vector with elements TBARA where $1 \leq i \leq N + M$ but i not equal to N + 1 and N + 3 to N + M - 1. Each element, $TBARA_i$, shows the ratio of export or import to the domestic consumption of the output of a sector (i) in Region p.

TBARB A column vector with elements $TBARB_i$ where $1 \leq i \leq N + M$ but i not equal to N + 1 and N + 3 to N + M - 1. Each element, $TBARB_i$, shows the ratio of export or import to the domestic consumption of the output of a sector (i) in Region q.

X A matrix with elements $X_{ij}$ where $1 \leq i \leq 2(N + M)$ and $1 \leq j \leq 2(N + K)$. X is a two-region inter-regional transactions table; its data are in dollars. X is made up of four matrices of dimensions N + M and N + K arranged as quadrants thus:

$$\begin{bmatrix} X^1 & X^2 \\ X^3 & X^4 \end{bmatrix} = [X]$$

$X^1$ with elements $X^1_{ij}$ shows transactions between sub-sectors in Region p with others in Region p; $X^2$ shows p with q transactions; $X^3$ shows q with p transactions; and $X^4$ shows q with q transactions.

K A matrix with elements $K_{ij}$ where $1 \leq i \leq 2(N + M)$ and $1 \leq j \leq 2(N + K)$ but $j \neq N + L, \ldots, 2N + K + 1$. K matrix is a two-region interregional technical coefficient table showing the direct requirements per dollar of output of sectors in Regions p and q from producing and primary input sectors in Region q. K is made up of four matrices dimensioned N + M bu N and arranged as quadrants:

$$\begin{bmatrix} K^1 & K^2 \\ K^3 & K^4 \end{bmatrix} = \begin{bmatrix} K \end{bmatrix}$$

$K^1$ with elements $K^1_{ij}$ shows direct requirements of industries in Region p from producing and primary input sectors in Region p, etc., for $K^2$, $K^3$, and $K^4$.

L A matrix with elements $L_{ij}$ where $1 \leq i \leq 2(N + M)$ and $N + 2 \leq j \leq 2(N + K)$ but $j \neq N + K + 1, \ldots, 2N + K + 1$. L is analagous to a two-region interregional technical coefficient table for elements of the final demand in Region p and Region q; it shows the distribution of a dollar of expenditure to final demand. L is made up of four matrices dimensioned N + M by K - 1 and arranged as quadrants:

$$\begin{bmatrix} L^1 & L^2 \\ L^3 & L^4 \end{bmatrix} = \begin{bmatrix} L \end{bmatrix}$$

$L^1$ with elements $L^1_{ij}$ shows the portion of one dollar expenditure by final demand in Region p that was spent in Region p, etc., for $L^2$, $L^3$, and $L^4$.

F A matrix with elements $F_{ij}$ where $1 \leq i \leq 2N + M$ but $i \neq N + 1, \ldots, N + M$ and where $1 \leq j \leq 2N + K$ but $j \neq N + 1, \ldots, N + K$. F represents the identity matrix minus the corresponding technical coefficient matrix represented by K above. Matrix F may be viewed as being made up of four N by N matrices arranged in quadrants thus:

$$\begin{bmatrix} F^1 & F^2 \\ F^3 & F^4 \end{bmatrix} = \begin{bmatrix} F \end{bmatrix}$$

H — A matrix with elements $H_{ij}$ where $1 \leq i \leq 2N + M$ but $i \neq N + L, \ldots, N + M$ and where $1 \leq j \leq 2N + K$ but $j \neq N + L, \ldots, N + K$. H is the inverse of F above and represents an interdependence coefficient table. H is divided into four matrices of dimension N by N arranged in quadrants thus:

$$\begin{bmatrix} H \end{bmatrix} = \begin{bmatrix} H^1 & H^2 \\ H^3 & H^4 \end{bmatrix}$$

FT — A row vector with elements $FT_j$ where $1 \leq j \leq 2N + K$ but $j \neq N + 1, \ldots, N + K$. Each element, $FT_j$, is the sum of a column in matrix F above.

