Staff Papers Series

Staff Paper P90-63 October 1990

MULTIDIMENSIONAL OUTPUT INDICES

Barbara J. Craig
Philip G. Pardey

Department of Agricultural and Applied Economics

University of Minnesota
Institute of Agriculture, Forestry and Home Economics
St. Paul, Minnesota 55108

MULTIDIMENSIONAL OUTPUT INDICES

Barbara J. Craig
Philip G. Pardey*

*Barbara Craig is an Assistant Professor, Department of Economics, Oberlin College. Philip Pardey is jointly an Assistant Professor, Department of Agricultural and Applied Economics, University of Minnesota and Senior Research Officer, International Service for National Agricultural Research, The Hague.

ABSTRACT

This article describes alternative output aggregates that provide both cross-sectional and temporal comparisons appropriate for the analysis of panel data sets. Several of these multidimensional output indices are constructed using detailed data on agricultural production to illustrate the effects of fixing price weights over time, employing sample average price weights, and choosing between alternative approximations of Divisia indices.

Keywords: index numbers, Divisia indices, multilateral indices,
panel data, international comparisons, output indices

JEL classification numbers: 211, 123, 022

Multidimensional Output Indices

Barbara J. Craig
Philip G. Pardey

The problem of constructing meaningful output aggregates is pervasive in applied economic analysis. A substantial body of literature has addressed issues relating to aggregate measures in a time series context while the increasing accessibility of international data sets has led to the development of various multilateral indices. As national and international panel data sets become more readily available the problems of constructing output aggregates that are amenable to comparative analysis in both the temporal and cross-sectional dimensions call for increased attention. After briefly recapping some salient aggregation issues we extend the existing aggregation procedures to a panel data context. Drawing on a panel data set consisting of observations across 48 agricultural commodities for the 48 contiguous states in the U.S. over the 1949-85 period we calculate a variety of *multidimensional* output indices in order to contrast the results from competing methodologies. The substantial variability in our data contrasts with the stability characterizing both relative prices and the composition of national accounts data that have been used in the past to assess the empirical implications of alternative aggregation procedures and so allows us to assess the sensitivity of competing methods.

Time Series Indices

The conventional approach to measurement of aggregate real output has always relied on index number construction where quantities are aggregated using value weights.

The most commonly used quantity index, the Laspeyres index, uses fixed weights in which base year prices are the values used to weight output in all periods being analyzed. The Paasche index is a logical variant which employs comparison period prices as the weights.

To fix these ideas, let P_t and Q_t represent m x 1 vectors of prices and quantities of time period t. Then these familiar indices have the following formulas when the base period is period b.

$$\textit{Laspeyres} \qquad I_t^L = \frac{P_b' Q_t}{P_b' Q_b} \tag{1}$$

$$\textit{Paasche} \qquad I_t^P = \frac{P_t' Q_t}{P_t' Q_b} \tag{2}$$

Both series are easy to compute and admit of a fairly simple interpretation since both indicate changes in aggregate output attributable to changes in quantity alone.

What is not so apparent, even though it has been frequently mentioned in the theoretical index number literature [see, for example, Richter (1966), Jorgenson and Griliches (1971)], is that these index numbers do not distinguish between changes in the product mix (substitution effects) and changes in the level of production (expansion effects). In an economy or sector with multiple outputs, aggregate quantity changes may reflect movements along an unchanged transformation surface or shifts in the transformation surface. Without precise knowledge of the transformation surface, we cannot construct an index number that discriminates between the two types of changes.

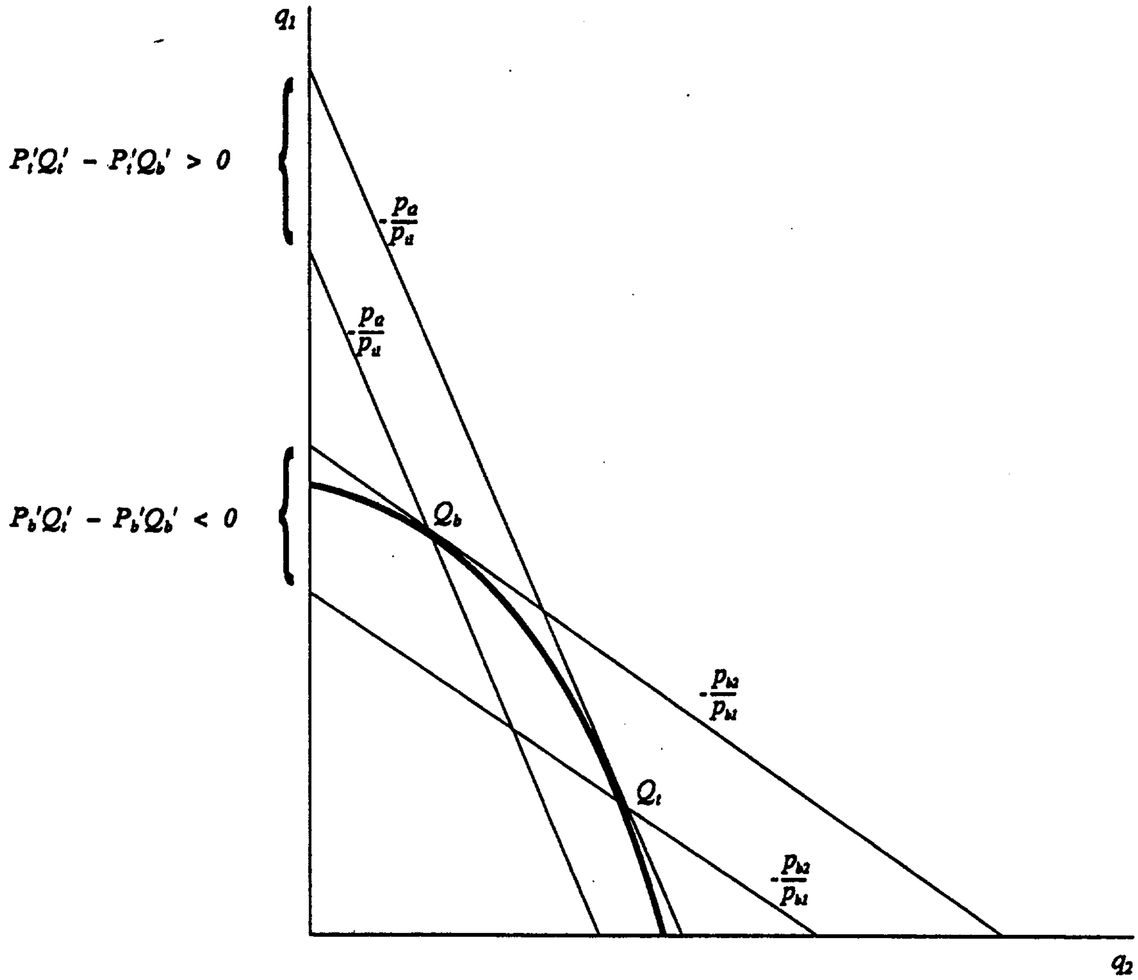
To illustrate the problem, refer to the transformation curve for an economy with two possible outputs given in figure 1. If Q_b represents the base period output and Q_t the comparison period's output, the Laspeyres index will indicate that aggregate real output -- evaluated at base period prices -- has fallen, whether the aggregate value of output is measured in units of currency or of either good. The Paasche index will tell us the opposite since the relative price change between periods would indicate that Q_t is a more highly valued output bundle than Q_b.

These two indices are in agreement as to the measured change in real aggregate output if relative prices are unchanged or relative quantities are unchanged or both. They give qualitatively and quantitatively different pictures of the same event if relative prices or quantities have changed. If relative prices do change over the period analyzed, one expects that optimizing producers in a competitive market will change the product mix. But, fixed weight indices will give us conflicting interpretations of the same behavior even if there is no change in the underlying technology governing resource use.

