Alternative Forms for Production Functions of Irrigated Crops

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Abstract. *The output elasticities of irrigation water are highly inelastic for every crop, indicating that reductions in water supply would have relatively small effects on crop production This article reports estimates of Cobb-Douglas and quadratic production functions for 13 irrigated crops in the 17 Western States Returns to scale, the output elasticity of irrigation water, and the marginal rate of substitution between water and land are estimated for each crop J-tests, used to test statistically which, if either, functional form is a correct specification, do not reject the Cobb-Douglas specification for four crops (barley, grain sorghum, potatoes, and rice) and do not reject the quadratic specification for three crops (cotton, dry beans, and potatoes)*

Keywords. *Production function, irrigation, output elasticity, model specification test, water conservation, Western United States*

As western water institutions adopt water management objectives to replace their traditional irrigation development mission, research on water use in irrigated agriculture takes on a new dimension Understanding the relationship between crop yield and water applications has contributed to public-sector irrigation development planning and private-sector water use decisions Private, State, and Federal institutions, however, are now designing methods and policies to enhance the efficiency of water use in irrigation (Smith, 1989, U S Dept Interior, 1987, Western Governors' Association, 1986) With increased intersectoral competition for surface-water resources and sustained mining of ground-water reserves, most irrigators in the Western United States will encounter water conservation incentives Estimating the relationship between irrigated crop production and input use, therefore, provides an important empirical basis for assessing the impact of irrigation water conservation on agricultural output and input use

This article presents estimates of production functions for 13 irrigated crops using two common functional forms, Cobb-Douglas and quadratic Whereas most previous studies use localized field-experiment data, the estimates reported here are based on survey data from the 1984 Farm and Ranch Irrigation Survey (FRIS) (U S Dept Commerce, 1986a) for the 17 Western States Broad geographic and crop coverage, uniform data sources, and uniform definition of variables across crops combine to produce a comprehensive, consistent econometric analysis of irrigated production The results establish a basis for evaluating four important water conservation alternatives applying less irrigation water, substituting irrigation technology for water, substituting land for water, and adopting more sophisticated techniques of irrigation scheduling By evaluating the performance of two common functional forms, the article also establishes a basis for discussing the merits of alternative functional forms for a large number of crops

Previous Research

Research on econometrically estimated production functions for irrigated agriculture can be divided into three categories The first category links agronomic concepts of nutrient intake, climate, and evapotranspiration with economic production analysis (for example, Yaron, 1967, Hexem and Heady, 1978) Based primarily on field-experiment data, this research estimates functions relating plant yield to water and, in some cases, fertilizer applications and weather [1] This research concludes that polynomial response functions (square root and quadratic functions) provide reasonable functional forms Polynomial response functions estimated for water and nutrients imply substitutability among inputs everywhere on the function

Researchers using the agronomic approach recently estimated von Liebig response functions for nutrients and water (Ackello-Ogutu, Paris, and Williams, 1985, Grimm, Paris, and Williams, 1987, Paris and Knapp, 1989) When an input is limiting, a von Liebig function behaves like a Leontief production function (with no substitutability among inputs) These authors discovered that, based on model specification tests,

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[1] Elaborate versions of the agronomically oriented research apply time-dated process models of plant growth to establish the effect of early and midseason growth and water stress on final yield (for example, Minhas Parikh and Srinivasan, 1974)

the von Liebig functions outperform polynomial functions

The second category of research, beginning with the seminal work of Ruttan (1965), concentrates on labor, farm machinery, and other inputs as substitutes for land and water (Brown and Beattie, 1975, Madariaga and McConnell, 1984) This research applies its analysis of the productivity of irrigated agriculture as a tool of rational regional and national planning for agricultural supply and resource use

The third category applies contemporary multioutput production methods to estimate production relationships for irrigated agriculture Research results include estimates of Cobb-Douglas and translog production functions for bell peppers, eggplant, melons, onions, and tomatoes with three inputs (Just, Zilberman, and Hochman, 1983, Chambers and Just, 1989) and estimates of conditional factor demand functions for irrigation water and three irrigation technologies (Nieswiadomy, 1988)

Data availability has repeatedly limited these three avenues of research Although agronomically sound, response functions estimated from field-experiment data cannot capture the substitution opportunities inherent in a full specification of farm inputs For instance, most response functions do not quantify tradeoffs between irrigation water and irrigation technology or between irrigation water and land The second research approach, in contrast, relies on farm-level data (rather than crop-specific data) that inevitably is aggregated to a county-level application using, for example, Census of Agriculture data The second approach solves the problem of aggregating farm output over different commodities by measuring output in dollar value However, aggregation obscures crop-specific relationships between output and inputs The third research approach generally relies on time-series data to create adequate price variation for application of duality theory These data requirements have restricted the number of crops studied and the geographic coverage

Production Function Specification

Characteristics of the data determined many of the fundamental modeling decisions in this study The primary data set is composed of cross-sectional microdata from the 1984 FRIS The core variables are crop-specific observations of output per acre, irrigation water applied per acre, land, and irrigation technology The core-variable data are the best available in terms of sample size and geographic and crop coverage

The comparative strengths of the data motivate three modeling decisions First, farm-level observations on crop-specific input and output quantities, rather than financial data, dictate a primal rather than a dual approach Unlike the dual approach, the production-function approach requires no behavioral assumptions That is, a production function is a purely physical relationship between inputs and output It is not an economic optimization problem requiring either a maximization assumption on producer behavior or the separation of inputs into fixed and variable inputs Second, without loss of generality, per-acre production functions are estimated rather than converting yield and water applied per acre to total output and water use to estimate conventional production functions (functions using total output as the dependent variable) [2] The per-acre and conventional production functions contain identical information in principle, but by using the per-acre data reported on the survey, we avoid introducing heteroskedastic error terms Third, von Liebig response functions are not estimated because the FRIS data do not meet their requirements for detailed field-level data on agronomic factors of plant growth Further, Berck and Helfand (1990) showed that, "even though an individual plant may actually grow via a von Liebig production function, in the aggregate a smooth concave function may provide a better approximation for actual crop yields" (p 990) This article's approach is consistent with their findings

For each crop, the per-acre production function for the Cobb-Douglas specification is [3]

$$y = A\, x_1^{\alpha}\, x_2^{\beta}\, x_3^{\gamma}\, x_4^{\delta} e^{\sum_{i=1}^{n} \rho_i z_i + \epsilon_i} , \qquad (1)$$

[2]The relationship between conventional production functions and per-acre production functions (namely, that per-acre functions are algebraically derived from conventional production functions) rarely is recognized explicitly Too frequently, researchers simply specify the output and input data on a per-acre basis and ignore land as an input, thereby imposing a constant returns-to-scale production function Per-acre yield response functions generally assume that crop yields can be replicated on each acre in effect assuming a production function that is multiplicatively separable in land

[3]Assuming input nonjointness in crop production, the per-acre, crop-specific Cobb-Douglas production function follows directly from a conventional production function, such as

$$q = B w^{\alpha} r^{\beta} c^{\gamma} n^{\phi}$$

where q is output of the crop B is a constant, w is water, r is rainfall, c is cooling degree days, and n is land As usual, returns to scale depend on summing the exponents, $\alpha+\beta+\gamma+\phi$ To convert to a per acre production function, divide both sides of the equation by n This simplifies to (see next page) where y is output per acre, x_1 is irrigation water applied per acre (acre-inches), x_2 is rainfall per acre (inches), x_3 is cooling degree days (days), x_4 is land (acres), e is the exponential function, z_i (i=1, ,n) are a series of qualitative variables representing irrigation technology, water management, farm structure, climate, and soil quality, ϵ_1 is an error term that captures the cumulative effect of all excluded variables, and A, α, β, γ, δ, and the ρ_i (i=1, ,n) are parameters to be estimated Because the functions are on a per-acre basis, the exponent on land measures returns to scale, rather than (as is conventional) the output elasticity of land The output elasticity of land can be computed from estimates of the Cobb-Douglas exponents in equation 1 Estimates of crop-specific returns to scale and the output elasticities of water and land provide new information on production functions for irrigated crops

