Some Empirical Implications of State-Contingent Production Models

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Perhaps the most important insight offered by the state-space approach to decisionmaking under uncertainty is that standard economic concepts and reasoning apply in an uncertain world. This insight stands in stark contrast to much of the existing literature in agricultural economics, where it has become commonplace to question even the most basic facts of rational economic behavior, for example, whether rational economic agents facing a stochastic technology minimize cost. Inevitably that latter world view has led agricultural economists to emphasize the apparently unique aspects of decisionmaking under uncertainty rather than its commonality with the rest of modern economics.

The state-space approach, however, reveals that the main thing that truly separates the theory of the nonstochastic producer from the theory of the stochastic producer is the partial nonlinearity (risk aversion) of the latter's objective function. So much emphasis has been placed on this aspect that the commonalities, which are far greater, have been ignored and even the differences that do exist have been overstated. This world view, in turn, has led agricultural economists to develop a very specialized set of models and jargon to analyze producer decisionmaking under uncertainty. Even a casual perusal of this literature would convince an unbiased reader that its intuition is grounded more firmly in regression analysis than in economics.

The point of this paper is to show that a simple economic concept, cost minimization, underlies virtually all existing models of rational producers facing stochastic technologies and stochastic markets. Recognizing that point, in turn, reveals a straightforward path to modeling stochastic technologies and farmer behavior that is closely associated with modern financial economics.

In what follows, I demonstrate how this basic economic principle can be used to model an important component of stochastic technologies and thereby to address two salient problems in the agricultural economics literature: do farmers hedge optimally,[1] and how to price crop insurance products for farmers?

[1] Carter points out that observed farmer behavior on hedging and production suggests that "...both the theoretical and empirical literature [on hedging] appear to contradict reality..." (p. 216)

1 The Theoretical Model

I study competitive farmers facing stochastic production and markets in a generalization of the framework considered by Danthine, Holthausen, Anderson and Danthine, and Rolfo. Throughout the paper, I speak in terms of farmers. However, it is apparent that the argument is general. It applies to any producer facing any mix of stochastic production, stochastic product prices, and stochastic financial markets.

Formally, there are two periods. The first period, t, is nonstochastic, and the second, $t+1$, is stochastic. The stochastic setting is modeled formally as a probability space (S, Ω, π) where S represents the set of states of 'Nature', π is a probability measure, and Ω represents the events (subsets of S) measurable with π.[2] Random variables are represented as bounded maps from S to the reals. Hence, random variable, $\tilde{f}$, can be thought of as the element of $\mathbb{R}^S$ defined by

$$\tilde{f} = \{f(s) : s \in S\},$$

where $f : S \to \mathbb{R}$ is the map defining the random variable (Savage, Duffie). Random variables will always be distinguished from their *ex post* values by a tilde $(\tilde{\ })$. Hence, $\tilde{f}$ represents the random variable, and $f(s)$ denotes the *ex post* (observed) outcome associated with Nature choosing s.

The stochastic production technology is represented by a single-product, input correspondence that maps a stochastic output, $\tilde{z} \in \mathbb{R}_+^S$, into sets of inputs that are capable of producing it. I operate with a single product technology solely because the empirical application that follows is for a single product specification. It is trivial to extend these ideas to a multiple output framework (Chambers and Quiggin, 2000).

Inputs are chosen in period t and are nonstochastic. Denote those inputs by $\mathbf{x} \in \mathbb{R}_+^N$ and their prices, which are nonstochastic, by $\mathbf{w} \in \mathbb{R}_+^N$. The stochastic output is also chosen in period t but realized or observed in period $t+1$. The period $t+1$ price of the output is stochastic and denoted by $\tilde{p} \in \mathbb{R}_{++}^S$. Notationally, therefore, if $\tilde{z}$ is chosen by the producer in period t and $s \in S$ is picked by Nature, then the *ex post* or observed output is $z(s)$ and the *ex post* (spot) output price is $p(s)$.

[2] S can be either finite or infinite.

The input correspondence describing the technology, $X : \mathbb{R}_+^S \to \mathbb{R}_+^N$, maps stochastic output into variable input sets according to:

$$X(\tilde{z}) = \{\mathbf{x} \in \mathbb{R}_+^N : \mathbf{x} \text{ can produce } \tilde{z}\}.$$

Intuitively, $X(\tilde{z})$ is associated with all of the variable-input combinations on or above the firm's production isoquant for $\tilde{z}$. The only technical requirement is that $X(\tilde{z})$ be closed. No curvature or disposability assumptions are imposed.

