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ON OLIGOPOLISTIC FARM INPUT MARKETS—
A THEORETICAL ANALYSIS

by

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INTRODUCTION

Literature on the performance of imperfectly competitive markets has focused almost exclusively on the classical profit-maximizing firm. But the profit maximization assumption seems quite inappropriate for at least one important class of firms—agricultural cooperatives. Cooperatives supposedly eschew the profit objective, adopting in its place the goal of providing goods and services to members at cost.

In light of their differing objectives, one would expect cooperatives to alter the familiar linkages between market structure and performance and to change market performance relative to what it would be if all firms behaved as profit-maximizers. This expectation has been theoretically confirmed for the case of agricultural marketing cooperatives by Peter Helmberger. Helmberger demonstrated that "...the welfare implications of cooperative marketing are favorable under some sets of market conditions but not under others" [10, p.616].

In this paper, the implications of agricultural supply cooperatives for the performance of imperfectly competitive farm input markets will be investigated. To this end, a simple model of an agricultural supply cooperative is presented in Section A. The proposed model portrays a single product farm supply cooperative in a static, partial equilibrium environment. While more general models of agricultural cooperatives exist,[1/] the model proposed in Section A lends itself more readily to the analysis of the impacts of supply cooperatives on the performance of farm input markets.

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1/ Bar [1]; Royer [17].

In Section B, theoretical relationships between market structure and market performance are reviewed. The focus is on oligopoly theory. Several models of oligopolistic behavior are investigated. Though each model has its own set of assumptions regarding the behavior of sellers, the various models share a common implication: oligopolistic structure may result in inefficient market performance. In some circumstances (e.g., natural monopoly) oligopolistic structure and inefficient performance may persist indefinately, even with free entry. These situations pose interesting questions for policymaking.

Whether or not, and under what circumstances, cooperatives can improve the performance of oligopolistic farm input markets is the subject of Section C. We argue that the greatest potential for beneficial impacts from cooperative production of farm inputs exist in markets where economies of size are prevalent. In such markets, the structure of the market is likely to be oligopolistic, and the performance of the market is likely to be suboptimal. Market performance can be improved by supplanting private firms with cooperatives; under some circumstances, the mere presence of a cooperative may lead to improved market performance.

In the concluding section, some suggestions for extension of this research are made, and limitations of the analysis are identified.

A. A MODEL OF A FARM SUPPLY COOPERATIVE

In this section, a model of a single-product agricultural supply cooperative is presented. The model is similar in a number of respects to a model of an agricultural marketing cooperative developed by Peter Helmberger and Sydney Hoos [12]. In the Helmberger and Hoos model, the cooperative marketed a raw product for its members. The objective of the cooperative was to maximize the price paid to members for the raw product. All members received the same price per unit of raw product; in this sense, benefits were shared in proportion to patronage. The cooperative was a nonprofit enterprise: it did not earn profits for itself as a distinct legal entity, nor for its members as investors. In the supply cooperative model presented below, the cooperative produces a farm input, and minimizes the price of the farm input to members. All members pay the same price for the farm input, and hence, costs are borne in proportion to patronage. The supply cooperative also operates on a nonprofit basis. Thus, the model of a supply cooperative presented here is a supply industry analogue to the marketing cooperative model presented by Helmberger and Hoos.

1. Technology and Cost Function

The technology employed by the cooperative is represented by a twice continuously differentiable production function having convex isoquants and positive but decreasing marginal physical products:

$$Q = F(X_1 \ldots\ldots X_N) \tag{1}$$

where Q is a farm input produced by the cooperative and sold to members, and X_i $(i=1\ldots\ldots N)$ are factors of production employed by the cooperative.

The cooperative is a price taker in markets where it buys factors of production. Prices of factors employed by the cooperative are denoted r_i $(i=1\ldots\ldots N)$. The cooperative has no monopsony power.

There may be other noncooperative supply firms in the market for farm input Q. All such firms produce an identical farm input, employ the same technology, and face the same vector of factor prices as the cooperative. Product differentiation is ruled out.

Since all supply firms use the same technology and face the same vector of input prices, their cost functions are identical. The cost function is defined as:

$$C(Q) = \text{MIN} \sum_{i=1}^{N} r_i X_i \tag{2}$$

$$\text{s.t.} \quad Q = F(X_1 \ldots\ldots X_N)$$

The necessary conditions for operating on the cost function are:

$$\frac{r_i}{r_j} = \frac{F_i}{F_j} \quad i,j=1\ldots\ldots N \quad i \neq j \tag{3}$$

and

$$Q = F(X_1 \ldots\ldots X_N) \tag{4}$$

To operate on the cost function, a supplier must (1) equate factor price ratios to ratios of marginal physical products for all pairs of factors, and (2) operate on the production possibility frontier. These conditions apply to cooperative and non-cooperative firms alike.

2. The Pricing Policy of the Cooperative

On the basis of the cooperative principle of operating on a nonprofit basis (alternately, on a "cost" basis), a pricing policy for the supply cooperative may be derived. The cooperative is a nonprofit enterprise.[2/] This is an important difference between cooperatives and other forms of business enterprise. If the cooperative is to provide service on a nonprofit basis, its profit must be zero. Let P_c be the cooperative's price, Q_c be its volume of output, and Π_c be its profit. Then:

$$\Pi_c = P_c Q_c - TC \tag{5}$$

where $TC = \sum_{i=1}^{N} r_i X_i$ is the cooperative's total cost. Clearly, if profit is to be zero, the cooperative's price must equal its average cost:

$$\Pi_c = 0 \Rightarrow P_c = TC/Q_c \tag{6}$$

3. The Membership and Output Policies of the Cooperative

It is assumed that the cooperative does not ration output among its members; nor does the cooperative impose quotas on members. Instead, the cooperative supplies whatever quantity its members wish to purchase. It is also assumed that the cooperative does business with members only. Let C denote the number of members and q_{ic} be the quantity demanded from the cooperative by the i^{th} member (i=1.....C). Then the quantity supplied by the cooperative is

$$Q_c = \sum_{i=1}^{C} q_{ic} \tag{7}$$

The cooperative can exercise control over its output indirectly, by regulating the number of members. Below, the advantages of membership restrictions will be described.

2/ Indeed, the Farmer Cooperative Service considers this part of the definition of a farmer cooperative. [20, p.4]. See also [18, p.191].

4. Member's Demand

It is assumed that the cooperative's members are profit maximizers with respect to their farm operations. The quantity of input Q demanded from the cooperative by a member is the quantity which maximizes his farm profit. The quantity of input Q demanded from the cooperative by a representative member i is a function of (1) the cooperative's price P_c; (2) the market price P_m--the market price is the prevailing market price charged by private (noncooperative) suppliers of input Q; and (3) other variables Z_j (j=1.....m)--the Z_j are demand shifters. The i^{th} member's demand function is:

$$q_{ic} = f(P_c, P_m, Z_1 \ldots . Z_m) \tag{8}$$

$$q_{ic} = \begin{Bmatrix} 0; \; P_c > P_m \\ q_i; \; P_c \leq P_m \end{Bmatrix}$$

$$\frac{\partial q_{ic}}{\partial P_c} \begin{Bmatrix} = 0; \; P_c > P_m \\ < 0; \; P_c \leq P_m \end{Bmatrix}$$

where q_i is the total quantity of input 0 bought by farmer i from the cooperative and/or from private suppliers, and q_{ic} is the amount bought from the cooperative. According to this specification, a member will patronize the cooperative exclusively if (and only if) the cooperative's price does not exceed the market price. If the cooperative's price exceeds the market price, the member will buy input Q from some other supplier at the lower market price.