OBAR — A column vector with elements $OBAR_i$ where $1 \leq i \leq 2N + M$ but $i \neq N + 1, \ldots, N + M$. OBAR is of dimension 2N. Each element, $OBAR_i$, represents the total dollar purchases of the output of a sector (i) in Region p for $1 \leq i \leq N$ and of the output of a sector in Region q for $N + M + 1 \leq i \leq 2N + M$.

FI — A matrix with elements $FI_{hj}$ where $N + 2 \leq h \leq 2(N + M)$ but $h \neq N + M + 1, \ldots, 2N + M + 1$ and where $1 \leq j \leq 2N + K$ but $j \neq N + 1, \ldots, N + K$. FI gives the interdependence coefficients of the producing sectors in Region p and q, from the primary input sectors in Regions p and q respectively.

$W^a$ — A matrix with elements $W^a_{ij}$ where $1 \leq i \leq 2N + M$ but $i \neq N + 1, \ldots, N + M$ and where $N + 2 \leq j \leq N + K$. Each element, $W^a_{ij}$ shows the percent of sector i's output accounted for directly by the increase in one of the K - 1 Final Demand sectors in Region p.

$W^{ac}$ — A matrix with elements $W^{ac}_{ij}$ where $1 \leq i \leq 2N + M$ but $i \neq N + 1, \ldots, N + M$ and where $N + 2 \leq j \leq N + K$. Each element, $W^{ac}_{ij}$, shows the percent of sector i's output accounted for indirectly by the increase in one of the K - 1 final demand sectors in Region p.

$W^{aa}$ A matrix with elements $W^{aa}_{ij}$ where $1 \leq i \leq 2N + M$ but $i \neq N + L, \ldots, N + M$ and where $N + 2 \leq j + N + K$. Each element, $W^{aa}_{ij}$, shows the percent of sector i's output accounted for directly and indirectly by the increase in one of the K - 1 final demand sectors in Region p.

$W^{b}$ A matrix with elements $W^{b}_{ij}$ where $1 \leq i \leq 2N + M$ but $i \neq N + L, \ldots, N + M$ and where $2N + K \leq 2 \leq j \leq 2(N + K)$. Each element, $W^{b}_{ij}$, shows the percent of sector i's total output accounted for directly by the increase in one of the K - 1 final demand sectors in Region q.

$W^{bc}$ A matrix with elements $W^{bc}_{ij}$ where $1 \leq i \leq 2N + M$ but $i \neq N + 1, \ldots, N + M$ and where $2N + K + 2 \leq j \leq 2(N + K)$. Each element $W^{bc}_{ij}$, shows the percent of sector i's output accounted for indirectly by the increase in one of the K-1 final demand sectors in Region q.

$W^{bb}$ A matrix with elements $W^{bb}_{ij}$ where $1 \leq i \leq 2N + M$ but $i \neq N + 1, \ldots, N + M$ and where $2N + K \leq 2 \leq j \leq 2(N + K)$. Each element, $W^{bb}_{ij}$, shows the percent of sector i's output accounted for directly and indirectly by the increase in one of the k-1 final demand sectors in Region q.

## 4.4. Macro-Flow Diagram of Computer Program

A gross view of computational flows of the two-region model using input data stored in tape unit 3 is depicted on the following diagrams:

FIGURE 4.2 Input Preparation Program for TRIO Based on U.S. Interindustry Transaction Table