The methods suggested for improving such bilateral comparisons are numerous but most implicitly recognize that it matters which value weights are employed. The aggregate Fisher's ideal index, for example, combines the Laspeyres and Paasche indices giving us:

$$I_t^F = \left[\left(\frac{P_b'Q_t}{P_b'Q_b}\right)\left(\frac{P_t'Q_t}{P_t'Q_b}\right)\right]^{1/2} = \left[I_t^L\right]^{1/2} \left[I_t^P\right]^{1/2} \qquad (3)$$

Figure 1: *Paasche versus Laspeyres measures of aggregate output*



Notes: $Q_b = (q_{b1}\ ,\ q_{b2})$ *while* $Q_t = (q_{t1}\ ,\ q_{t2})$

If there are no relative price changes, all three indices are equal. If there are relative price changes, the contrary indications of the Paasche and Laspeyres index may cancel each other since the Fisher ideal index is a geometric average.

The choice of value weights is thus critical for calculation of explicit aggregate quantity indices and it is precisely the possibility of relative price variability either over time or across different entities being compared that complicates our measurement of real quantity changes. The method proposed for minimizing the errors in forming an aggregate quantity index over an extended *time* period is the use of Divisia indices.

The index proposed by Divisia (1928) and analyzed, amongst others, by Richter (1966) and Hulten (1973) is desirable because of its invariance property: if nothing real has changed (i.e., the only quantity changes involve movements along an unchanged transformation surface, along an unchanged isoquant, or along an unchanged indifference curve), the index itself is unchanged. The formula for the index is

$$I_t^D = I_b^D \exp \int_b^t \frac{P_s' \Delta Q_s}{P_s' Q_s} \, ds \tag{4}$$

where I_b^D is the index value of the base period.

If the economy is moving along an unchanged transformation surface, the changes in output weighted by current prices will be approximately zero; the index will be unchanged. If the economy's transformation surface is shifting, current value weighted changes will be different from zero leading to changes in the index value. This invariance property is, one should note, dependent upon a maintained assumption of optimizing agents.

Unfortunately, the calculation of a Divisia index requires continuous measurement of values and quantities. In any discrete approximation, some information is lost, but the advantage of using a chained index always reduces to the notion that recent quantity changes are weighted by the most recently observed values. Intuitively, these indices are attempting to evaluate current behavior in the light of current prices. In proceeding from the base period to some distant period t, each small step is chained together to minimize the measurement error possible when only base period and period t prices are used to evaluate real quantity changes.

There are, of course, many possible discrete approximations to the Divisia index. Richter (1966) proposes what others have called the Laspeyres approximation:

$$I_t^{DL} = I_{t-1}^{DL} \left[1 + \frac{P_{t-1}^{\prime}(Q_t - Q_{t-1})}{P_{t-1}^{\prime} Q_{t-1}} \right] = I_{t-1}^{DL} \frac{P_{t-1}^{\prime} Q_t}{P_{t-1}^{\prime} Q_{t-1}} \tag{5}$$

In a similar way, we could define an approximation of the Paasche variety:

$$I_t^{DP} = I_{t-1}^{DP} \left[1 + \frac{P_t^{\prime}(Q_t - Q_{t-1})}{P_{t-1}^{\prime} Q_{t-1}} \right] = I_{t-1}^{DP} \frac{P_t^{\prime} Q_t}{P_t^{\prime} Q_{t-1}} \tag{6}$$

or of the Fisher ideal variety:

$$I_t^{DF} = I_{t-1}^{DF} \left[\frac{P_{t-1}^{\prime} Q_t}{P_{t-1}^{\prime} Q_{t-1}} \right]^{1/2} \left[\frac{P_t^{\prime} Q_t}{P_t^{\prime} Q_{t-1}} \right]^{1/2} \tag{7}$$

Another approximation, called the Törnqvist (1936) or Törnqvist-Theil approximation uses both current and previous period value shares in weighting quantity changes yielding:

$$I_t^{DT} = I_{t-1}^{DT} \prod_{i=1}^{m} \left[\frac{Q_{it}}{Q_{it-1}} \right]^{\bar{w}_i} \tag{8a}$$

$$\text{where} \quad \bar{w}_i = 1/2 \left(\frac{P_{it}Q_{it}}{P_t^{\prime}Q_t} + \frac{P_{it-1}Q_{it-1}}{P_{t-1}^{\prime}Q_{t-1}} \right) \tag{8b}$$

The advantage offered by any of these approximate Divisia indices is that substantial drift in relative prices over time will be accommodated by rolling weights. In addition, theoretical work on superlative index numbers by Diewert (1976) has established that the Divisia indices are exact for specific aggregator functions. If vectors of output are appropriately aggregated with linear functions, the Laspeyres and Paasche approximations of the Divisia offer exact measures of real quantity changes. The Fisher approximation is exact for quadratic aggregator functions. The Törnqvist Divisia index is exact for a more general class of aggregator functions, namely translog aggregator functions.

In the context of constructing real income indexes for Canada, Diewert (1978) demonstrated that fixed weight indices understate rates of change and that there is little difference between alternative chained indices. If disaggregated data is difficult to obtain, we may be forced to use fixed-weight indices and live with any resulting biases.[1] However, the same amount of information is required to construct the alternative chained indices, so what basis do we have for deciding which of the Divisia approximations to use?

If we return to the aggregator functions for which the various indices are exact, we may find some guidance. If we deem the translog function to be the appropriate aggregator, we are implicitly taking every output type to be in some sense essential to our aggregate since the translog function is technically undefined when any one of the possible outputs is zero.[2] In aggregating national accounts, the categories are typically so broadly defined that this is not an issue. If, however, we have finely disaggregated information on output, corner solutions in which some commodities are not produced over part of the sample are quite likely. A linear or quadratic aggregator function which implicitly allows for partial or complete specialization is defined as long as at least one commodity is produced.

Another practical consideration is the degree to which the approximation method provides some smoothing of price weights. When aggregating commodities whose prices vary widely from period to period but whose quantity responses may lag one or more periods, there may be less economic sense to employing weighting schemes that make use of only one period's prices. The property of characteristicity emphasized by Drechsler (1973) would imply using the price weights most specific to the economic activity being measured. In this respect, the Törnqvist approximation may be more appropriate than the Laspeyres for aggregating quantities when we have reason to expect that producers are reacting to local prices but cannot do so instantaneously. The Törnqvist approximation implicitly smoothes prices by averaging current and previous value shares when each value share is calculated using contemporaneous prices and quantities.

Multilateral Indices

The usefulness of Divisia indices in a cross-section context is not so obvious. There is no logical ordering of distinct firms, states or countries in the same way that dated observations on a single economic entity may be ordered over time. And, as Kloek and Theil (1965) point out, the conditions under which Divisia indices are exact for particular aggregator functions are more likely to be violated in cross-section comparisons than in temporal ones.[3]

We can, however, construct bilateral indices for a group of n distinct subjects or agents taking a particular agent as the base with a mechanistic substitution of individual rather than time subscripts. In this context, the approximations reduce to:

$$I_j^{DL} = \frac{P_b^{\prime}Q_j}{P_b^{\prime}Q_b} \qquad j = 1, ..., n \tag{9}$$

$$I_j^{DP} = \frac{P_j^{\prime}Q_j}{P_j^{\prime}Q_b} \qquad j = 1, ..., n \tag{10}$$

$$I_j^{DF} = \left[\frac{P_b^{\prime}Q_j}{P_b^{\prime}Q_b}\right]^{1/2} \left[\frac{P_j^{\prime}Q_j}{P_j^{\prime}Q_b}\right]^{1/2} = (I_j^{DL})^{1/2}\,(I_j^{DP})^{1/2} \quad j = 1, ..., n \tag{11}$$

$$I_j^{DT} = \prod_{i=1}^{m} \left[\frac{Q_{ij}}{Q_{ib}}\right]^{\bar{w}_i} \quad where\ \bar{w}_i = 1/2\left(\frac{P_{ij}Q_{ij}}{P_j^{\prime}Q_j} + \frac{P_{ib}Q_{ib}}{P_b^{\prime}Q_b}\right) j = 1, ..., n \tag{12}$$

In these expressions, the b subscript indicates the base firm or country and j indexes the comparison firm or country. In this context, the Laspeyres, Paasche and Fisher approximation of a Divisia index are simple fixed-weight bilateral indices.

The calculation of these measures still requires one to choose economically meaningful value weights. If there is no relative price dispersion across the sample, all of these indices yield exactly the same cardinal ranking for the n agents being compared regardless of which unit is chosen as the base.