The specification of the quadratic production function, following the approach of Caswell and Zilberman (1986, p 800-2), is

$$y = a + \sum_{i=1}^{3} b_i x_i + \sum_{i=1}^{3} c_i x_i^2 + \sum_{i=1}^{n} d_i z_i + \epsilon_2, \qquad (2)$$

where a, b_i (i=1,2,3), c_i (i=1,2,3), and d_i (i=1, ,n) are parameters to be estimated, ϵ_2 is an error term, and the remaining variables are defined as before Cross-product interaction variables are not included in the quadratic specification for reasons discussed in footnote 8 Land is not an argument in the function because the quadratic specification imposes constant returns to scale

Data and Variables

The primary data set is composed of 8,009 FRIS observations from the 17 Western States[4] The FRIS survey instrument emphasizes irrigation-related decisions and contains no information on other purchased inputs and human capital Crop-specific data are available for 13 crops alfalfa hay, barley, corn silage, cotton, dry beans, grain corn, grain sorghum, other hay (other than alfalfa), potatoes, rice, soybeans, sugar beets, and wheat For each crop, the survey reports output per acre, irrigation water applied per acre, harvested acreage, and irrigation technology (table 1)[5] Two dummy variables describe the irrigation technology used for water application The impact on yield of sprinkler technology and subirrigation technologies are measured relative to gravity systems, the omitted irrigation technology The appendix defines these and subsequent variables in more detail

The FRIS survey includes several questions that permit construction of variables measuring the effects of farm-level water management decisions Data from a question on irrigation scheduling ("the method of deciding when to apply water") were divided into two dummy variables (see the appendix) More sophisticated methods of irrigation scheduling should increase crop yields, other things equal The FRIS also identifies the source of irrigation water on the farm One hypothesis is that, because ground water typically provides more flexibility in timing of use than surface water, relying solely on surface water reduces yield A confounding factor, though, is that surface and ground-water quality may differ, with ground water more saline than surface water in some regions (for instance, the San Joaquin Valley of California) However, data are not available to control for water quality Finally, information on whether irrigation was discontinued because of unanticipated events forms the final irrigation-related dummy variable Unanticipated discontinuation of irrigation for any reason should depress yields

A set of variables not associated directly with onfarm irrigation practices is included to control for physical and structural characteristics of the farm These 21 variables include four categories of information farm structure variables, weather variables, climate variables, and soil quality variables The appendix defines the variables and also describes a prior expectation for each variable's

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$$\frac{q}{n} = B\left(\frac{w}{n}\right)^{\alpha}\left(\frac{r}{n}\right)^{\beta}\left(\frac{c}{n}\right)^{\gamma} n^{(\alpha+\beta+\gamma+\phi-1)},$$

or, converting to the notation used in the text

$$y = Ax_1^{\alpha} x_2^{\beta} x_3^{\gamma} x_4^{\delta},$$

where $\delta = (\alpha+\beta+\gamma+\phi-1)$ The econometric analysis estimates the returns to scale, δ, directly Production exhibits constant, decreasing, or increasing returns as δ is equal to, less than, or greater than zero The output elasticity of land, ϕ, can be calculated from the estimates in equation 1 as $\phi = \delta+1-\alpha-\beta-\gamma$

[4]The 17 Western States are Arizona, California, Colorado, Idaho, Kansas, Montana, Nebraska, Nevada, New Mexico, North Dakota, Oklahoma, Oregon, South Dakota, Texas, Utah, Washington, and Wyoming

[5]Water prices must be high enough for producers to have the incentive to apply water at a rate other than the yield-maximizing water application rate That is, the data should not be used to estimate a function if water prices are negligible and every producer is at the yield-maximizing point of a response function Such an estimated function would simply trace out the envelope of a series of yield-maximizing points Two published papers that utilize data from the 1984 FRIS (Negri and Brooks, 1990, Ogg and Gollehon, 1989) provide evidence that sufficiently high water prices and sufficient water-price variation exist to induce the observed variation in water application rates In both papers, a water-price variable was highly significant in explaining irrigation technology choice and irrigation water demand respectively

Table 1—Characteristics of crop-specific variables[1]

Crop	Units	Irrigated crop yield		Irrigation water-application-rate[2]		Irrigated acres		Percentage of observations by irrigation technology[3]	
								Gravity	Sprinkler
		Mean	std dev	Mean	std dev	Mean	std dev	Percent	Percent
Alfalfa	tons	4 33	2 05	29 1	19 5	263	560	59 7	39 6
Barley	bu	79 5	26 5	20 7	14 5	253	515	51 3	47 7
Corn silage	tons	20 40	5 69	24 0	14 3	158	291	70 0	29 7
Cotton	lbs	916	371	32 5	20 5	936	2,727	86 5	12 8
Dry beans	cwt	20 43	6 03	22 6	13 6	200	286	57 9	41 9
Grain corn	bu	132 1	31 9	22 1	13 3	606	1,085	53 9	45 6
Grain sorghum	bu	86 0	27 3	16 9	10 0	328	397	71 9	27 6
Other hay	tons	2 14	1 21	22 9	16 7	594	1,381	78 5	19 4
Potatoes	cwt	348	121	28 2	16 1	447	707	18 3	80 4
Rice	cwt	67 8	13 0	62 5	21 2	856	1,033	100 0	0 0
Soybeans	bu	37 9	10 9	12 1	8 1	196	226	50 5	49 5
Sugar beets	tons	23 4	5 7	32 9	16 6	319	386	74 7	25 3
Wheat	bu	73 6	26 9	19 1	13 2	441	921	53 4	46 6

[1]Space limitations do not permit listing of the statistical characteristics or percentages of remaining variables used in the analysis They are available from the authors

[2]Irrigation water application rate measured as acre-inches per acre

[3]The percentage of observations in other irrigation technologies is the difference between 100 percent and the percentages in gravity and sprinkler technologies

sign This extensive effort to account for as many production factors as possible was critical given the broad geographic coverage of the research

Econometric Results

The alternative forms of per-acre production functions in equations 1 and 2 are estimated using ordinary least squares, with the Cobb-Douglas functions estimated in a linear-in-logarithms form The number of estimated parameters changes by crop because the definitions of weather, climate, and soil-quality variables remain identical across crops Thus, crops produced in diverse physical conditions have many parameters estimated The most parameter estimates, 31, are with the quadratic forms for the alfalfa hay and other hay equations In contrast, crops produced in more homogeneous conditions have fewer parameters estimated The fewest estimates, 17, are with the Cobb-Douglas rice equation

Assessing the Cobb-Douglas specification, significant parameters (at the 0 05 level) range from 19 of 27 estimated parameters for wheat and 18 of 26 for grain corn to 3 of 17 for rice and 4 of 22 for soybeans (table 2) The number of observations for each crop, which varies from 142 for rice to 3,516 for alfalfa, explains in part the range in performance across crops Results for crops with many insignificant coefficients, like soybeans, do provide statistically significant information on the relationship between irrigation water and yield The results are comparable for the quadratic specification, although table 3 reports results only for a selected set of independent variables

The adjusted R^2's in the Cobb-Douglas case range from 0 603 for cotton and 0 539 for rice to 0 096 for soybeans and 0 094 for dry beans [6] Rice is an interesting case of a relatively high adjusted R^2 associated with only a few statistically significant variables This indicates multicollinearity, and multicollinearity diagnostics confirmed this [7] With the quadratic form, adjusted R^2's are generally similar to the Cobb-Douglas case (table 3)

Since evaluating and selecting functional specification based on R^2's is inappropriate (Davidson and MacKinnon, 1981), we conduct non-nested specification tests A subsequent section reports the test results

[6]Adjusted R^2's reported in previous research for regression results using experimental data are higher than the adjusted R^2's reported here using survey data, see, for example, Grimm and others, or Yaron The higher R^2's using experimental data are not surprising for two reasons First, survey data are inherently noisy Second, field experiments control for inputs other than water and nitrogen and the FRIS does not contain data on several inputs

[7]With every regression equation, variance inflation factors (a multicollinearity diagnostic) were computed for each variable to indicate whether sufficient multicollinearity was present to potentially affect t-statistics In general, multicollinearity is not a problem in this data set Variance inflation factors are less than 10 except for the cases discussed explicitly in the text The single systematic exception to the general rule is the weather variables Footnote 10 describes multicollinearity in this context more fully