The (period t) minimal cost of producing the stochastic output, $\tilde{z}$, is given by the production cost function

$$c(\mathbf{w},\tilde{z}) = \min\{\mathbf{w}'\mathbf{x} : \mathbf{x} \in X(\tilde{z})\},$$

if $X(\tilde{z})$ is nonempty and ∞ otherwise. As usual, $c(\mathbf{w},\tilde{z})$ is nondecreasing, positively linearly homogeneous and concave in $\mathbf{w}$. The proof of these properties is standard and, therefore, omitted.

The only restriction on the farmer's *ex ante* (period t) preferences is that he or she strictly prefers more period t consumption to less and at least weakly prefers more period $t+1$ consumption to less. More formally, if we denote the farmer's *ex ante* preferences over period t consumption, q_t, and period $t+1$ consumption, $\tilde{q}_{t+1}$, by $W : \mathbb{R}_+ \times \mathbb{R}_+^S \to \mathbb{R}$, then

$$q_t^* > q_t \Rightarrow W(q_t^*, \tilde{q}_{t+1}) > W(q_t, \tilde{q}_{t+1}),$$

and

$$\tilde{q}_{t+1}^* \geq \tilde{q}_{t+1} \Rightarrow W\left(q_t, \tilde{q}_{t+1}^*\right) \geq W(q_t, \tilde{q}_{t+1}).$$

The farmer can also transform period t income into period $t+1$ consumption by investing in financial markets. These markets include but are not restricted to futures and forward markets for agricultural products. These markets are frictionless but stochastic, and the *ex ante* financial security payoffs are given by the $S \times J$ matrix $\mathbf{A}$ (a matrix of J random variables). The stochastic payout on the jth financial asset is denoted $\tilde{A}_j \in \mathbb{R}^S$, and its period t price is denoted v_j. The firm's portfolio vector, corresponding to the period t purchases of the financial assets, is denoted $\mathbf{h} \in \mathbb{R}^J$. Denote the jth stochastic return by $\tilde{R}_j = \frac{\tilde{A}_j}{v_j}$. In what follows, we shall refer to the farmer's choice of $\mathbf{h}$ as his or her *hedge* even though it may include investment in assets that have nothing to do with agriculture or the commodity produced.

2 Equilibrium Production and Hedging Behavior

We start by demonstrating a basic result that characterizes equilibrium production and hedging behavior.

Proposition 1 *Given any stochastic consumption, $\tilde{q}_{t+1}$, the farmer solves:*

$$C\left(\tilde{q}_{t+1}\right)=\inf_{\tilde{z},\mathbf{h}}\left\{c\left(\mathbf{w},\tilde{z}\right)+\mathbf{v}'\mathbf{h}:\mathbf{Ah}+\tilde{p}\tilde{z}\geq\tilde{q}_{t+1}\right\},$$

where $\tilde{p}\tilde{z}$ denotes the random variable whose ex post realization is $p\left(s\right)z\left(s\right)$.

Proof. *Suppose to the contrary that for given level of stochastic consumption, $\tilde{q}_{t+1}$, the farmer chooses $\tilde{z}$ and $\mathbf{h}$ that are not cost minimizing as claimed, but that yield $\tilde{q}_{t+1}$. Denote these choices by $\tilde{z}^0$ and $\mathbf{h}^0$. This cannot be optimal because by choosing $\inf_{\tilde{z},\mathbf{h}}\left\{c\left(\mathbf{w},\tilde{z}\right)+\mathbf{v}'\mathbf{h}:\mathbf{Ah}+\tilde{p}\tilde{z}\geq\tilde{q}_{t+1}\right\}$ the farmer saves*

$$c\left(\mathbf{w},\tilde{z}^0\right)+\mathbf{v}'\mathbf{h}^0-\inf_{\tilde{z},\mathbf{h}}\left\{c\left(\mathbf{w},\tilde{z}\right)+\mathbf{v}'\mathbf{h}:\mathbf{Ah}+\tilde{p}\tilde{z}\geq\tilde{q}_{t+1}\right\}>0,$$

which can be used to strictly increase period t consumption. ■

Proposition 1 can be explained simply in terms of eliminating arbitrages (Chambers and Quiggin, 2005), the Fisher separation theorem, or current period profit maximization. To emphasize the formal similarity with ideas originally articulated by Holbrook Working over a half century ago, I choose the last.