How important is cross-section relative price dispersion? In some applications, such as comparison of competitive agents operating in the same regional market, there is little reason to expect agents to face significantly different prices of inputs or outputs. When the units of analysis are countries or even states, some commodities may not enter trade across administrative units or may enter trade with varying transportation costs. In this case there is more reason to be concerned about relative price dispersion and consequently, more reason to question the particular method of aggregation.

If there is relative price dispersion, how do we justify any particular aggregation procedure? To mimic what a Divisia index does in a time series context we would want to form links between agents who have a common aggregator function. This requires that agents be near one another in a complex sense: they must face similar relative price vectors for inputs and outputs and, at the same time, operate under the same technology. To minimize the bias in forming chained indices when these conditions are not met, would involve inching out from an arbitrary base forming the smallest possible discrete links.

In a time series Divisia index, distant commodity bundles are not directly compared with base bundles; they are only indirectly compared through intermediate aggregates lying along the shortest path from the base. For the time series index, that path is naturally defined by the calendar. In a cross-section context, the appropriate path may not be obvious, and, as with any chained index, the choice of the path will influence the resulting bilateral comparisons.

In the growing literature on international comparisons, the concern over construction of comparable currency units naturally focuses the discussion on differences in average price levels, distracting analysts from the ever present issue of relative price dispersion. What has emerged as the dominant method in cross-country comparisons draws on "international" price calculations developed by Geary (1958) and Khamis (1970,1972). Instead of using base and comparison country prices, a synthetic international price vector, Π, is developed that is used to weight quantities for all countries. The resulting index is a simple Laspeyres index with a specially developed set of base weights.

$$I_j^{GK} = \frac{\Pi' Q_j}{\Pi' Q_b} \qquad j = 1,\ldots., n - 1 \tag{13}$$

The appeal of the Geary-Khamis method is the common sense approach to deriving international unit values or prices. The international price of commodity i, Π_i, is the weighted average price of the n country-specific prices, P_{ij} where country prices are converted to a common currency using implicit exchange rates and then weighted by the physical share of country j in total output of commodity i. The implicit exchange rate or purchasing power parity for country j, PPP_j, is defined as the ratio of its aggregate output weighted by international prices to its aggregate output evaluated at domestic prices. The m international prices and n purchasing power parities are calculated simultaneously by solving a system of $m + n - 1$ equations once one country's currency is chosen as numeraire. The equations to be solved are:

$$\Pi_i = \sum_{j=1}^{n} \frac{P_{ij}\, Q_{ij}}{PPP_j \sum_{k=1}^{n} Q_{ik}} \qquad i = 1, ..., m \tag{14a}$$

$$PPP_j = \frac{\Pi' Q_j}{P_j' Q_j} \qquad j = 1, ..., n-1 \tag{14b}$$

As emphasized in Kravis, Heston and Summers (1982), the international or Geary-Khamis prices give rise to quantity indices which are "base-country invariant"; the choice of numeraire has no effect on cardinal rankings of real quantity aggregates since the numeraire choice affects only the value of the *PPP* and not the international price relativities. This invariance along with the resolution of problems of non-comparable currency units are appealing features of using Geary-Khamis prices. However, a quantity index constructed using fixed weights across countries still suffers from the problem of any fixed weight index. The ranking of countries using this index will still confound substitution effects on the commodity mix with level effects to the extent that producers in different countries respond to local prices and not the fixed international ones. The primary shortcoming of these fixed international prices is the fact that they provide no guarantee that the price relativities are representative of any country, even the base country.

Multidimensional Indices

To extend the use of index numbers to make real output comparisons in a panel data context, the issues of relative price dispersion and drift continue to plague us. Constructing Divisia indices for each time series reduces aggregation errors when local relative prices are changing over the time period sampled, but these leave us with no cross-sectional scaling. Constructing cross-section indices for each time period will leave us with no time-series measurement for the individual entities.

Again, the literature on cross-country comparisons has generated some procedural suggestions to deal with the rapidly developing panel data. Khamis (1988) has suggested constructing a single set of international prices which averages prices over the whole period. This requires the calculation of implicit exchange rates for each country and time period. The solution of the resulting $m + nT - 1$ equation system may, however, be unusually cumbersome.[4] As it remains, at base, a method which yields fixed value weights, it does not resolve the essential problem of price dispersion and leaves us with a synthetic price series which again may not be characteristic of any country at any time.

Caves, Christensen, and Diewert (1982) suggest, instead, a modification of the Törnqvist Divisia index in which the T different time series observations on each of the n-1 countries are compared with one base country's output vector at a specific date. If the base country is, in fact, a hypothetical country calculated as the geometric mean of all sample observations, then this index provides nT direct bilateral comparisons with the constructed base. Indirect, multilateral comparisons are still possible because linking all units to the common base yields transitive measures of output. We would argue, however, that this multidimensional index lacks the appealing features of Divisia indices in that it no longer links observations of the same economic entity over time but rather links observations to a base where economic behavior provides weaker natural bridges.

The strongest argument for chained indices hinges on the desirability of exploiting economic behavior in order to distinguish substitution effects from expansion or contraction of output. We have a stronger notion of economies evolving over time than over space. It therefore seems a waste to sacrifice the temporal structure of the data in order to treat all observations symmetrically.

In panel data regression analysis, one often begins by treating all observations symmetrically, but there are generally efficiency gains to be had by exploiting the most likely structural relationships between observations. While the analogy is not perfect, it lends some support to the idea that we minimize the bias inherent in multidimensional aggregation by using the most direct behavioral links available. This reasoning suggests it is advisable to first construct chained temporal indices which can then be scaled in the base year to account for cross-sectional differences. We suspect that in most applications, the resulting ordinal rankings are relatively insensitive to the choice of base-year value weights provided that, for the units being compared, the basket of commodities being aggregated is not highly variable and the weights are broadly representative. It is, however, impossible to escape the problems of obtaining cardinal rankings which are insensitive to alternative cross-sectional scaling techniques.

The Data Set

The real quantity indices discussed below are constructed using a detailed data set of agricultural commodity production and prices received by farmers. The indices are aggregates over commodity baskets produced annually in each of the 48 contiguous states from 1949 to 1985. These figures were generally taken from annual volumes of the USDA's Agricultural Statistics.[5]

The 48 commodities included (table 1.1) account for approximately 90% of the national value of agricultural output and thus provide reasonable coverage. Commodities omitted include such things as ornamental trees and shrubs, flowers, seeds and buffalo. While these account for only a small fraction of national production, their exclusion will lead to a bias in estimates of real output growth in the few states in which these commodities are important. However, since they are omitted from all indices we have constructed, they should not affect the state by state comparison of these indices.

The competing price series used as aggregating weights are all derived from the same source but represent different degrees of aggregation. State-specific prices (SS) are the annual average prices received by farmers in a given state over the relevant calendar year. The national unit value prices (UV) are weighted averages of the annual state prices with weights given by state shares in the total quantity produced in the 48 state sample. Geary-Khamis (GK) prices are the alternative national prices calculated for each year of the sample by solving equation system 14. These national prices will be identical to the simple unit values in any given year if and only if one dollar can acquire an identical basket of agricultural commodities in each state in that year.

Taking the Minnesota agricultural dollar as numeraire, the implicit agricultural exchange rate series calculated for this sample indicate some significant geographical differences in purchasing power. Broadly speaking, the further a state is geographically from Minnesota, the lower is the purchasing power of the Minnesota dollar and so the higher is its state exchange rate. Over the years of the sample, differences in implicit exchange rates across states diminished, although this trend is most pronounced in the eastern states [table 1.2].

In the northeastern states, the average exchange rate varied from 1.4 in the early years of the sample to 1.2 in recent years implying significantly higher prices for a broad range of agricultural commodites in the northeast as compared with Minnesota. Exchange rate averages for the southeastern states varied from 1.2 in the early years to 1.05 in the 1980s. Central and midwestern exchange rates averaged 1.1 or less in all decades, and average exchange rates for western states ranged from 1.18 in the early years to 1.15 over the last six years. Given the variability in the implicit exchange rates in this sample, we should be able to gain some insight into the problems of constructing international unit value price series when there are potential currency conversion measurement errors in the range of 5% to 40%.

The Geary-Khamis prices, whose units are Minnesota agricultural dollars, are consistently lower than the simple unit value prices for all observations in our sample.