Implicit in this specification is the assumption that members are not obligated (e.g., by contract) to patronize the cooperative. Members are free to take their patronage elsewhere, and will do so if the cooperative's price exceeds the market price. As a consequence, if $P_c > P_m$, members' demand from the cooperative is zero.

5. The Objective of the Cooperative

The objective of the cooperative is to minimize the price of farm input Q to members. Since the cooperative must operate on a nonprofit basis, its price must equal its average cost. Thus, in minimizing the price to members, the cooperative must minimize its average cost.

The quantity to be sold is exogenous to the cooperative—the cooperative must produce whatever quantity its members wish to purchase. With output predetermined, minimization of average cost is equivalent to minimization of total cost, so the problem of the cooperative may be stated as

$$\underset{\text{w.r.t. } X_1 \ldots\ldots X_N}{\text{MINIMIZE}} \qquad TC = \sum_{i=1}^{N} r_i X_i \tag{9}$$

$$\text{s.t. } Q_c = \sum_{i=1}^{C} q_{ic}$$

$$Q_c = F(X_1 \ldots\ldots X_N)$$

The Lagrangian for this problem is

$$L = \sum_{i=1}^{N} r_i X_i + \lambda \left[\sum_{i=1}^{C} q_{ic} - F(X_1 \ldots\ldots X_N) \right] \tag{10}$$

where λ, the Lagrangian multiplier, may be interpreted as the reciprocal of the cooperative's marginal cost [12, p.66]. The necessary conditions for this problem are equations (3) and (4), i.e., the cooperative must (1) equate ratios of factor prices to ratios of marginal physical products for all pairs of factors, and (2) must operate on the production possibility frontier. In short, the cooperative must operate on its cost function.

6. The Equilibrium of the Cooperative

The equilibrium of the cooperative is determined at the intersection of its demand and average cost curves. Formally, we may define the equilibrium of the cooperative as a price and quantity $[P_c, Q_c]$ such that

(i) $Q_c = \sum_{i=1}^{C} q_{ic}$

(ii) $q_{ic} = f(P_c, P_m, Z_1 \ldots\ldots Z_m)$

(iii) $P_c = C(Q_c)/Q_c$

The equilibrium quantity equals the quantity demanded by members when they are maximizing their profits, and the equilibrium price equals the value of the average cost function at that quantity.

7. The Rationale for Membership Restrictions

If the cooperative is operating in a region where returns to size are diminishing, there is a rationale for membership restrictions. Figure 1 illustrates why this is true. A U-shaped average cost curve for a hypothetical cooperative is shown in Figure 1. Also shown are various demand curves D^i, corresponding to various numbers of member C^i. As the number of members grows, the demand curve shifts to the right. Corresponding to each particular number of members C^i is a cooperative equilibrium $[P_c^i, Q_c^i]$.

(Notice that the demand curves D^i do not extend above the market price P_m. If the cooperative's price exceeded the market price, its demand would be zero.)

If the cooperative currently has C^1 members, the admission of new members is advantageous. As new members are admitted and volume increases, economies of size are captured and the price to the original C^1 members is reduced. However, when the number of members reaches C^2, economies of size are exhausted and diseconomies set in. No longer would it be advantageous from the current members perspectives' to admit new members. If new members were admitted, the price to the current members would rise, imposing a penalty on them. As Trifon noted, there may be a range of output over which the penalty of expansion is unnoticeable to the current members, "...but as it becomes noticeable, an inclination may develop among patrons to regulate patronage." [19, p.223].

For the cooperative depicted in Figure 1, it would seem appropriate to restrict the number of members to C^2. At this number, the cooperative's (equilibrium) average cost is minimized, and in this sense C^2 is the "optimal" number of members. Expansion of membership beyond C^2 could be inimical to the interest of current members.

Several writers have argued tht cooperatives should have an open-membership policy as a matter of principle.[3/] Others, noting the disadvan-

3/ See Gislason [9, p.37]; and Nourse [15, p.173].

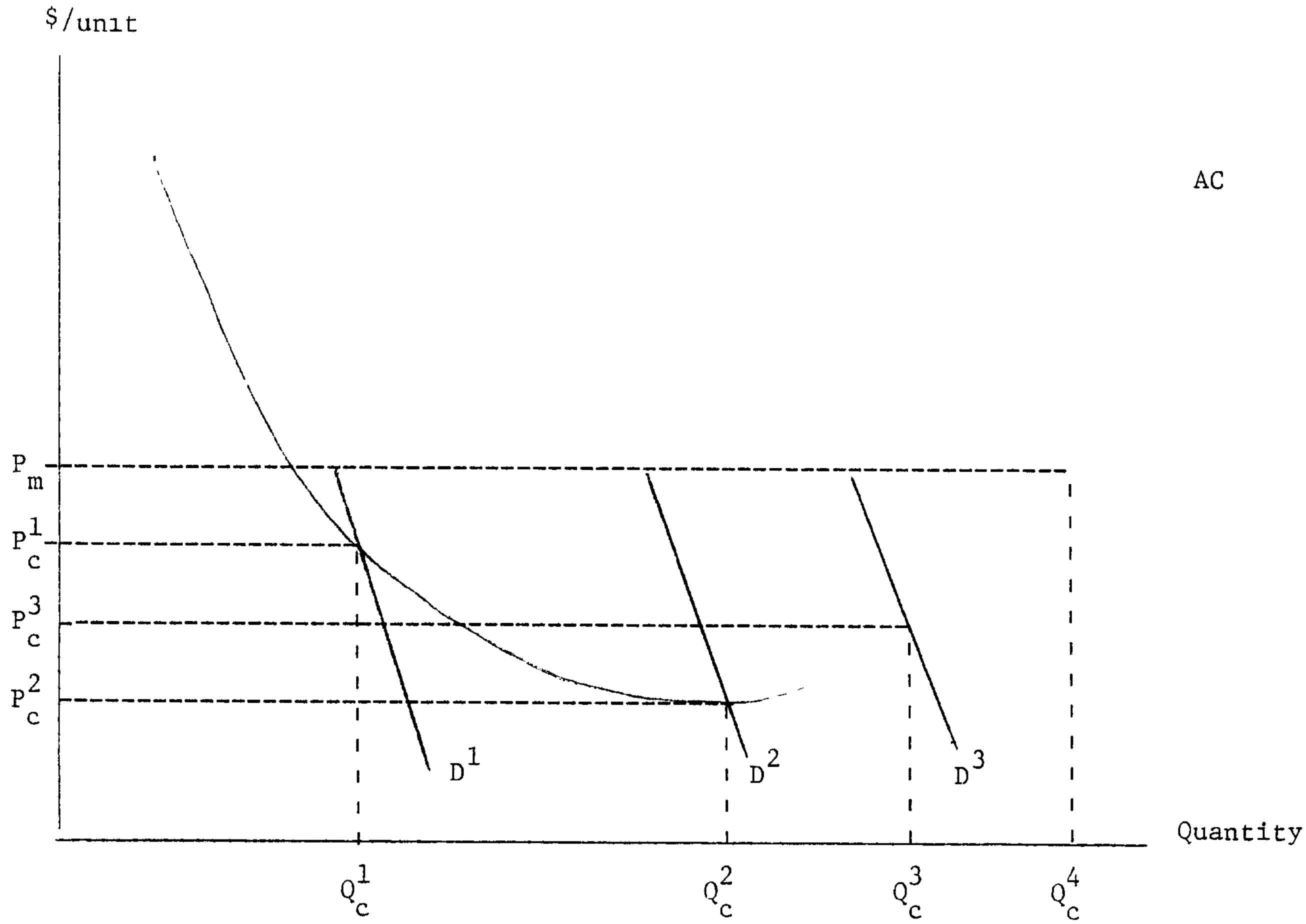

Figure 1. The rationale for membership restrictions. With C^1 members, demand is D^1; with C^2 members, demand is D^2; with C^3 members, demand is D^3. Once the membership reaches C^2, economies of size are exhausted, and increases in membership (e.g. to C^3) cause increases in the cooperative price (from P_c^2 to P_c^3). By restricting its membership to C^2, the cooperative can minimize the price paid by members.

tages of an open-membership policy in the presence of diseconomies of size, have argued in defense of a restricted-membership policy.[4/] Our position is that a cooperative can and should impose membership restrictions when it is operating under diminishing returns to size. Failure to impose membership restrictions under these conditions will reduce or eliminate the benefits to current members. For example, in Figure 1, an open-membership policy would lead to an equilibrium [P_m, Q_c^4], at which there is no advantage to belonging to the cooperative.[5/] The benefits of cooperation would be dissipated if the cooperative followed an open-membership policy.