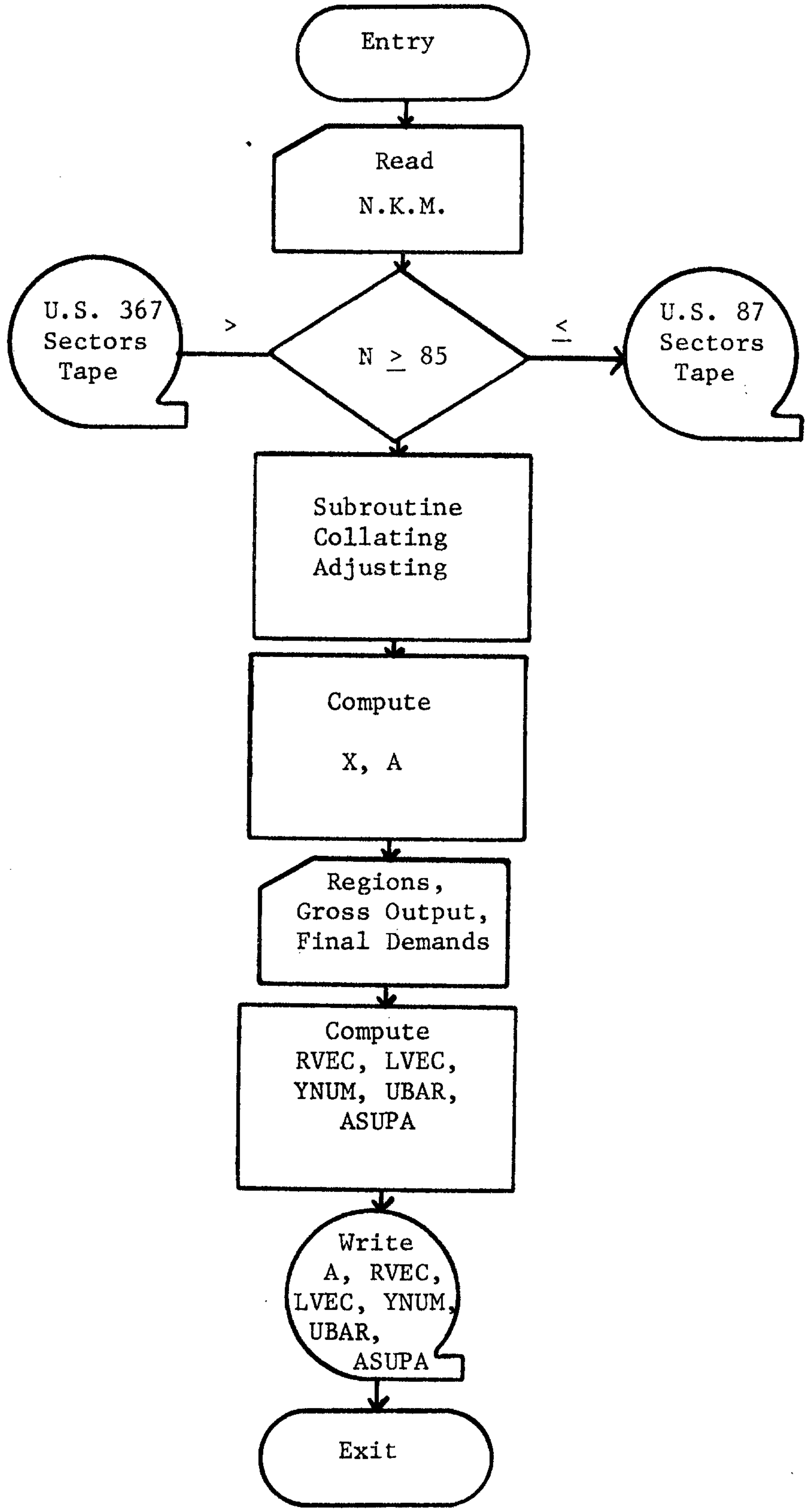


FIGURE 4.3 Two Region Input-Output Main Program