Table 2—Crop-water production function estimates, Cobb-Douglas specification

Independent variable	Irrigated crop				
	Alfalfa	Barley	Corn silage	Cotton	Dry beans
	Tons	*Bu*	*Tons*	*Pounds*	*Cwt*
Crop-specific					
Log IRRWATER	0 1382	0 0201	0 0856	0 1263	0 0257
(ac-in/ac)	(9 66)[1]	(1 13)	(4 20)	(5 23)	(0 92)
Log LAND[2]	0 0338	0 0146	0 0298	0 0123	-0 0032
(acres)	(4 73)	(1 49)	(2 84)	(1 38)	(-0 22)
SPKLRTECH	0 0713	0 0110	-0 0332	-0 1266	-0 0674
(d v)[3]	(3 33)	(0 40)	(-1 06)	(-3 36)	(-1 92)
SUBTECH	-0 0062	NA[4]	NA	NA	NA
(d v)	(-0 06)				
Farm-level					
HIGHMGMT	0 1531	0 1113	0 0851	0 0342	0 0377
(d v)	(5 54)	(3 75)	(2 94)	(1 24)	(1 08)
LOWMGMT	-0 1603	-0 1020	NA	NA	NA
(d v)	(-4 28)	(-1 72)			
SURFACE	0 0034	-0 0154	-0 0492	-0 0168	-0 0011
(d v)	(0 15)	(-0 56)	(-1 78)	(-0 62)	(-0 03)
DSCNTN	-0 1298	-0 0914	-0 0530	-0 0291	-0 0703
(d v)	(-5 72)	(-3 10)	(-1 82)	(-1 00)	(-1 70)
LRGDRYLND	0 0039	-0 0975	-0 0276	-0 0679	0 0218
(d v)	(0 14)	(-2 75)	(-0 70)	(-1 77)	(0 45)
NONFAMILY	-0 0184	0 0411	0 0030	0 0520	0 0581
(d v)	(-0 76)	(1 44)	(0 11)	(1 95)	(1 57)
Weather					
Log RAIN	-0 1119	-0 0398	0 0241	-0 0270	0 0603
(ac-in/ac)	(-5 40)	(-1 36)	(1 02)	(-1 24)	(1 61)
Log CDD	0 0178	-0 0262	0 0401	0 7598	0 1647
(days/ac)	(0 91)	(-1 15)	(0 92)	(7 26)	(3 24)
HRDRAIN	0 0435	-0 0573	-0 0469	-0 0189	-0 0744
(days)	(4 15)	(-2 91)	(-4 57)	(-1 02)	(-2 58)
HEAT90	0 0005	0 0002	-0 0007	-0 0080	-0 0025
(days)	(0 55)	(0 17)	(-0 59)	(-3 49)	(-1 26)
Climate					
VERYDRY	-0 0150	0 0213	-0 0026	0 2669	0 1108
(d v)	(-0 47)	(0 46)	(-0 06)	(4 81)	(1 94)
DRY	0 0048	-0 0370	-0 0100	0 1285	0 1106
(d v)	(0 16)	(-0 86)	(-0 27)	(2 12)	(2 02)
WET	-0 1082	0 0678	0 0516	0 0074	NA
(d v)	(-1 84)	(0 75)	(0 84)	(0 13)	
VERYWET	0 0592	NA	0 2423	NA	NA
(d v)	(0 75)		(3 40)		
COLD	-0 0645	-0 0712	0 0113	NA	NA
(d v)	(-1 33)	(-1 22)	(0 12)		
COOL	-0 0322	-0 0047	-0 0805	NA	-0 0354
(d v)	(-0 99)	(-0 11)	(-2 06)		(-0 62)
WARM	0 1398	-0 2563	-0 1042	0 0327	-0 1508
(d v)	(3 31)	(-3 96)	(-2 34)	(0 43)	(-2 08)
HOT	0 1780	-0 0770	-0 0651	0 0010	-0 0102
(d v)	(4 06)	(-1 13)	(-1 06)	(0 01)	(-0 13)
Soil quality					
LNDCLASSA	0 0633	0 1157	0 0263	0 0864	-0 0789
(d v)	(1 78)	(1 85)	(0 64)	(2 92)	(-1 22)
LNDCLASSC	-0 1354	-0 0595	-0 0309	-0 2316	0 0517
(d v)	(-5 78)	(-1 20)	(-0 89)	(-3 12)	(0 93)
SANDY	-0 0429	-0 0788	-0 0366	-0 0349	0 2277
(d v)	(-1 25)	(-1 36)	(-0 82)	(0 73)	(3 68)
CLAYEY	0 0032	-0 0448	-0 1034	0 0450	0 0032
(d v)	(0 08)	(-0 94)	(-2 28)	(1 20)	(0 05)
SLOPE	0 0202	0 0127	0 0142	0 0325	0 0040
(% slope)	(3 76)	(1 90)	(1 71)	(1 74)	(0 36)
Intercept	0 7819	4 4707	2 3971	0 9157	1 7854
	(5 42)	(24 19)	(8 02)	(1 43)	(5 37)
Adjusted R^2	0 193	0 104	0 110	0 603	0 094
No observations	3,516	1,169	734	411	748

(continued)

Table 2—Crop-water production function estimates, Cobb-Douglas specification (continued)

Independent variable	Irrigated crop			
	Grain corn	Grain sorghum	Other hay	Potatoes
	Bu	*Bu*	*Tons*	*Cwt*
Crop-specific				
Log IRRWATER	0 0641	0 1147	0 0779	0 1145
(ac-in/ac)	(4 72)	(4 50)	(4 49)	(3 92)
Log LAND²	0 0478	0 0360	–0 0332	0 0301
(acres)	(9 01)	(2 97)	(–3 72)	(2 63)
SPKLRTECH	–0 0357	–0 0492	0 1599	0 0305
(d v)	(–2 20)	(–1 35)	(4 07)	(0 69)
SUBTECH	NA	NA	–0 0835	NA
(d v)			(–0 91)	
Farm-level				
HIGHMGMT	0 0545	0 0341	0 1283	0 0645
(d v)	(3 82)	(1 02)	(2 49)	(1 94)
LOWMGMT	–0 0602	NA	–0 1009	NA
(d v)	(–1 28)		(–2 33)	
SURFACE	–0 0472	–0 0896	–0 0137	0 0223
(d v)	(–2 73)	(–1 77)	(–0 38)	(0 62)
DSCNTN	–0 0671	–0 1149	–0 0666	–0 0583
(d v)	(–3 94)	(–3 57)	(–2 03)	(–1 25)
LRGDRYLND	–0 0450	0 0339	0 0557	–0 0397
(d v)	(–2 43)	(1 01)	(1 18)	(–0 81)
NONFAMILY	0 0295	–0 0052	–0 0502	–0 0263
(d v)	(1 87)	(–0 14)	(–1 49)	(–0 79)
Weather				
Log RAIN	0 0328	0 0901	–0 1191	–0 2311
(ac-in/ac)	(2 15)	(2 49)	(–3 34)	(–5 24)
Log CDD	0 1379	0 0943	0 0439	–0 0463
(days/ac)	(4 20)	(0 87)	(2 30)	(–1 52)
HRDRAIN	–0 0118	0 0031	–0 0048	0 0317
(days)	(–2 40)	(0 26)	(–0 27)	(0 98)
HEAT90	–0 0019	–0 0015	–0 0007	–0 0015
(days)	(–2 59)	(–0 76)	(–0 46)	(0 81)
Climate				
VERYDRY	0 0105	–0 1047	0 0173	0 0466
(d v)	(0 41)	(–1 62)	(0 40)	(0 64)
DRY	0 0513	–0 1182	0 0784	0 1130
(d v)	(2 78)	(–2 84)	(1 89)	(1 65)
WET	–0 0408	–0 0081	0 0419	NA
(d v)	(–1 64)	(–0 14)	(0 49)	
VERYWET	–0 0451	–0 0056	–0 0323	NA
(d v)	(–1 17)	(–0 06)	(–0 45)	
COLD	NA	NA	0 0667	–0 1037
(d v)			(0 92)	(–1 22)
COOL	–0 0687	0 0622	0 0668	0 0812
(d v)	(–3 15)	(0 52)	(1 12)	(1 37)
WARM	0 0318	–0 0913	0 2920	–0 1448
(d v)	(1 41)	(–2 08)	(3 54)	(–1 27)
HOT	–0 1974	–0 2430	0 2225	NA
(d v)	(–5 36)	(–3 21)	(2 71)	
Soil quality				
LNDCLASSA	0 0178	0 0183	0 1436	–0 0638
(d v)	(0 83)	(0 42)	(1 63)	(–0 53)
LNDCLASSC	–0 0489	–0 2820	–0 1216	–0 0573
(d v)	(–2 29)	(–4 24)	(–3 67)	(–1 41)
SANDY	0 0227	–0 0649	–0 0307	–0 0511
(d v)	(1 10)	(–1 27)	(–0 60)	(–0 88)
CLAYEY	–0 0551	–0 0147	0 0126	NA
(d v)	(–2 52)	(–0 35)	(0 19)	
SLOPE	0 0157	–0 0293	0 0232	0 0256
(% slope)	(3 23)	(–1 10)	(3 41)	(3 00)
Intercept	3 5033	3 3371	0 4746	5 7665
	(15 83)	(4 56)	(2 67)	(22 86)
Adjusted R^2	0 226	0 194	0 172	0 312
No observations	1,485	623	1,492	393

