Determination of a Variable Price Support Schedule as Applied to Agricultural Production Control

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Abstract *Adoption of variable price support (VPS) schedules could be effective in controlling agricultural production and targeting program benefits to specific farm groups The design of a VPS program would require determination of price schedules for farm-level production decisions that satisfy both farmer and program objectives We applied a primal-dual mathematical programming model to the determination of a VPS program for production control of U S corn, wheat, and soybeans We show that government program costs under the VPS program would decline to $15 billion from $26 8 billion under a comparably scaled mandatory production control program The program benefits to a 120-acre farm would increase 80 percent to $18,000 from $10,000, while the benefits to a 2,500-acre farm would fall 82 percent to $40,000*

U S agricultural commodity markets frequently experience excess supply or surplus, especially when prices are supported above the market price Stock accumulations resulting from surpluses lead to increased government expenditures on farm support programs and a depressed farm economy Continuous technological innovation in U S agricultural production, unstable export demand (*11*), and slow domestic demand growth aggravate the surplus problem [1] To deal with this problem, the U S Government has often attempted to control agricultural supply by requiring commodity program participants to place cropland into set-aside programs or paid land diversion programs or by restricting farm sales of commodities The success of these controls has been mixed (*2*)

A Variable Price Support Program

A promising alternative to mandatory production control systems is the variable price support (VPS) program (*4*) Rather than restricting production on an individual farm basis, participating farms would face a set of declining support prices for the program crops Under these support prices, a farmer would receive a monotonically declining price as his/her output of a particular crop increased The price received for the initial units of production would not be affected by the total quantity produced Figure 1 illustrates this con-

cept The highest support price, (IP_1), is paid to a farmer for the production of the first unit (w_1) and the next highest price is paid for the next unit (w_2), and the declining price continues A VPS schedule that sets the support price for the last unit of the commodity produced (w_n) equal to or below the expected market price would induce farmers to base marginal production (beyond w_n) on the market price Therefore, marginal production would be governed by market prices instead of the support prices A farmer with an initial production level, q_u^0, under current programs, responds to the market price by producing q_u^1 under the VSP program, a reduction of w_s Because a VPS program allows market prices to prevail, chronic surpluses disappear

The basic reasoning behind the use of a VPS can be compared with the logic of increasing block rate structure used by electric power companies and municipal water authorities The utility rate structure is designed to discourage excessive electricity (water) consumption by consumers, and the VPS discourages excess production by farms While the utility companies are concerned with finding a price schedule that leads to the efficient utilization of its physical capacity, the VPS is concerned with finding a price schedule that leads to a more efficient allocation of resources within the agricultural sector Such an allocation would reduce the social welfare deadweight loss associated with excess production

The objective of this article is to investigate methods of obtaining a set of VPS schedules to achieve predetermined national production levels Two methods for estimating the farm-level price schedules will be presented The first method employs an iterative procedure, while the second method uses a primal-dual (PD) programming model The iterative method is presented because it illustrates the problem to be solved The programming model is a generalized procedure Both methods use a farm production decision model to estimate the production response at the farm level The estimated commodity price schedules are declining functions of the quantity of the commodity produced on a farm

The PD formulation has the advantage of expressing the problem in a concise manner For a simple farm decision model, the PD formulation can determine the price schedule in one iteration If the PD formulation solves the problem, the farm decision model must be simple (linear in its constraints) to obtain a solution The iterative procedure on the other hand is less elegant in its formulation but has the advantage of

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[1]Italicized numbers in parentheses cite sources listed in the References section at the end of the article

