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Dynamic Recontracting of Water Rights*

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Abstract

Scarcity of water has become a major issue facing many nations around the world. To improve the efficiency of water usage there has been considerable interest in recent years in trading water. A major issue in trading water rights is the problem of how an allocation system can be designed in perpetuity that also has desirable properties at each point of time. This is an isssue of the time consistency of the contract to trade water. In this paper we develop a model of dynamic recontracting of water rights and study time consistency properties of the resultant contracts using the ideas of Filar and Petrosjan [7].

JEL Classification: C71,C73

1 Introduction

Scarcity of water has become a major issue facing many nations around the world. To improve the efficiency of water usage there has been considerable interest in recent years in using market based instruments to trade water. The introduction of markets for water was first suggested in the literature by Burness and Quirk [2, 3]. Since then the literature has expanded to include a number of studies including Howe, Schurmeier and Shaw [11], Provencher [16] and Provencher and Burt [17]. The analysis of dynamic aspects of water trading is still relatively new. The main contributions so far being Lahmandi-Ayed and Matoussi [12] and Freebairn and Quiggin [8].

A major issue in trading water rights is the problem of how an allocation system can be designed in perpetuity that also has desirable properties at each point of time. This is an issue of the time consistency of the contract to trade water. Lahmandi-Ayed and Matoussi have examined the impact of water markets on investment in water-saving technology using a theoretical model. They find that water markets have an unclear impact on investment in new technology. One interpretation of their result is that of time-inconsistency in water trading and this in turn leads to the focus of this paper in identifying time-consistency conditions for contracts for water rights.

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Freebairn and Quiggin are concerned with the impact of highly variable rainfall on markets for water and develope an analysis of water trading using contingent contracts in a state-contingent asset pricing model. The concern with highly variable rainfall also lies at the center of our paper. Unlike Freebairn and Quiggin we attempt to reconcile perpetual contracts for water with temporary trading. Freebairn and Quiggin recognize the difference in the nature of these two contracts and attempt to address this by allowing short-selling as a means to capture temporary trades. Nevertheless they completely ignore the issue of the compatibility of the permanent contract with the temporary contract which is the focus of our paper.

Time consistency of the solution is a requirement that needs to be imposed on contracts for permanent markets in water to be of practical use. We apply Filar and Petrosjans [7] dynamic extension of cooperative games in characteristic function form to the problem of recontracting water rights in the presence of fluctuations in rainfall. The model extends Ambec and Sprumont [1], who model the sharing of water resources along a river to a dynamic cooperative game setting.

Using the ideas of Filar and Petrosjan [7] we develop a model of dynamic recontracting of water rights and study time consistency properties of the resultant contracts. We study the evolution of the characteristic function over time when the solution concept employed is that of *downstream incremental distribution*. The downstream incremental distribution is a compromise solution concept due to Ambec and Sprumont [1] that captures the interests of both upstream and downstream users of a river. Implications for contract and market design for water markets are then discussed.

The remainder of the paper is organised as follows, section 2 presents the model. Section 3 discusses dynamics, section 4 the time consistency of water contracts, section 5 the coalitional τ-value and bargaining for water, and section 6 dropout monotonicity. Section 7 presents a numerical example of the computation of the characteristic function over time using an artificial dataset. Section 8 concludes.

2 The Model

We follow Ambec and Sprumont [1] and consider a river which flows through a number of jurisdictions, the set of all jurisdictions is the set of agents $N = \{1, ..., n\}$. The ordering of players along the river may be modelled in a number of ways, one way would be via a graph and and a graph restricted game along the lines of Myerson [14], another approach is to view the ordering along the river as restricting coalitions to a-priori unions [4]. In the specific case here it is modelled either as a consecutive game or as a type of sequencing game.

Agents are linearly ordered along a river so that agent $i < j$ means i is upstream from j. For any two coalitions $S, T \subset N$ then $S < T$ if $i \in S, j \in T$ $i < j$. $\min S$ is the smallest member of coalition S and $\max S$ the largest member of coalition S. The set of predecessors of agent i is defined as $P_i = \{j \in N : j \leq i\}$ and the set of strict predecessors of agent i is given by $P_i^0 = P_i \setminus \{i\}$.