(continued)

Table 2—Crop-water production function estimates, Cobb-Douglas specification (continued)

Independent variable	Irrigated crop			
	Rice	Soybeans	Sugar beets	Wheat
	Cwt	*Bu*	*Tons*	*Bu*
Crop-specific				
Log IRRWATER	0 0868	0 0938	0 0549	0 0833
(ac-in/ac)	(2 25)	(2 48)	(1 96)	(6 66)
Log LAND[2]	–0 0107	–0 0151	–0 0246	0 0257
(acres)	(–0 85)	(–0 86)	(–1 91)	(4 13)
SPKLRTECH	NA	–0 0337	0 0091	–0 0289
(d v)		(–0 79)	(0 26)	(–1 53)
SUBTECH	NA	NA	NA	NA
(d v)				
Farm-level				
HIGHMGMT	0 0241	0 1231	0 0163	0 0694
(d v)	(0 63)	(3 24)	(0 54)	(3 97)
LOWMGMT	NA	NA	NA	–0 1268
(d v)				(–1 99)
SURFACE	–0 0226	–0 2068	–0 0779	0 0465
(d v)	(–0 88)	(–2 80)	(–2 62)	(2 45)
DSCNTN	NA	–0 0260	–0 0899	–0 0913
(d v)		(–0 52)	(–2 13)	(–4 71)
LRGDRYLND	–0 0481	0 0172	–0 1295	–0 0751
(d v)	(–1 38)	(0 32)	(–3 22)	(–3 37)
NONFAMILY	0 0026	0 0594	0 0222	0 0374
(d v)	(0 09)	(1 28)	(0 75)	(2 08)
Weather				
Log RAIN	0 1216	0 2070	–0 0594	–0 0659
(ac-in/ac)	(1 60)	(1 82)	(–2 24)	(–3 97)
Log CDD	–0 2268	0 3706	0 2116	–0 0603
(days/ac)	(–0 84)	(1 78)	(2 33)	(–2 83)
HRDRAIN	–0 1394	–0 0329	–0 1085	–0 0264
(days)	(–5 25)	(–2 17)	(–3 79)	(–3 27)
HEAT90	0 0042	–0 0017	–0 0030	0 0014
(days)	(0 81)	(–0 52)	(–1 52)	(1 70)
Climate				
VERYDRY	NA	NA	0 0878	0 1887
(d v)			(1 80)	(6 66)
DRY	0 0186	–0 0701	0 1153	–0 0049
(d v)	(0 28)	(–0 10)	(2 57)	(–0 20)
WET	–0 0853	–0 0095	NA	–0 1584
(d v)	(–1 01)	(–0 16)		(–3 28)
VERYWET	NA	–0 0727	NA	–0 1830
(d v)		(–0 92)		(–2 43)
COLD	NA	NA	NA	–0 0715
(d v)				(–1 60)
COOL	NA	0 0681	–0 1184	–0 0188
(d v)		(0 60)	(–2 14)	(–0 68)
WARM	NA	–0 0820	–0 1000	–0 0958
(d v)		(–1 23)	(–1 81)	(–3 53)
HOT	–0 0097	NA	–0 1282	–0 0996
(d v)	(–0 09)		(–1 71)	(–2 74)
Soil quality				
LNDCLASSA	0 0080	0 0413	0 0436	–0 0104
(d v)	(0 15)	(0 83)	(0 64)	(–0 34)
LNDCLASSC	NA	NA	–0 0885	–0 1210
(d v)			(–1 88)	(–4 81)
SANDY	NA	0 0084	NA	–0 0070
(d v)		(0 14)		(–0 24)
CLAYEY	0 0057	–0 0111	–0 0285	0 0268
(d v)	(0 14)	(–0 17)	(–0 72)	(1 05)
SLOPE	–0 0038	–0 0034	0 0191	0 0254
(% slope)	(–0 13)	(–0 16)	(1 70)	(5 44)
Intercept	5 2658	0 4506	1 9147	4 2803
	(2 86)	(0 35)	(3 26)	(28 57)

(continued)

Table 2—Crop-water production function estimates, Cobb-Douglas specification (continued)

Independent variable	Irrigated crop			
	Grain corn	Grain sorghum	Other hay	Potatoes
	Bu	*Bu*	*Tons*	*Cwt*
Adjusted R^2	0 539	0 096	0 369	0 373
No observations	142	333	288	1,923

[1]Numbers in parentheses are t-statistics
[2]As described in the text, coefficients on the land variable measure returns to scale because the production functions are on a per-acre basis
[3]d v = dummy variable
[4]NA = insufficient observations available to estimate the variable

Table 3—Crop-water production function estimates, selected variables in quadratic specification

Crop	IRRWATER (ac-in/ac)	IRRWATER SQUARED (ac-in/ac sq)	SPKLRTECH (d v)[1]	HIGHMGMT (d v)	LOWMGMT (d v)	SURFACE (d v)	DSCNTN (d v)	Adjusted R^2
Alfalfa	0 0268	–0 000094	0 2707	0 4687	–0 3586	–0 0896	–0 3483	0 342
(tons)	(6 12)[2]	(–2 02)	(4 06)	(5 36)	(–3 04)	(–1 29)	(–4 91)	
Barley	0 0941	–0 0009	0 6658	9 6629	–5 0046	–2 1349	–7 1485	0 141
(bu)	(0 74)	(–0 59)	(0 38)	(5 10)	(–1 33)	(–1 22)	(–3 80)	
Corn silage	0 2016	–0 0021	–0 1522	1 3428	NA[3]	–0 6422	–1 0608	0 116
(lbs)	(3 97)	(–2 88)	(–0 28)	(2 58)		(–1 30)	(–2 06)	
Cotton	5 6867	–0 0358	–84 7650	24 1197	NA	–14 1238	–19 1478	0 626
(lbs)	(3 10)	(–2 08)	(–2 92)	(1 10)		(–0 65)	(–0 83)	
Dry beans	0 1253	–0 0016	–1 2499	0 4497	NA	0 3610	–1 0272	0 080
(cwt)	(2 08)	(–2 11)	(–1 81)	(0 65)		(0 52)	(–1 25)	
Grain corn	0 8884	–0 0097	–0 4071	7 3661	–7 6723	–5 6206	–7 5591	0 220
(bu)	(4 92)	(–4 08)	(–0 22)	(4 42)	(–1 40)	(–2 77)	(–3 81)	
Grain sorghum	1 1294	–0 0158	–3 2451	4 1714	NA	–7 5860	–8 5330	0 204
(bu)	(3 78)	(–2 70)	(–1 26)	(1 76)		(–2 06)	(–3 73)	
Other hay	0 0086	–0 000013	0 4161	0 2330	–0 0889	0 0015	–0 1478	0 205
(tons)	(1 96)	(–0 26)	(4 91)	(2 09)	(–0 95)	(0 02)	(–2 09)	
Potato	2 9106	–0 0220	10 9760	26 0361	NA	15 7978	–23 2053	0 369
(cwt)	(3 31)	(–2 52)	(0 76)	(2 43)		(1 36)	(–1 56)	
Rice	–0 0606	0 0014	NA	2 1920	NA	–0 4599	NA	0 412
(cwt)	(–0 32)	(0 98)		(0 81)		(–0 24)		
Soybeans	0 4013	–0 0052	–0 1076	3 1563	NA	–4 1481	–0 9587	0 133
(bu)	(2 14)	(–1 56)	(–0 08)	(2 62)		(–1 76)	(–0 62)	
Sugar beets	0 1161	–0 0010	–0 0142	–0 0012	NA	–1 1596	–2 1096	0 379
(tons)	(1 80)	(–1 36)	(–0 02)	(–0 002)		(–1 73)	(–2 29)	
Wheat	0 5035	–0 0048	–1 2573	5 0418	–6 4452	2 9350	–5 6806	0 422
(bu)	(5 48)	(–3 93)	(–1 06)	(4 61)	(–1 61)	(2 43)	(–4 65)	