Figure 1

A linear declining price support schedule to reduce single-farm crop production
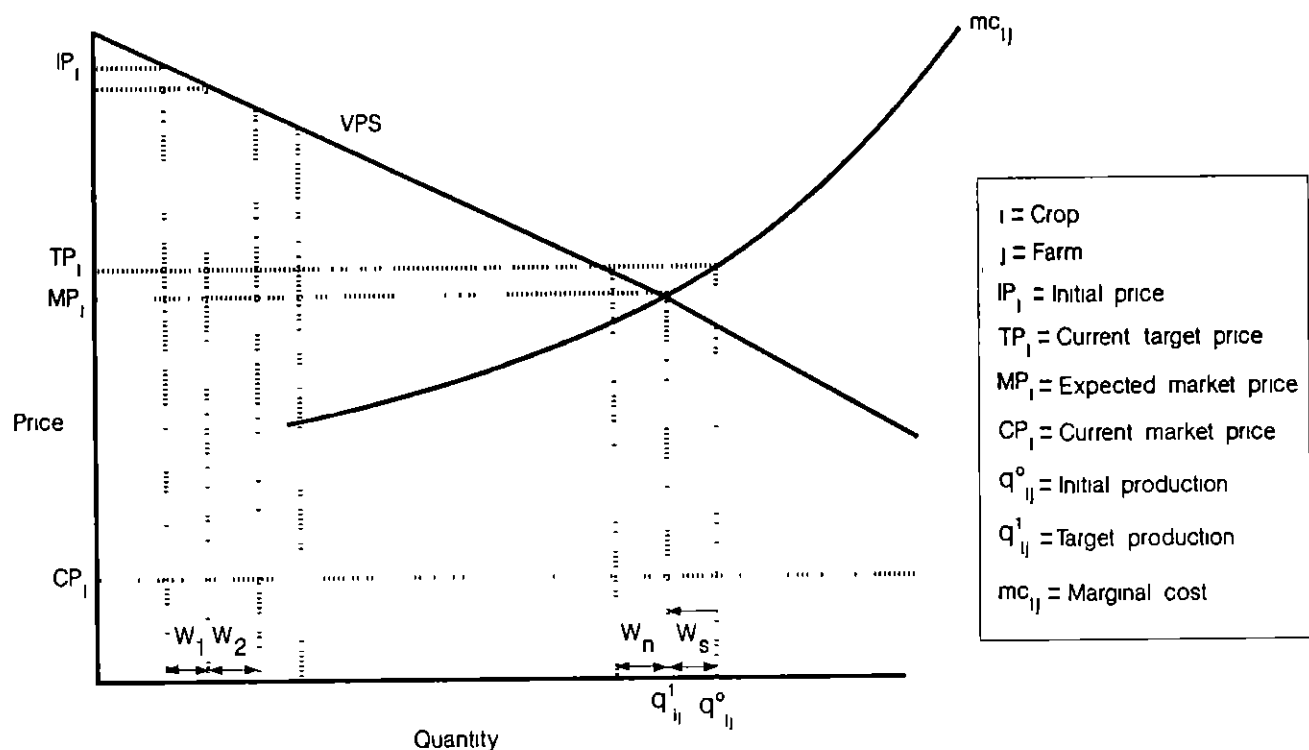
being able to estimate the price schedule when non-linear constraints are imposed Thus, the iterative procedure can be used to determine the farm production response for a farm decision model that includes a risk component or has nonlinear constraints

The Problem

The problem can be formally stated as follows given a desired aggregate production level, TQ_i, find a set of farm-level, declining support price schedules, P_i (a_i, q_{ij}), such that the sum of the production of crop i over all farms is equal to TQ_i That is, $\Sigma\ q_{ij} = TQ_i$ for all i In this problem, q_{ij} is the commodity i produced by the farm j, and a_i is the parameter to be estimated It is assumed that a farm produces a set of crops in a manner that maximizes net farm revenue

The production decision model for farm j can be formulated as

(Problem 1)

$$\underset{q_{ij}}{\text{Max }} Z_i = \Sigma \int_0^{q_{ij}} [P_i\ (\hat{a}_i,\ q) - \max\ (mp_i,\ c_{ij})]\ dq, \quad (1)$$

which is subject to the resource constraint

$$\Sigma\ d_{ij}\ q_{ij} \le L_j, \quad (2)$$

where P_i ($\hat{a}_i$, q) is the given government price support function for crop i, q is the integration (dummy) variable for q_{ij} to be determined, mp_i is the expected market price for crop i, c_{ij} is the production cost for crop i on farm j, L_j is the land available for crop production, and d_{ij} is the portion of an acre on farm j required to produce one unit of crop i $\hat{a}_i$ is the parameter estimate that defines the support price function

