CAPITAL MARKETS AND THE STABILITY OF THE GROWTH PROGRESS

By

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I. Introduction

Increasing attention has been paid to providing a
dynamic macro-monetary framework since the publication of
Tobin's _Econometrica_ article "Money and Economic Growth"
in 1965. However, these discussions have in general been
limited to an investigation of the impact of introducing
money into the neoclassical aggregate growth model. These
attempts at modifying and placing the Tobin model on a more
rigorous basis contain in general one severe shortcoming--
they lack stability.[1] Furthermore, the very limited nature
of the portfolio behavior involved, the choice between money
and real capital, does not provide a mechanism for separating

[1] See Keizo Nagatani [11], and for an expanded proof
Mathew Shane, _Capital Markets and The Process of Economic
Growth_, unpublished dissertation, Purdue University, June 1969.

the investment and saving process which is such a fundamental part of the neo-keynesian and neoclassical frameworks. One further shortcoming uniformly appears in these attempts at creating a dynamic macroeconomic analysis. Although the monetary problem has been considered in a growth context, no effort has been made to describe the adjustment process of the economy, except on the growth, i.e., equilibrium, path.

Recently, in this journal [16] Sidrauski and Foley considered a growth model with an alternative financial asset to money. However, the debt asset plays no important role in their analysis and they in no way consider the financial underpinnings of investment and saving behavior and how this relates to the dynamic adjustment process. Enthoven [3] and Stein [18] also include a debt asset, but the role of the asset is not of fundamental importance to the results.

In this paper, I expand the stock-flow framework developed by Clower and Horwich [1,2,5,6,7] into a dynamic macro-monetary theory. The crucial role of the capital markets must be stressed,[2/] for the capital markets become the critical link between real and monetary behavior. It is through the capital markets that saving-investment decisions are separated, and that the process by which monetary policy

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[2/]This crucial role has been largely overlooked in the literature. The major exception is the work of George Horwich cited above, which provides the security oriented framework of this paper.

changes induce real behavioral changes occurs. This is the
very basis of the macro-monetary adjustment process.

The paper will start with a presentation of the model
which includes a stock and flow supply and demand equation
for real capital, money, and securities. The analysis will
then continue on to a presentation of the equilibrium and
stability properties of the model.

II. The Capital Market Growth Model

The model has three markets, (1) a neoclassical pro-
duction and factor market, (2) an existing asset market for
real capital, securities and real balances, and (3) a flow
supply and demand for the three assets derived from saving-
investment behavior. Associated with the three market sec-
tors are three behavioral units: (1) households who hold
all of the securities which firms and government issue, and
whose saving is the flow demand for new securities and real
balances, (2) firms who hold the entire capital stock and
issue securities to finance the accumulation of capital
stock and real balances and (3) the government which finances
its budget deficit by issuing securities and increases the
stock of money through open market operations. As a means
of simplifying the analysis, it will be assumed that house-
holds do not distinguish between the securities issued by
the government and those by firms. Furthermore, all

government expenditures will be treated as consumption.[3]/
Given this basic framework, an explicit statement of the
model will now be presented.

It will be assumed that a single good (Y) is produced
in the economy by a neoclassical production function of the
two factors labor (L) and capital (K).

$$(II.1) \quad Y = F(L,K) \text{ [4]}/$$

This production function satisfies the following well-known
properties:

$$(II.1.a) \quad F_L, F_K > 0 \quad - \text{ Positive Factor Products}$$

$$(II.1.b) \quad F_{LL}, F_{KK} < 0 \quad - \text{ Active Diminishing Returns}$$

$$(II.1.c) \quad \emptyset Y = F(\emptyset L, \emptyset K) \quad - \text{ Linear Homogeneity}$$

The labor force is assumed to be growing at some constant
exogenous rate n.

$$(II.2) \quad \frac{\dot{L}}{L} = n$$

Further, it is assumed that at some initial point (t_o) both
the capital stock and labor force are given:

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[3]/This clearly does not have to be the case. Government
could obviously utilize its expenditures for capital accumu-
lation in the production of public goods.

[4]/It is assumed that each of the production function
variables has a time dimension. They are omitted here for
notational convenience.

$$(\text{II.1.d}) \qquad K(t_o) = K_o$$

$$(\text{II.1.e}) \qquad L(t_o) = L_o$$

An exposition of the flow relationships will now be presented.