As the river flows downstream the volume of water flowing through the river increases through inflows from tributaries and run-off from surrounding land. the flow of water at the source is given by $e_1 > 0$. Land is assumed to be privately owned and divided up amongst agents, so that the inflow of water from surrounding land can be attributed to individual agents e_i.

This leads to the following consecutive game $\Gamma = (N, e, b)$, where $e = (e_1, ..., e_n)$ and $b = (b_1, ..., b_n)$. Agents' utilities are assumed to be of the form:

$$U_i(x_i, t_i) = b_i(x_i) + t_i \tag{1}$$

where x_i is the i-th agents consumption of water and t_i is a net transfer of money $t_i < 0$ corresponding to a payment and $t_i > 0$ corresponding to the receipt of money.

Now following Filar and Petrosjan [7] we consider a family of such consecutive games $\Gamma_k = (N, e_k, b_k)$, where $k = 0, 1, ..., m$. Given a characteristic function $v_k(S), S \subset N(v_k \geq 0)$ we consider a solution concept $C(v_k) \in \Gamma_k$. Filar and Petrosjan note that this solution concept may be any known solution concepts from static cooperative game theory. We will first consider the case of Ambec and Sprumont's downstream incremental distribution.

First solve the following constrained optimization problem to obtain the optimal feasible consumption plan and maximal welfare for all players:

$$\max \sum_{k=0}^{m} \sum_{i \in N} U_i(x_i(N), t_{ik})$$

subject to

$$\sum_{k=0}^{m} \sum_{i \in N} t_{ik} \leq 0$$

and

$$\sum_{k=0}^{m} \sum_{i \in Pj} (x_{ik} - e_{ik}) \leq 0, \forall j \in N$$

The solution of this is $(x^*(N), t^*(N))$. The amount given by $\sum_{k=0}^{K} \sum_{i \in N} b_i(x^*_{ik}(N))$ gives the agents' maximal welfare to be split using the downstream incremental distribution.

Now consider the consumption plan

$$x^*(S) = argmax \sum_{i \in S} b_i(x_i) \tag{2}$$

subject to $\sum_{i \in P_j \cap T}(x_i - e_i) \leq 0 \, \forall j \in T$ and $T \in \mathcal{T}$, where T denotes a consecutive coaltion. Then the secure benefit of S is given by $v(S) = \sum_{i \in S} b_i(x^*_i(S))$.

The aspiration welfare of an arbitrary coalition S is the highest welfare it could achieve in the absence of $N \setminus S$. It is obtained by choosing a consumption plan

$$x^{**}(S) = argmax \sum_{i \in S} b_i(x_i)$$

subject to $\sum_{i \in P_j \cap S} x_i \leq \sum_{i \in P_j} e_i \, \forall j \in S$.

Ambec and Sprumont introduce the concept of downstream incrmental distribution to capture the marginal value of each agent's contribution to the coaltion of upstream agents. More formally, the downstream incremental distribution can be defined as follows:

Definition 1 (Downstream incremental distribution) $v_i^* = v(P_i) - v(P_i^0)$ *or* $v_i^* = w(P_i) - w(P_i^0)$

Downstream incremental distribution is an example of a compromise solution concept. Ambec and Sprumont [1] show that the downstream incremental distribution is the unique distribution satisfying the core lower bounds and the aspiration upper bounds. They note that it represents a compromise between the legal principles of absolute territorial integrity and unlimited terrritorial integrity.

3 Dynamics

We now extend the consecutive game considered by Ambec and Sprumont and embed it within a dynamic game along the lines of Filar and Petrosjan [7]. The way to think about this is that there are two types of dynamics occuring here. The first involves the flow of water downstream and is modelled as a consecutive game. The second involves the recharging of the river through periodic rainfall events leading to the beginning of a new consecutive game at each stage.