Table 1.1 reports the average ratio of these two national prices for four subperiods in the sample. These alternative prices for commodities such as citrus fruits and sugar cane which are produced in only a few states differ by as much as 20%. Commodities whose production is dominated by states in the upper midwest, where implicit exchange rates are very close to one, are the only ones with Geary-Khamis prices which very nearly match the simple unit values.

Empirical Results

For each state, nine real quantity indicies with base year 1980 were constructed. Three of the indices are fixed-weight Laspeyres indices each with a different set of 1980 prices: state-specific (SSFL), unit value (UVFL) and Geary-Khamis (GKFL). The entire time series of these same three types of prices were used to construct Divisia indices using two different approximations: the Laspyeres approximation (SSDL, UVDL, GKDL) and the Törnqvist-Theil approximation (SSDT, UVDT, GKDT).

Regressions of various pairings of the logged versions of these nine different indices allowed us to test for significant differences in implied growth rates when using alternative value weights and indexing procedures.[6] First, for each method we contrasted the real growth rates obtained when using state-specific prices with those obtained using national prices in order to understand the consequences of using nonrepresentative price weights in aggregation. Second, for each method we compared the real growth rates implied by indices constructed using national unit value prices and those indices constructed with Geary-Khamis national prices. Thirdly, we contrasted indices which use the same weights but different aggregation methods. Here we can examine both the differences in measured growth rates using fixed instead of Divisia indices and when using competing approximations of the Divisia index.

State-specific prices versus national prices

Pooling the indices for all 48 states, we found that indices constructed using state-specific prices as weights yield significantly different real growth rates than did indices constructed using national prices [table 2.1]. The nature of the bias did, however differ from method to method. Fixed weight indices constructed using national price weights led to higher measured growth rates than quantity indices using state-specific prices. The Divisia indices gave the opposite result.

State by state comparisons indicate that the choice of price weights almost always affects measurement of growth rates although it affects different states in different ways [table 2.2]. While it is difficult to say, a priori, what pattern will emerge, for the majority of states in our sample, the use of national prices rather than local ones gave lower estimated rates of change regardless of the aggregation method. The fact that this pattern is most pronounced for the Divisia Laspeyres index appears to be related to the price and quantity variability in this sample. Year to year variability in state level prices can be "smoothed" by aggregating to form national prices, by averaging value weights as the Törnqvist approximation does, or by simply using one (base) year's prices. The formula for the Laspyeres approximation provides no smoothing of state price shocks and so yields the most ragged quantity index. The fluctuations in this quantity index are amplified when state rather than national prices are used as value weights.

Geary-Khamis versus unit value prices

Even though the Geary-Khamis prices represent a weighted average national price, the use of this set of price weights does produce significantly different output indices than obtained using national unit value prices [tables 2.3 and 2.4]. Once again the nature of the bias differs across index types and states.

When used in a fixed weight Laspeyres or Divisia Törnqvist index, the Geary-Khamis weights yield higher rates of change in output for the sample as a whole. When used in a Divisia Laspeyres index, Geary-Khamis weights imply slightly lower rates of change.

Regardless of the method used, the majority of states have higher implied growth rates when Geary-Khamis prices are used than when unit values serve as value weights. A look at table 2.4 should, however, make clear that this is much less systematic when Divisia indices are used than when fixed weight indices are constructed. With a fixed weight index, the absolute and relative price differences between alternative national prices are fixed for the whole sample, so any bias in the weights used will be stable over the sample. With moving weights, systematic biases are less likely.

Fixed versus chained indices

Fixing the weights, using either state or national prices, leads to broadly lower measured growth rates in output. In tables 2.5 and 2.6 this pattern is displayed both in the sample taken as a whole and when analyzed state by state. These results reaffirm the findings of Diewert (1978). Fixing weights over a sample will result in an easily predictable pattern of bias, no matter how representative the base year weights happen to be, if the sample spans years with relative price variability.

Divisia approximations

One of the most surprising results of these index number comparisons is that the two Divisia approximations yield significantly different results. It matters crucially which approximation is used regardless of the price weights being employed. When all observations are pooled, our analysis indicates that the Törnqvist approximation yields indices with lower implied rates of change in output than does the Laspeyres approximation [table 2.7]. In table 2.8 we report results of state by state comparisons and find that at least 90% of the states display this same pattern.

As discussed earlier, the Divisia Laspeyres index provides the least smoothing of price and quantity changes of all the index types. Given the year-to-year variability of price and quantity data for individual agricultural commodities, it is not surprising that an index which uses last period's prices to weight current quantities will tend to amplify measured changes in the commodity basket produced. Diewert's (1978) comparisons of alternative Divisia approximations based on national accounts data did not show a marked difference, probably because the quantities being aggregated and their relative prices were far more stable than those in our sample.

Cross-section comparisons

The discussion to this point has focussed on the effect of different measurement techniques on measured growth rates of output in individual states. However, the indices discussed so far provide no basis for comparing levels of output across states. To contrast the effects of alternative cross-sectional scaling techniques, we will discuss only the Divisia Törnqvist index constructed with state-specific prices. To compare rankings of states when various scaling factors are applied, Minnesota's 1980 output level is taken as the numeraire.

The choice of a scaling factor once again involves the selection of appropriate value weights. We can evaluate the base year output of state *j* at local base year prices and divide it by the base states' output in the same year to get the first scaling factor:

$$STSF_j = \frac{P'_{jb}Q_{jb}}{P'_{bb}Q_{bb}} \tag{15}$$

Alternatively, we could evaluate these same quantities at either of the national prices, unit value or Geary-Khamis, to get two more scaling factors:

$$UVSF_j = \frac{P'_{b}Q_{jb}}{P'_{b}Q_{bb}} \tag{16}$$

$$GKSF_j = \frac{\Pi'_{b}Q_{jb}}{\Pi'_{b}Q_{bb}} \tag{17}$$

International studies of agriculture commonly measure output in wheat equivalent units to finesse the problem of currency conversion.[7] We include this possibility by constructing an additional wheat equivalent scaling factor:

$$WESF_j = \frac{R_{jb}^{\prime} Q_{jb}}{R_{bb}^{\prime} Q_{bb}} \tag{18}$$

In this formula, R_{jb} is the vector of relative prices for state j in base year b where wheat is the numeraire commodity. This scaling factor is based on local prices since these are the only prices for which the price relativities can vary across states.[8]

The ranking of states by real output is hardly affected by the choice of scaling factors [table 3.1]. The exceptions to this generalization are states whose commodity basket contains relatively few items. These same states have rankings which are quite variable over time as well since their lack of diversification typically results in more erratic real output indices.

The actual index values did differ in systematic ways according to the value weights used to perform the cross-section scaling. Because the Geary-Khamis prices denominated in Minnesota agricultural dollars are systematically lower than the unit values, almost every state outside of the corn belt is given a lower valued output index when scaled in these prices [table 3.2].

The use of either national price series in calculating scaling factors results in a lower valued output index than did the use of state-specific absolute prices or wheat relativities [tables 3.3 and 3.4]. The fact that this pattern is displayed by at least 94% of the states in the sample helps explain why the scaling factors can make a difference in the actual values of the indices and yet leave ordinal rankings in tact.

Conclusions

Economic behavior rarely provides us with only one explicit functional form for aggregation, but it can inform the choice of aggregation method. The use of sample average prices, however sophisticated the algorithm by which they are derived, can lead to significant biases in real output indices relative to aggregates derived using local prices. Economic theory would lead one to use the most representative prices available as the value weights. By the same reasoning, fixing weights over a long time series should be avoided if there is temporal variability in relative prices. Chained indices are preferred for time series, but choosing amongst them depends upon the nature of the data at hand. When both prices and quantitites are volatile, as in our data set, the Törnqvist-Theil approximation will result in a smoother index of real output than will the Laspeyres approximation since with the Törnqvist-Theil index price changes are muted through the value weighting technique.

Any multidimensional index still faces the intractable index number problem. The procedure used here of scaling the base year observation of a chained time series to provide cross-sectional comparisons has two advantages over other proposed multidimensional indices. First, the chained time series methodology which is suggested by economic behavior is not sacrificed in order to obtain multidimensional indices. This method leaves one with a reasonably clear picture of real individual growth rates. Second, the method reduces the problem of cross-sectional comparisons to the calculation of scaling factors for only the base year. While this is a nontrivial task, it reduces the computational burden of calculating simple average prices or Geary-Khamis prices. Neither this procedure nor others suggested in the literature relieves researchers of the need to be cautious when interpreting cardinal measures of real output in panel data, but what we find is that ordinal rankings are fairly robust to alternative scaling procedures.