Given a set of estimated coefficients, $\hat{a}_i$, the optimal production response, q^*_{ij}, for farm j can be obtained By solving problem 1 for all farms and summing q^*_{ij}, the aggregate production level, Q^*_i, can be determined In practice, the problem for all the farms is solved simultaneously rather than repeatedly solving the problem for each farm This is done by solving

(Problem 2)

$$\underset{q_{ij}}{\text{Max }} Z_2 = \Sigma\ \Sigma \int_0^{q_{ij}} [P_i\ (\hat{a}_i,\ q) - \max\ (mp_i,\ c_{ij})]dq, \quad (3)$$

which is subject to

$$\Sigma\ d_{ij}\ q_{ij} \le L_j \text{ for all } j \quad (4)$$

The aggregate and farm-level production figures obtained from problem 2 will be identical to Q^*_i and q^*_{ij} obtained from problem 1 The Kuhn-Tucker neces-

sary conditions (9) for the optimal solution of problem 2 are given by the following relations

$$P_i(\hat{a}, q^*_{ij}) - \max(mp_i, c_{ij}) - \mu^*_j d_{ij} \leq 0, \text{ for all } i \text{ and } j, \quad (5)$$

$$[P_i(\hat{a}, q^*_{ij}) - \max(mp_i, c_{ij}) - \mu^*_j d_{ij}] q^*_{ij} = 0,$$

$$\text{for } i \text{ and } j, \quad (6)$$

$$\sum_i d_{ij} q^*_{ij} - L_j \leq 0, \text{ for } j, \text{ and} \quad (7)$$

$$\left[\sum_i d_{ij} q^*_{ij} - L_j\right] \mu_j = 0, \text{ for } j, \quad (8)$$

where $q^*_{ij} \geq 0$ and $\mu^*_j (\geq 0)$ is the shadow price of the resource L_j

Given a production level, TQ_i, the set of parameters, $\hat{a}_i$, in the price function, $P_i(\hat{a}_i, q)$, should be estimated such that the sum of the commodity production over individual farms j, Q^*_i, is equal to this targeted level To solve this problem, an iterative procedure and a PD mathematical programming model can be employed

An Iterative Procedure

This procedure uses the farm production decision model and the iterative estimation method outlined in the flowchart (fig 2) The first step is to select a func-

Figure 2
Procedure to determine a VPS schedule targeted for a given production level

Select a functional form
$P_i(a_i \; q)$

Use the best guess to determine initial values for $\hat{a}_i$

Determine q_{ij} which maximizes objective function

$$\sum_i \sum_j \int_0^{q_{ij}} [P_i(\hat{a}_i \; q) - \max(mp_i \; c_{ij})]d_q$$

Subject to

$$\sum_i d_{ij} q_{ij} \leq L_j \text{ for all } i$$

Adjust $\hat{a}_i$

Calculate aggregate production
$$Q^*_i = \sum_{j=1} q^*_{ij}$$

Is Q^*_i equal to the target production TQ_i

No

Yes

A set of VPS schedule $P_i(\hat{a}_i \; q_{ij})$ is found

tional form for the VSP schedule that is able to direct government benefits to the farm groups the program is designed to help Two characteristics of the functional form that affect the program's ability to direct government benefits to a given farm size group are its general shape and its slope The general shape of the functional form determines how the government support is distributed among farm size groups The current farm program has a single flat support price which provides greater support to large farms because all farm production receives the same support and the larger farms produce more An exponentially decreasing schedule, on the other hand, would provide a high support price to the first units of production, but the support would decline rapidly as onfarm production increased The exponential function directs a much greater proportion of benefits to smaller farms A linear declining support price can direct a greater proportion of farm support payments to smaller farms than can current programs but not as much as under the exponentially declining rate