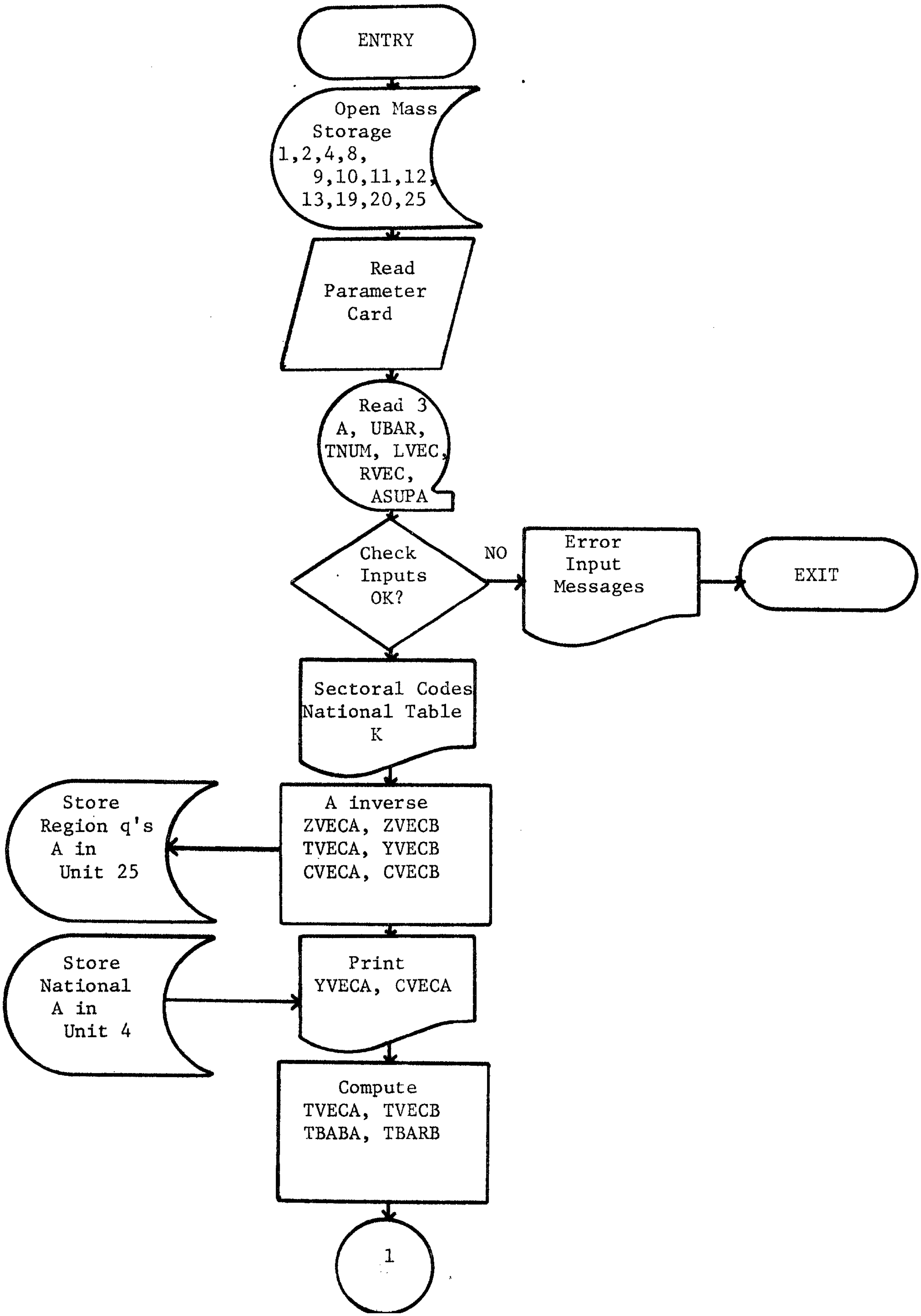


Figure 4.3 Two-Region Input-Output Main Program (Continued)