Following Filar and Petrosjan the dynamics of the characteristic function are given by

$$v_{k+1} = f(v_k, \alpha_k), k = 0, ..., m-1 \tag{3}$$
$$v_0 = v, \alpha_k \in C(v_k), v_k \in \Gamma_k$$

The dynamics of the characteristic function for the downstream incremental distribution are given by the following recursive scheme:

$$v_{k+1} = v_k + c(v_k, \alpha_k) \sum_i v_{ik}^* \tag{4}$$

or if $c(v_k, v_{ik}^*, i)$ then

$$v_{k+1} = v_k + \sum_i c(v_k, v_{ik}^*, i) v_{ik}^* \tag{5}$$

and

$$c(v_k, v_{ik}^*) = \begin{cases} -t_{ik}^*, \text{ if } \frac{|S|}{N}\Delta(t) > \sum_{i \in S} v_{ik}^* \\ t_{ik}^*, \text{ if } \frac{|S|}{N}\Delta(t) \leq \sum_{i \in S} v_{ik}^* \end{cases} \tag{6}$$

with $v_0 = \sum_{i \in S} b_i(x_{i0}^{**}(S))$ and $v_{i0} = w_0(P_i) - w_0(P_i^0)$. x_{i0}^{**} is obtained by solving $x_S^{**} = argmax \sum_{i \in S} b_i(x_{i0})$ subject to

$$\sum_{i \in P_j \cap S} x_{i0} \leq \sum_{i \in P_j} e_{i0} \, \forall j \in S$$

Note that $\Delta(t) = v_k(N) - \sum_{i \in N} v_k(\{i\})$.

This system may be solved numerically by first solving the intertemporal programming problem to find the sequence of transfers between agents. Then solving for the initial conditions of the recursive scheme before finally solving for the time path of the characteristic equation.

4 Time Consistency of Water Contracts

An important question in negotiating water contracts, is how to allocate these contracts in perpetuity, this is essentially a question of how to design time consistent contracts.

First we introduce a number of definitions drawn from Filar and Petrosjan [7].

Definition 2 (Summed solution set) *With every solution concept $C(v_k)$ in Γ_k associate the summed solution set $\bar{C}$ defined by*

$$\bar{C} = C(v_0) \oplus C(v_1) \oplus ... \oplus C(v_m)$$

as a corresponding solution concept of G. Where $\oplus$ is defined as set addition.

Definition 3 (Game Sequence) *For a fixed sequence $\bar{\alpha} = \{\bar{\alpha}_0, ..., \bar{\alpha}_m\}$, where $\bar{\alpha}_k \in C(\bar{v}_k)$. Let $\bar{v}_0, \bar{v}_1, ..., \bar{v}_m$ be the corresponding characteristic function defined by the scheme in the previous section. Now consider a sequence of one stage games $\bar{\Gamma}_k$ with characteristic function $\bar{v}_k$ and the cooperative game $\bar{G}$ starting from stage k in $\bar{G}$ with the initial stage game $\bar{\Gamma}_k$ with characteristic function $\bar{v}_k$ and the characteristic function of $\bar{G}_k$ defined by*

$$\bar{v}^k(S) = \sum_{l=k}^{m} \bar{v}_l(S)$$

with this notation $\bar{G}$ is $\bar{G}_0$ and $\bar{v}$ is $\bar{v}^0$.

Now we introduce time consistency following Petrosjan and Zenkevich [15]

Definition 4 (Time Consistency) *A solution concept is time consistent if for each $\alpha \in \bar{C}$ there exists $\beta = (\beta_0, ..., \beta_m), \beta_k \geq 0, k = 0, 1, ..., m$ such that $\alpha = \sum \beta_j$ and*

$$\alpha^k = \sum \beta_j \in \bar{C}^k$$

As pointed out by Filar and Petrosjan this definition of time consistency can have problems in the case of non-unique solution concepts [7, p. 54]. However, both downstream incremental distribution and the coalitional τ-value are unique so we do not need to consider the question of internal time-consistency. It will therefore suffice to show that the downstream incremental distribution is time consistent.