Once the function, $P_i(a_i, q)$, has been selected, a preliminary VPS function, $P_i(\hat{a}_i, q)$, is specified using a set of starting values for $\hat{a}_i$ The production level, q^*_{ij}, for each farm is then determined using the farm decision model By summing q^*_{ij} over all farms, Q^*_i is obtained This level of production is then compared with the target production level TQ_i If the difference for each commodity is not significant, the final set of $\hat{a}_i$ has been found Otherwise, the $\hat{a}_i$ are adjusted and the set of adjusted $\hat{a}_i$ is used to generate a new production estimate A new set of $\hat{a}_i$ can be computed on the basis of the inverse relationship between the value of $\hat{a}_i$ and the quantity produced The iterative process continues until $Q^*_i = TQ_i$, indicating a suitable set of $\hat{a}_i$

The procedure can be extended to estimate an efficient support price schedule given both a government farm program budget and target production levels The procedure to find $\hat{a}_i$, however, becomes more complicated as additional restrictions are added A PD programming model can be formulated as an alternative method to determine the VPS schedule

A Primal-Dual Programming Model

This approach uses the fact that the solution of a PD programming model is also the solution to the corresponding primal formulation Thus, if the price schedule obtained by the PD model is used in the corresponding primal formulation, the production pattern and targeted aggregate production level, Q^*_i, obtained from the primal formulation will be identical to those produced by the PD model

The PD problem is derived as follows (7)

1 A new objective function is constructed The new objective function is the difference between the
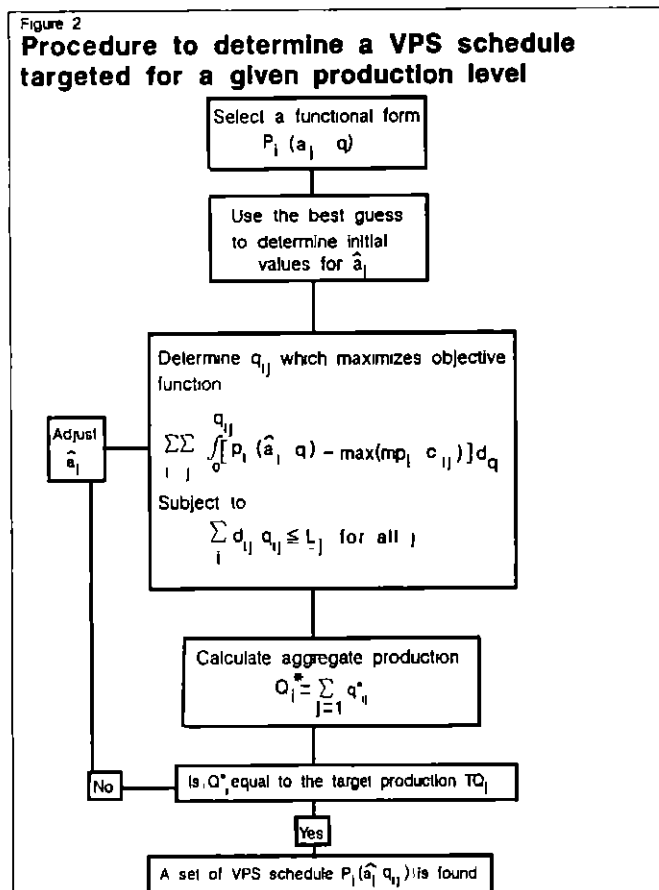
objective functions of the primal and the dual formulations