Suppose in period t that the farmer chooses a production/hedging position of $(\tilde{z}^0,\mathbf{h}^0)$ that achieves $\tilde{q}_{t+1}$, and that he or she is considering another position $(\tilde{z}^1,\mathbf{h}^1)$ that also achieves $\tilde{q}_{t+1}$. By changing the hedge from $\mathbf{h}^0$ to $\mathbf{h}^1$, the farmer effectively sells (shorts) the composed asset $\mathbf{A}\left(\mathbf{h}^0-\mathbf{h}^1\right)$ for a return of $\mathbf{v}'\left(\mathbf{h}^0-\mathbf{h}^1\right)$. To replace the stochastic consumption lost by selling the composed asset, the producer reallocates stochastic production from $\tilde{z}^0$ to $\tilde{z}^1$ at a marginal cost of $c\left(\mathbf{w},\tilde{z}^1\right)-c\left(\mathbf{w},\tilde{z}^0\right)$. If $\mathbf{v}'\left(\mathbf{h}^0-\mathbf{h}^1\right)-\left[c\left(\mathbf{w},\tilde{z}^1\right)-c\left(\mathbf{w},\tilde{z}^0\right)\right]>0$, a strict period t profit is realized from this change while not sacrificing any stochastic consumption loss in period $t+1$. By the assumed monotonicity of preferences, all such changes will be made until none remain available.[3] Thus, position $(\tilde{z}^0,\mathbf{h}^0)$ is optimal if and only if there is

[3] If $\mathbf{v}'\left(\mathbf{h}^0-\mathbf{h}^1\right)>0$ and $\mathbf{A}\left(\mathbf{h}^1-\mathbf{h}^0\right)\geq\mathbf{0}$, an arbitrage exists. It is then trivial to establish that $C\left(\tilde{q}_{t+1}\right)=-\infty$ implying that an infinite period t profit is available from operating in financial markets. This represents a money pump and cannot exist in any well-defined equilibrium

no feasible alternative $(\tilde{z}^1, \mathbf{h}^1)$ satisfying

$$\mathbf{v}'\mathbf{h}^0 + c\left(\mathbf{w},\tilde{z}^0\right) > \mathbf{v}'\mathbf{h}^1 + c\left(\mathbf{w},\tilde{z}^1\right),$$

which trivially requires

$$\mathbf{v}'\mathbf{h}^0 + c\left(\mathbf{w},\tilde{z}^0\right) \leq \mathbf{v}'\mathbf{h}^1 + c\left(\mathbf{w},\tilde{z}^1\right).$$

Proposition 1 is trivial.[4] Analytically, once the model is set up, it certainly is. But so is the no-arbitrage notion that underlies much of modern financial economics. And that notion has rich empirical implications that have laid the foundation for many developments in financial economics. Proposition 1 extends that notion to encompass arbitrary stochastic production technologies for stochastic markets in a simple, and thoroughly understood, economic decision model. It also makes a prediction that can help one determine empirically whether farmers rationally produce and hedge without requiring any assumptions on the farmer's risk attitudes.

In developing that implication, it eases exposition if we can use differential arguments. Because we work with potentially infinite dimensional objects, the usual notions of partial derivatives and gradients may not be available. Define the (one-sided) directional derivative of $c(\mathbf{w},\tilde{z})$ in the direction $\tilde{n} \in \mathbb{R}^S$ by

$$c'(\mathbf{w},\tilde{z};\tilde{n}) = \lim_{t \to 0^+} \left\{ \frac{c(\mathbf{w},\tilde{z} + t\tilde{n}) - c(\mathbf{w},\tilde{z})}{t} \right\},$$

and assume that this limit exists. c is *Gateaux differentiable* if there exists $\partial c(\mathbf{w},\tilde{z}) \in \mathbb{R}^S$ such that

$$c'(\mathbf{w},\tilde{z};\tilde{n}) = E\left[\partial c(\mathbf{w},\tilde{z})' \tilde{n}\right], \quad \text{for all } \tilde{n} \in \mathbb{R}^S$$

[4] The only reason that I present a formal proof is that some authors have maintained the notion that cost minimization is not consistent with decision making under uncertainty except under very restrictive assumptions on the decision maker's risk preferences.

where E denotes expectation taken with respect to π.[5] $\partial c(\mathbf{w},\tilde{z})$ is referred to as the *Gateaux derivative*.[6]

Given $\tilde{q}_{t+1}$, Proposition 1 allows us to restrict attention to cost minimization in determining the optimal hedge. A detailed examination of the solution to the problem is available by the use of standard (although possibly infinite dimensional) optimization arguments (Clarke). That manipulation is left to the interested reader. The fundamental prediction that emerges can be established directly.

Let

$$(\tilde{z}^*, \mathbf{h}^*) \in \arg\min \left\{c(\mathbf{w},\tilde{z}) + \mathbf{v}'\mathbf{h} : \mathbf{A}\mathbf{h}+\tilde{p}\tilde{z} \geq \tilde{q}_{t+1}\right\},$$