Of course, diminishing returns to size need never be encountered. Instead of being U-shaped as in Figure 1, the cooperative's average cost curve may be L-shaped, or horizontal. If there are no diminishing returns to size, the admission of new members does not penalize current members, so an open-membership policy is presumably acceptable.

In analyzing the impacts of supply cooperatives on the performance of farm input markets, it will be assumed that cooperatives restrict membership when diseconomies of size are encountered. Otherwise, membership is assumed open.

8. The Rationale for Forming a Supply Cooperative

In the context of this model, there is but one reason to form a cooperative to supply farm input Q: to obtain this input at a lower price. If by so doing, the price of Q cannot be reduced, there is no reason to form an input Q supply cooperative.

Evidently, if the market for input Q was perfectly competitive, there would be no reason to form an input Q supply cooperative. In competitive equilibrium, the price would equal the minimum average cost of producing Q. A cooperative could not sell Q at a price lower than this, so there would be no reason to form a Q supply coopertive.

4/ See Helmberger and Hoos [11, p. 288]; Trifon [19, p. 217]; Clark [4, p. 405], [3, p. 37].

5/ Implicit in this statement, as well as in Figure 1, is the assumption that P_m is independent of C. This assumption would be valid if the cooperative was small relative to the market as a whole.

But the market for Q may not be perfectly competitive. In particular, it may be an oligopoly, in which case the price may exceed the minimum average cost of producing Q. If the market for Q is an oligopoly, there may be a rationale for forming a supply cooperative. Accordingly, we turn now to an investigation of price and output determination in oligopoly markets.

B. OLIGOPOLY, PRICE, AND MARKET PERFORMANCE

An oligopoly is generally defined as a market in which there are many buyers but relatively few sellers. This sort of structure is characteristic of farm input markets—particularly at the local market level. Three questions which arise in relation to oligopoly markets are: (1) What are the causes of oligopoly? (2) Does an equilibrium exist, and if so, what are its characteristics? and (3) What are the implications for market performance and social welfare? This section summarizes—albeit only partially—answers which economic theory has provided to these questions. Although some minor extensions of oligopoly theory are presented, the major purpose of this section is to provide background information for the subsequent investigation of the impacts of supply cooperatives on the performance of farm input markets.

1. Three Causes of Oligopoly

For discussion purposes, three causes of oligopoly are identified: barriers to entry; competitive or warlike behavior among established firms; and economies of size.

Barriers to entry include: restrictions on entry imposed by government through licensure, franchise, or regulation; ownership of essential factors of production by established firms; imperfect knowledge regarding technology, demand, or cost conditions; and inability to finance entry owing to capital market imperfections. When entry is not impeded by any such barriers, there is said to be "free entry."

A second cause of oligopoly is competitive or warlike behavior among established firms. Even if they are few in number, established firms may behave in a competitive or warlike manner, making entry unattractive for newcomers. Under these conditions, the market may retain its oligopoly structure even though barriers to entry and economies of size are not present. Warlike behavior is characteristic of Bishop's duopoly warfare model, and will be described in some detail below.

Finally, economies of size are an important cause of oligopoly. In the presence of economies of size, large firms can produce more efficiently (i.e., at lower average cost) than small firms, and there is an incentive for firm growth and increased concentration. Economies of size pose somewhat of a dilemma for policymaking: to impose an atomistic structure on an industry characterized by economies of size is wasteful. Yet unfettered competition may lead to concentration and non-competitive behavior, resulting in suboptimal market performance. Below, it will be argued that cooperatives are a way of dealing with this dilemma.

2. Price and Output Determination in Three Oligopoly Models

We shall discuss price and output determination in three oligopoly models: the Cournot model, the Chamberlin model, and Bishop's duopoly warfare model. These models encompass a wide range of behavioral assumptions and market outcomes, and provide an indication of the indeterminateness oligopoly engenders.

We shall limit our discussion to cases in which: (1) each seller's cost function is identical; (2) the product is homogeneous; (3) firms adjust quantity rather than price; and (4) the market demand curve is linear.

The market demand function is written:

$$P = A - BQ \qquad A, B > 0 \tag{11}$$

where P is market price and Q is quantity traded.

For generality, the Cournot and Chamberlin models will be solved for three alternative assumptions regarding cost conditions: constant returns to size (CRS); increasing returns to size (IRS); and first increasing then decreasing returns to size (ITDRS). The cost functions which correspond to these alternative cases are respectively:

$$C(Q) = bQ \qquad b > 0 \tag{12}$$

$$\text{IRS: } C(Q) = \begin{cases} a + bQ, & Q > 0 \\ 0 & , Q = 0 \end{cases} \quad a,b > 0 \tag{13}$$

$$\text{ITDRS: } C(Q) = \begin{cases} a + bQ + CQ^2 + dQ^3, & Q > 0 \\ 0 & , Q = 0 \end{cases} \quad \begin{matrix} a,b,d > 0 \\ c < 0 \\ c^2 < 3bd \end{matrix} \tag{14}$$

The average cost curves are respectively horizontal, L-shaped, and U-shaped. The constraints imposed on the coefficients ensure that average, marginal, and total cost are nonnegative when output is nonnegative. In each case, $C(0) = 0$, implying that firms can incur zero cost by producing zero output. The absence of fixed cost implies that the lower bound on profit is zero; this gives the analysis a long fun flavor.

The Cournot model is solved first for an arbitrary number M of firms. Then the following issue is addressed; assuming free entry and exit, and that potential entrants exist, is there an equilibrium number of firms? An equilibrium number of firms is defined as an integer M^* such that profit in nonnegative for each of the $M = M^*$ firms in market equilibrium, while the profit anticipated by the $(M + 1)^{st}$ potential entrant is zero or negative. If the market is in equilibrium and the number of firms is an equilibrium number, neither entry nor exit will occur, and the structure of the market will not change. We demonstrate that an equilibrium number of Cournot oligopolists exists, but that this number is generally not unique.

The Chamberlin model is then solved for an arbitrary number of firms. The following question is then considered: Is there an equilibrium number of Chamberlin oligopolists? We show that there is an equilibrium number of Chamberlin oligopolists, and that this number is generally unique.

Finally, Bishop's duopoly warfare model is described.

a. The Cournot Model

A Cournot oligopolist perceives that changes in his output affect the market price. For this reason, he does not behave as a price-taker, as would a perfectly competitive firm. An important behavioral assumption is that a Cournot oligopolist expects his rivals to maintain their current levels of output regardless of his output decision. In general, this expectation is incorrect. This assumption is often criticized as being naive [5, p.46]. Nevertheless, we proceed to solve the Cournot model.

Let there be M sellers, each a Cournot oligopolist. Profit for the i^{th} seller $(i = 1.....M)$ is:

$$\Pi_i = [A - B \sum_{j=1}^{M} Q_j]\, Q_i - C(Q_i) \tag{15}$$

where subscripts i and j denote the i^{th} or j^{th} firm.