2   The constraints in the PD model include all constraints specified in the primal and dual formulations

Problem 2 can also be expressed using a dual formulation $(6, 12)$

(*Problem 3*)

$$\underset{q_{ij},\ \mu_j}{\text{Min}}\ Z_3 = Z_2 - \sum_i \sum_j [p_i\,(\hat{a}_i,\ q_{ij}) - \max(mp_i,\ c_{ij})]\,q_{ij}$$

$$+ \sum_j \mu_j\,L_j, \tag{9}$$

subject to

$$P_i\,(\hat{a}_i,\ q_{ij}) - \max(mp_i,\ c_{ij}) - \mu_j\,d_{ij} \le 0, \text{ for all } i \text{ and } j, \tag{10}$$

where the term Z_2 in equation 9 is the objective function of problem 2

Given the primal and dual formulations and the above description of the PD formulation, the PD formulation for problem 2 is

(*Problem 4*)

$$\underset{q_{ij},\ \mu_j}{\text{Max}}\ Z_4 = \sum_i \sum_j [P_i\,(\hat{a}_i,\ q_{ij}) - \max(mp_i,\ c_{ij})]\,q_{ij}$$

$$- \sum_j \mu_j\,L_j, \tag{11}$$

subject to

$$\sum_i d_{ij}\,q_{ij} \le L_j, \text{ for all } j, \text{ and} \tag{12}$$

$$P_i\,(\hat{a}_i,\ q_{ij}) - \max(mp_i,\ c_{ij}) - \mu_j\,d_{ij} \le 0, \text{ for all } i \text{ and } j \tag{13}$$

A solution to problem 4, if it exists, will be a solution to problem 2 $(7,\ 12)$

Using problem 4 as the basic framework, the problem can be reformulated to find a set of proper values, $\hat{a}_i$, to determine a set of farm-level support price schedules This is done by maximizing Z_5 (problem 5) with respect to the parameters, a_i, as well as q_{ij} and μ_j It should be noted that in problem 5, a_i is a variable parameter

(*Problem 5*)

$$\underset{a_i,\ q_{ij}\ \mu_j}{\text{Max}}\ Z_5 = \sum_i \sum_j [P_i\,(a_i,\ q_{ij}) - \max(mp_i,\ c_{ij})]\,q_{ij}$$

$$- \sum_j \mu_j\,L_j, \tag{14}$$

subject to

$$\sum_i d_{ij}\,q_{ij} \le L_j, \text{ for all } j, \text{ and} \tag{15}$$

$$P_i\,(a_i,\ q_{ij}) - \max(mp_i,\ c_{ij}) - \mu_j\,d_{ij} \le 0, \text{ for all } i \text{ and } j \tag{16}$$

$P_i(a_i,\ q_{ij})$ is the price function that must satisfy the following two sets of conditions

$$\partial\ P_i(a_i,\ q_{ij})/\partial\ q_{ij} \le 0, \text{ for all } i \text{ and } j, \tag{17}$$

and

$$P_i\,(a_i,\ 0) \le b_i, \text{ for all } i, \tag{18}$$

where b_i is a constant for crop i Condition 17 states that a support price cannot increase as production of a commodity on a farm increases Equation 17 results in the declining support prices The relationship in equation 18 sets the initial maximum support price for each commodity A solution $(a^*_i,\ q^*_{ij})$ that satisfies problem 5 also satisfies problems 4 and 2 To incorporate supply control, aggregate production constraints are added to problem 5

$$\sum_j q_{ij} = TQ_i, \text{ for all } i \tag{19}$$

The addition of production constraints 19 to problem 5 changes the slope of the price schedule $P_i(a^*_i,\ q_{ij})$ [2] However, the condition that the q_{ij} obtained from problem 5 is the solution to the individual farm's revenue maximization problem (problem 2) still holds The condition holds because any set of values $(a^*_i,\ q_{ij})$ obtained from problem 5 with the production constraints 19 will also satisfy the Kuhn-Tucker conditions (relations 5 to 8) associated with problem 2 Thus, the price function obtained from this formulation can be substituted in problem 2 to obtain an identical production pattern