Household saving, which in this economy is the only saving, is assumed to be a constant proportion (s) of disposable income (Y_d).

$$(\text{II.3}) \quad S = sY_d$$

Disposable income is equal to total income (Y) minus taxes (gY) plus transfer payments which are assumed to be equal to the entire government deficit $(\dot{G}_s)$ which is simply the new issues of government securities.[5] Thus:

$$(\text{II.4}) \quad Y_d = Y(1-g) + \dot{G}_s \; .$$

At the same time, this saving flow is equal to the new household demand for real balances (S_m) and the new demand for securities (S_s). The flow demand for real balances is a positive function of income, a negative function of the interest rate (r), and a positive function of the return to balances $(-\frac{\dot{P}}{P})$:

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[5] If the government runs a surplus, those funds will be used to buy back some of the outstanding securities. This can be thought of as in fact a negative transfer payment.

$$(\text{II.5}) \quad S_m = S_m(\overset{+}{Y}, \ \overset{-}{r}, \ \overset{\cdot}{\tfrac{P}{P}})$$

The new demand for securities can be derived from (II.3) and (II.5):

$$(\text{II.6}) \quad S_s = S - S_m$$

The rate of issuance of government securities is exogenously fixed to be:

$$(\text{II.7}) \quad \frac{\overset{\cdot}{G_s}}{G_s} = \delta$$

It will be implicitly assumed that all household and government consumption is satisfied. Thus the actual amount of capital accumulation ($\overset{\cdot}{K}$) is equal to real saving. This is determined to be total saving from production sources $[sY(1-g)]$ minus that amount of additional consumption generated by transfer payments $[(1-s)\overset{\cdot}{G}_s]$. Thus we get:

$$(\text{II.8}) \quad \overset{\cdot}{K} = sY(1-g) - (1-s)\overset{\cdot}{G}_s \ .$$

At the same time, the business sector's demand for capital accumulation (I) is a positive function of output, a negative function of the cost of investment (the interest rate), a positive function of the marginal product of capital $[\rho = F_K = f'(k)]$, and a negative function of the return to holding balances $(- \tfrac{\overset{\cdot}{P}}{P})$. Thus,

$$(\text{II.9}) \quad I = I(\overset{+}{Y}, \ \overset{-}{r}, \ \overset{+}{\rho}, \ \overset{+}{\tfrac{\overset{\cdot}{P}}{P}}) \ .$$

It should be clear that the investment demand is not necessarily equal to the supply of new capital except at the long run equilibrium. In addition to investment demand, the firm has a demand to add to its cash balances. The firm holds balances to meet its transaction demand, to pay its labor and to maintain reserves for investment. Thus the firm's flow demand for balances (I_m) is a positive function of output, a negative function of its cost (the interest rate), a negative function of its opportunity cost (the marginal product of capital), and a positive function of the return to holding balances (the negative of the rate of inflation). Thus,

$$(II.10) \quad I_m = I_m(\overset{+}{Y}, \overset{-}{r}, \overset{-}{\rho}, \overset{-}{\frac{\dot{P}}{P}}) .$$

Assuming the business sector pays out all its earnings in the form of dividend payments, then it must finance the funds for both accumulation of balances and capital investment by issuing new securities (I_s). Thus we get:

$$(II.11) \quad I_s = I + I_m .$$

The quantity of issues can be determined by dividing I_s by the price of securities, p_s, or

$$(II.12) \quad I_s = I_q \cdot p_s .$$

The present value of past issues does not necessarily equal the value raised by the business sector. The value of the

existing stock of securities is always evaluated at current price although issued at past prices. Thus, it is not true that the value of the capital stock plus the value of real balances held by the sector is equal to the value of securities outstanding.[6]/

A presentation of the flow supply of money will complete a specification of the flow functions. It will be assumed that the flow supply of money ($\dot{M}$) is issued through open market operations and that the government maintains some constant rate of money issuance u. Thus:

$$(II.13) \quad \frac{\dot{M}}{M} = u.$$

However, since this is achieved through open market purchases, this implies that the government has a flow demand for securities ($\dot{G}_d$) as well as a flow supply. This gives us from (II.13):

$$(II.13.a) \quad \frac{\dot{G}_d}{M} = u.$$

We may now summarize the flow relationships. There are two flow supplies of securities I_s from firms and $\dot{G}_s$ from the government; there are also two flow demands, S_s from households and $\dot{G}_d$ from government. There is a supply and demand

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[6]/This would only be true if one of the two following cases occur: (1) there has been no previous fluctuation in security prices, i.e., the present price is the same as it always has been, (2) the present price is somehow a price which averages the value of all previous issues.

for capital accumulation by firms, $\dot{K}$ and I. Further, there is an exogenous supply of new money, $\dot{M}$, and two new demands for real balances, I_m and S_m, from firms and households respectively. We will now derive the existing asset market relationship from the flow functions.