Footnotes

* This work was supported by Interregional Hatch Project 6 (IR-6) funds made available to the Minnesota Agricultural Experiment station. The authors thank Michelle Hallaway and Kirstie Hallaway for their tireless and accurate assistance in compiling the data set.

[1]We discuss the compromises forced on analysts who must use preaggregated data in Pardey, Roseboom and Craig (1991).

[2]In calculating the Törnqvist approximation, outputs that are zero at any point in time can of course be omitted from the index entirely, or they can be introduced anytime they are produced in two successive periods. In our empirical results below, we have used the latter option since commodities generally appear and disappear from the reported statistics in years when they constitute a relatively small part of any state's annual output.

[3]In the construction of price (as opposed to quantity) indices, Divisia indices provide local approximations of constant utility price indices if real income is constant across units or periods being compared. This, as Kloek and Theil (1965) point out, is much more likely to hold in successive periods than across regions.

[4]In the context of state level agricultural production, this is really impractical since it requires the inversion of an 1822 x 1822 matrix.

[5]For several years in the early 1980s, a small set of prices and quantities had to be derived from unpublished USDA data because federal budget cuts led to reduced commodity coverage in the USDA's publications.

[6]All regressions take the form of $ln[I] = \alpha + \beta\, ln[I^*]$ where I and I^* are alternative output indices. When the pair of indices indicate virtually identical growth rates, the regression parameters α and β should be insignificantly different from 0 and 1, respectively. If the parameter β is significantly greater than 1, then we can conclude that annual rates of change in output indicated by index I are significantly greater than those for I^*. The converse is true when β is significantly less than 1. If β is not significantly different from 1, then the intercept indicates differences in the sample average growth rates. For example, if α is significantly less than zero, then the sample average growth rate implied by index I is less than that implied by index I^*.

[7]A wheat standard was used by FAO until recently and is the basis of the influential development work of Hayami and Ruttan (1971,1985). Our version differs from these in that *local* price relativities are used instead of employing the price relativities of selected countries or regions. For a more detailed discussion see Craig, Pardey and Roseboom (1991).

[8]For states which report no wheat production in 1980, the national unit value price of wheat was used. This affects seven states: Connecticut, Florida, Maine, Massachusetts, New Hampshire, Rhode Island and Vermont.

References

Caves, D. W., Christensen, L. R., and Diewert, W. E. (1982). 'Multilateral Comparisons of Output, Input and Productivity using Superlative Index Numbers.' *Economic Journal*, Vol. 92, pp. 73-86.

Craig, B. J., Pardey, P. G., and Roseboom, J. (forthcoming 1991). 'Internationally Comparable Growth, Development, and Research Measures.' In *Agricultural Research Policy: International Quantitative Perspectives* (ed. P. G. Pardey, J. Roseboom, and J. R. Anderson). Cambridge: Cambridge University Press.

Diewert, W. E. (1978). 'Superlative Index Numbers and Consistency in Aggregation.' *Econometrica*, Vol. 46, pp. 883-900.

Diewert, W. E. (1976). 'Exact and Superlative Index Numbers.' *Journal of Econometrics*, Vol. 4, pp. 115-45.

Divisia, F. (1928). Economique Rationnelle. Paris: Gaston Doin.

Drechsler, L. (1973). 'Weighting of Index Numbers in Multilateral International Comparisons.' *Review of Income and Wealth*, Series 19, pp. 17-35.

Geary, R. C. (1958). 'A Note on Comparisons of Exchange Rates and Purchasing Power between Countries.' *Journal of the Royal Statistical Society*, Vol. 121, pp. 97-9.

Hayami, Y. and Ruttan, V. W. (1985). *Agricultural Development: An International Perspective*. Baltimore: The Johns Hopkins University Press.

Hayami, Y. and Ruttan. V. W. (1971). *Agricultural Development: An International Perspective*. Baltimore: The Johns Hopkins University Press.

Hulten, C. R. (1973). 'Divisia Index Numbers.' *Econometrica*, Vol. 41, pp. 1017-25.

Jorgenson, D. W. and Griliches, Z. (1971). 'Divisia Index Numbers and Productivity Measurement.' *Review of Income and Wealth*, Vol. 17, pp. 227-9.

Khamis, S. H. (1970). 'Properties and Conditions for the Existence of a New Type of Index Numbers.' *Sankhya*, Vol. 32, pp. 81-98.

Khamis, S. H. (1972). 'A New System of Index Numbers for National and International Purposes.' *Journal of the Royal Statistial Society*, Vol. 135, pp. 96-121.

Khamis, S. H. (1988). 'Suggested Methods for Consistent Temporal-Spatial Comparisons.' In *World Comparison of Incomes, Prices and Product* (ed. J. Salazar-Carillo and D. S. Prasada Rao). Amsterdam: North-Holland.

Kravis, I. B., Heston, A. W., and Summers, R. (1982). *World Product and Income: International Comparisons of Real Gross Product*. Baltimore: The Johns Hopkins University Press.

Kloek, T. and Theil, H. (1965). 'International Comparisons of Prices and Quantities Consumed.' *Econometrica*, Vol. 33, pp. 535-56.

Pardey, P. G., Roseboom, J., and Craig, B.J. (forthcoming 1991). 'A Yardstick for International Comparisons: An Application to National Agricultural Research Expenditures.' *Economic Development and Cultural Change*.

Richter, M. K. (1966). 'Invariance Axioms and Economic Indexes.' *Econometrica*, Vol. 34, pp. 739-55.

Theil, H. (1965). 'The Information Approach to Demand Analysis.' *Econometrica*, Vol. 33, pp. 67-87.

Törnqvist, L. (1936). 'The Bank of Finland's Consumption Price Index.' *Bank of Finland Monthly Bulletin*, Vol. XVI, pp. 27-34.

Table 1.1 Ratio of Geary-Khamis Prices to Unit Value [v/q] Prices

COMMODITY	Average 1949-59	Average 1960-69	Average 1970-79	Average 1980-85
Crops				
Barley	0.91	0.90	0.93	0.96
Corn	0.91	0.92	0.94	0.96
Cotton	0.86	0.86	0.91	0.91
Flax	1.00	0.99	0.98	1.00
Hay	0.88	0.87	0.85	0.88
Oats	0.93	0.95	0.96	0.97
Peanuts	0.83	0.85	0.91	0.94
Potatoes	0.91	0.97	0.98	0.95
Rice	0.86	0.87	0.92	0.93
Rye	0.94	0.94	0.96	0.98
Sorghum	0.90	0.90	0.93	0.91
Soybeans	0.91	0.91	0.94	0.95
Sugar beets	0.89	0.90	0.93	0.97
Sugar cane	0.84	0.84	0.88	0.87
Tobacco	0.81	0.83	0.91	0.94
Wheat	0.92	0.93	0.94	0.95
Livestock products				
Broilers	0.83	0.84	0.91	0.93
Cattle	0.91	0.91	0.93	0.94
Eggs	0.86	0.86	0.90	0.93
Hogs	0.92	0.92	0.94	0.96
Honey	0.88	0.89	0.92	0.94
Milk	0.89	0.89	0.92	0.95
Sheep	0.90	0.91	0.92	0.94
Turkeys	0.88	0.89	0.93	0.96
Wool	0.90	0.90	0.92	0.93
Fruit				
Apples	0.83	0.90	0.90	0.93
Apricots	0.82	0.83	0.90	0.98
Cherries	0.87	0.89	0.92	0.96
Grapefruit	0.80	0.81	0.86	0.89
Grapes	0.81	0.84	0.90	0.98
Lemons	0.82	0.81	0.88	0.96
Oranges	0.81	0.81	0.86	0.90
Peaches	0.82	0.83	0.93	0.95
Pears	0.84	0.86	0.86	0.96
Pecans	0.86	0.86	0.91	0.92
Strawberries	0.84	0.85	0.83	0.97
Vegetables				
Beans	0.85	0.88	0.92	0.94
Carrots	0.83	0.85	0.92	0.99
Cauliflower	0.84	0.84	0.89	0.97
Celery	0.82	0.83	0.89	0.96
Cucumbers, processed	0.88	0.92	0.93	0.95
Lettuce	0.81	0.82	0.88	0.95
Onions	0.87	0.88	0.90	0.94
Peas	0.91	0.93	0.95	0.96
Sweet corn, fresh	0.80	0.81	0.86	0.90
Sweet corn, processed	0.92	0.94	0.95	0.96
Tomatoes, fresh	0.82	0.82	0.88	0.91
Tomatoes, processed	0.83	0.84	0.90	0.98