The necessary condition for profit maximization is:

$$\frac{\partial \Pi_i}{\partial Q_i} = A - B \sum_{j \neq i}^{M} Q_j - 2BQ_i - BQ_i \sum_{j \neq i}^{M} \left[\partial Q_j / \partial Q_i\right]^* - C' = 0 \qquad (16)$$

where the terms $\left[\partial Q_j / \partial Q_i\right]^*$ are called "conjectural variations" and indicate how i expects j's output to change in response to changes in his (i's) output. A Cournot firm expects his rivals to keep their current levels of output regardless of his output decisions, so that the conjectural variations terms are zero for all i and j. Under this assumption, the necessary condition reduces to:

$$\frac{\partial \Pi_i}{\partial Q_j} = A - B \sum_{j \neq i}^{M} Q_j - 2BQ_i - C' = 0 \qquad (17)$$

and the sufficient condition is

$$\frac{\partial^2 \Pi_i}{\partial Q_i^2} = -2B - C'' < 0 \qquad (18)$$

If the cost function is either (12) or (13), the necessary condition is:

$$A - B \sum_{j \neq i}^{M} Q_j - 2B\, Q_i - b = 0$$

which may be written

$$Q_i = (A - b)/2B - \frac{1}{2} \left[\sum_{j \neq i}^{M} Q_j\right] \qquad (19)$$

Equation (19) is called a "reaction function," and shows the level at which i will set his output, given the output of his rivals. The sufficient condition for profit maximization with cost functions (12) and (13) is $-2B < 0$, which is certainly satisfied.

With cost function (14), the necessary condition is:

$$A - B \sum_{j \neq i}^{M} Q_j - 2BQ_i - b - 2cQ_i - 3dQ_i^2 = 0$$

or

$$Q_i = \{-2(B + c) \pm [(B + c)^2 + 12d(A - B \sum_{j \neq i}^{M} Q_j - b)]^{\frac{1}{2}} \}/6d$$

and the sufficient condition is

$$-2B - 2c - 6dQ_i < 0$$

or

$$Q_i > -2(B + c)/6d$$

Of the two values which satisfy the necessary condition, only the following satisfies the sufficient condition as well:

$$Q_i = \{-2(B + c) + \left[4(B + c)^2 + 12d(A - B \sum_{j \neq i}^{M} Q_j - b\right]^{\frac{1}{2}} \}/6d \qquad (20)$$

Equation (20) is therefore the reaction function of Cournot firm i associated with cost function (14).

If the behavior of each Cournot oligopolist is to be consistent with the behavior of his rivals, the reaction functions must hold simultaneously for all M firms. The reaction functions will all hold if and only if the variation terms $\partial Q_j/\partial Q_i$ are in fact zero as conjectured for all i and j, i.e. if and only if the ex ante conjectural variations are valid ex post relations. A situation in which the firms' reaction functions simultaneously hold is an equilibrium, and is known as a "Cournot-Nash" equilibrium.

The properties of Cournot-Nash equilibrium for alternative cost functions, for an arbitrary linear demand function, and for an arbitrary number of firms, are displayed in Table 1. The properties shown are Cournot-Nash equilibrium values for representative firm i's quantity, the market quantity, and the market price, and firm i's profit. Shown in the last row are restrictions on M needed to ensure that equilibrium profit is nonnegative. An equilibrium number of firms must satisfy these restrictions. What other restrictions must an equilibrium number of firms satisfy?

Entry, Exit, and the Equilibrium Number of Cournot Oligopolists.

Suppose that entry and exit are free, and that there exist potential entrants. The equilibrium number of Cournot oligopolists is a number M^* such that each of the M^* firms earns nonnegative profit in Cournot-Nash

Table 1. Properties of Cournot-Nash Equilibrium

PROPERTY [b]	COST FUNCTION [a] CRS: $C = bQ$	IRS: $C = a + bQ$	ITDRS: $C = a + bQ + cQ^2 + dQ^3$
Representative Firm's Equilibrium Output Q_1	$\frac{(A-b)}{B(M+1)}$	$\frac{(A-b)}{B(M+1)}$	$\frac{1}{6d}\{-[2(B+c)+B(M-1)] + [[2(B+c)+B(M-1)]^2 + 12d(A-b)]^{\frac{1}{2}}\}$
Equilibrium Market Output $Q = \sum_{i=1}^{M} Q_i$	$\frac{M(A-b)}{B(M+1)}$	$\frac{M(A-b)}{B(M+1)}$	$\frac{M}{6d}\{-[2(B+c)+B(M-1)] + [[2(B+c)+B(M-1)]^2 + 12d(A-b)]^{\frac{1}{2}}\}$
Equilibrium Market Price P	$\frac{(A+Mb)}{(M+1)}$	$\frac{(A+Mb)}{(M+1)}$	$A - \frac{BM}{6d}\{-[2(B+c)+B(M-1)] + [[2(B+c)+B(M-1)]^2 + 12d(A-b)]^{\frac{1}{2}}\}$
Representative Firm's Equilibrium Profit Π_1	$\frac{(A-b)^2}{B(M+1)^2}$	$\frac{(A-b)^2}{B(M+1)^2} - a$	$[A - MBQ_i - \frac{a}{Q_i} - b - cQ_i - dQ_i^2]Q_1$
Restrictions on M To Ensure $\Pi_i \geq 0$	NONE	$M \leq \frac{(A-b)}{\sqrt{aB}} - 1$	M Must Satisfy $[A - MBQ_i - \frac{a}{Q_i} - b - cQ_i - dQ_i^2] \geq 0$

[a] The parameters of the cost functions must satisfy: $a,b,d > 0$; $c < 0$; and $c^2 < 3bd$.

[b] The market demand equation is $P = A - BQ$; the number of participating Cournot oligopolists is M.

equilibrium, while the profit anticipated by the $(M^* + 1)^{st}$ potential entrant is zero or negative.

Suppose there are presently M firms operating and that the market is in Cournot-Nash equilibrium. Suppose further that the cost function is either (12) or (13). Assuming that the $(M + 1)^{st}$ potential entrant has zero conjectural variations, the profit he anticipates earning upon entering is

$$\Pi^*_{M+1} = [A - B \sum_{i=1}^{M} Q_i - BQ_{M+1}]\ Q_{M+1} - C(Q_{M+1}) \tag{21}$$

where, by the assumption that the market is in equilibrium and the cost function is either (12) or (13), $\sum_{i=1}^{M} Q_i = M(A - b)/B(M + 1)$. If the $(M+1)^{st}$ firm enters, he will do so in an amount which maximizes (21). The amount which maximizes (21) is:

$$Q_{M+1} = (A - b)/2B(M+1) \tag{22}$$

If the cost function is (12), the profit anticipated by the $(M+1)^{st}$ potential entrant is:

$$\Pi^*_{M+1} = \frac{(A - b)^2}{4B(M+1)^2} \tag{23}$$

Clearly, in this instance, Π^*_{M+1} is always positive. Thus, under CRS, the $(M+1)^{st}$ potential entrant will always anticipate earning positive profit upon entering a market with M firms that is in Cournot-Nash equilibrium. Conceptually, if there are a sufficient number of potential entrants, entry could continue until the number of participating firms was very large. Of some interest is the fact that as M grows, the properties of the Cournot-Nash equilibrium begin to resemble ever more closely the properties of a competitive equilibrium: Q_i becomes very small, Π_i approaches zero for all i, and the market price approaches the constant marginal cost b, which is the price associated with perfect competition.