The business sector's total asset value at time T, $[W_f(T)]$, is equal to the value of real balances (M_f/P) held by this sector plus the capital stock. Since the current price of securities is not necessarily equal to the issue price, $W_f(T)$ is not equal to the current value of securities.[7]/ Thus,

$$(II.14) \quad W_f(T) = \frac{M_f}{P}(T) + K(T).$$

The stock of capital at time T is equal to its initial value (K_o) plus the integral from t_o to T of capital accumulation, $\dot{K}$. This is:

$$(II.15) \quad K(T) = K_o + \int_{to}^{T} \dot{K} dt.$$

The current value of the firm's real balances, $(M_f/P)(T)$, is equal to the value at some initial time t_o plus the

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[7]/There is a further reason for it being "unrealistic" to assume that the asset value of firms is equal to the value of the liability against them. If we asserted this equality, we would then be overlooking the very special property of the business sector, that its assets are used in production. Since the performance of firms depends on how the assets are organized and managed, it would certainly be an over-simplification to assume that the organization of the factors did not add value to the sector.

change in value from that initial time to the present (T).
This has two components, that which accrues to the sector
through the issue of securities, but not spent for invest-
ment, and that which is due to the change in the value of
real balances held by the firm. Analytically, this is:

$$(II.16) \quad \frac{M_f}{P}(T) = \frac{M_f}{P}(t_o) + \int_{to}^{T} (I_s - \dot{K} - \frac{\dot{P}}{P} \frac{M_f}{P}) dt$$

The determination of the rate of inflation will be considered
after the firm's stock demand functions.

At t_o, the initial point in time, the firm has a given
demand for real balances which, like the flow demand, is
positively related to income, negatively related to the
interest rate, negatively related to the marginal product
of capital, and negatively related to the rate of inflation.
Thus,

$$(II.17) \quad (\frac{M_f}{P})^d(t_o) = m_{fo}(\overset{+}{Y}, \overset{-}{r}, \overset{-}{\rho}, \overset{\overset{-}{\cdot}}{\frac{\dot{P}}{P}}).$$

At some time T after t_o, the firm's stock demand for real
balances can be determined by adding to the initial demand
function, m_{fo}, the integral from t_o to T of the firm's flow
demand for real balances, I_m. This gives:

$$(II.18) \quad (\frac{M_f}{P})^d(T) = m_{fo} + \int_{to}^{T} I_m dt.$$

At every point in time the above statement (II.18) defines
the firm's stock demand function for real balances. This
is independent of I_m since the value of the integral is

zero for every instant of time. On the other hand, I_m completely defines the shift in the demand for real balances, since the initial function m_{fo} does not change over time. Thus, in contrast to previous integral representations [such as (II.15)] which define only a particular value at each point in time, (II.18) defines a movement of a function over time. To define the path of the value of the firm's demand for real balances one must not only take into account the movement of the function defined by the integral of I_m, but also the change in the value of m_{fo} itself as the dependent variables change. This implies that although the value of the stock functions can be defined equal at all points in time, the flow functions do not necessarily have to be equal. The significance of this, lies in its ramifications for defining a nonequilibrium adjustment process.