Table 1.2: *Implicit Agricultural Exchange Rates Minnesota as numeraire*

	Average 1949-59	Average 1960-69	Average 1970-79	Average 1980-85
Northeast				
Connecticut (CT)	1.64	1.72	1.53	1.38
Delaware (DE)	1.21	1.24	1.13	1.15
Maine (ME)	1.21	1.20	1.20	1.10
Maryland (MD)	1.25	1.25	1.14	1.12
Massachusetts (MA)	1.56	1.58	1.38	1.29
New Hampshire (NH)	1.47	1.43	1.28	1.21
New Jersey (NJ)	1.42	1.32	1.34	1.18
New York (NY)	1.19	1.17	1.14	1.06
Pennsylvania (PA)	1.27	1.30	1.16	1.11
Rhode Island (RI)	1.60	1.44	1.49	1.21
Vermont (VT)	1.32	1.28	1.21	1.10
Corn Belt				
Illinois (IL)	1.10	1.10	1.08	1.06
Indiana (IN)	1.10	1.09	1.07	1.04
Iowa (IA)	1.06	1.07	1.05	1.03
Missouri (MO)	1.09	1.09	1.06	1.05
Ohio (OH)	1.14	1.12	1.08	1.05
Lake States				
Michigan (MI)	1.08	1.07	1.04	0.99
Minnesota (MN)	1.00	1.00	1.00	1.00
Wisconsin (WI)	1.00	0.99	1.00	1.02
Northern Plains				
Kansas (KS)	1.07	1.07	1.05	1.07
Nebraska (NE)	1.07	1.08	1.04	1.05
North Dakota (ND)	0.98	0.99	1.03	1.01
South Dakota (SD)	1.00	1.03	1.02	1.01
Appalachian States				
Kentucky (KY)	1.16	1.15	1.08	1.06
North Carolina (NC)	1.25	1.21	1.11	1.06
Tennessee (TN)	1.16	1.14	1.06	1.02
Virginia (VA)	1.24	1.22	1.09	1.06
West Virginia (WV)	1.26	1.25	1.09	1.04
Southeast				
Alabama (AL)	1.18	1.17	1.08	1.06
Florida (FL)	1.27	1.25	1.19	1.15
Georgia (GA)	1.21	1.20	1.11	1.06
South Carolina (SC)	1.26	1.22	1.11	1.06

Table 1.2: *Implicit Agricultural Exchange Rates Minnesota as numeraire (Contd.)*

	Average 1949-59	Average 1960-69	Average 1970-79	Average 1980-85
Delta States				
Arkansas (AR)	1.13	1.15	1.09	1.07
Louisiana (LA)	1.17	1.17	1.07	1.09
Mississippi (MS)	1.15	1.16	1.09	1.12
Southern Plains				
Oklahoma (OK)	1.08	1.11	1.08	1.06
Texas (TX)	1.13	1.11	1.09	1.15
Mountain				
Arizona (AZ)	1.26	1.26	1.18	1.22
Colorado (CO)	1.15	1.17	1.13	1.10
Idaho (ID)	1.03	1.02	1.03	1.03
Montana (MT)	1.07	1.08	1.09	1.04
Nevada (NV)	1.14	1.12	1.13	1.08
New Mexico (NM)	1.20	1.21	1.19	1.12
Utah (UT)	1.12	1.11	1.09	1.05
Woming (WY)	1.11	1.12	1.12	1.08
Pacific				
California (CA)	1.22	1.21	1.12	1.01
Oregon (OR)	1.18	1.13	1.10	1.04
Washington (WA)	1.18	1.12	1.11	1.08

Note: State-specific exchange rates represent Geary-Khamis purchasing power parity indices derived by simultaneously solving equations 14a and 14b for each year 1949-1985.

Table 2.1: *Comparison of National versus State Weights Pooled 4 -state regressions*

Regression	α	β
$ln[UVFL] = \alpha + \beta\ ln[SSFL]$	.010 (.001)[a]	1.016 (.003)
$ln[GKFL] = \alpha + \beta\ ln[SSFL]$	.009 (.001)	1.020 (.003)
$ln[UVDL] = \alpha + ln[SSDL]$	.001 (.001)	.995 (.003)
$ln[GKDL] = \alpha + ln[SSDL]$	-.001 (.001)	.954 (.003)
$ln[UVDT] = \alpha + \beta\ ln[SSDT]$	-.021 (.001)	.912 (.003)
$ln[GKDT] = \alpha + \beta\ ln[SSDT]$	-.008 (.001)	.968 (.002)

[a]Figures in brackets are standard errors.

Table 2.2: *Comparison of National versus State Weights; State-specific regression*

Slope	Intercept $\alpha < 0$	$\alpha = 0$	$\alpha > 0$	Percent of States
				%
ln[UVFL] $= \alpha + \beta$ *ln[SSFL]*				
$\beta < 1$	AZ,AR,MD,NV,NM, NY,NC,PA,UT,WV	CT,DE,GA,IN,IA,KS ,MD,MT,NE,OH,TX, VT	IL,KY,LA,MA,MN,MS ,ND,SC,SD,WY	66
$\beta = 1$	CO,WI	MI,TN	FL,ID,OR,WA	17
$\beta > 1$	AL,VA	CA,NH,OK	ME,NJ,RI	17
ln[GKFL] $= \alpha + \beta$ *ln [SSFL]*				
$\beta < 1$	AZ,MD,MO,NV,PA	CT,DE,KS,KY,MN, NY,TX,UT,VT,WY	FL,IL,IN,LA,MA,MS, MT,ND,SD	50
$\beta = 1$	AR,CO,NM,TN,WI	TA,ID,IA,NE,SC,W V	OR,WA	27
$\beta > 1$	AL,NC,OK,VA	CA,MI,NH,NJ,OH	ME,RI	23
ln[UVDL] $\alpha + \beta$ *ln[SSDL]*				
$\beta < 1$	CA,CO,FL,MD,NY, NC,PA,UT,VA,WI	CT,GA,ID,IL,IA,KS, MI,MN,MO,NE,NJ, OH,OK,OR,SC,TN, TX,WA,WY	KY,LA,ME,MA,MT, NM,ND,SD	77
$\beta = 1$	AZ,DE,IN,VT	AR,WV		13
$\beta > 1$	AL,NV	MS,NH	RI	10
ln[GKDL] $\alpha + \beta$ *ln[SSDL]*				
$\beta < 1$	CA,FL,MD,NY,OK, PA,UT,VA,WI	AR,CO,CT,GA,IL,IA ,KS,KY,MI,MN,MS, MO,NE,NJ,OH,SC, TN,WY	ID,LA,ME,MA,MT, NM,ND,OR,SD,TX, WA	79
$\beta = 1$	DE,IN,NC,VT	AZ,RI		13
$\beta > 1$	AL,NV,NH		WV	8

Table 2.2: *Comparison of National versus State Weights; State-specific regresion (Contd.)*

Slope	Intercept $\alpha < 0$	$\alpha = 0$	$\alpha > 0$	Percent of States
				%
ln[UVDT] $\alpha + \beta$ *ln[SSDT]*				
$\beta > 1$	AZ,CA,CO,CT,DE, FL,MD,MO,NH,NC, WV	IL,KY,NE,NJ,NM, OH,PA,WY	GA,LA,ME,MA,ND, OR,RI	54
$\beta = 1$	NV,NY,VT,VA	IA,KS,TX	MI,MN,OH	21
$\beta > 1$	AL,IN,MS,SC,UT	AR,SD,TN,WI	ID,MT,WA	25
ln[GKDT] $\alpha + \beta$ *ln[SSDT]*				
$\beta < 1$	MD,MO,NV,OK,PA, WI	IL,IA,KS,MN,NE,NJ ,WY	ME,MA,MI,MT,ND,RI ,SD	42
$\beta = 1$	CA,NH,NM,TN,VT, VA	GA,ID,KY,LA,WV	OR	25
$\beta > 1$	AL,AZ,CO,CT,DE, FL,MS,NY,NC,SC, UT	AR,IN,TX	OH,WA	33

Table 2.3: *Comparison of Geary-Khamis versus Value Weights Pooled 4 -state regressions*

Regression	α	β
$ln[GKFL] = \alpha + \beta\ ln[UVFL]$	-.002 (.0001)[a]	1.004 (.0003)
$ln[GKDL] = \alpha + \beta\ ln[UVDL]$	-.002 (.0002)	.998 (.0005)
$ln[GKDT] = \alpha + \beta\ ln[UVDT]$	-.013 (.0002)	1.052 (.0021)

[a]Figures in brackets are standard errors.