[1]d v = dummy variable
[2]Numbers in parentheses are t-statistics
[3]NA = insufficient observations available to estimate the variable

Irrigation Water

Irrigation water is a highly significant determinant of crop output regardless of the functional form, with most t-statistics exceeding significance at the 0 01 level (tables 2 and 3) Only a few coefficients are insignificant barley, in both specifications, despite substantial variation in water application rates (table 1), dry beans in the Cobb-Douglas specification, and rice and sugar beets in the quadratic specification With rice and sugar beets, the insignificance is due to multicollinearity between the linear and squared terms for irrigation water A joint test of the significance of both coefficients shows significance at the 0 05 and 0 10 levels in the rice and sugar beets equations, respectively

The parameter estimates also indicate the diminishing marginal productivity of irrigation water for all 13 crops Although the quadratic function does not impose concavity, the function is concave in water for every crop but rice.[8] The Cobb-Douglas function imposes concavity, with significant coefficients on the water variable providing statistical confirmation of diminishing marginal productivity.

The Cobb-Douglas results are only somewhat comparable to previous empirical estimates. The coefficient estimates on the irrigation water variable range from 0.020 (barley) to 0.138 (alfalfa). Previous estimates for five vegetables varied from 0.005 to 0.079 (Just and others) and for wheat from 0.041 to 0.241 depending on the model and the econometric technique (Antle and Hatchett, 1986). For the majority of crops, the Cobb-Douglas results provide new empirical estimates.

Comparing the quadratic functions to previous research reveals distinct differences in yield-maximizing water application rates. Yield-maximizing rates reported here are significantly higher than previous results for the five crops for which comparisons can be made (Grimm and others, p. 188).[9] The differences can be attributed to data sources. Previous research relies on data from field experiments (experimental test plots), producing two substantive implications for the comparisons. First, field experiments typically involve relatively uniform water applications, while water applications are nonuniform in actual production activity. Thus, maximum yield estimates based on actual behavior occur at higher water application rates than in field experiments. Second, field experiments are designed in part to characterize maximum yield. With survey data, yield-maximizing irrigation rates will tend to be outside the range of most of the observed data because rational profit-maximizing growers do not produce where the marginal product of water is zero or negative. This article's quadratic functions, while concave in irrigation water, generate near-linear functions for many of the crops. Thus, survey data may not accurately characterize maximum yield.

Land

With the Cobb-Douglas functions specified on a per-acre basis, estimates of the coefficient on land must be computed from the estimated parameters (see footnote 3). For all crops except cotton and soybeans, land coefficients range between 0.73 and 1.20. Table 4 reports the coefficients as output elasticities of land. The land coefficient estimates are roughly an order of magnitude greater than the estimates for irrigation water, indicating that land overshadows irrigation water as a production input.

Land coefficient estimates are not available from the quadratic functions because the land input cancels out when specifying the function on a per-acre basis.

Water Management Variables

Modern irrigation technologies either increase water application efficiency *per se* or substitute for poorer quality land, like sandy soil or relatively sloped topography (Caswell and Zilberman 1986, Lichtenberg, 1989). Both roles are expected to increase crop yields provided that other variables control for land quality. The dummy variable indicating the presence of sprinkler irrigation, SPKLRTECH, has the expected sign and significance with alfalfa hay and other hay, increasing yields by 0.27 ton and 0.42 ton, respectively, in the quadratic specification (table 3). With the Cobb-Douglas form, coefficients on SPKLRTECH have significant, negative signs with cotton, dry beans, and grain corn. SPKLRTECH frequently is insignificant with the remaining crops.

Evidence from previous research that irrigation technology augments low-quality land explains in part the weak results for irrigation technology. If sprinklers tend to be located in fields with relatively sandy soil or sloped topography, they serve incidentally as a field-level proxy for poor land quality. The field level is a finer degree of geographic detail than the county-level soil quality variables used in the estimation. Thus, the two functions of irrigation technology—reducing water application rates versus augmenting land quality—cannot be accurately isolated given the current land-quality variables.

The other irrigation-related variables generally have the anticipated signs. Relying on more

[8]Cross-product interaction variables are not included in the quadratic specification for two empirical reasons. First, interaction variables between irrigation water and irrigation technology and between irrigation water and water management introduced serious multicollinearity into the analysis. The consequence was inefficient estimates of irrigation water coefficients without the interaction variables, 10 of 13 linear terms for the water variables are statistically significant, with the variables, this drops to 2 of 13 significant variables. Second, the weather, climate, and soil quality variables are included as general indicators of physical conditions. These variables are not used as determinants of water productivity (via cross-product interaction terms with irrigation water) because, as county level data, they are not sufficiently accurate for that purpose. Any information added by this type of interaction variables would be suspect.

[9]Grimm and others computed yield-maximizing water levels for von Liebig and polynomial functions for five crops. Our results for the same five crops find that, with the exception of corn silage our estimates of quadratic functions require more water to maximize yield than their polynomial functions. Comparing results in terms of acre-inches per acre (with the Grimm and others results presented first), the relations include corn silage, 54.7 versus 47.5, cotton, 37.7 versus 79.5, grain corn, 24.9 versus 45.6, sugar beets, 50.2 versus 56.6, and wheat, 33.8 versus 52.4.

Table 4—Output elasticity measures of irrigation water and land

Crop	Output elasticity of irrigation water		Output elasticity of land	
	Cobb-Douglas	Quadratic[1]	Cobb-Douglas	Quadratic[1]
Alfalfa	0 138	0 145	0 990	0 901
Barley	0 020	0 014	1 061	1 164
Corn silage	0 086	0 118	0 880	1 022
Cotton	0 126	0 115	0 153	0 435
Dry beans	0 030	0 061	0 746	0 709
Grain corn	0 064	0 070	0 813	0 797
Grain sorghum	0 115	0 112	0 737	1 003
Other hay	0 078	0 112	0 964	1 428
Potatoes	0 114	0 128	1 193	0 885
Rice	0 087	0 107	1 008	1 464
Soybeans	0 094	0 088	0 313	0 767
Sugar beets	0 055	0 064	0 768	1 206
Wheat	0 083	0 082	1 069	1 011

[1]Elasticity measures for the quadratic functions are evaluated at mean input levels of the data

sophisticated techniques of irrigation scheduling (HIGHMGMT) improves yields for 8 of 13 crops (alfalfa, barley, corn silage, grain corn, other hay, potatoes, soybeans, and wheat) Relying on fewer sophisticated techniques (LOWMGMT) depresses yields for three of five crops in the Cobb-Douglas form (alfalfa, other hay, and wheat), but only one of five crops in the quadratic form (alfalfa) The irrigation scheduling coefficients indicate that, with some crops, managerial inputs can successfully substitute for irrigation water Discontinuing irrigation for a period of the growing season (DSCNTN) depresses yields for 7 of 12 crops Finally, farms with surface water as their only source (SURFACE) experience lower yields of grain corn, grain sorghum, soybeans, and sugar beets in at least one specification, and a higher yield of wheat in both specifications The tendency for lower yields with SURFACE is consistent with the hypothesis that complete reliance on surface water constrains irrigation flexibility For most crops, though, relying solely on surface water does not constrain options enough to influence yield

Weather and Other Variables

While rainfall was expected to increase yields, the results suggest otherwise Coefficients on RAIN are positive and significant only with the Cobb-Douglas specification for grain corn and sorghum and the quadratic specification for dry beans [10] RAIN coefficients are negative and significant with five crops, and otherwise are not significantly different from zero One plausible explanation is that, while rainfall contributes water for plant growth it can be both untimely and excessive More detailed data on the timing and volume of irrigations, rainfall, and plant growth would be required to distinguish the different effects of rainfall

Energy availability for plant growth, as measured by cooling degree days (CDD), contributed positively to yields of seven crops [11] The common elements for most of these crops are either an agronomic requirement for a relatively long growing season (cotton, grain corn, and sugar beets) or an opportunity for multiple harvests (alfalfa hay and other hay) (Hagan, Haise, and Edminster, 1967, Jensen, 1969) CDD affected the yields of wheat and barley negatively for at least one functional form The negative coefficients can be interpreted as excessive heat Small grains, such as wheat and barley, can suffer heat stress both in the spring (early growth stages) and midsummer (late stages) (Ash and Lin, 1987)