The demand for the capital stock at the initial point is a positive function of income, a negative function of the interest rate, a positive function of the marginal product of capital, and a positive function of the rate of inflation.

$$(II.19) \quad K^d(t_o) = K^d(Y, r, \rho, \frac{\dot{P}}{P}).$$

The demand for capital function at a time T greater than t_o is equal to the initial function plus the evaluated integral of investment between t_o and T.

$$(II.20) \quad K^d(T) = K^d{}_o + \int_{to}^{T} I dt.$$

At all points in time there is an actual portfolio ratio $\frac{M_f}{PK}(T)$ and a desired ratio $(\frac{M_f}{PK})^d(T)$. It is assumed that the rate of inflation $\frac{\dot{P}}{P}$ is such that the actual and desired ratios are always equal.

$$(II.21) \quad \frac{M_f}{PK}(T) = (\frac{M_f}{PK})^d(T)$$

Equation (II.21) implicitly assumes that the inverse function exists with respect to $\frac{\dot{P}}{P}$. Since $(\frac{M_f}{PK})^d(T)$ is a function of Y, r, ρ and $\frac{\dot{P}}{P}$, this means that expressed as an inverse, $\frac{\dot{P}}{P}$ becomes a function of Y, r, ρ and the equality (II.21). Thus,

$$(II.22) \quad \frac{\dot{P}}{P} = \frac{\dot{P}}{P}[\overset{+}{Y}, \overset{-}{r}, \overset{-}{\rho}, \frac{M_f}{PK}(T) = (\frac{M_f}{PK})^d(T)]$$

The total household portfolio (W_n) consists of real balances and the stock of securities outstanding (S_E):

$$(II.23) \quad W_h(T) = \frac{M_h}{P}(T) + S_E(T).$$

The value of securities at time T is equal to the quantity of securities outstanding at time t_o (S_{qo}) times the current price of securities plus the integral of the flow quantity of securities issued between t_o and T times the current price of securities $[p_s(T)]$.

$$(II.24) \quad S_E(T) = S_{qo} \cdot p_s(T) + p_s(T) \int_{to}^{T} I_q dt.$$

The quantity of new securities issued at any time $[I_q(t)]$ is equal to the number of securities issued by firms $[1_q^f(t)]$ plus the number issued by government $[\dot{G}_s = I_q^g \cdot p_s(t)]$, less the amount purchased in open market operations $[\dot{G}_d = I_q^r \cdot p_s(t)]$. Thus,

$$(II.25) \quad I_q = I_q^f + I_q^g - I_q^r$$

The household's demand for real balances at time t_o is equal to a positive function of income, a negative function of the interest rate, and a negative function of the rate of inflation.

$$(II.26) \quad (\frac{M_h}{P})^d(t_o) = m_h^d \ (\overset{+}{Y}, \ \overset{-}{r}, \ \overset{-}{\frac{\dot{P}}{P}})$$

At time T after t_o, it is equal to its initial function m_h^d plus the integral of S_m from t_o to T. Analytically this is:

$$(II.27) \quad (\frac{M_h}{P})^d(T) = m_h^d + \int_{to}^{T} S_m \ dt.$$

The demand for securities in value units (D_E) at any time t between t_o and T is equal to total household wealth minus the demand for balances.

$$(II.28) \quad D_E(t) = W_h(t) - (\frac{M_h}{P})^d(t).$$

The interest rate is equal to the earnings price ratio, since it is assumed that all profits are paid out in the form of dividends. Thus at an arbitrary time t between t_o and T:

$$(II.29) \quad r(t) = \frac{\rho(t) \cdot K(t)}{P_s(t) \cdot S_{qt}}$$

The price of securities is always such that the relative supply and demand for household portfolios is equal.

$$(II.30) \quad \frac{M_h}{PS_E}(t) = (\frac{M_h}{PD_E})^d(t) \text{ for all } t.$$

From (II.30) and (II.29), we can explicitly derive the partial inverse fundtion for the interest rate.

$$(II.31) \quad r(t) = r[Y, \frac{\dot{P}}{P}; \frac{M_h}{PS_E} = (\frac{M_h}{PD_E})^d]$$

This completes a presentation of the model. In the next section, equilibrium and stability properties of the model will be investigated.

III. The Equilibrium in the Short and Long-Run

Before proceeding into a demonstration of existence, the concept of short-run and long-run equilibrium will be explained. Equilibrium, in the traditional partial sense, is determined by the price which equates the supply and demand for a particular good. General equilibrium is the existence of a price vector where consumers are maximizing their satisfaction given their preferences, while producers are maximizing their profits. With the introduction of stock-flow dynamics, we must further distinguish between short-run and long-run equilibrium.