Table 2.4: *Comparison of Geary-Khamis versus Unit Value Weights State-specific regressions*

Slope	Intercept $\alpha < 0$	$\alpha = 0$	$\alpha > 0$	Percent of States
				%
$ln[GKFL] = \alpha + \beta\, ln[UVFL]$				
$\beta < 1$	NJ	CT,NV,NH	FL,MT,UT	14
$\beta = 1$	OK,SD,TX,WY	MA,RI,WI	OR	17
$\beta > 1$	AL,GA,KS,KY,LA, ME,MD,MI,MN,MS, MO,NM,ND,PA,SC, TN,VT,VA	AZ,AR,CO,DE,ID,IL, IN,IA,NE,NY,NC,OH, WA	CA,WV	69
$ln[GKDL]\ \alpha + \beta\, ln[UVDL]$				
$\beta < 1$	GA,NJ,OH	AL,AR,CT,FL,ME, MD,MA,MS,RI	DE,OR,VT	31
$\beta = 1$	MO,ND,SD	PA,VA	MT	13
$\beta > 1$	KS,KY,LA,MN,OK, TN,VT,WY	CO,IL,IN,IA,MI,NV, NH,NM,NY,NC,SC, TX	AZ,CA,ID,NE,WA, WV,WI	56
$ln[GKDT]\ \alpha + \beta\, ln[UVDL]$				
$\beta < 1$	IA,MI,MN,MO,NV	AR,KS,MT,NE,TN,WI	ID,ME,ND,SD,UT	33
$\beta = 1$		AL,VA,WY	IN	8
$\beta > 1$	GA,IL,LA,MA,NM, OK,OR,SC	FL,KY,MS,NH,NJ,NY, NC,OH,PA,RI,TX	AZ,CA,CO,CT,DE, MD,VT,WA,WV	59

Table 2.5: *Comparison of Fixed versus Chained Indices Pooled 4 -state regressions*

Regression	α	β
$ln[SSFL]\ \alpha + \beta\ ln[SSDL]$	.025 (.002)[a]	.819 (.004)
$ln[SSFL] = \alpha + \beta\ ln[SSDT]$	.007 (.001)	.915 (.004)
$ln[UVFL] = \alpha + \beta\ ln[UVDL]$	.037 (.002)	.875 (.003)
$ln[UVFL] = \alpha + \beta\ ln[UVDT]$	.036 (.001)	1.009 (.005)
$ln[GKFL] = \alpha + \beta\ ln[GKDL]$	.036 (.001)	.88 (.004)
$ln[GKFL] = \alpha + \beta\ ln[GKDT]$	.024 (.001)	.965 (.004)

[a]Figures in brackets are standard errors.

Table 2.6: *Comparison of Fixed versus Chained Indices State-specific regressions*

Slope	Intercept $\alpha < 0$	$\alpha = 0$	$\alpha > 0$	Percent of States
				%
$ln[SSFL] = \alpha + \beta\ ln[SSDL]$				
$\beta < 1$	CA,WA,WI	AZ,CO,DE,FL,ID,IN, IA,MD,MI,NV,NH,NJ, NY,NC,OH,OK,OR, PA,UT	AL,AR,GA,IL,KS,KY, LA,ME,MN,MS,MO, MT,NE,NM,ND,SC, SD,TN,TX,VA,WY	90
$\beta = 1$		MA		2
$\beta > 1$	VT	CT	RI,WV	8
$ln[SSFL] = \alpha + \beta\ ln[SSDT]$				
$\beta < 1$	CA	CO,ID,MA,MI,NH,NJ, NC,OK	AL,AR,FL,GA,KS,LA, ME,MD,MN,MS,MT, NE,NM,ND,OR,PA, TN,TX,WA	58
$\beta = 1$	VT	AZ,CT,DE,NY,OH,WI	IL,MO,SD,VA,WY	25
$\beta > 1$		IA,NV,UT	IN,KY,RI,SC,WV	17
$ln[UVFL] = \alpha + \beta\ ln[UVDL]$				
$\beta < 1$	CA,WI	AZ,CO,FL,ID,IA,MI, NH,NY,NC,OH,OR, PA,UT	AL,AR,DE,GA,IL,IN, KS,KY,LA,ME,MD, MN,MS,MO,MT,NE, NV,NM,ND,OK,SC, SD,TN,TX,VA,WA, WY	88
$\beta = 1$	VT	MA,NJ		6
$\beta > 1$		CT	RI,WV	6
$ln[UVFL] = \alpha + \beta\ ln[UVDT]$				
$\beta < 1$	CA,CO	AZ,DE,FL,ID,IN,IA, MD,MI,NJ,NY,NC, OH,OK,OR,PA,UT	AL,AR,GA,IL,KS,KY, LA,ME,MA,MN,MS, MO,MT,NE,NM,ND, SC,SD,TN,TX,VA, WA,WI,WY	88
$\beta = 1$	VT	NV,NH		6
$\beta > 1$		CT	RI,WV	6

Table 2.6: *Comparison of Fixed versus Chained Indices State-specific regressions (Contd.)*

Slope	Intercept $\alpha < 0$	$\alpha = 0$	$\alpha > 0$	Percent of States
				%
$ln[GKFL] = \alpha + \beta\ ln[GKDL]$				
$\beta > 1$	CA,WI	AZ;CO,ID,IA,MI,NH, NY,NC,OH,OK,OR, PA	AL,AR,DE,FL,GA,IL, IN,KS,KY,LA,ME, MD,MN,MS,MO,MT, NE,NV,NM,ND,SC, SD,TN,TX,UT,VA, WA,WY	88
$\beta = 1$		MA,NJ		4
$\beta < 1$		CT,VT	RI,WV	8
$ln[GKFL] = \alpha + \beta\ ln[GKDT]$				
$\beta < 1$	CA	AZ,CO,DE,GA,ID,MI, NJ,NY,NC,OH,OK, OR,UT	AL,AR,KS,ME,MD, MA,MN,MS,MT,NM, ND,PA,SC,TN,TX, WA	62
$\beta = 1$		VT,WI	FL,IN,LA,MO,NE, NH,SC,VA,WY	23
$\beta > 1$		IA,NV	CT,IL,KY,RI,WV	15

Table 2.7: *Comparions of Divisia Approximations Pooled 4 -state regressions*

Regression	α	β
$ln[SSDT] = \alpha + \beta\, ln[SSDL]$	.019 (.001)[a]	.888 (.003)
$ln[UVDT] = \alpha + \beta\, ln[UVDL]$	-.005 (.001)	.849 (.003)
$ln[GKDT] = \alpha + \beta\, ln[GKDL]$	.010 (.001)	.899 (.003)

[a]Figures in brackets are standard errors.