The remaining variables include control variables for general farm characteristics, extreme weather events, climate, and soil quality Variables in these four categories frequently are statistically significant determinants of crop output The variables performing best include HRDRAIN, LNDCLASSC, SLOPE, and IRRSHARE (the own crop's share of total irrigated cropland) They are significant in explaining output of 7-8 of the 13 crops, and their signs conform to expectations On the other hand, HEAT90 had little explanatory value

[10]According to the multicollinearity diagnostics the linear and squared terms for the weather variables in the quadratic regressions have a high degree of multicollinearity Despite the multicollinearity, t-statistics are statistically significant at the 0 10 level or better for the weather variables with 9 of the 13 crops A problem of multicollinearity resulting in insignificant parameter estimates on the weather variables appears to have occurred only with the quadratic regressions for corn silage, grain sorghum, rice and sugar beets Multicollinearity probably occurs with these crops because they are produced in relatively small geographic areas under relatively homogeneous weather conditions

[11]The contribution of CDD to cotton output is notably large With the Cobb-Douglas form, the elasticity of output with respect to CDD is 0 7598 This is substantially larger than the combined contribution of irrigation water and land to cotton production

While a complete discussion of these results is omitted to conserve space, either the previous section or the appendix contains expectations for the variables' signs, and table 2 reports their coefficient estimates for the Cobb-Douglas function Although not reported, results for these variables with the quadratic form are very consistent with the Cobb-Douglas form An extended discussion of these variables and a complete set of results for the quadratic form are available from the authors

General Production Characteristics

The econometric results also produce information on general characteristics of the production functions This section focuses on returns to scale, output elasticities of irrigation water and land, and substitutability of irrigation water and land

Returns to Scale

With the specification of per-acre production functions, the estimated coefficient on the land variable in the Cobb-Douglas form directly measures returns to scale in irrigated production (table 2) (The quadratic function imposes constant returns to scale) A significant positive coefficient on the land variable indicates increasing returns to scale, a significant negative coefficient indicates decreasing returns to scale, and a coefficient not significantly different from zero indicates constant returns to scale Adding 1 0 to the land coefficient produces the conventional measure of returns to scale, the resulting number indicates homogeneity of degree k, where $k = 1$ is linear homogeneity The results range from $k = 1\ 048$ for grain corn to $k = 0\ 967$ for other hay crops Alfalfa, corn silage, grain corn, grain sorghum, potatoes, and wheat exhibit increasing returns to scale technologies Barley, cotton, dry beans, rice, soybeans, and sugar beets exhibit constant returns to scale Only other hay crops exhibit decreasing returns to scale While the results for wheat are consistent with Antle and Hatchett's, comparable information does not exist on crop-specific returns to scale for the other 12 irrigated crops The results suggest that, while imposing constant returns to scale is not necessarily an accurate assumption, it may be defensible for many purposes because the deviations from constant returns to scale are minor

Output Elasticities of Irrigation Water

The output elasticity of irrigation water provides a common measure across crops and functional forms of the effect of irrigation water on output (table 4) The estimated elasticities consistently are very inelastic across crops and functional forms Across crops, the elasticities fall in a fairly narrow range of 0 014 (barley) to 0 145 (alfalfa) Across functional forms for the same crop, other hay crops show the largest disparity in estimated elasticities, with a difference of only 0 034 The small differences across functional forms lend credibility to the results

The elasticities give insight into the production consequences of irrigation water conservation The results provide persuasive evidence that, within the broad range of water application rates observed in the data (table 1), output is very inelastic with respect to water In a period of competition for existing Western water supplies with no new supplies on the horizon, the elasticities imply that reductions in production associated with diminished irrigation water supply would be much smaller, proportionately, than the water supply reductions For these 13 irrigated crops, for example, a 10-percent reduction in water use would induce at most a 1 5-percent reduction in output, *ceteris paribus* Using mean water application rates (table 1), a 10-percent reduction in water equals 3 25 acre-inches per acre and 2 91 acre-inches per acre on cotton and alfalfa, respectively This translates into an average per-acre decline of 10 5 pounds in cotton production and 0 063 tons in alfalfa production using the quadratic elasticities Given the mean yields of 916 pounds per acre of cotton and 4 326 tons per acre of alfalfa, the output reductions are relatively minor [12]

Output Elasticities of Land

Compared with irrigation water, output elasticities of land show relatively greater elasticity and a wider range of estimates (table 4) [13] The range across crops shows that land differs markedly in its contribution to crop output Regardless of functional form, the barley, rice, and wheat elasticities are relatively elastic, while the soybean and cotton elasticities are quite inelastic Elasticities for alfalfa, dry beans, grain corn, grain sorghum, and silage corn are slightly inelastic to

[12]These calculations can be used to predict the impact of a 10-percent reduction in irrigation water on average per-farm cotton output and alfalfa output Based on the mean irrigated cotton acreage on a cotton-producing farm, 936 acres, and mean irrigated alfalfa acreage on an alfalfa-producing farm, 263 acres (table 1), a 10-percent reduction in irrigation water use translates into 254 acre-feet of conserved water and 9,828 pounds of forgone cotton output or, for alfalfa, 65 acre-feet of conserved water and 16 6 tons of forgone alfalfa output

[13]Transforming the per-acre quadratic function to a standard quadratic production function, accomplished by multiplying through by land, permits computation of land's output elasticity for the quadratic form The Cobb-Douglas elasticities are computed using the relationship in footnote 3 and information from table 2

unitary elastic, falling in the range of 0 7 to 1 0 for both functional forms Unlike the output elasticities of water, land's output elasticities are not sufficiently uniform to draw general policy-oriented conclusions

Technical Substitution of Irrigation Water and Land

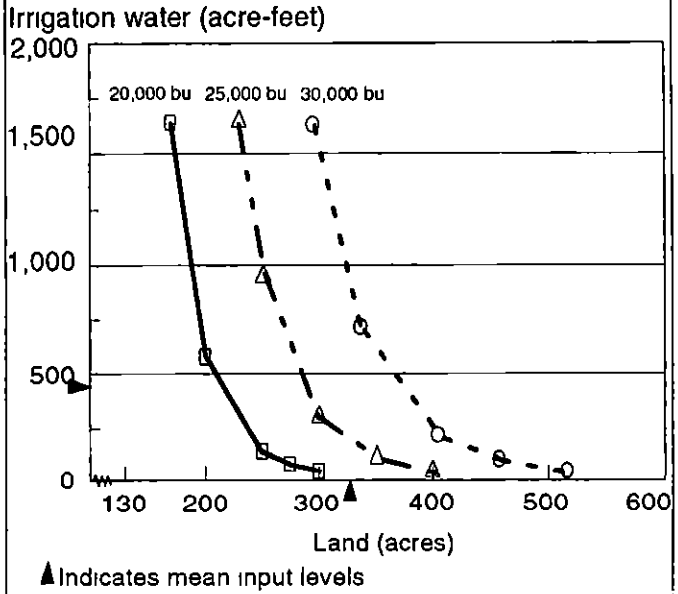
While the elasticities indicate that land is more important to production than irrigation water, the relative contribution of the two inputs can be analyzed most effectively by assessing their substitutability Two procedures demonstrate substitutability, the marginal rate of technical substitution between water and land (MRTS) and water-land isoquants Estimates of the Cobb-Douglas case are presented because of their computational simplicity

The MRTS between irrigation water and land measures the volume of water required to substitute for an acre of land to hold output constant (that is, at a point on an isoquant) The Cobb-Douglas MRTS equals $(-\phi/\alpha)$ (w/n), where w is water, n is land, α is water's exponent, and ϕ is land's exponent Evaluated at the crops' mean water and land input levels, selected MRTS levels are alfalfa –20, barley –85, cotton –4, grain corn –22, grain sorghum –9, potatoes –23, and wheat –20 Highly inelastic output elasticities on water or land explain the extreme cases of barley and cotton Since barley's output elasticity of water is so inelastic (α equals 0 020), a large volume of water is required to substitute for land Similarly, since cotton's output elasticity of land is so inelastic (ϕ equals 0 153), a relatively small volume of water is required to substitute for land