Proposition 1 *The downstream incremental distribution is time consistent.*

Proof Set $\beta_k = v^*_{ik}$. Now consider $\alpha \in \bar{C}$ represented as $\alpha = \sum_{k=0}^m \alpha_k, \alpha_k \in C(\bar{v}_k)$. Let $\beta_k = \alpha_k$ then $\alpha^k = \sum_{j=k}^m \beta_j = \sum_{j=k}^m \alpha_j \in \bar{C}^k$. Linearity of the downstream incremental distribution implies

$$v^*_i = w(Pi) - w(Pi^0) = \sum_{i \in Pi} b_i(x^{**}_i(Pi)) - (\sum_{i \in Pi} b_i(x^{**}_i(Pi^0)))$$

Because we know that $v(S) = \sum_{k=0}^m v_k(S)$ [7, p. 51] and we know that $v(S) = \sum_{i \in S} b_i(x^*_i(S))$ [1, p. 7]. Then the right hand side of v^*_i may be expanded

$$\begin{aligned} \sum_{i \in Pi} b_i(x^{**}_i(Pi)) - (\sum_{i \in Pi} b_i(x^{**}_i(Pi^0))) &= \sum_{k=0}^m b_i(x^{**}_{ik}(Pi)) - \sum_{k=0}^m (b_i(x^{**}_{ik}(Pi^0))) \\ &= \sum_{k=0}^m v^*_{ik} \qquad (7) \end{aligned}$$

QED.

This result implies that if downstream incremental distribution were to be implemented as a water rights allocation system, then this compromise solution will be consistent through time. The recharge of the water supply through rainfall events would not require a reallocation of water rights as long as users of water are optimizing their use at each point in time. Furthermore it provides us with a way of reducing the computational burden that would be required in order to calculate the size of the surplus needed for redistribution. As long as the path of v_k is optimal, which will be the case as long as the transfers are given by an optimal sequence and as long as the initial v_0 is optimal, we need not solve a mathematical programming problem at each time-step.

Are other compromise solution concepts perhaps equally plausible and how do they relate to downstream incremental distribution? Another possible candidate solution concept is the coalitional τ-value [4]. This is an extension of the τ-value to games with coalition structures. We now turn to the question of time consistency of the coalitional τ-value.

5 The Coalitional τ-value and Bargaining for Water

Following Casas-Mendez et al. [4] we now introduce a game with a-priori unions or coalition structure. Such a game $\Gamma_S = (N, v, P)$ where $P = \{P_1, ..., P_m\}$ is a partition of the player set. A partition of the players may be defined that corresponds to linear river of Ambec and Sprumont, so that P_i, P^0_i defines a partition. Consequently, the consecutive game of Ambec and Sprumont may be formulated as a game with a priori unions or coalition structures. It is also straightforward to write P in terms of predecessor and strict predecessor sets. It is straightforward to extend the model by defining other partitions to branching river networks. So this setting appears to have no disadvantage over that employed by Ambec and Sprumont.

Now introduce the quotient game $v^P \in G(M)$ where $v^P = v(\cup_{k \in L} P_k)$ for all $L \subset M$ and $G(M)$ is the The set of unions is defined to be $M := \{1, ..., m\}$ which is the index set of the partitions P_k, i.e. $k \in M$. set of games defined on unions in M.

Definition 5 (Utopia Payoff) $M_i(v, P) := v(N) - v(N \setminus i)$

Note that the utopia payoff of the i-th player would correspond to the downstream incremental distribution of the n-th player. Casa-Mendez also note that the utopia payoff $M_i(v, P) = v^P(M) - v^P(M \setminus \{k\}) - (v^P_{-i}(M) - v^P_{-i}(M \setminus \{k\})$.

Definition 6 (Minimal Right)

$$m_i(v, P) := \max_{S \in P(k): i \in S} (v(S) - \sum_{j \in S \setminus \{i\}} M_j(v, P))$$

.

Definition 7 (Quasi-Balanced Game) *A game $v \in G(N)$ that satisfies:*

i) $m(v) \leq M(v)$

ii) $\sum_{i \in N} m_i(v) \leq v(N) \leq \sum_{i \in N} M_i(v)$

The class of quasi balanced games with player set N is denoted $QBG(N)$.