and consider moving from $(\tilde{z}^*, \mathbf{h}^*)$ to $\left(\tilde{z}^* + \delta\frac{\tilde{A}_j}{\tilde{p}}, h_1^*, .., h_{j-1}^*, h_j^* - \delta, h_{j+1}^*, ..., h_J^*\right)$ for δ small but positive, where $\frac{\tilde{A}_j}{\tilde{p}}$ denotes the random variable whose *ex post* realization in state s is $\frac{A_j(s)}{p(s)}$. In words, consider selling off a small amount of the jth asset and replacing it by producing an additional amount of the physical commodity in each state of Nature, with the value of the additional output equal to the amount of the foregone payout from the jth asset in that state, and then selling the extra output produced in the spot market. Because this change maintains $\tilde{q}_{t+1}$, it is feasible. The associated change in the objective function at the margin is

$$\lim_{\delta\to 0}\left\{\frac{c\left(\mathbf{w},\tilde{z}^* + \delta\frac{\tilde{A}_j}{\tilde{p}}\right) - v_j\delta - c(\mathbf{w},\tilde{z}^*)}{\delta}\right\} = c'\left(\mathbf{w},\tilde{z}^*;\frac{\tilde{A}_j}{\tilde{p}}\right) - v_j.$$

[5] These assumptions can be relaxed to only require that c be locally Lipschitzian in $\tilde{z}$, and analysis can then be based on the generalized directional derivative introduced by Clarke. In that case, the results derived for the Gateaux derivative would apply to any element of the generalized gradient defined by Clarke.

[6] Suppose that S is finite dimensional and that $c(\mathbf{w},z(1), z(2), ..., z(S))$ is differentiable in the usual sense in each $z(s)$ with the corresponding partial derivative being denoted $c_s(\mathbf{w},\tilde{z})$. Then using the definition

$$\begin{aligned} c'(\mathbf{w},\tilde{z};\tilde{n}) &= \sum_s c_s(\mathbf{w},\tilde{z})\, n(s) \\ &= \sum_s \pi_s \frac{c_s(\mathbf{w},\tilde{z})}{\pi_s} n(s) \\ &= E\left[\partial c(\mathbf{w},\tilde{z})'\,\tilde{n}\right], \end{aligned}$$

where

$$\partial c(\mathbf{w},\tilde{z})' = \left[\frac{c_1(\mathbf{w},\tilde{z})}{\pi_1}, ..., \frac{c_S(\mathbf{w},\tilde{z})}{\pi_S}\right].$$

If this expression is negative, a marginal change in the farmer's production/hedging behavior reduces period t cost while maintaining $\tilde{q}_{t+1}$. This contradicts the assumed optimality of $(\tilde{z}^*, \mathbf{h}^*)$. Thus, I have established:

Proposition 2 $(\tilde{z}^*, \mathbf{h}^*)$ *is optimal only if*

$$c'\left(\mathbf{w},\tilde{z}^*;\frac{\tilde{A}_j}{\tilde{p}}\right) - v_j \geq 0, \quad j = 1, ..., J.$$

By Proposition 2, a strictly interior solution exists only if

$$c'\left(\mathbf{w},\tilde{z}^*;\frac{\tilde{A}_j}{\tilde{p}}\right) = v_j, \quad j = 1, ..., J. \tag{1}$$

Expression (1) is my key analytic expression. It also serves as the basis for the empirical work that follows. In simple economic terms, $c'\left(\mathbf{w},\tilde{z}^*;\frac{\tilde{A}_j}{\tilde{p}}\right)$ is the marginal cost to the farmer of replicating $\frac{\tilde{A}_j}{\tilde{p}}$ (the marginal cost of replicating the jth asset physically). Expression (1) thus requires that the price of any asset equals its marginal cost of production—the most basic requirement for profit maximization. The profit motive requires that production and hedging behavior are chosen to exploit or eliminate any arbitrage opportunities between the the farmer's physical technology and financial markets. This argument applies regardless of the farmer's risk preferences. Thus, the first order of business for a producer is not, as many models presume, to smooth risky consumption. Rather, as Working pointed out over a half century ago, optimal behavior is fundamentally driven by the profit motive. Rational farmers will exploit any opportunity to raise profit nonstochastically.

On the other hand, expression (1) does not imply that risk attitudes are irrelevant. Remember, the behavior described by Propositions 1 and 2 is conditioned by $\tilde{q}_{t+1}$. That choice hinges importantly on the individual's attitudes towards risk. This is the essential point. *It is $\tilde{q}_{t+1}$ that reflects the farmer's risk attitudes directly, not the hedge.* The hedge and the production are tools that the farmer uses to achieve the desired $\tilde{q}_{t+1}$. They are not ends in themselves. In essence, they are derived and not direct demands. The principle of eliminating any potential for sure profit thus drives choices between feasible alternatives. Working states this idea better than I: "...any curtailment of risk may be only an incidental advantage gained, not a primary or even very important incentive to hedging".