Table 2.8: *Comparison of Divisia Approximations State-specific regressions*

Slope	Intercept $\alpha < 0$	$\alpha = 0$	$\alpha > 0$	Percent of States
				%
ln[SSDT] = $\alpha + \beta$ *ln[SSDL]*				
$\beta < 1$	CA,CT,FL,ME,MD, OH,OR,PA,WA,WI	AZ,CO,DE,ID,IN,IA, KS,MI,MN,NY,NC, OK,UT,VT	AL,AR,GA,IL,KY,LA, MS,MO,MT,NE,NV, NM,ND,SC,SD,TN,TX ,VA,WY	90
$\beta = 1$			RI	2
$\beta > 1$	MA,NH	NJ	WV	8
ln[UVDT] = $\alpha + \beta$ *ln[UVDL]*				
$\beta < 1$	CA,CT,ME,NH,OH, WA	AZ,CO,FL,IN,IA,MD, MA,MI,MN,NJ,NM, NY,NC,OK,OR,PA, VT,VA,WV,WI	AL,AR,DE,GA,IL,KS, KY,LA,MS,MO,MT, NE,NV,ND,RI,SC,SD, TN,TX,UT,WY	98
$\beta = 1$		ID		2
$\beta > 1$				0
ln[GKDT] = $\alpha + \beta$ *ln[GKDL]*				
$\beta < 1$	CA,ME,NH,PA,WA, WI	AZ,CO,FL,ID,IN,IA, MD,MI,MN,NV,NM, NY,NC,OH,OK,OR, RI,VT,VA	AL,AR,DE,GA,IL,KS, KY,LA,MS,MO,MT, NE,ND,SC,SD,TN,TX, UT,WY	92
$\beta = 1$				0
$\beta > 1$	CT	MA,NJ	WV	8

Table 3.1: *Ranking of States Based on Scaled Divisia Törnqvist Output Indices*

	Geary-Khamis		Unit Value		State Price		Wheat Equivalent	
	Average	Variance	Average	Variance	Average	Variance	Average	Variance
Alabama	28.05	2.16	28.22	2.22	27.86	2.66	27.65	2.77
Arizona	33.54	0.90	33.46	0.90	33.24	1.10	33.46	1.11
Arkansas	17.78	19.79	18.08	21.48	18.27	21.33	17.76	21.16
California	2.68	0.33	2.41	0.35	2.22	0.44	2.97	0.57
Colorado	23.86	5.63	23.78	5.68	23.49	4.84	21.59	6.73
Connecticut	44.49	0.30	44.49	0.30	43.81	1.18	44.00	0.86
Delaware	43.14	4.87	43.24	4.78	43.24	5.32	41.76	5.97
Florida	22.65	21.85	21.24	22.67	19.38	19.69	19.81	20.32
Georgia	19.38	11.96	19.76	12.08	20.16	12.08	18.22	11.74
Idaho	26.11	3.56	26.11	3.61	27.14	4.01	25.78	3.79
Illinois	2.57	0.62	2.84	0.62	2.95	0.54	2.81	0.86
Indiana	8.76	0.99	8.92	0.89	8.89	1.02	9.14	0.93
Iowa	1.03	0.03	1.03	0.03	1.11	0.10	1.03	0.03
Kansas	9.38	2.67	9.32	2.76	9.22	2.87	9.11	2.96
Kentucky	18.14	4.39	18.30	4.53	18.30	5.07	17.62	6.13
Louisiana	30.00	1.19	29.89	1.39	30.00	1.35	29.65	1.58
Maine	40.78	1.90	40.84	1.81	40.57	3.00	40.81	2.42
Maryland	35.16	0.24	35.19	0.26	35.16	0.30	35.16	0.35
Massachusetts	44.41	2.62	44.43	2.68	44.57	2.41	44.70	2.64
Michigan	16.68	6.76	17.00	8.38	19.27	9.49	16.97	9.38
Minnesota	4.86	0.22	5.00	0.22	5.00	0.11	5.35	0.28
Mississippi	23.49	3.71	23.73	3.98	23.16	3.60	24.84	4.30
Missouri	9.08	1.05	8.97	1.00	8.76	1.16	8.59	0.94
Montana	28.62	4.18	28.46	5.06	28.24	4.56	29.57	2.41
Nebraska	7.19	1.61	7.19	1.56	7.27	2.09	6.84	1.97
Nevada	44.76	1.21	44.70	1.13	44.68	1.52	44.70	2.43
New Hampshire	47.00	0.00	47.00	0.00	47.00	0.00	47.00	0.00
New Jersey	39.08	12.67	39.05	12.86	38.68	11.41	38.86	11.52
New Mexico	36.95	1.51	37.05	1.51	36.95	1.40	36.92	1.43
New York	15.57	13.06	15.49	12.57	15.65	13.69	15.81	15.50
North Carolina	12.51	0.79	12.51	0.79	12.51	0.74	12.00	0.59
North Dakota	18.00	7.68	17.97	8.08	17.27	6.36	20.35	6.98
Ohio	10.54	1.17	10.57	1.16	10.57	1.16	10.89	0.75
Oklahoma	19.00	4.27	19.00	3.68	19.14	3.90	18.68	4.92
Oregon	32.00	0.32	32.00	0.27	32.14	0.39	32.24	0.40
Pennsylvania	17.68	5.19	17.68	5.35	16.49	4.09	18.22	6.76
Rhode Island	48.00	0.00	48.00	0.00	48.00	0.00	48.00	0.00
South Carolina	33.41	0.73	33.54	0.52	33.62	0.72	33.19	1.67
South Dakota	14.24	6.02	14.22	6.12	14.24	5.48	15.84	7.76
Tennessee	24.30	11.13	24.41	11.32	24.86	9.79	24.27	9.28
Texas	4.11	1.02	4.03	0.78	3.78	0.28	3.22	0.66
Utah	37.81	0.32	37.76	0.35	38.03	0.30	38.11	0.31
Vermont	40.11	0.42	40.05	0.48	40.24	0.24	40.65	0.39
Virginia	27.95	14.59	28.05	14.54	28.27	12.25	28.22	12.39
Washington	25.95	5.94	26.00	5.46	26.24	5.05	27.08	4.02
West Virginia	41.70	0.80	41.62	0.72	42.51	0.79	42.92	1.05
Wisconsin	6.11	0.75	6.03	0.84	6.51	0.79	6.46	1.33
Wyoming	37.43	0.62	37.38	0.72	37.35	0.55	37.19	0.37

Note: Table reports the average cross-sectional ranking and variance across the 1949-1985 sample for each of four scaling techniques.

Table 3.2: *Comparison of Unit Value versus Geary-Khamis Scaling Factors*

Slope	Intercept $\alpha < 0$	Intercept $\alpha = 0$	Intercept $\alpha > 0$	Percent of States
				%
$\beta < 1$		DE,IL,IN,IA,MD,NC,OH		15
$\beta = 1$		MN,RI		4
$\beta > 1$	KS	AZ,AR,CA,CO,CT,FL,GA, ID,KY,LA,ME,MA,MI,MS, MD,NE,NV,NH,NM,NY,ND, OK,OR,PA,SC,SD,TN,UT, VT,VA,WV,WI,WY	AL,MT,NJ,TX, WA	81

Note: The scaled output index SSDT*UVSF is regressed upon a constant and the scaled output index SSDT*GKSF.

Table 3.3: *Comparison of Unit Value and State Specific Scaling Factors*

Slope	Intercept $\alpha < 0$	Intercept $\alpha = 0$	Intercept $\alpha > 0$	Percent of States
				%
$\beta < 1$		AL,AZ,AR,CA,CO,CT,DE, FL,GA,ID,IL,IN,IA,KS,KY, LA,ME,MD,MA,MS,MO, MT,NE,NV,NH,NM,NY,NC, ND,OH,OK,OR,PA,RI,SC, SD,TN,UT,VA,WA,WV,WY	NJ,TX,VT	94
$\beta = 1$		MN		2
$\beta > 1$		MI,WI		4

Note: The scaled output index SSDT*UVSF is regressed upon a constant and the scaled output index SSDT*STSF.

Table 3.4: *Comparison of Unit Value and Wheat Equivalent Scaling Factors*

Slope	Intercept $\alpha < 0$	Intercept $\alpha = 0$	Intercept $\alpha > 0$	Percent of States
				%
$\beta < 1$	NE	AZ,AR,CA,CO,CT,DE,FL, GA,ID,IL,IN,IA,KS,KY,LA, ME,MD,MA,MI,MS,DO, MT,NV,NH,NY,NM,NY, NC,ND,OH,OK,OR,PA,RI, SC,TN,UT,VA,WA,WV,WI, WY	AL,SD,TX	98
$\beta = 1$		MN		2
$\beta > 1$				0

Note: The scaled output index SSDT*UVSF is regressed upon a constant and the scaled output index SSDT*WESF.