We can now introduce quasi-balanced games with a-priori unions.

Definition 8 (Quasi-Balanced Game with a-priori Unions) *A game with a -priori unions $(v, P) \in U(N)$ is said to be* **quasi-balanced** *iff the following three conditions are satisfied:*

i) $v^P \in QBG(M)$,

ii) $m(v, P) \leq M(v, P)$,

iii) $\sum_{i \in P_k} m_i(v, P) \leq \sum_{i \in P_k} \tau_i(v^P) \leq \sum_{i \in P_k} M_i(v, P)$ *for all* $P_k \in P$.

The class of quasi balanced games with a-priori unions and player set N is denoted $QBU(N)$.

We are now ready to introduce the coalitional τ-value.

Definition 9 (Coalitional tau-value) *The coalitional τ-value is a mpa τ : $QBU(N) \to R^N$ which assigns to every $(v, P) \in QBU(N)$ the vector $(\tau_i(v, P))_{i \in N}$ such that, for all $P_k \in P$ and all $i \in P_k$,*

$$\tau_i(v, P) := m_i(v, P) + \alpha_{k'}(M_i(v, P) - m_i(v, P)),$$

where , for each $k \in M, \alpha_{k'}$ is such that $\sum_{i \in P_k} \tau_i(v, P) = \tau_k(v^P)$.

The coalitional τ-value is a compromise between the upper and lower vectors defined by the utopia payoff and the minimal right of a player. Compare this to the aspiration upper bounds of Ambec and Sprumont.

Proposition 2 *The coalitional-τ-value is time consistent.*

Proof Set $\beta_k = \tau^*_{ik}$. Now consider $\alpha \in \bar{C}$ represented as $\alpha = \sum_{k=0}^{m} \alpha_k, \alpha_k \in C(\bar{v}_k)$. Let $\beta_k = \alpha_k$ then $\alpha^k = \sum_{j=k}^{m} \beta_j = \sum_{j=k}^{m} \alpha_j \in \bar{C}^k$. Linearity of the downstream incremental distribution implies

$$\begin{aligned}
\tau^*_i &= m_i(v,P) + \alpha_{k'}(M_i(v,P) - m_i(v,P)) && (8)\\
&= (1-\alpha_{k'})(v(S^*) - \sum_{j\in S\setminus\{i\}} (v(N) - v(N\setminus\{j\}))) \\
&\quad +\alpha_{k'}(v(N) - v(N\setminus\{i\}))
\end{aligned}$$

Because we know that $v(S) = \sum_{k=0}^{m} v_k(S)$ (Filar and Petrosjan [7, p. 51]). Then the right hand side of τ^*_i may be expanded

$$\begin{aligned}
\tau^*_i &= (1-\alpha_{'k})(\sum_{k=0} v_k(S^*) - \sum_{k=0}\sum_{j\in S^*\setminus\{i\}} (v_k(N) - v_k(N\setminus\{i\}))) \\
&\quad +\alpha_{k'}\sum_{k=0}(v_k(N) - v_k(N\setminus\{i\})) \\
&= \sum_{k=0}[m_{ik}(v,P) + \alpha_{k'}(M_{ik}(v,P) - m_{ik}(v,P))] \\
&= \sum_{k=0}\tau_{ik}(v,P) && (9)
\end{aligned}$$

QED.

By theorem 3 of Casas-Mendez et al. [4, p. 501] the coalitional τ-value is unique so that we need not consider internal time consistency.

It would seem that both downstream incremental distribution and the coalitional τ-value are time consistent and both possible candidates for perpetual contracts in water rights. How then can we choose between these two solution concepts?

Hendrickx [10] has shown that the model of Ambec and Sprumont is related to sequencing games. Sequencing games were first introduced by Curiel, Pederzoli and Tijs [5] to study the problem of sharing cost-savings in queueing. Hendrickx shows that downstream incremental distribution, which he refers to as the μ-rule, captures the concept of drop-out monotonicity, in sequencing games with regular cost functions. Drop-out monotonicity is a requirement that costs (or values) are not reduced (increased) when an agent drops out of the queue.