Calculating the water application rate implied by the MRTS levels provides another perspective For instance, sorghum's MRTS means that 9 acre-feet of water must be applied to compensate for a marginal decline in sorghum acreage Since farms growing sorghum average 328 acres in the crop (table 1), applying 9 acre-feet over 328 acres minus the marginal decline in acreage results in a change in application rate of 0 33 acre-inch per acre That is, increasing water use by 0 33 acre-inch per acre on the remaining sorghum acres keeps sorghum output constant For other crops, the increases in water application rates (in acre-inches per acre) that hold output levels constant are alfalfa 0 93, barley 4 02, cotton 0 05, grain corn 0 44, potatoes 0 61, and wheat 0 54

Isoquants graphically represent a continuum of rates of input substitution Figure 1 illustrates water-land isoquants for sorghum's Cobb-Douglas

Figure 1
Grain sorghum isoquants, Cobb-Douglas specification[1]



[1] Isoquants are drawn while holding all other inputs constant Returns to scale consequently cannot be displayed via a ray through the origin (as in the classic textbook display)

production function [14] For example, 25,000 bushels of sorghum can be produced by applying either 168 acre-feet to 328 acres (a water application rate of 0 51 acre-feet per acre) or 460 acre-feet to 280 acres (a water application rate of 1 64 acre-feet per acre) Both water application rates are well within two standard deviations of sorghum's mean rate (see table 1)

The water-land isoquant and MRTS levels demonstrate a final critical point water and land *do* substitute For crops with Cobb-Douglas or quadratic production functions, irrigated production does not occur with Leontief, fixed-coefficient technologies in water and land inputs As microeconomic principles suggest, the optimal water-land input combinations depend on relative prices of water and land (among other factors)

Specification Tests

To test if either the quadratic model or the Cobb-Douglas model is correctly specified, we apply the non-nested J-test (Davidson and MacKinnon, 1981) Unlike ordinal measures that select one model in preference to another (such as R^2), non-nested hypothesis tests attempt to establish the "validity" of one or more alternative specifications The test is conducted in two stages In the first

[14] Sorghum is presented because it is one of the crops for which the J-test results do not reject the Cobb-Douglas form The J-test results are described in the next section

stage, the J-test designates one model as the null hypothesis and the competing model as the alternative hypothesis The test involves using the predicted values from the alternative model as an explanatory variable in the null-hypothesis model If the coefficient on the predicted-value variable is statistically different from zero, the test rejects the null-hypothesis model as the "true" specification In the second stage, the roles of the models are reversed and the test procedure repeated Thus, the J-test may reject both specifications, accept both specifications, or accept one specification

The J-test should not be construed as determining the statistical validity of particular coefficient estimates and their related output elasticity measures As a specification test, J-test results do not affect the interpretation given above of the econometric results

At the 0 05 level of significance, the J-test rejects both the Cobb-Douglas and quadratic specifications for seven crops alfalfa, corn silage, grain corn, other hay, soybeans, sugar beets, and wheat (table 5) The test also accepts the Cobb-Douglas specification for barley, grain sorghum, and rice, accepts the quadratic specification for cotton and dry beans and accepts both specifications for potatoes Rejecting both specifications for seven crops is not surprising given the complexity of "true" yield response functions

In three cases, the J-test results can be compared with prior research For grain corn and wheat, non-nested hypothesis tests by Grimm and others rejected quadratic specifications and accepted von

Table 5—J-test results

Crop	H_0 Cobb-Douglas H_1 Quadratic	H_0 Quadratic H_1 Cobb-Douglas	Conclusion[1]
Alfalfa	8 14[2]	6 56	Reject both
Barley	1 61	2 53	Accept Cobb-Douglas
Corn silage	2 24	3 48	Reject both
Cotton	4 71	−1 77	Accept quadratic
Dry beans	3 29	0 33	Accept quadratic
Grain corn	2 99	7 51	Reject both
Grain sorghum	1 21	2 82	Accept Cobb-Douglas
Other hay	4 82	2 71	Reject both
Potatoes	1 62	1 62	Accept both
Rice	1 30	3 85	Accept Cobb-Douglas
Soybeans	3 86	1 97	Reject both
Sugar beets	2 88	3 89	Reject both
Wheat	2 26	7 02	Reject both

[1] J-test results produced from applying a t-test at a 5-percent significance level

[2] Entries are t-statistics from the coefficients on the additional variables used to create the J-test's artificial nesting model

Liebig functions (p 190) By rejecting both the quadratic and the Cobb-Douglas forms for these two crops, our results are not inconsistent with their results For sugar beets, they accepted the quadratic form while we reject it

Three conclusions can be drawn from the specification tests One, the J-test results demonstrate that selecting functional forms based on measures of fit can lead to erroneous conclusions While measures of fit, such as adjusted R^2 or mean square error (MSE), always find a "better" specification, the J-test procedure chose a "true" specification on only 5 of the 13 crops evaluated here Rice is the only crop for which the J-test, adjusted R^2, and MSE all select the Cobb-Douglas specification While MSE or adjusted R^2 criteria prefer the quadratic specification on 10 of 13 crops, the J-test selects the quadratic as the "true" form on only cotton and dry beans The J-test also fails to reject the hypothesis that the Cobb-Douglas specification correctly describes grain sorghum production even though the adjusted R^2 is larger with the quadratic function Non-nested hypothesis testing thus provides an important decisionmaking tool when theoretical considerations do not dictate correct functional specifications

Two, the J-test results underscore the difference between agronomic and economic criteria for choosing functional form Based on agronomic principles, Hexem and Heady ruled out Cobb-Douglas functions *a priori* (p 36) The Cobb-Douglas form contradicts agronomic principles because its total physical product in an input never achieves a maximum In economic terms, however, the negative portion of a marginal physical product function—the portion beyond maximum yield—is irrelevant since profits cannot be maximized in that region Because the FRIS data are based on actual production decisions rather than field experiments, the J-test demonstrates that the Cobb-Douglas function is well suited to economic evaluation of some crops Functional forms that preclude negative marginal product should not be rejected *a priori* when the estimates evaluate the behavior of economic agents

Three, other functional forms should be studied when data availablity does not restrict options The J-test results indicate that the restrictions imposed by the Cobb-Douglas and quadratic functions frequently limit the analysis With a set of detailed agronomic variables, the von Liebig specification should be evaluated With more information on other purchased inputs (like labor, capital, and chemicals) or more variation in price data, more flexible functional forms such as the translog function should be evaluated This paper's results—based on application to 13 irrigated crops of the two most common forms of production functions—create a strong basis for future dialogue on the merits of various functional forms

Summary and Conclusions

As the West moves fully into an era of water management and conservation, economic analysis of irrigated agriculture will continue to inform debate and decisions concerning Western water policy Over 25 years ago, Ruttan's seminal research established a solid econometric foundation for assessing the profitability of regional irrigation development in the United States His focus was on the extensive margin of irrigation development what is the value of additional irrigated acreage in a region? In the emerging water-management era, assessment of regional water policy remains an important component of economic research For example, U S Bureau of Reclamation policy, Federal law on interstate water marketing, and the possibility of climate change have implications for Western irrigated agriculture in its entirety At the same time, the research focus has changed to the intensive margin how will agricultural output be affected by input substitution and a reduction in irrigation water application rates?