If the cost structure is Gateaux differentiable, (1) can be rewritten in a perhaps more familiar format:

$$E\left[\frac{\partial c\left(\mathbf{w},\tilde{z}\right)'}{\tilde{p}}\tilde{A}_j\right] = v_j, \quad j = 1, 2, ..., J. \tag{2}$$

In equilibrium, the expected value of the product of $\frac{\partial c(\mathbf{w},\tilde{z})}{\tilde{p}}$ and $\tilde{A}$ equals the acquisition price of the asset.[7] Thus, $\frac{\partial c(\mathbf{w},\tilde{z})}{\tilde{p}}$, which is a random variable, is interpretable as a stochastic discount factor.[8] Expression (2) requires that the discounted value of the stochastic payouts from the assets equal their acquisition price.

3 A Test of the Theory

If farmers behave according to this theory, (2) must apply for Gateaux differentiable technologies. A straightforward and well understood test for optimal producer behavior, based on the generalized method of moments (GMM) tests for nonlinear model identification (Hansen 1982; Hansen and Singleton, 1983; Hansen and Jagannathan, 1991,1997; Campbell, Lo, and MacKinlay, 1997; Cochrane, 2000), is thus available. Because time-series data are to be used, it is appropriate to be more careful with subscripts. Henceforth, I subscript all period t variables with a t and all random variables chosen in period t but whose realization occurs in $t+1$ with $t+1$. At time period t, (2), in returns notation, then requires

$$E_t\left[\frac{\partial c_t\left(\mathbf{w}_t,\tilde{z}_{t+1}\right)'}{\tilde{p}_{t+1}}\tilde{R}_{jt+1}\right] = 1, \quad j = 1, 2, ..., J \tag{3}$$

where E_t denotes the expectation conditional on information available at time t.

Theoretically, $\mathbf{w}_t$ has been treated as nonrandom. Econometrically, however, $\mathbf{w}_t$ is predetermined at time, t. Because the theory requires that (3) holds exactly, it is also true for

[7] As far as I am aware, Chambers and Quiggin (1997) were the first to derive an expression of this form explicitly in terms of a cost function for a stochastic technology. They assumed that the objective was to maximize the expected utility of profit. They later generalized this to general preference structures over net returns (Chambers and Quiggin, 2000). Versions of (2) for nonstochastic technologies and no basis risk, albeit in a very disguised form, are also implicit in results reported in Danthine and Holthausen. Both studies assume expected utility maximization.

[8] Here we follow the terminology of Cochrane. However, stochastic discount factors go by other names including the pricing kernel (for obvious reasons), risk-neutral probabilities, the state-claim densities, and the ideal portfolio.

any other predetermined random variable, say v_t, that

$$v_t E_t \left[\frac{\partial c_t \left(\mathbf{w}_t, \tilde{z}_{t+1} \right)'}{\tilde{p}_{t+1}} \tilde{R}_{jt+1} \right] = v_t, \quad j = 1, 2, ..., J. \tag{4}$$

The law of iterated expectations applied to (3) and (4) gives the unconditional expectations

$$h \equiv E \left[\frac{\partial c_t \left(\mathbf{w}_t, \tilde{z}_{t+1} \right)'}{\tilde{p}_{t+1}} \tilde{R}_{jt+1} \right] - 1 = 0, \quad j = 1, 2, ..., J, \tag{5}$$

and

$$g \equiv E \left[v_t \frac{\partial c_t \left(\mathbf{w}_t, \tilde{z}_{t+1} \right)'}{\tilde{p}_{t+1}} \tilde{R}_{jt+1} \right] - E \left[v_t \right] = 0 \quad j = 1, 2, ..., J. \tag{6}$$

These expressions offer a way to check the consistency of a given body of data with (2).

The test that I use chooses a parametric specification for the stochastic discount factor, $\frac{\partial c_t(\mathbf{w}_t, \tilde{z}_{t+1})}{\tilde{p}_{t+1}}$, replaces (5) and (6) with sample moments (denoted, respectively, by h_T and g_T) for a given data set, and then derives estimates of the parameters of the stochastic discount factor by minimizing the criterion function

$$J_T = [g_T, h_T] \Sigma^{-1} [g_T, h_T]', \tag{7}$$

where Σ is the spectral density matrix for the implied pricing errors associated with (5) and (6). As is well-known, $T J_T$ is distributed as χ^2 with degrees of freedom equal to the number of moment conditions less the number of estimated parameters, where T is the sample size. The theory predicts that $T J_T = 0$. Thus, replacing $T J_T$ with a consistent estimate and comparing it with tabulated values of the χ^2 distribution offers an asymptotically appropriate test of the validity of (2).

4 Data and Empirical Model

To implement a test, I use annual US data (1957 to 1997) that are publicly available but from different sources. The agricultural production, price, and input data are aggregate annual data taken from the United States Department of Agriculture's total factor productivity data base.[9] The output data correspond to an (implicit) aggregate agricultural output index,[10]

[9] These data are publicly available at http://www.ers.usda.gov/Data/AgProductivity/.