6 Drop-Out Monotonicity

We now modify the model of Ambec and Sprumont to make the connection to sequencing games more explicit. A sequencing situation consists of a queue of n players whose positions in the queue are described by a permutation σ of the player set N. For example $\sigma(i) = j$ denotes that player i holds position j in the queue. A given permutation of the player set corresponds to a particular ordering of players The predecessor set $P(\sigma, i) := \{j \in N | \sigma(j) < \sigma(i)\}$ is equivalent to the predecessor set of Ambec and Sprumont for a given permutation. A given permutation of the player set corresponds to a particular ordering of players along the river. Each player has a utility function employed above associated with them. We now introduce the following definition of a sequencing situation modified for water flow.

Definition 10 (Sequencing situation) *A sequencing situation (for water flow) is an ordered triple (σ, b, x) consisting of where $\sigma \in \Pi_N$ is the set of permutations of N, b is the utility of a player and x is the water consumed by a player.*

The water x consumed by an agent located along the river takes the place of the service time in the standard from of the sequencing game. The way to think about this is that both are resources consumed by agents in sequence.

Curiel, Pederzoli and Tijs [5] define the τ-value of a sequencing situation to be the τ-value of the coresponding sequencing game. Note that a given permuation of the player set defines a particular P_k of the set of a-priori unions of the game with a-priori unions and that consequently the coalitional τ-value corresponds to the τ-value of a sequencing game for a given permutation of the player set. Essentially sequencing games may be viewed as a special case of a game with a-priori unions in which the partition of the player set posseses a sequential order.

The question is whether or not the τ-value is drop-out monotonic? We now formally define drop-out monotonicity following Hendrickx [10].

Definition 11 (Drop-out Monotonicity) *An allocation rule is called drop-out monotonic if for all (σ, b, x) and all $q \in N$*

$$\alpha_j((\sigma, b, x)) \leq \alpha_j((\sigma, b, x)^{-q}), \alpha \in C$$

for all $j \in N \setminus \{q\}$, where $(\sigma, b, x)^{-q} = (N \setminus \{q\}, (x_i)_{i \in N \setminus \{q\}}, (b_i)_{i \in N \setminus \{q\}})$

Note that the inequality is reversed here compared with Hendrickx because we are dealing with utility functions not cost functions.

Proposition 3 *The τ-value is not drop-out monotonic and stable*

Proof: This is a corollary of the fact (shown by Hendrickx [10, p. 42] Theorem 4.4.2 that downstream incremental distribution is the uniquely stable and drop-out monotonic allocation rule for sequencing games. QED.

One way to think about drop-out monotonicity is as a coalitional analogue of an individual rationality constraint. If it is satisfied players would have an incentive to force player q not to participate in the contract. This could be considered anti-social behaviour. Consequently there is a trade-off between the self-enforcement characteristics of the τ-value (individual rationality) and the fairness element of drop-out monotonicity. It would seem difficult to resolve this tension embedded in the concept.

7 An Example: Numerical Solution of Characteristic Function for Downstream Incremental Distribution

In this section we consider the problem of numerically computing the evolution of the characteristic function for downstream incremental distribution over time. For a discrete time problem this can lead to the problem of the characteristic function taking on negative values. This problem has been addressed by the literature on regularization in multistage cooperative games [6]. However as can bee seen in what follows we do not encounter this problem here.