It has already been assumed that existing asset market
is always in equilibrium. It was in this way that we de-
fined both the rate of inflation and the rate of interest.
However, because the system is always in existing-asset
equilibrium in no way guarantees that the system is in long-
run equilibrium. If the excess flow demands are not zero,
the existing-asset functions are shifting by different
amounts implying a cumulative change in the rate of inter-
est and inflation. Thus primary to the concept of long-run
equilibrium is the "stationary" nature of the existing-asset
and production markets.

The proof of existence which follows is restricted to
demonstrating that an interest rate, rate of inflation and
marginal product of capital exist such that the flow supplies
of securities, money and capital are just equal to their flow
demands.[8] Because these functions are assumed to be con-
tinuous Brouwer's Fixed Point Theorem will be used.[9] The
proof will involve defining a function which satisfies
Brouwer's conditions.

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[8]The marginal product of capital is not independent
of output. Thus with the rates determined, Y is also
determined and therefore it is not necessary to explicitly
include it.

[9]Brouwer's Fixed Point Theorem states that if f is
a continuous function from a closed bounded convex set of
Euclidean space C into itself, then there exists X* element
of C, such that $f(X^*) = X^*$.

Theorem 1. There exists a triple $(r, \rho, \frac{\dot{P}}{P})$ such that for every t between to and T

(Th 1.1) $Is + \dot{G}s - \dot{G}d = Ss$

(Th 1.2) $\dot{K} = I$

(Th 1.3) $(M/\dot{P}) = Sm + Im.$

Proof: For notational convenience, let

$$P = (r, \rho, \frac{\dot{P}}{P})$$

$$S = \begin{bmatrix} Is + \dot{G}s - \dot{G}d \\ \dot{K} \\ M/\dot{P} \end{bmatrix}$$

$$D = \begin{bmatrix} Ss \\ I \\ Sm + Im \end{bmatrix}$$

We will first show that D can be represented by:

(Th.1.4) $D = f(r, \rho, \frac{\dot{P}}{P})$

and that the inverse of S exists with respect to P, i.e.:

(Th.1.5) $P = g^{-1}(S).$

By construction, it will then be shown that a bound can be determined such that over the bounded set defined the domain of (Th.1.4) is equal to the range of (Th.1.5)

and that the range of (Th.1.4) is equal to the domain of (Th.1.5). If it can be shown that such a bound is possible, then a mapping is easily defined which satisfies Brouwer's conditions and existence of long-run equilibrium is demonstrated.

(A) It will now be shown that the system of flow equations can be represented in the form of (Th.1.4) and (Th.1.5).

The system of flow demand equations can be represented as follows:

(Th.2.1) $Ss = s(1-g)Lg_2(\bar{\rho}) + s\dot{G}s - f_1(\bar{r}, \frac{\dot{\bar{P}}}{P})$ $\underline{10/}$

(Th.2.2) $I = f_2(\bar{r}, \overset{+}{\rho}, \frac{\dot{P}}{P})$

(Th.2.3) $Im + Sm = f_3(\bar{r}, \bar{\rho}, \frac{\dot{\bar{P}}}{P})$

$\underline{10/}$This result can be derived as follows:

(7.1) $Ss = S - Sm$

$\dot{S} = s(1-g)Y + s\dot{G}s$

$\dot{S} = s(1-g)Lf(k) + s\dot{G}s$

$\rho = f'(k)$

$k = f'^{-1}(\rho)$

By substitution, we get:

$\dot{S} = s(1-g)Lf[f'^{-1}(\rho)] + s\dot{G}s$

Letting

$f[f'^{-1}(\rho)] = g_2(\bar{\rho})$ and $S_m = f_1(r, \frac{\dot{P}}{P})$

gives the noted result.

Equations (Th.2.1) - (Th.2.3) can be represented as (Th.1.4).

The supply equations can be represented as follows:

(Th.3.1) $\dot{I}s + \dot{G}s - \dot{G}d = g_1(r, \rho, \frac{\dot{P}}{P}) + \dot{G}s - \dot{G}d$

(Th.3.2) $\dot{K} = s(1-g)Lg_2(\bar{\rho}) - (1-s)\dot{G}s$

(Th.3.3) $\dot{(M/P)} = u\frac{M}{P} - \frac{\dot{P}}{P}\frac{M}{P} = u\frac{M}{P} - g_3(\overset{+}{\frac{\dot{P}}{P}})$

To derive the inverse system, we do the following:

(1)   Solve for $\dot{P}/P$ in (Th.3.3)

(Th.3.3*) $\frac{\dot{P}}{P} = -\frac{\dot{(M/P)}}{M/P} + u$

(2)   solve for   from (Th.3.2)

(Th.3.2*) $\rho = g_2^{-1} [\dot{K}; (1-s)\dot{G}s]$

(3)   substituting (Th.3.3*) and (Th.3.2*) into (Th.3.1) and solving for r gives the last inverse equation,

(Th.3.1*) $r = g_1^{-1} [Is, \dot{K}, \dot{(M/P)}; (1-s)\dot{G}s]$

Thus equations (Th.3.1*) - (Th.3.3*) satisfy the equation (Th.1.5).

(B)   We will now define bounds on the functions (Th.1.4) and (Th.1.5) as derived above such that they are continuous over a parallelepiped.[11]

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[11] A parallelepiped is a general rectangle in n-space which satisfies the needed convexity condition.

Assuming non-negativity of both the supply and demand functions and considering just those portions of the returns that are positive, i.e. $(r, \rho, - \dot{P}/P) > 0$, then an obvious lower bound is the zero vector.

Given this lower bound, it is possible to construct an upper bound such that the average partial slopes of the functions are equal. This bounded set enclosed in the upper and lower bounds satisfies the convexity condition needed.

(C) We must only define the proper functional mapping, Φ, to complete the proof. It is readily seen that the following is a continuous function from a closed bounded convex set into itself:

$$(Th.5.1) \qquad (D,P) = \Phi[f(P), g^{-1}(S)].$$

Thus existence is demonstrated.

$$Q.E.D.$$

IV. The Stability of the System

In the introduction it was asserted that the important result of this paper, i.e. of introducing a capital market into the money and growth framework, was the stability which results. In this section, the stability of the system will be demonstrated. As in the previous section, the critical factors determining stability involve the flow functions.

The question of stability is one of determining that given any initial point (vector of returns), that the system will achieve an equilibrium (a point in time where the vector of returns does not change). Although the interest rate, the rate of inflation and the marginal product of capital are determined by the stock functions, it is the excess flow demands which determine the relative movement of the stock functions and thus the long-run adjustment of the system. It is only because of the separation of the equilibrium of the "stock" markets and that of the flow markets that it is possible to describe the adjust process in terms of excess demand functions.[12]

First, the system of excess demand function will be derived. Then the matrix of partial derivatives of excess demand functions with respect to the rates of return.

A. The Flow Excess Demand Functions

There are three flow excess demand functions: (1) securities, (2) capital accumulation, and (3) real balances. Analytically, these can be represented by

$$(IV.1.1) \quad Xd_1 = Ss - Is - \dot{G}s + \dot{G}d$$

$$(IV.1.2) \quad Xd_2 = I - \dot{K}$$

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[12] The error of previous research in the money and growth area has been not separating the concept of long and short run equilibrium, i.e. stock and flow equilibrium and thus not obtaining a "traditional" adjustment process.

$$(IV.1.3) \quad Xd_3 = Sm + Im - (M/\dot{P})$$

Substituting the functional forms derived in section III gives us:

$$(IV.2.1) \quad Xd_1 = Kg_2(\bar{\rho}) - f_1(\bar{r}, \frac{\dot{P}}{P}) - f_2(\bar{r}, \rho, \frac{\dot{P}}{P})$$

$$- Im(\bar{r}, \bar{\rho}, \frac{\dot{P}}{P}) - (1-s)\dot{G}s + \dot{G}d$$

$$(IV.2.2) \quad Xd_2 = f_2(r, \rho, \frac{\dot{P}}{P}) - Kg_2(\rho) + (1-s)\dot{G}s$$

$$(IV.2.3) \quad Xd_3 = f_1(r, \rho, \frac{\dot{P}}{P}) - u\frac{M}{P} + \frac{\dot{P}}{P}\frac{M}{P}$$

Thus the system (IV.2.1) - (IV.2.3) can be represented by the general system Xd:

$$(IV.3) \quad Xd = f(r, \rho, \frac{\dot{P}}{P})$$

where Xd, $f \in R^3$. Since Walras Law can be shown to hold in the flow markets, we only have to concern ourselves with two of the equations and two of the returns. For convenience we will choose Xd_1 and Xd_3.