The PD approach can lead to a nonlinear programming problem that becomes difficult to solve For instance use of a nonlinear price function or a budget constraint to control total program expenditures makes obtaining an optimal solution difficult In some situations, the combination of a PD formulation with an iterative procedure is the only method to obtain the optimal VPS schedule to control production

A PD Model With Linear Price Schedules

We used a linear price function to design a declining support price schedule to control crop production A PD model with the linear price function is formulated We compared the results from the PD model with the results from a mandatory production control (MPC) program, assuming the market price to be less than the production costs for each crop

We constructed a PD model with a set of production constraints and a linear price function A linear,

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[2]For example, with a linear functional form for the price support and a declining quantity of TQ_i, the slope becomes steeper

declining price function is used because it has high initial support prices which are an advantage to small farms The price function is expressed as

$$P_i(a_i, q_{ij}) = b_i - a_i q_{ij}, \text{ for all } i \text{ and } j, \quad (20)$$

where b_i is a given positive constant and a_i is a positive parameter to be solved for

Problem 5, containing a linear declining function 20, can be reformulated as

(Problem 6)

$$\underset{a_i, q_{ij}, \mu_j}{\text{Max}} \quad Z_6 = \sum_i \sum_j [-a_i q_{ij}^2 + (b_i - c_{ij}) q_{ij}] - \sum_j \mu_j L_j, \quad (21)$$

subject to

$$\sum_i d_{ij} q_{ij} \le L_j, \text{ for all } j \quad (22)$$

$$b_i - a_i q_i - c_{ij} - \mu_j d_{ij} \le 0, \text{ for all } i \text{ and } j \quad (23)$$

We add a set of constraints to control aggregate production

$$\sum_j q_{ij} = Q_i, \text{ for all } i \quad (24)$$

An Application

The VPS program was compared with an MPC program similar to that proposed by Byrd and Harkin (*1*) An MPC program can be characterized as offering a flat support price for the controlled commodities while limiting the cropland acreage available for production on each farm Each farm idles the same proportion of its cropland under an MPC program, and the quantity of a farmer's production does not affect the support price received for that commodity For this reason, the shape of the rate schedule is horizontal or flat

In this case study, a farm is considered a production unit of 100 acres or larger which can annually grow corn, soybeans, wheat, or a combination of these three crops U S farms are divided into eight (j = 1, .8) groups according to size with the farms in each group assumed to be identical Costs of production for each commodity reflect economies of scale (table 1) We used the 1982 census data to estimate the number of farms and the average crop yield by farm size class

To provide adequate income to each farm under the VPS program, we assume the initial prices (b,) for the 1986 crop year approached 80 percent of parity prices (the support price level proposed by Byrd and Harkin (*1*) Table 2 carries the initial prices The equilibrium production associated with these prices came from the FAPSIM model (*10*) When production is reduced to the target level, the market prices, theoretically, equal the support prices There would be no government payment to farms at these price levels The government would have to pay farms participating in the VPS program only if the final market prices fall below the support prices Under the MPC program, table 2's prices and production represent support prices and the quantities of production to be controlled To achieve the targeted production level, each farm, regardless of its size, must idle the same proportion of land

The Generalized Algebraic Modeling System (GAMS) (*5*), a nonlinear, quadratic programming package, estimated the price schedules and associated production response of individual farms under the VPS program A linear programming model determined the production levels and farm incomes under the MPC program

Results

The estimated price functions for corn, soybeans, and wheat are $P_1 = 3\,95 - 0\,000077q_{1j}$, $P_2 = 9\,76 - 0\,00065q_{2j}$, and $P_3 = 5\,36 - 0\,000155q_{3j}$, respectively These price schedules produce higher net incomes for small farms with no increase in government expenditures relative to the MPC program Under an MPC program, which has a single support price, a small

Table 1—Number of farms, acreage, crop yield, and production cost per farm size group