In contrast, if the cost function is (13), there are IRS, and the $(M+1)^{st}$ firm may expect negative profit upon entering. In particular, the profit anticipated by the $(M+1)^{st}$ potential entrant in this case is

$$\Pi^*_{M+1} = \left[(A - b)^2/4B(M+1)^2\right] - a \tag{24}$$

which is negative if and only if

$$M > (A - b)/2\sqrt{aB} - 1 \tag{25}$$

From Table 1, the profit of the M currently participating firms is non-negative if and only if:

$$M \leq (A - b)/\sqrt{aB} - 1 \tag{26}$$

Therefore, the M^{th} firm will not exit, and the $(M+1)^{st}$ firm will not enter, if M satisfies:

$$(A - b)/2\sqrt{aB} - 1 < M \leq (A - b)/\sqrt{aB} - 1 \tag{27}$$

Any such M is on M^* for a Cournot oligopoly in which each firm's cost function is (13). In general M^* is not unique. For example, if $A = 6$, $a = b = B = 1$, then $M^* = 2$, 3, or 4.

Similarly, if the cost function is (14), an equilibrium number of Cournot firms will again exist, but in general, this number will not be unique.

In summary, with CRS, the equilibrium number of Cournot firms is as large as the number of potential entrants. With either IRS or ITDRS, an equilibrium number of Cournot firms generally exists, but this number is generally not unique.

b. <u>The Chamberlin Model</u>

The profit each firm earns in Cournot-Nash equilibrium could generally be increased through collusion. Especially for small M, firms would likely recognize this. Chamberlin proposed a model in which firms not only recognize the feasibility of collusion, but in fact collude [5, pp.46-51].

Collusion requires an agreement—either tacit or explicit—regarding the division of industry profit among participating firms. In Chamberlin's model, the firms tacitly agree to divide jointly maximized profit equally.

Chamberlin limits his analysis to the case of CRS for which (12) is the relevant cost function. In this case, a reaction function which is compatible with Chamberlin's model is:

$$Q_i = \begin{cases} (A-b)/2MB \text{ if } \sum_{j \neq i}^{M} Q_j = (A-b)(M-1)/2MB \\ (A-b)/B - \sum_{j \neq i}^{M} Q_j \text{ if } \sum_{j \neq i}^{M} Q_j > (A-b)(M-1)/2MB \\ (A-b)/2B - \frac{1}{2} \sum_{j \neq i}^{M} Q_j \text{ if } \sum_{j \neq i}^{M} Q_j < (A-b)(M-1)/2MB \end{cases} \tag{28}$$

This reaction function exhibits three branches. The first branch indicates that firm i is willing to restrict his output to a level consistent with equal division of jointly maximized profit; i.e., firm i is willing to collude symetrically. The second branch indicates that if his rivals set their output at a level in excess of the amount that is consistent with symetric collusion, firm i will retaliate by setting his output at a level which drives profit for all firms to zero. In other words, this branch indicates that firm i is unwilling to accept anything less than an equal share of the jointly maximized profit. The third branch shows that firm i will gladly accept more than an equal share of jointly maximized profit.

If all M firms have similar reaction functions, and each firm believes this is the case, then each firm will believe it is in his best interest to collude symetrically, and the Chamberlin equilibrium will be established. The Chamberlin equilibrium could be established with or without verbal communication, either spontaneously (as in Chamberlin's example), or after a period of experimentation in which the firm's "learn" the reaction functions of their rivals.

If the cost function is (13), there are IRS, and the Chamberlin equilibrium will depend on whether side payments are allowed. If side payments are permitted, efficient collusion requires one firm to produce the entire industry output. Side payments would then be used to bring about an equal or satisfactory division of profit amongst the participants.

Ruling out side payments, the Chamberlin equilibrium finds each firm with an equal market share, and an equal share of joint profit. The equilibrium market price and quantity are the same as for a multiplant monopolist.

Dewey proposed a simple method for solving the Chamberlin model [7, p.70]. If there are M firms and the market demand equation is (11), Dewey shows that each colluding oligopolist will maximize his profit with respect to a demand equation of the form:

$$P = A - MBQ_1 \tag{29}$$

Properties of Chamberlin equilibrium were derived for three alternative cost functions, an arbitrary linear demand function, and an arbitrary number of firms. These properties are summarized in Table 2. As in the Cournot model, certain restriction must be imposed on the number of firms M to ensure that profit is nonnegative in Chamberlin equilibrium.

A notable contrast between the Cournot and Chamberlin models is that in the former, equilibrium P declines as M grows, whereas in the latter, P is independent of M. As Chamberlin noted [p.48], in his model "there is no gradual descent to a purely competitive price with increase of numbers, as in Cournot's solution." Even with free entry and CRS, the Chamberlin equilibrium price is at the level associated with monopoly.

Entry, Exit, and the Equilibrium Number of Chamberlin Oligopolists Assume entry and exit are free, and that potential entrants exist. Firms will enter a Chamberlin market if, upon entering, they expect to earn positive profits; otherwise, they will not enter.

With CRS, profits anticipated by potential entrants will always be positive. In principle, entry could continue until all potential entrants had entered. However, the Chamberlin equilibrium price would remain at the level associated with monopoly.

In contrast, if there are IRS, the profit anticipated by the $(M+1)^{st}$ potential entrant—assuming he expects a Chamberlin equilibrium to be established after he enters—is

$$\Pi^*_{M+1} = \frac{(A-b)^2}{4B(M+1)} - a \tag{30}$$

Clearly, the $(M+1)^{st}$ firm's anticipated profit is positive if and only if

$$M < \left[(A-b)^2/4Ba\right] - 1 \tag{31}$$

Table 2. Properties of Chamberlin Equilibrium

PROPERTY [b]	COST FUNCTION [a] CRS: $C = bQ$	IRS: $C = a + bQ$	ITDRS: $C = a + bQ + cQ^2 + dQ^3$
Representative Firm's Equilibrium Output Q_i	$\frac{A-b}{2MB}$	$\frac{A-b}{2MB}$	$\frac{-2(MB+c) + [4(MB+c)^2 + 12d(A-b)]^{\frac{1}{2}}}{6d}$
Equilibrium Market Output $Q = \sum_{i=1}^{M} Q_i$	$\frac{A-b}{2B}$	$\frac{A-b}{2B}$	$\frac{M[-2(MB+c) + [4(MB+c)^2 + 12d(A-b)]^{\frac{1}{2}}]}{6d}$
Equilibrium Market Price P	$\frac{A+b}{2}$	$\frac{A+b}{2}$	$A - \frac{MB[-2(MB+c) + [4(MB+c)^2 + 12d(A-b)]\ ]}{6d}$
Representative Firm's Equilibrium Profit Π_i	$\frac{(A-b)^2}{4MB}$	$\frac{(A-b)^2}{4MB} - a$	$[A - MBQ_i - \frac{a}{Q_i} - b - cQ_i - dQ_i^2]Q_i$
Restrictions on M To Ensure $\Pi \geq 0$	None	$M \leq \frac{(A-b)^2}{4Ba}$	$[A - MBQ_i - \frac{a}{Q_i} - b - cQ_i - dQ_i^2]Q_i \geq 0$

[a] The parameters of the cost functions must satisfy: $a,b,d > 0$; $c < 0$; and $c^2 < 3bd$.

[b] The market demand equation in $P = A - BQ$; the number of participating Chamberlin firms is M.

Only if this inequality holds will the $(M+1)^{st}$ Chamberlin firm enter.

For the M Chamberlin firms that are presently operating, equilibrium profit is nonnegative if and only if

$$M \leq (A-b)^2/4Ba \tag{32}$$

Presumably, if this weak inequality were not satisfied, some firm (without loss of generality, the M^{th}) would exit. Therefore, any M which satisfies

$$\frac{(A-b)^2}{4Ba} -1 < M \leq \frac{(A-b)^2}{4Ba} \tag{33}$$

is an M^* in a Chamberlin oligopoly with cost function (13). M^* is unique in this case.