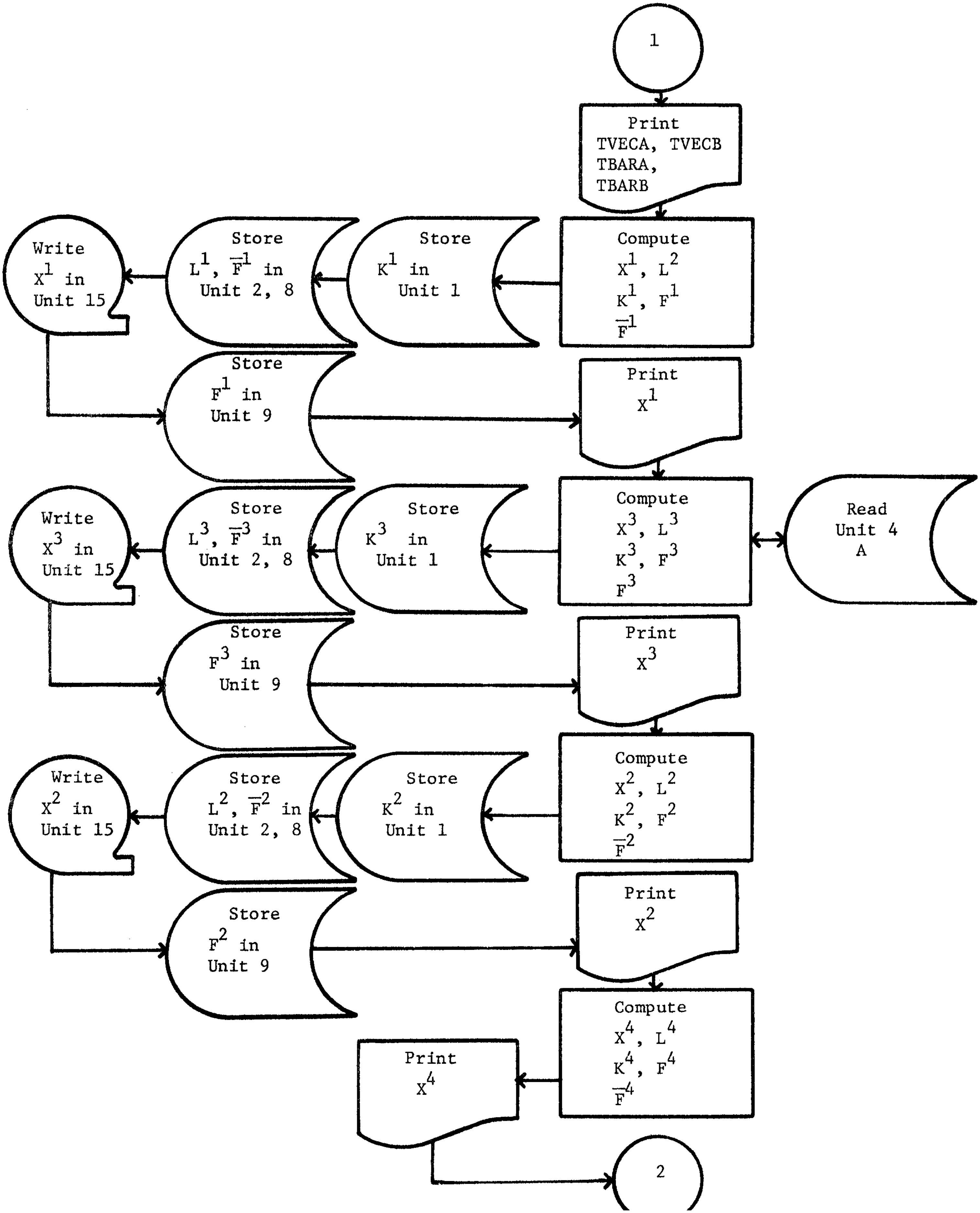


FIGURE 4.3 Two-Region Input-Output Main Program (Continued)