Farm size group (acres)[1]	Average acreage per farm, L_j	Number of farms, N_j	Yield, Y_{ij}			Production cost, C_{ij}[2]		
			Corn	Soybeans	Wheat	Corn	Soybeans	Wheat
	Acres	*Thousands*	---- *Bu /acre*-------------			----------*Dollars/bu* ------------		
100-139	120	67	101	28	34	1 87	3 99	3 19
140-179	160	69	102	30	33	1 84	3 91	3 12
180-219	200	50	102	30	35	1 80	3 82	3 06
220-259	240	48	104	31	34	1 76	3 73	2 98
260-499	280	161	107	27	34	1 72	3 64	2 91
500-999	750	97	110	31	34	1 68	3 57	2 86
1,000-1,999	1,500	57	111	30	35	1 65	3 50	2 80
2,000 and more	2,500	32	110	28	33	1 65	3 50	2 80

[1]It is assumed that only full-time farms can participate in the VPS Excluded are farms with fewer than 100 acres because they are likely to be part-time farms with substantial off-farm income

[2]The production costs are derived from a 1982 base solution of the National Linear Program LP model (*3*), adjusted for farm size from the study by Miller and Rodewald (*8*)

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farm with 120 acres would receive $10,500 in government support payments, while a large farm with 2,500 acres would receive more than $220,000 in benefits The VPS schedule directs more benefits to small farms A small farm would receive $18,500 in government benefits, while the maximum payment received by a large farm falls below $40,000 Thus, net incomes for small farms increase substantially with a VPS program, while transfers to large farms decline

The distribution of program benefits under an MPC program demonstrates the difficulty of flat support price schedules in supporting small-farm income Because of their large production levels, large farms receive most of the government benefits In addition, a set of flat support prices fixed above prevailing market prices will encourage profit-maximizing producers to increase production, creating excess commodity supplies and increasing program payments and government storage costs The VPS program is designed to discourage excess production by removing program production incentives beyond some targeted production level, TQ_i Production beyond TQ_i will be eligible to receive only a support price that is below the market price, resulting in marginal production decisions that are based on market prices

If market prices are above the target price, program costs for both the MPC and VPS programs will be zero Program costs will increase as the market prices slip from the target price For example, if market prices (factored with the 1986 support loans) are $1 98 for corn, $4 88 for soybeans, and $2 68 for wheat, an MPC program would cost $26 8 billion, while a VPS program would cost the government $15 billion

Government expense under the VPS program is lower because the support price is monotonically reduced for each additional unit of production With the marginal support price below the expected market price, production that exceeds the target quantity would not require storage Government expense for the storage of commodity surpluses would decline Administrative costs would remain constant because a VPS program could use existing program yields and program enrollment procedures So, the VPS could diminish program costs by reducing the amount of production receiving support payments, the size of the marginal support payment, and storage costs

Conclusions

The VPS program would enable the government to control agricultural program spending while meeting commodity program objectives The marginal support price at the target production level should be set below the expected market equilibrium price in designing a support price schedule Government expenditures would be reduced whenever the marginal support price fell below the market price

Table 2—Target quantities and expected market prices for 1986 crop year under a mandatory supply control program

Crop	Production, TQ_i	Prices, b_i
	Million bushels	*Dollars/bushel*
Corn	6,161	3 95
Soybeans	1,836	9 76
Wheat	2,136	5 36

The VPS program requires the estimation of price schedules that will lead to farm production decisions that satisfy both farmer and commodity program objectives Both the iterative and PD mathematical programming procedures are useful tools for generating price schedules appropriate for a given VPS program These modeling systems can design a VPS price schedule that achieves both national and farm-level commodity goals The iterative procedure is a relatively simple procedure (compared with the PD method) which does not require an advanced modeling technique The procedure, however, can limit determining an optimal support price schedule when production restrictions are added The PD approach, on the other hand, can be used for the situation with multiple production restrictions, albeit requiring an advanced modeling technique to set up a PD problem that can lead to a difficult-to-solve nonlinear programming problem In some situations, a combination of a PD formulation with an iterative procedure is the only way to obtain the optimal VPS schedule

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