[10] The output data were divided by 100,000 to ensure conformability of units in estimation.

the output price variable is the associated price index, and the input price is an input price index for agricultural inputs.

If the theory is correct, it should apply for any financial asset and, by the law of one price, to any composed asset derived from marketed assets. In the empirical analysis, I consider two financial assets. The first is a returns measure constructed from the Commodity Research Bureau's monthly futures price index for the United States.[11] The second measures annual returns on commercial paper in the United States.[12]

The next step is to specify a parametric form for the stochastic discount factor. Because the theoretical model imposes no structure on $X(\tilde{z})$, other than closedness, the only functional restriction inherited by the stochastic discount factor from the cost structure is superlinearity (positive linear homogeneity and concavity) in input prices. Thus, I opt for a simple linear representation:

$$\frac{\partial c_t\left(\mathbf{w}_t, \tilde{z}_{t+1}\right)}{\tilde{p}_{t+1}} = \frac{\phi\left(\mathbf{w}_t\right)}{\tilde{p}_{t+1}}\left[\alpha + \beta\left(\tilde{z}_{t+1} - z_t\right)\right], \tag{8}$$

where $\phi(\mathbf{w}_t)$ is understood to correspond to the input price index.[13] The choice of form is based on several considerations. Most important is simplicity. The linear form appears to be the simplest, nontrivial representation of the stochastic discount factor possible. Output differences are used for two reasons. First is the practical reason of ensuring stationarity in the data. Second, the stochastic discount factor at time t corresponds to the 'marginal cost' of stochastic output for the technology available at time t. Because we deal with time-series data, it is implausible that that marginal cost did not change over the sample period. The difference specification accommodates a very simple (and practical) form of learning by doing (Chambers; Berndt) that does not involve the standard (but nonstationary) artifice of introducing a time trend into the technical specification.

[11] These data are publicly available, for example, at https://www.economy.com/freelunch/. The returns measure was constructed using the April futures price index for each year. This corresponds roughly to the early part of the crop season for most field crops in the United States.

[12] The return on commercial paper was drawn from http://kuznets.fas.harvard.edu/~campbell/data.htm.

[13] A note on timing. The data were constructed so that the input price data and the output price data for the same year correspond to the planning and received prices. Therefore, the observed values of these prices for year t and $t+1$ respectively are taken from the same year.

5 Results

In the empirical analysis, I fit two versions of (5) and (6) using the parametric specification (8). In both cases, the estimation procedure is iterated GMM with an ideal weighting matrix (Σ^{-1}) which involves iteratively minimizing (7) after replacing Σ with a consistent estimate from the previous step. Σ was estimated using the Hansen procedure with lag length set to 3.[14]

In the first version, I fit (5) and (6) for a single asset, the constructed commodity futures return. In constructing (6) two instruments were used, the lagged commodity future return and the input price index. Both are reasonably presumed to be predetermined at time t. Overall there are three moment conditions and two parameters, resulting in one degree of freedom. The parameter estimates as well as the computed $J-$statistic are reported in Table 1.

Table 1:	**CRB**				
		α	β		TJ_T
Estimate		2.308095	-24.650112		.0579
t		7.52	-2.46		
deg. freedom					1
P-value					.8113

Next (5) and (6) were fit using the commodity future returns data as well as the data on returns on commercial paper. The instruments were lagged returns on commodity futures, lagged returns on commercial paper, and the input price index. There are eight moment conditions resulting in six degrees of freedom. The results are summarized in Table 2.

[14] All estimation was done in a Matlab framework using the publicly available GMM program library developed and maintained by M. T. Cliff. Different procedures were used for estimating Σ with little change in calculated results.

Table 2:	CRB and Commercial Paper				
		α	β		TJ_T
Estimate		2.069348	-17.7163		2.980
t		11.46	-5.26		
deg. freedom					6
P-value					.8112

The estimation results reported in Tables 1 and 2 are similar. In particular, the parameter estimates reported in Table 2 fall within the implied confidence intervals for the parameter estimates in Table 1. The reported J statistics are quite similar. Both provide statistical support for (2). The parameters are estimated precisely, and the degree of pricing error as measured by the J statistic seems acceptably small.

The results reported in Table 1 and Table 2 are supportive of (2). Strictly speaking, however, (2) need not hold exactly, and farmers could still behave optimally. By Proposition 2, optimality requires that the marginal cost of replicating the asset at least equal its market price. Because physical production is involved, one can easily imagine situations (for example, corner solutions) where requiring equality in (2) might entail the producer moving in infeasible (negative real output) directions. For that reason, the sample average pricing errors corresponding to (2) are of interest. Although not significantly different from zero, they are all positive as required by Proposition 2. For the model reported in Table 1, the sample average pricing error on the CRB futures return measure is .017 with a standard error of .072. For the model reported in Table 2, the sample average pricing error on the CRB futures return is .0346 with a standard error of .075, while the sample average pricing error on the commercial paper return is .0338 with a standard error of .070.[15] On the basis of these results, it appears that observed production and hedging behavior as it is embedded in these agricultural output choices and these agricultural prices is not inconsistent with Proposition 2.