By taking the partial derivatives of Xd_1 and Xd_3 with respect to r and $\frac{\dot{P}}{P}$, we can determine the stability conditions of the system. For consistency, the negative of the partial derivatives will be taken with respect to $\dot{P}/P$. (IV.4) below is the matrix of these partials $(\partial Xd/\partial P)$:

$$(IV.4) \quad \frac{\partial Xd}{\partial P} = \begin{bmatrix} \dfrac{\partial Xd_1}{\partial r} & \dfrac{\partial Xd_1}{\partial(\dot{P}/P)} \\ \dfrac{\partial Xd_3}{\partial r} & \dfrac{\partial Xd_3}{\partial(\dot{P}/P)} \end{bmatrix} = \begin{bmatrix} + & ? \\ - & ? \end{bmatrix}$$

As can be seen, the partials with respect to r are determinate, while those with respect to $\frac{\dot{P}}{P}$ are not.

Several additional conditions on (IV.4) will guarantee local stability:

(IV.4.1)   if $\frac{\partial Xd}{\partial P}$ is quasipositive definite

(IV.4.2)   if $\frac{\partial Xd}{\partial P}$ is quasidominant diagonal.[13]

Since for the purposes of this paper, conditions (IV.4.1) and (IV.4.2) are equivalent, only the implications of (IV.4.1) will be discussed.

Condition (IV.4.1) will be satisfied if for all y_1 and y_2 different from zero, the following quadratic equation is greater than zero:

$$(IV.5) \quad y_1^2 \left(\frac{\partial Xd_1}{\partial r}\right) + y_1 y_2 \left[\frac{\partial Xd_3}{\partial r} + \frac{\partial Xd_1}{\partial(\dot{P}/P)}\right]$$
$$+ y_2^2 \left(\frac{\partial Xd_3}{\partial(\dot{P}/P)}\right) > 0$$

This will hold if the characteristic roots of (IV.5) are positive which is guaranteed if

$$(IV.5.1) \quad \frac{\partial Xd_1}{\partial r} + \frac{\partial Xd_3}{\partial(\dot{P}/P)} > 0$$

and

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[13]The quasi in these conditions means that the matrix plus its transpose satisfy the given condition.

$$(IV.5.2) \quad 4\frac{\partial Xd_1}{\partial r} \cdot \frac{\partial Xd_3}{\partial(\dot{P}/P)} \qquad [\frac{\partial Xd_3}{\partial r} + \frac{\partial Xd_1}{\partial(\dot{P}/P)}]^2$$

Several conditions on the partials of Xd with respect to $\dot{P}/P$ would satisfy (IV.5.1) and (IV.5.2). One possible stable solution would be obtained if

$$(IV.6.1) \quad \frac{\partial Xd_1}{\partial(\dot{P}/P)} < -\frac{\partial Xd_3}{\partial r}$$

and

$$(IV.6.2) \quad \frac{\partial Xd_3}{\partial(\dot{P}/P)} > 0$$

subject to (IV.5.2). Stability of the system could be obtained under other similar conditions. The important conclusion is that under a wide range of situations the system will prove to be stable.

V. Conclusions

In this paper, the equilibrium and stability properties of a capital market growth model have been investigated. It was found that under the conditions assumed that the system contained an equilibrium point and under a wide range of

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[14]/ If we let $2\frac{\partial Xd_1}{\partial r} = a$, $\frac{\partial Xd_3}{\partial r} + \frac{\partial Xd_1}{\partial(\dot{P}/P)} = b$ and

$2\frac{\partial Xd_3}{\partial(\dot{P}/P)} = c$, then the characteristic roots of (IV.5) are

determined by: $\lambda = \dfrac{a + c \pm \sqrt{(a+c)^2 - 4(ac-b^2)}}{2}$

and $\lambda > 0$ when $ac > b^2$.

situations equilibrium would be locally stable. This result
must be compared to that obtained in a simple Tobin money
and growth model where the equilibrium except for a finite
number of paths was unstable [11].

We can interpret this difference in result to mean that
in a simple money and economic growth model without securities,
the goods market, through a rapid change in prices, is forced
to take the full impact of any disturbance. However, the
introduction of a security market, by providing a buffer be-
tween monetary disturbances and real changes, reduces the
adjustment role of the rate of inflation and thus provides
a stabilizing influence on the model.

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