Similarly, with cost function (14), the equilibrium number of Chamberlin oligopolists in unique.

Now consider the situation in Figure 2. Shown there are an average cost curve (AC) for a hypothetical Chamberlin oligopolist, and various demand curves D^1, D^2, and D^3. The curve D^M (M=1,2,3) is the demand curve with respect to which the Chamberlin firm maximizes his profit when there are M Chamberlin firms in the market; i.e., D^M is the graph of (29) for that value of M. The Chamberlin equilibrium market price is P_{CH} and the market quantity is Q. The i^{th} firm's market share is $Q_i = Q/M$ (i=1...M). For M=1, the monopolist's profit is the diagonally shaded area. For M=2, the duopolist's profit is the cross-hatched area. Notice that the sum of two cross-hatched areas is less than one diagonally shaded area, reflecting the advantage of having only one firm produce the total output in the presence of economies of size. Assume side payments are ruled out and M=2. Suppose a third Chamberlin firm wishes to enter. The third firm can foresee that when M=3, the properties of the Chamberlin equilibrium are $Q_i = Q/3$ and $\Pi_i = 0$ (i=1....3). If he believes that upon entering, the Chamberlin equilibrium will be established, the third firm has no incentive to enter. Presumably, he will not enter.

Another possibility is that the third firm will enter in the hope of convincing the first or second to exit. Why, one must ask, should the third firm be content to earn zero profit when the first and second firms are earning positive profit?

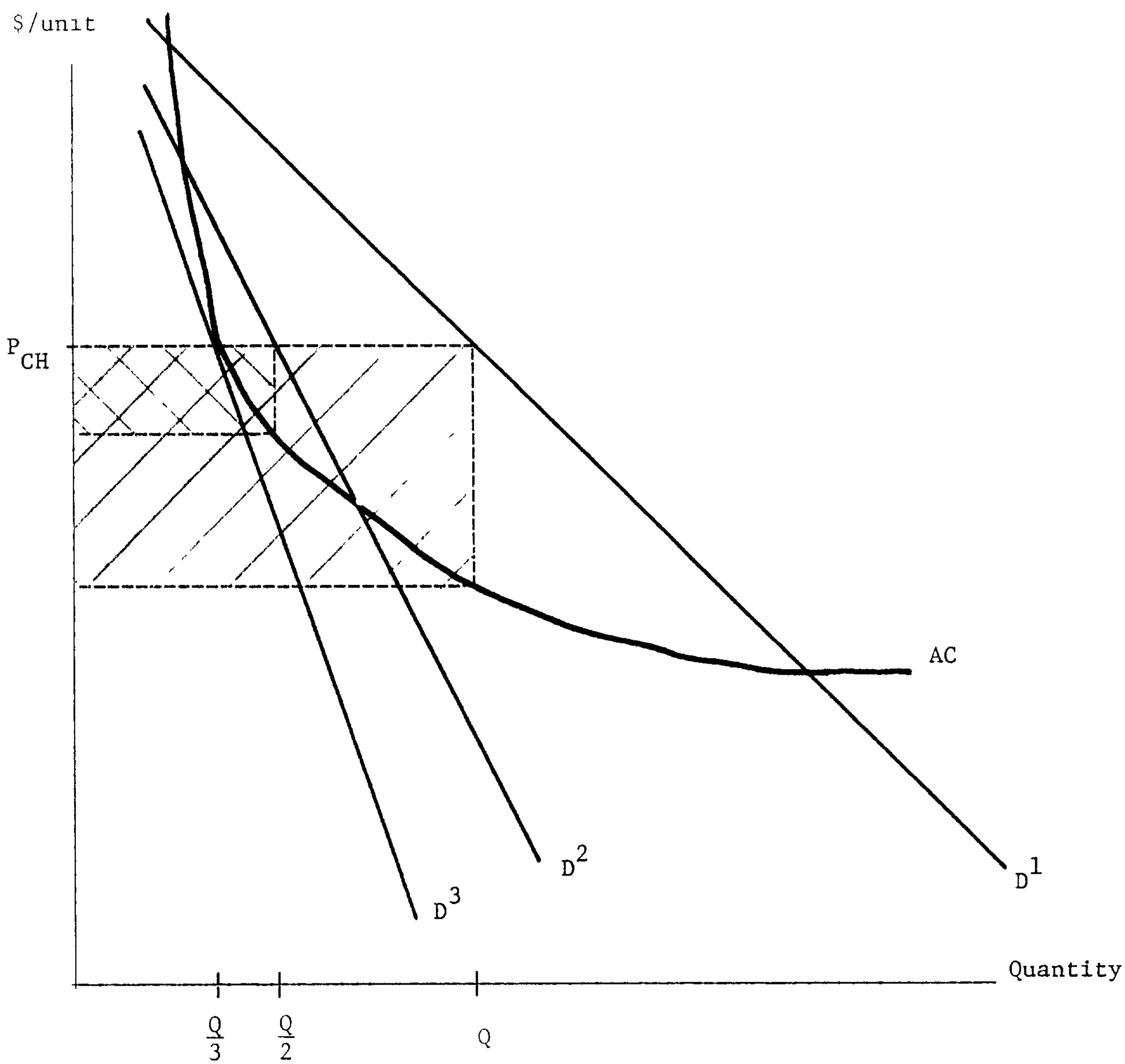

Figure 2. Chamberlin equilibrium with alternative numbers of Chamberlin firms under increasing returns to size. The Chamberlin equilibrium price P_{CH} is independent of the number of firms as is the market quantity Q. When there are one, two, or three firms, each firm's equilibrium output is Q, Q/2, or Q/3 respectively. With one firm, profit is the diagonally shaded area. With two firms, each firm's profit is the cross-hatched area. With three firms, P = AC and each firm's profit is zero.

Carrying this line of inquiry one more step, suppose there are only two prospective producers in Figure 2 (i.e., M=2 and entry is barred). If the first firm has any hope of driving the second out of business, and thereby securing a monopoly, would he be content with collusion, or would he try to drive his rival out of business? This brings us to another aspect of oligopoly theory: the prospect of economic warfare.

c. Bishop's Duopoly Warfare Model

If an oligopolist believes he can force or otherwise convince his rivals to content themselves with less-than-equal shares of jointly maximized profits, he will not be satisfied with the profit he earns in the symetric Chamberlin equilibrium. Instead, he will press for a greater-than-equal share of profits. This belief could be based upon unequal cost or wealth conditions among participants: if he has lower unit costs or greater wealth, he might press for a larger-than-equal share of jointly maximized profit. His rivals might concede in this case, because his threat of economic warfare is backed by a superior ability to take and to inflict economic losses. If, on the other hand, cost and wealth conditions are symmetrical, and this is known to each firm, then such a belief must rest on his appraisal of the psychological properties of his rivals, and in particular, on his appraisal of their "toughness."

Fellner [p.28] defines toughness as "...unwillingness to yield in a range in which the other party is expected to yield if one fails to do so." The oligopolist in question would evince toughness because he believes his rivals lack toughness. Fellner [p.29] continues:

> "Errors of appraisal that lead to the assumption that the other party will yield, when he will not, produce a stalemate. ...under oligopoly, (a stalemate) means cutthroat competition, that is, attempts at hurting one's rival even at the cost of a short-run sacrifice.... A permanent stalemate would have to rest on a series of mutual errors.... At least two parties must be mutually underestimating each other's toughness. It is unlikely that this would persist."

An opposite view is exemplified by Bishop's model of duopoly warfare. In his model, "...one possible result is a more or less permanent state of economic war" [2, p.943 Emphasis added].

In our notation, the case considered by Bishop is:

$M=2$; $A-b = 24$; $B=1$; $a=c=d=0$.