[15] In fact, for all versions of the model estimated all estimated moment conditions were slightly positive.

6 An Inherently Risky Technology?

To this point, (2) has been viewed solely as a means of characterizing and testing optimal producer behavior. However, the estimated stochastic discount factor also conveys structural information on $c(\mathbf{w},\tilde{z})$.

Table 1 and Table 2 suggest that, holding higher moments constant, increasing mean output increases cost. On the other hand, holding mean output constant, decreasing dispersion raises cost. Thus, the estimated structure is consistent with a technology that is "inherently risky" in the sense of Chambers and Quiggin (2000). That is, introducing a mean preserving spread of the output distribution reduces period t cost. Given the nature of agricultural technologies, this is very reasonable because it implies that, in grappling with Nature, farmers must incur significant costs to mitigate production risk.

However, there is an empirical issue. The β parameter does double duty. It also measures "size" effects.[16] And when interpreted in that light, it suggests that significant economies of size are present. Most observers argue that economies of size in US agriculture, as measured by average cost, are either positive and significant or positive but small. The stylized fact, as derived from a broad range of empirical studies, is a "sagging" L-shaped average cost curve (Hallam) that is never positively sloped. This stylized fact is consistent with the evidence here, but for the specification chosen, the results also suggest that marginal cost (in an appropriately defined sense) is decreasing in output. This will strike many as intuitively implausible. I agree. However, I also hasten to add that the theoretical development does not require marginal cost to be positive much less increasing. Neither for that matter does basic production economics unless explicit structure is placed upon $X(\tilde{z})$ (free disposability of output and convexity of the graph, respectively (Chambers)).

That said, the problem is that the empirical specification confounds two effects. For purely time-series data, these two effects are not simply identifiable independently for much the same reasons that Diamond and McFadden have argued that size and technical change cannot be disentangled. Sorting these effects appropriately requires either more data or

[16] Actually, β does triple duty because it also captures the 'learning-by-doing' effect. See Chambers and Quiggin (2005) for a discussion of this problem in terms of aggregate US production behavior and asset pricing.

more data and a much richer technical specification under a stronger set of stationarity and identifying assumptions. I leave its resolution to future research. For the present, an appropriate and intuitive interpretation is that the point estimates suggest that the risk-mitigation effect dominates the size effect. Given the positive measured returns to size that are typical in US agricultural studies, this is very reasonable.

7 Willingness to Pay for Insurance Products

Expression (2) also suggests a cost-based asset pricing model that potentially could explain asset price behavior under the presumption that agricultural output is predetermined (and predictable) much as the consumption-based capital asset pricing model (e.g., Hansen and Singleton, 1983; Cochrane, 2000; Campbell, 2003) is used to explain asset price behavior in terms of aggregate consumption patterns. Chambers and Quiggin (2005) study such a cost-based asset pricing model for equity and commercial paper returns data using a macroeconomic data set. There the stochastic discount factor is a function of aggregate wages, aggregate output (real GDP), aggregate investment, and aggregate capital stock.

It stretches credulity, however, to suggest that agricultural output fluctuations are a serious causal factor driving returns on financial assets that are only remotely related to agricultural markets. Thus, it seems to me that interpreting (2) as a model potentially capable of explaining a broad spectrum of security returns is, at best, ptolemaic, and, more likely, just wrong headed. What (2) does offer is a method for virtual (shadow) valuation of financial assets for agricultural producers.

Consider, for example, the empirical problem of pricing agricultural insurance products. Because these products are publicly supported in the United States, a growing (and somewhat controversial) literature has emerged on calculating actuarially fair insurance rates (Skees, Reed, and Barnett; Goodwin and Ker, 1998; Ker and Goodwin, 2000; Babcock, Hart, and Hayes). Although the technical details differ, the basic approach is to characterize the yield distribution empirically in a parametric or non parametric framework, and then combine that estimate with a premium/indemnity formula to arrive at actuarially fair rates.[17]

[17] Not surprisingly, given the approach that has been taken, most of the controversy in this literature

As such, it is mainly an exercise in rate setting for a hypothetical insurer that presumes that the yield distribution is stable enough to be estimated and characterized empirically. More to the point, it's an attempt to calculate an actuarially fair marginal willingness to sell for an insurance provider.