Consider a river with five agents spread along the river. We will assume a quasi-linear utility function of the form $U(x,t) = \sqrt{x}+t$. the Pareto efficient problem is then given by the following optimization problem:

$$\max \sum_{k=0}^{9} \sum_{i \in N} \sqrt{x_{ik}} + t_{ik}$$

subject to

$$\sum_{k=0}^{9} \sum_{i \in N} t_{ik} \leq 0$$

and

$$\sum_{k=0}^{9} \sum_{i \in P_5} (x_{ik} - e_{ik}) \leq 0$$

$$\sum_{k=0}^{9} \sum_{i \in P_4} (x_{ik} - e_{ik}) \leq 0$$

$$\sum_{k=0}^{9} \sum_{i \in P_3} (x_{ik} - e_{ik}) \leq 0$$

$$\sum_{k=0}^{9} \sum_{i \in P_2} (x_{ik} - e_{ik}) \leq 0$$

$$\sum_{k=0}^{9} \sum_{i \in P_1} (x_{ik} - e_{ik}) \leq 0$$

Note that the inflows into the river are shown in table 1. No units are specified as these figures are simply to illustrate the technique. Each column represents a different point of inflow along the river and each row a different point in time. Location 5 is furthest upstream and and hence has the least accumulated water.

We first need to compute the Pareto efficient allocation of water over time. This is done by solving the above nonlinear programming problem. The results are presented in the following tables for water consumption (table 2) and transfers between agents (table 3).

Table 1: Water Flows at Each Point in Time and at Each Location along River

k	1	2	3	4	5
0	78.12643233	54.68850263	38.28195184	26.79736629	18.7581564
1	80.2682291	56.18776037	39.33143226	27.53200258	19.27240181
2	35.41070132	24.78749093	17.35124365	12.14587055	8.502109387
3	89.63038574	62.74127002	43.91888901	30.74322231	21.52025562
4	81.15103529	56.8057247	39.76400729	27.8348051	19.48436357
5	5.450252794	3.815176956	2.670623869	1.869436708	1.308605696
6	48.06539021	33.64577314	23.5520412	16.48642884	11.54050019
7	71.32991207	49.93093845	34.95165691	24.46615984	17.12631189
8	52.19912309	36.53938616	25.57757031	17.90429922	12.53300945
9	68.72056912	48.10439839	33.67307887	23.57115521	16.49980865

The Pareto efficient level of welfare was 282.2127. This welfare needs to be distributed in a fair way between the agents along the river and over time. A water trading contract needs to be designed that achieves such a division of water in a fair way both initially and in perpetuity. We therefore need to compute both initial allocations and using a time consistent imputation distribution principle (IDP) compute the evolution of the characteristic function of each coalition over time.

Table 2: Pareto Efficient Allocation of Water over Time

k	1	2	3	4	5
0	61.03520296	42.72464202	29.90724963	20.93509878	14.65454699
1	61.03520351	42.72464258	29.9072499	20.93507188	14.65455281
2	61.03520296	42.72464202	29.90724934	20.93507188	14.65455281
3	61.03520296	42.72464202	29.90724934	20.93507188	14.65455281
4	61.03520296	42.72464202	29.90724919	20.93507173	14.65455266
5	61.03520296	42.72464202	29.90724919	20.93507173	14.65455266
6	61.03520296	42.72464202	29.90724966	20.93507219	14.65455298
7	61.03520327	42.72464234	29.90724966	20.93507219	14.65455298
8	61.03520327	42.72464234	29.90724966	20.93507219	14.65455298
9	61.03520327	42.72464234	29.90724966	20.93507219	14.65455298

For each coalition S $\{5,4,3,2,1\},\{5,4,3,2\},\{5,4,3\},\{5,4\},\{5\})$ we need to solve a non-linear programming problem to compute the initial value of the characteristic function at time $k=0$.

We now solve the following non-linear programming problem for each of these coalitions:

$$x_S^{**} = argmax \sum_{i\in S} b_i(x_{i0})$$

subject to

$$\sum_{i\in P_j\cap S} x_{i0} \leq \sum_{i\in P_j} e_{i0}\, \forall j \in S$$

This results in the following initial values of the characteristic function corre-