This research establishes a partial foundation for evaluating irrigation water conservation and input substitution by estimating irrigated crop production functions using farm-level observations from the 1984 Farm and Ranch Irrigation Survey The analysis is comprehensive, involving coverage of 13 irrigated crops with data from the 17 Western States The analysis also is consistent, with uniformity across crops in functional specifications, data sources, and variable definitions The combination of comprehensiveness and consistency creates the potential to use these results in further analysis of irrigated agriculture in the Western United States and, perhaps, other regions of the world

For each of the 13 crops, the estimates capture the diminishing marginal productivity of irrigation water Although certainly not surprising, this had not been established econometrically for many of the crops The results also produce new information for these crops on the output elasticities of irrigation water, returns to scale, and the marginal rate of technical substitution between land and water

J-test results fail to reject the Cobb-Douglas specification as the correct function for four crops and fail to reject the quadratic function for three crops The acceptance of the Cobb-Douglas function for some crops implies that, when estimating a production function with survey data rather than field experiment data, functional forms without the ability to estimate maximum yield should not be excluded for consideration *a priori* They are particularly suitable for evaluating behavior with economic content, which typically does not include applying water to the point of zero or negative marginal physical product

The results of this research contain one immediate policy implication for water conservation in the West Because the output elasticities of irrigation water are highly inelastic for every crop examined, producers should be able to mitigate many of the production impacts of water conservation efforts This holds regardless of whether the conservation occurs from voluntary efforts, such as water marketing, or through policy-imposed restrictions in irrigation water supply

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Appendix I—Variable Descriptions and Definitions

The 1984 Farm and Ranch Irrigation Survey (U S Department of Commerce, Bureau of the Census, 1986a) is a 6-percent stratified random sample of irrigated farms from the *1982 Census of Agriculture* Weather, climate, and soil-quality variables also are emerged with the FRIS variables to capture the impact of the physical environment on crop yields Hexem and Heady (chap 10) pioneered the use of physical variables in estimating production functions for irrigated crops

Crop-Specific Variables

YIELD	– Per-acre crop output
IRRWATER	– Per-acre irrigation water application by crop

LAND – Acres of the crop irrigated

SPKLRTECH – Dummy variable that is 1 if the crop is irrigated with sprinkler technology and 0 if the technology is gravity or SUBTECH

SUBTECH – Dummy variable that is 1 if the crop is irrigated with subirrigation technology and 0 if the technology is gravity or sprinkler

All crop-specific variables are from 1984 FRIS

Farm-Level Variables

HIGHMGMT – Dummy variable that is 1 if the irrigation decision considers any advanced irrigation management method (media reports, soil moisture sensing devices, or commercial scheduling) and 0 if the look of the crop, feel of the soil, or any LOWMGMT method was the basis for decision

LOWMGMT – Dummy variable that is 1 if the irrigation decision is made on either a fixed time schedule method (calendar schedule) or the producer had no choice in when to irrigate (water delivered by irrigation organization in turn) and 0 if the look of the crop, feel of the soil, or any HIGHMGMT method was the basis for decision

SURFACE – Dummy variable that is 1 if surface water is the sole water source and 0 if ground water is available

DSCNTN – Dummy variable that is 1 if producers indicated that irrigation was discontinued long enough to affect yields and 0 otherwise

LRGDRYLND – Dummy variable that is 1 if the farm has a relatively large nonirrigated acreage and 0 otherwise Threshold levels for "relatively large" varied by crop but placed about 15 percent of the farms in the large class for each crop

SMLIRRLND – Dummy variable that is 1 if the irrigated portion of the farm is relatively small and 0 otherwise Threshold levels for "relatively small" varied by crop but placed about 15 percent of the farms in the small class for each crop

IRRSHARE – The crop's share of total irrigated acres on the farm

NONFAMILY – Dummy variable that is 1 if the farm is in estate or trust, prison farm, Indian reservation, or incorporated under State law and 0 if the farm is a family or partnership operation

All farm-level variables except NONFAMILY are from the 1984 FRIS NONFAMILY is from the *1982 Census of Agriculture* for the FRIS farms The main text describes expectations for the performance of the variables directly associated with irrigation Expected performances of the remaining farm-level variables are farms with a large area in nonirrigated production (LRGDRYLND) have lower yields since managerial talent is diverted from irrigated crops and farm machinery conforms less to irrigated crop needs farms with a small area in irrigated production (SMLIRRLND) have lower yields because these farms are more likely to be part-time operations, the crop's share of total irrigated acreage (IRRSHARE) can increase or decrease yields depending on whether it indicates that the crop is the focus of the operation or suggests that the farm is a monoculture operation receiving lower yields from failure to rotate crops, nonfamily ownership (NONFAMILY) increases yields if professional management improves crop output IRRSHARE and SMLIRRLND are excluded from the Cobb-Douglas specification because they are highly collinear with the crop acreage variable, LAND

Weather Variables

RAIN – The sum of April through September precipitation

CDD – The sum of April through September base 65 cooling degree days Daily cooling degree values represent the number of degrees Fahrenheit that the average temperature exceeds the base

HRDRAIN – The number of days in the months April through June in which rainfall exceeded 1 inch

HEAT90 – The number of days in June, July, and August that the maximum temperature exceeded 90 degrees

All weather variables are from 1984 weather records for cooperative weather stations (U S Dept Commerce, National Climatic Data Center, 1986c) that are selected to be representative of county conditions RAIN measures the water available for plant growth in addition to irrigation water, while CDD measures solar energy availability RAIN and CDD are continuous variables modeled as primary inputs (as in Madariaga and

McConnell) while HRDRAIN and HEAT90 are qualitative variables indicating extreme weather events Among weather variables, we expect RAIN and CDD to affect yields positively and HRDRAIN and HEAT90 to affect yields negatively

Climate Variables

VERYDRY – Dummy variable that is 1 if the average annual precipitation is less than 12 inches and 0 otherwise

DRY – Dummy variable that is 1 if the average annual precipitation is 12 inches or greater but less than 18 inches and 0 otherwise

WET – Dummy variable that is 1 if the average annual precipitation is 24 inches or greater but less than 30 inches and 0 otherwise

VERYWET – Dummy variable that is 1 if the average annual precipitation is 30 inches or greater and 0 otherwise

COLD – Dummy variable that is 1 if the average annual base 65 cooling degree days is less than 300 units and 0 otherwise

COOL – Dummy variable that is 1 if the average annual base 65 CDD is 300 units or greater but less than 800 units and 0 otherwise

WARM – Dummy variable that is 1 if the average annual base 65 CDD is 1,300 units or greater but less than 1,800 units and 0 otherwise

HOT – Dummy variable that is 1 if the average annual base 65 CDD is 1,800 units or greater and 0 otherwise

All climate variables are based on 1951-80 average climatic conditions for cooperative stations (U S Dept Commerce, National Climatic Data Center, 1986b) that are selected to be representative of county conditions The climate variables serve as proxies for unobserved producer decisions affected by climate but made prior to the observation of the production season's weather For example, choice of seed type or crop rotation practices depend on climate, not weather Given a certain seed variety, then, weather conditions during the growing season help to determine crop yield Regional dummy variables are not included because climate variables likely capture most of the important regional differences in the study area

The average precipitation and cooling degree day variables are specified as a series of dummy variables to minimize collinearity with the weather variables Precipitation dummy variables measure the impact on yield relative to the omitted midrange condition of 18-24 inches Similarly, as a surrogate for radiant energy available for plant growth, dummy variables for cooling degree days measure yield relative to the omitted midrange condition of 800-1,300 CDD We expect positive coefficients on the rain variables (given the arid and semi-arid conditions of the study area) negative coefficients on COLD and COOL, and positive coefficients on WARM and HOT The magnitude and significance of the coefficients should vary across crops

Soil-Quality Variables

LNDCLASSA – Dummy variable that is 1 if the soil capability class is 2 25 or less (1 to 8 scale) and 0 otherwise

LNDCLASSC – Dummy variable that is 1 if the soil capability class is 3 5 or greater (1 to 8 scale) and 0 otherwise

SANDY – Dummy variable that is 1 if the soil type is 2 50 or less (1 to 5 scale) and 0 otherwise

CLAYEY – Dummy variable that is 1 if the soil type is 3 75 or greater (1 to 5 scale) and 0 otherwise

SLOPE – Average soil gradient in percent

All soil-quality variables are average county values from the 1982 Natural Resources Inventory conducted by the Soil Conservation Service, USDA (Goebel and Dorsch, 1986) We expect that coefficients on the variables will be significant when crops have inflexible agronomic needs for certain soil conditions Variables for land class and soil texture are constructed as dummy variables Land Class B (2 25 to 3 5) serves as the omitted land class, with LNDCLASSA and LNDCLASSC serving as the extremes As land classes tend to reflect soil productivity, the sign on LNDCLASSA should be positive and LNDCLASSC should be negative Loamy soil is the omitted soil texture (2 5 to 3 75), with SANDY and CLAYEY serving as the extremes SANDY and CLAYEY should typically have negative signs because they were defined to represent extreme conditions Crops that either adapt easily to a variety of soil textures or prefer an extreme soil for agronomic reasons may be exceptions For instance, rice plants prefer clayey soil while potatoes prefer sandy soil Finally, in the observed range of the SLOPE data, topography of the land reflects the beneficial effect of slight slope, which promotes an even application of water, rather than the detrimental effect of extremely sloped topography