Expression (2) offers a method for empirically investigating the other side of the market, the farmer's marginal willingness to pay for crop insurance. Purely for the sake of illustration, suppose that the empirical version of (2) associated in Table 2 is credible. Then at time t, an estimate of a representative farmer's marginal willingness to pay for an insurance product $\tilde{A}^*_{t+1}$ is

$$v^*_t = \phi(\mathbf{w}_t)\left[2.07E_t\left[\frac{\tilde{A}^*_{t+1}}{\tilde{p}_{t+1}}\right] - 17.72E_t\left[\frac{\tilde{A}^*_{t+1}}{\tilde{p}_{t+1}}(\tilde{z}_{t+1} - z_t)\right]\right]. \tag{9}$$

Hence, if $\tilde{A}^*_{t+1}$ covaries negatively with $\tilde{z}_{t+1}$, the farmer has a higher willingness to pay for it than if it covaried positively.

It is here that the interpretation of β as capturing the inherent riskiness of agricultural technologies comes into play economically. If the technology is inherently risky, farmers pay a premium to avoid dispersion in their output. So if an asset naturally covaries negatively with $\tilde{z}_{t+1}$, it should be more valuable to the farmer *not because purchasing it helps stabilizes random consumption* (as, for example, in the consumption-based capital asset pricing model), but because it allows the adoption of less costly production practices in achieving $\tilde{q}_{t+1}$. The cost saving in production realized at the margin, *ceteris paribus,* enables the farmer to profitably pay a higher price for the asset.

Conceptually, this point becomes more transparent when one realizes that Proposition 1 represents a situation where a farmer uses *two stochastic technologies* to achieve a desired $\tilde{q}_{t+1}$. The first technology is his or her physical production technology, the second is the linear constant returns technology associated with the span of the financial market. The second technology, in fact, consists of J stochastic linear technologies. If one component of the financial technology (i.e., one asset) reduces the physical technology's marginal cost (in usual parlance, is complementary but in a stochastic sense), it should be more valuable to

has nothing to do with the economics of the problem. Instead, it focuses almost exclusively on the proper statistical estimation of the yield distribution.

the farmer than an asset that raises that marginal cost.

Perhaps the greatest empirical puzzle of agricultural insurance valuation is the large subsidy that is required to entice presumably risk-averse farmers to purchase the various types of marketed insurance products (Hennessy and Moschini). *If farmers are so risk averse, why are they so insurance-averse?* Truly risk averse farmers would buy actuarially fair insurance. In fact, they would happily pay a premium for it. In a sense, this is *the* central challenge to the whole rate-setting literature. So far, despite a number of attempted explanations, most of which focus on market failures (moral hazard, adverse selection), this puzzle remains unresolved.[18] The model here presented advances an, as yet, unexplored further possibility. Agricultural insurance products, as they currently exist, may not complement the farmer's ability to deal with the production and revenue risk associated with his or her physical technology.

I do not mean to suggest that expressions of the form (9) should replace the rate setting exercises I described above. Those (rate setting) are fundamentally important. They measure what the insurer needs to break even. But the buying price is equally important. This is doubly so since it is routinely argued (and routinely accepted) that crop insurance is not commercially viable in the United States. Given that farmers do have access to other financial markets (which they certainly do), and that they appear to be using those markets in a fashion that approaches optimality as the empirical results here suggest, then the presence of a significant gap between the marginal willingness to sell and the marginal willingness to buy the insurance product would be empirically-based evidence that the product is, indeed, not viable. The question of whether the government should be in the business of subsidizing nonviable financial products is best left to other arenas. The important point is that the approach here outlined offers the promise of an empirical approach to grounding the debate in measurable economic magnitudes.

[18] One challenge to these explanations is why agricultural-insurance should not be able to deal more effectively in a commercial setting with problems faced by other insurance products. Any objective reading of this literature reveals that it is typically grounded in less than compelling *a*empirical assertions about the 'special nature' of agriculture.

8 Concluding Remarks

In evaluating the empirical results, there are numerous reasons for caution. First, the data series are quite short[19] and were constructed at a very high level of aggregation. Second, the stochastic discount factor assumes a particularly simple form that imposes a fair amount of structure upon the underlying technology.[20] Third, although the range of financial assets covered in the empirical analysis is broader than usually considered in empirical hedging models, it is much smaller than the class of assets to which it should apply. In principle, it should apply to any asset lying in the span of $\mathbf{A}$. Thus, even though some of these caveats are usually encountered as apologies for empirical results do not coincide with theoretical predictions, final empirical acceptance of the behavior depicted by (2) must await a broader and more thorough empirical validation of its implications.

[19] They are shortened even further by the requirement to splice different series together for a common period.

[20] Notice, however, that the linear stochastic discount factor used here can be manipulated to produce a factor-based asset pricing model of the type that is routinely used in empirical financial economics (Cochrane, 2001; Chambers and Quiggin, 2005).

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