That is, the situation is duopoly, there are CRS and the cost function is (12), the duopolists' positions are symmetrical and the demand curve is linear. Making use of Table 2, we find that the properties of the Chamberlin equilibrium, where joint profit is maximized and equally divided, are:

$Q_1 = Q_2 = 6$; and $\Pi_1 = \Pi_2 = 72$.

In Bishop's model, the reaction function of the first duopolist is

$$Q_1 = \begin{cases} 12 - Q_2 & \text{if } Q_2 \leq 6 \\ Q_2 & \text{if } 6 \leq Q_2 \leq 12 \\ 12 & \text{if } Q_2 \geq 12 \end{cases} \tag{34}$$

This three-branched reaction schedule, though different from the one we postulated for Chamberlin oligopolists in equation (28), is nevertheless compatible with Chamberlin's model. Firm 1's reaction schedule shows that he is willing to collude symmetrically, but unwilling to collude asymmetrically unless he is the recipient of the largest share of the unequally divided joint profit. The second duopolist, unlike the first, is unwilling to collude symmetrically. Instead, he sets his sights on a profit of 96. Consistent with his profit goal is a reaction function as follows:

$$Q_2 = \begin{cases} 12 - Q_1 & \text{if } Q_1 \leq 4 \\ 2Q_1 & \text{if } 4 \leq Q_1 \leq 6 \\ 12 & \text{if } Q_1 \geq 6 \end{cases} \tag{35}$$

As long as firm 1 refuses to concede while firm 2 continues to demand more profits, then as Bishop states, "...warfare is the only possible outcome; for they will quickly gravitate toward the unique warfare point where $Q_1 = Q_2 = 12$ and $\Pi_1 = \Pi_2 = 0$" [2, p.942]. The point stressed by Bishop is that neither firm will concede if he expects his rival to concede. Hence, he argues, the war may be "more or less permanent."

The position taken here on the duration of warfare is similar to Fellner's: permanent warfare requires each firm to continually expect victory while continually being proven wrong. This irrational expectation would sooner or later be modified, and a collusive agreement reached. This is not to say that economic warfare does not or cannot arise in oligopoly. Nor do we deny that warfare can be frequent or persistent. Intermittant periods of warfare are likely as firms test the reactions of rivals—especially new entrants. But the prospect of "permanent" warfare seems doubtful because it requires expectations to be optimistically biased, i.e. irrational. We conclude that warfare is a temporary disequilibrium situation in oligopoly, and that Bishop's model is untenable.

3. Welfare Implications of Oligopoly

To this point, nothing has been said about the implications of oligopoly for "market performance" or social welfare. In this section a criterion for judging market performance is suggested, and the various oligopoly models are evaluated according to this criterion.

The welfare criterion used to evaluate market performance is the standard partial equilibrium comparative static Marshallian surplus criterion, i.e.

$$S = CS + \Pi \tag{36}$$

where S is total surplus, CS is consumer surplus, and Π is producer surplus or profit. This criterion is well known for its usefulness in applied welfare economics, as well as for its limitations. For a discussion of the theoretical foundations of this criterion, the reader is referred to Currie, Martin, and Schmitz [6].

With the linear market demand equation (11), consumer surplus is

$$CS = \frac{1}{2}(A-P)Q = BQ^2/2 \tag{37}$$

Producer surplus is the sum of the individual firm's profits:

$$\Pi = \sum_{i=1}^{M} \Pi_i = \sum_{i=1}^{M} \left[\left(A - B\sum_{j=1}^{M} Q_j\right) Q_i - C(Q_i)\right] \tag{38}$$

Total surplus is therefore

$$S = BQ^2/2 + \sum_{i=1}^{M} [(A-B \sum_{j=1}^{M} Q_j)Q_i - C(Q_i)] \tag{39}$$

a. Necessary and Sufficient Conditions for Surplus Maximization

In general, total surplus depends upon Q_i ($i=1...M$), and on M:

$$S = S(Q_i ... Q_M, M) \tag{40}$$

Let MAX(S|M) be maximum surplus when there are M firms such that $Q_i > 0$ ($i=1...M$). MAX(S|M) is the "conditional surplus maximum," i.e., the maximum value of S subject to $Q_i > 0$ ($i=1...M$).

Let $\hat{M}$ be the number of firms such that MAX(S|M) is a maximum for $M=\hat{M}$. $\hat{M}$ is called the "optimal number of firms;" MAX(S) = MAX(S|$\hat{M}$) is called the "unconditional surplus maximum." The following conditions are necessary for conditional maximization of surplus:

$$\frac{\partial S}{\partial Q_i} = A-B \sum_{i=1}^{M} Q_i - C_i' = 0 \quad (i=1...M) \tag{41}$$

where C_i' is the i^{th} firm's marginal cost. In addition to conditions (41), the following condition is also necessary for unconditional surplus maximization:

$$M = \hat{M} \tag{42}$$

In words, the necessary conditions for unconditional surplus maximization are that each firm must produce a quantity such that its marginal cost equals the market price; and, the number of firms must be optimal.

The sufficient conditions for unconditional surplus maximization are that the principle minors of the Hessian matrix H - where $H_{ij} = \partial^2 S/\partial Q_i \partial Q_j$, i, j=1...$\hat{M}$ - must alternate in sign starting with negative. That is, if H_k is the k^{th} principle minor of H, the sufficient conditions are

$$(-1)^k H_k > 0 \quad i = 1...\hat{M} \tag{43}$$

b. Maximum Surplus Under Alternative Cost Conditions

MAX(S) — the unconditional surplus maximum—will now be computed for alternative cost conditions.

Case 1: Constant Returns to Size. Under CRS, the cost function is of the form (12). In this case there is only one linearly independent necessary condition for conditional surplus maximization:

$$A - B\sum_{j=1}^{M} Q_j - b = 0 \tag{44}$$

or equivalently,

$$Q = (A-b)/B \tag{45}$$

The total surplus if Q = (A-b)/B is

$$\mathrm{MAX}(S|M) = (A-b)^2/2B \tag{46}$$

In this case, MAX(S) = MAX(S|M); i.e., the unconditional maximum equals the conditional maximum, because MAX(S|M) is independent of M.

Case 2: Increasing Returns to Size. If the cost function is (13), there are IRS at all levels of output, and the market is a natural monopoly.

For a conditional maximum, the necessary condition is

$$A-B \sum_{j=1}^{M} Q_j - b = 0 \tag{47}$$

or

$$Q = (A-b)/B \tag{48}$$

The conditional maximum is

$$\mathrm{MAX}(S|M) = (A-b)^2/2B - Ma \tag{49}$$

In this case, the conditional maximum is a declining function of the number of firms. Therefore, the optimal number of firms is $\hat{M}=1$; 6/ the corresponding unconditional surplus maximum is

$$\mathrm{MAX}(S) = (A-b)^2/2B - a \tag{50}$$

Case 3: Increasing Then Decreasing Returns to Size. For cost function (14), there are ITDRS, and the necessary conditions for a conditional maximum form a simultaneous system of nonlinear equations:

$$A-B \sum_{j=1}^{M} Q_j - b - 2cQ_i - 3dQ_i^2 = 0 \quad (i=1\ldots M) \tag{51}$$

6/ We ignore the uninteresting case where $(A-b)^2/2B - a<0$. In this case, $\hat{M}=0$, and the product should not be produced.