Table 3: Pareto Efficient Transfers between Agents over Time

k	1	2	3	4	5
0	0.001385928	-1E-06	-1E-06	-1E-06	-1E-06
1	-1E-06	-6.62812E-05	-6.62812E-05	-6.62812E-05	-6.62812E-05
2	-6.62812E-05	-6.62812E-05	-1E-06	-1E-06	3.72327E-05
3	3.72327E-05	3.08195E-05	3.08195E-05	3.08195E-05	3.08195E-05
4	3.08195E-05	3.08195E-05	3.08195E-05	0.000142285	0.000142285
5	-3.44617E-05	-3.44617E-05	-3.44617E-05	-7.26944E-05	-7.26944E-05
6	-7.26944E-05	-7.26944E-05	-7.26944E-05	-6.37378E-05	-6.37378E-05
7	-6.37378E-05	-6.37378E-05	-6.37378E-05	-6.37378E-05	-6.37378E-05
8	-6.37378E-05	-6.37378E-05	-6.37378E-05	-6.37378E-05	-6.37378E-05
9	-7.48603E-05	-7.48603E-05	-7.48603E-05	-7.48603E-05	-7.48603E-05

sponding to each coalition $(36.92901, 27.09009, 18.69493, 11.50769, 5.331069)$.

These can now be used to compute the evolution of the characteristic function at each point along the river, i.e. for each subcoalition or predecessor set of agents, over time.

This is done by implementing the recursive schemediscussed in section 3 above. Beginning with the initial values computed above. The solution of this recursive scheme is show in table 4:

Table 4: Evolution of Characteristic Function over Time

k	$v(N,t)$	$v(\{5,4,3,2\},t)$	$v(\{5,4,3\},t)$	$v(\{5,4\},t)$	$v(\{5\},t)$
0	36.92901	27.09009	18.69493	11.50769	5.331069
1	36.98019094	27.09005307	18.69489307	11.50765307	5.331032071
2	36.98015396	27.08760198	18.69244198	11.50520198	5.328580979
3	36.97770287	27.08515089	18.692405	11.505165	5.329957851
4	36.97907965	27.08629052	18.69354463	11.50630463	5.331097485
5	36.98021933	27.0874302	18.69468431	11.51156621	5.33635906
6	36.97894493	27.0861558	18.69340991	11.50887795	5.333670804
7	36.97625677	27.08346764	18.69072175	11.50652099	5.331313846
8	36.97389998	27.08111085	18.68836496	11.50416421	5.328957059
9	36.97154334	27.07875421	18.68600832	11.50180757	5.326600422

Variation in the characteristic function over time is largely dependent on the scale and variation of the riverflow at each location and each point in time. If the scale is insufficiently large there will be little change in the characteristic function, because there will be insuffient variation in transfers being made between agents at each point in time and at each location.

It should be noted that because we have employed Pareto optimal transfers at each point in time we have not encountered problems with negativity of the value function that are typical for discrete-time multistage coalitional form games. Consequently, we have not had to resort to regularization methods in computing the evolution of the characteristic function.

8 Conclusion

In this paper we have examined time consistency properties of compromise solutions concepts for water contracts along a river. We have found that both downstream incremental distribution and the coalitional τ-value are time consistent but that the desirability of the τ-value is reduced compared to that of downstream incremental distribution because the τ-value of the corresponding sequencing game does not satisfy drop-out monotonicity. Consequently, if players were to source water from somewhere other than the river, i.e. drop-out of the queue for water, other players would benefit.

In choosing between these two solution concepts downstream incremental distribution is to be preferred as a principle for defining water contracts because it possesses less anti-social properties compared with the τ-value.

We computed an example of the evolution of the characteristic function over time for downstream incremental distribution. To illustrate the way in which the method works. The computation of fair and time consistent contracts for water is quite demanding, involving the solution of multiple non-linear programming problems. For real riverine systems it would be considerably more demanding than for the example presented here. Nevertheless, time consistency goes some way to reducing the computational burden. Altogether we solved 6 non-linear programming problems to solve the contract design problem in the example. If we had not been able to resort to time consistency we would have had to solve 51 separate optimization problems. This is a considerable reduction in computational burden.

Possible extensions of the work include considering additional compromise solution concepts. Exploring the conditions under which coalitions reform over time. As pointed out by Ambec and Sprumont this model can be easily extended to consider the case of branching rivers and to consider the case of sharing the costs of pollutants such as sediments and and agricultural run-off as they flow down-river.

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