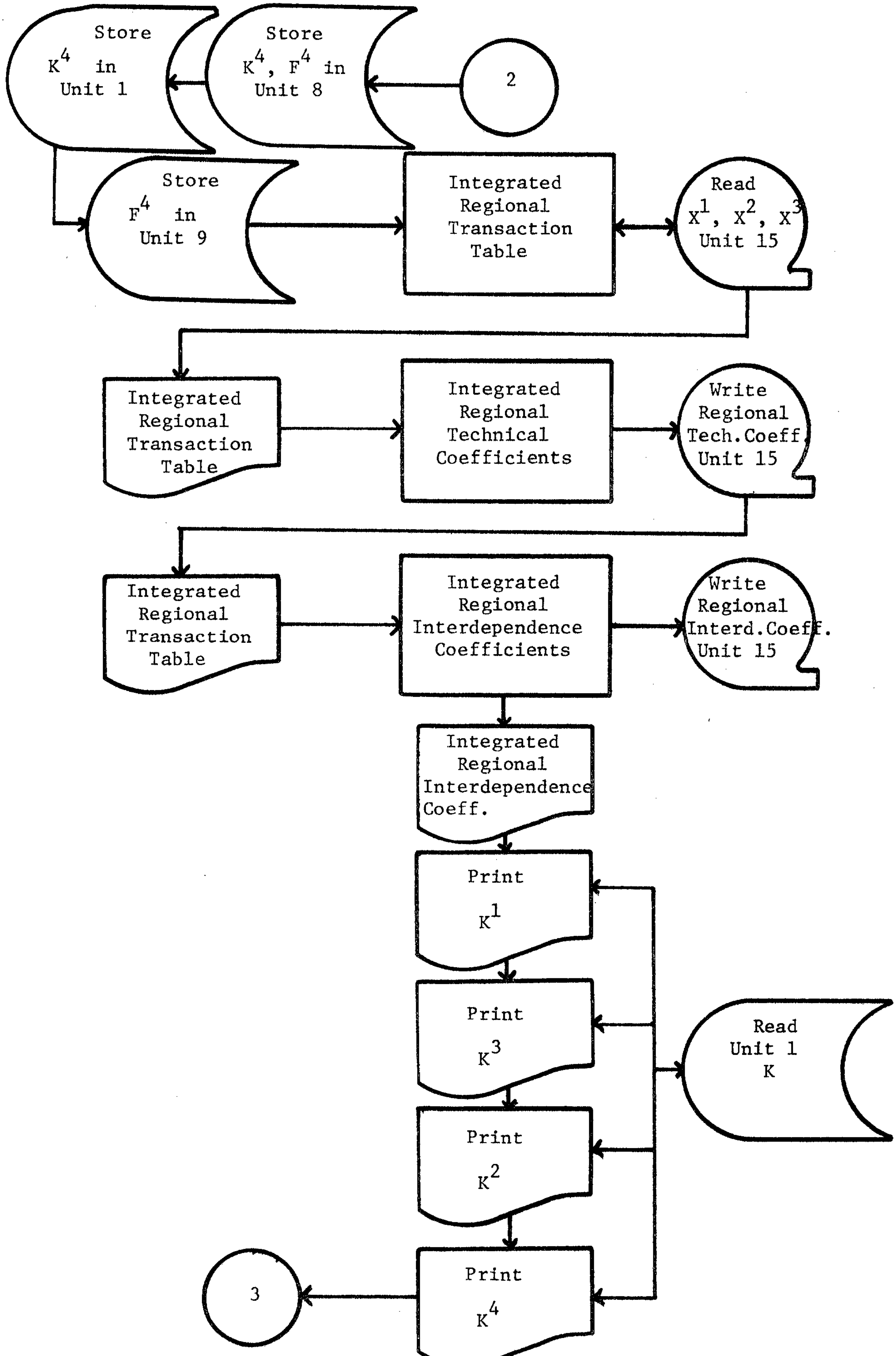


FIGURE 4.3 Two-Region Input-Output Main Program (Continued)

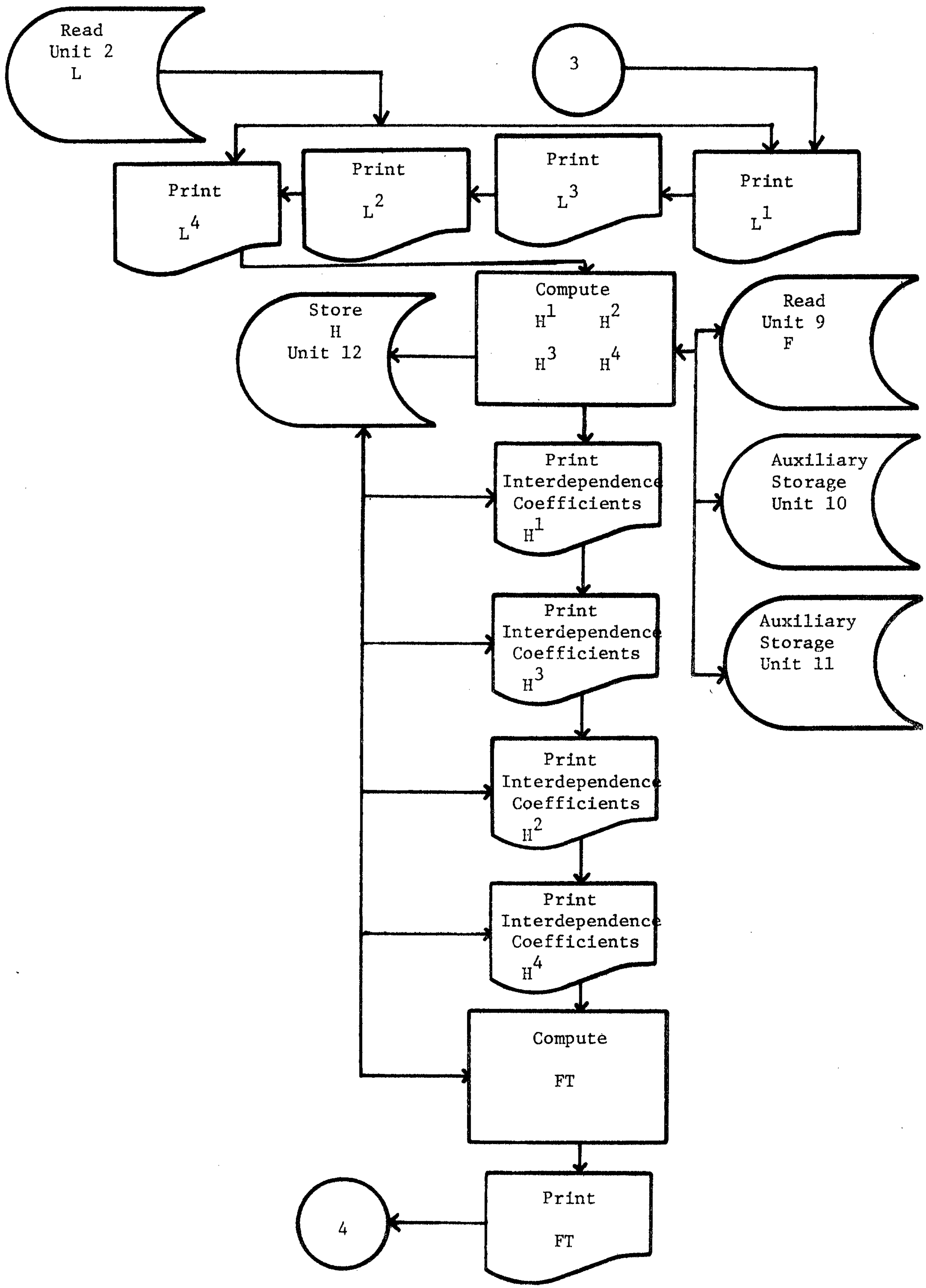
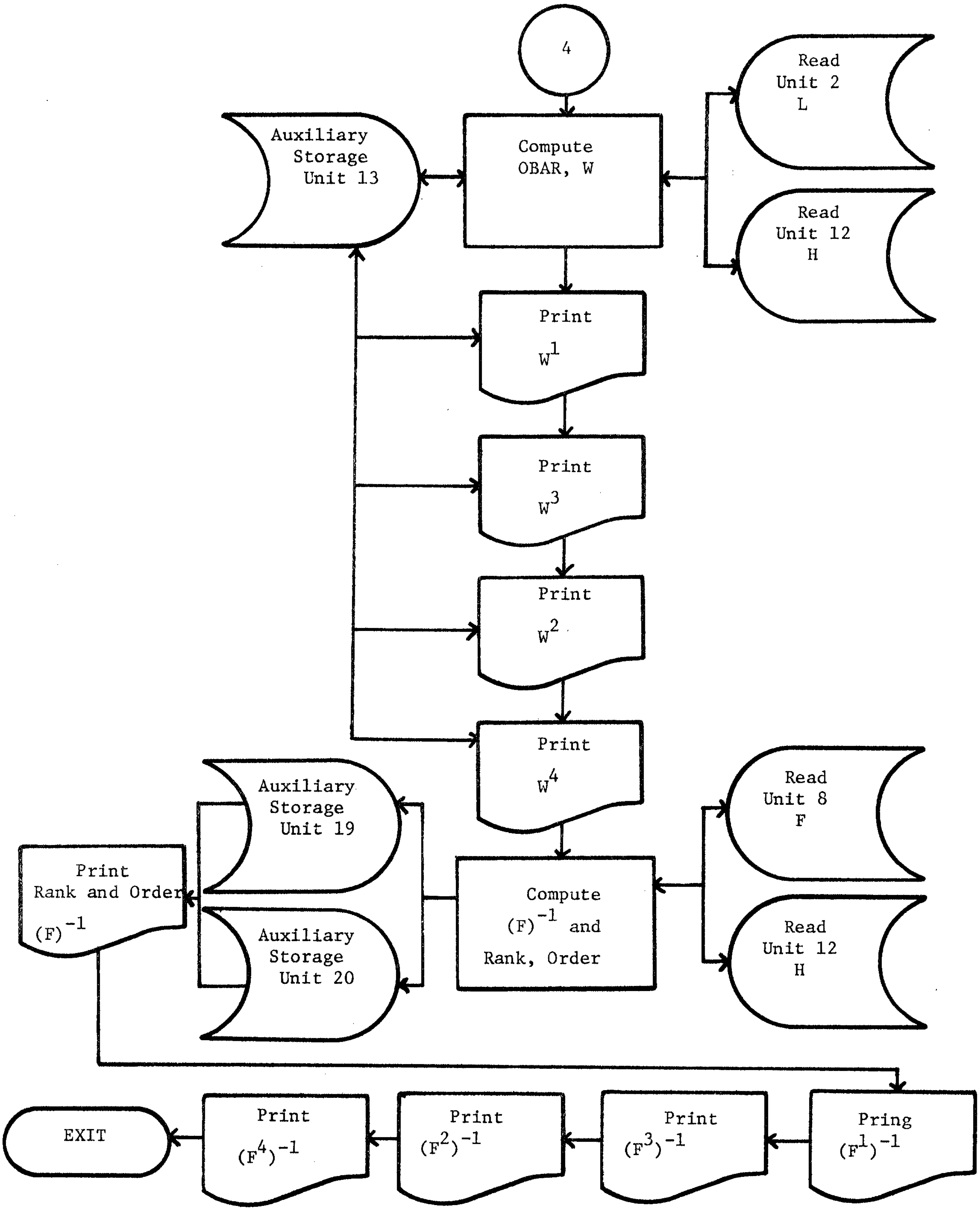


FIGURE 4.3 Two-Region Input-Output Main Program (Continued)



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APPENDIX: SUMMARY LIST OF TWO-REGION INPUT-OUTPUT TABLES FOR 35-, 55-, 87-, 95-, and 109- INDUSTRY MODELS

Table