A method for solving such a general system does not exist. However, for specific parameter values, iterative techniques such as the Gauss-Seidel, Newton, and Brown methods can be used to solve this sytem. For illustrative purposes we shall consider a specific example in which A=4, B=a=b=d=1, and c=-1. For this set of parameters, the necessary conditions for conditional surplus maximization are

$$3-\sum_{j\neq i}^{M} Q_j + Q_i - 3Q_i^2 = 0 \quad (i=1...M) \tag{52}$$

Solutions to this system for various values of M are as follows:

(53)

Value of M	Solution to System Q_i (i=1...M)
1	1.18
2	1.0
3	.847
4	.721

Not only do these solutions satisfy the necessary conditions for conditional surplus maximization, but they satisfy the sufficient conditions as well.[7/] Therefore, these are the unique solutions to the problem of maximizing conditional surplus.

The corresponding conditional surplus maxima are as follows:

M	MAX(S|M)	CS	Π
1	1.593	.696	.897
2	2.0	2.0	0
3	1.724	3.229	-1.505
4	1.076	4.156	-3.080

7/ The sufficient conditions in this case are

$$(-1)^k \begin{vmatrix} 1-6Q_1 & -1 & \dots -1 \\ -1 & 1-6Q_2 & \dots -1 \\ \vdots & \vdots & \vdots \\ -1 & -1 & \dots -1-6Q_k \end{vmatrix} > 0, \quad k=1...M$$

Apparently, for this example, $\hat{M}=2$ and MAX(S) = 2.0. In the surplus maximizing situation, profits are zero and average cost, marginal cost, and price are equal. In these respects, the surplus maximizing situation resembles a long run competitive equilibrium. However, because the optimum number of firms is 2 in this example, it is doubtful that a competitive equilibrium would be established. This market is a natural oligopoly.

c. Welfare Implications of Cournot and Chamberlin Oligopoly

Using Tables 1 and 2 and equation (39), we have computed the total surplus at the Cournot-Nash and Chamberlin equilibria. Table 3 compares the surplus attained in Cournot-Nash equilibrium and Chamberlin equilibrium with the maximum attainable surplus. Three cases are shown: CRS, IRS, and ITDRS with parameters A=4, B=a=b=d=1, c=-1. In the case of ITDRS, the surplus values shown for the Cournot-Nash and Chamberlin equilibria are based on the assumption that M=2. For the parameters A=4, B=a=b=d=1, and c=-1, M=2 is an equilibrium number of firms (i.e., and M^*) for the Cournot and Chamberlin models.

For all three cost conditions, the maximum attainable surplus exceeds the surplus achieved in Cournot-Nash and Chamberlin equilibrium. This is to be expected, because a representative Chamberlin or Cournot oligopolist always produces a quantity such that its marginal cost is less than the market price in equilibrium, and this violates the necessary conditions for surplus maximization. Moreover, in Cournot-Nash or Chamberlin equilibrium, there may be a nonoptimal number of firms. One may conclude that Cournot and Chamberlin oligopolies have nonoptimal equilibria by the surplus criterion. Can cooperatives improve the performance of such oligopoly markets?

C. IMPACTS OF SUPPLY COOPERATIVES ON THE PERFORMANCE OF OLIGOPOLISTIC FARM INPUT MARKETS

Using the total surplus criterion, we shall investigate the impacts of supply cooperatives on the performance of oligopolistic farm input markets. This analysis is organized into three subsections, corresponding to three alternative assumptions with respect to cost conditions for farm supply firms: CRS, IRS, ITDRS.

Table 3. Total surplus in Cournot-Nash and Chamberlin equilibria, and comparison with maximum attainable surplus.

Cost Condition	Total Surplus[a/] Cournot-Nash Equilibrium	Chamberlin Equilibrium	Maximum Attainable
CRS: $C=bQ$ $b>0$	$\frac{M(M+2)(A-b)^2}{2B(M+1)^2}$	$\frac{3(A-b)^2}{8B}$	$\frac{(A-b)^2}{2B}$
IRS: $C=a+bQ$ $a,b>0$	$\frac{M(M+2)(A-b)^2}{2B(M+1)^2}-Ma$	$\frac{3(A-b)^2}{8B}-Ma$	$\frac{(A-b)^2}{2B}-a$
ITDRS:[b/] $C=1+Q-Q^2+Q^3$	1.945	1.784	2.0

a/ The market demand equation is $P = A-BQ$, and the number of firms is M.

b/ In this case, $A=4$, $B=1$, $M=2$

1. Constant Returns to Size

Assume that all firms face cost function (12). Let $[P_{C-N}, Q_{C-N}]$ be the Cournot-Nash equilibrium price and quantity, $[P_{CH}, Q_{CH}]$ be the Chamberlin equilibrium price and quantity, and $[\hat{P}, \hat{Q}]$ be the surplus - maximizing price and quantity. In this case,

$$[P_{C-N}, Q_{C-N}] = [(A+Mb)/(M+1),\ M(A-b)/B(M+1)]$$

$$[P_{CH}, Q_{CH}] = [(A+b)/2,\ (A-b)/2B]$$

$$[\hat{P}, \hat{Q}] = [b,\ (A-b)/B]$$

Let S_{C-N} be the surplus attained in Cournot-Nash equilibrium, S_{CH} be the surplus in Chamberlin equilibrium, and MAX(S) be maximum attainable surplus. As shown in Table 3, MAX(S) > S_{C-N} and MAX(S) > S_{CH}. The Cournot-Nash and Chamberlin equilibria are nonoptimal according to the surplus criteria.

Apparently, with free entry, a nonoptimal equilibrium could not be sustained. If entry were free, any farmer could integrate backward into the farm supply industry and produce input Q for himself at a unit cost of $\hat{P} = b$. We conclude that the market equilibrium could be nonoptimal only if there were barriers to entry.

It may be that there are barriers to entry for individual farmers which could be overcome through collective action, e.g., by forming a cooperative.[8/] Therefore, imagine that a supply cooperative is introduced into this oligopolistic market. The equilibrium price for the cooperative is $P_c = b$, which is also the surplus-maximizing price. With a cooperative present, other firms would either have to meet the cooperative price, or lose all of their customers to the cooperative. In the presence of a supply cooperative, the market price could not exceed the cooperative price - unless farmers were unaware of the discrepancy between the cooperative price and the market price. With free information regarding prices, the presence of

8/ For example, individual farmers may be denied access to the capital markets, whereas a cooperative may be able to obtain capital at a comparable cost to private firms.

a cooperative should be sufficient to keep the market price at the optimum $\hat{P}$. The resulting surplus should therefore be a maximum.

With CRS, there is a rationale for forming a supply cooperative only if entry is barred to individual farmers but open to groups of cooperating farmers. The presence of a cooperative should eliminate any gap between the market price and the surplus-maximizing price. The cooperative can act as a "competitive yardstick" in CRS industries.

2. Increasing Returns to Size

In this case each supply firm faces cost function (13). Assume that entry and exit are free. Then,

$$[P_{C-N}, Q_{C-N}] = [(A+bM)/(M+1),\ M(A-b)/B(M+1)]$$

$$(A-b)/2\sqrt{ab} - 1 < M \leq (A-b)/\sqrt{aB} - 1$$

$$[P_{CH}, Q_{CH}] = [(A+b)/2,\ (A-b)/2B]$$

$$(A-b)^2/4Ba - 1 < M \leq (A-b)^2/4Ba$$

$$[\hat{P}, \hat{Q}] = [b,\ (A-b)/B]$$

$$\hat{M} = 1$$

In this case, the industry is a natural monopoly. To maximize surplus, only one firm must be allowed to produce. However, the firm would require a subsidy to achieve the surplus-maximizing results, because at $[\hat{P}, \hat{Q}]$, the firm's profit would be negative. Without a subsidy, a surplus-maximizing firm would go broke in a natural monopoly industry.

If subsidies are ruled out, the following problem is of interest:

$$\text{MAXIMIZE w.r.t. } Q \qquad S = (A-b)Q - BQ^2/2 - a \quad \text{s.t. } \Pi \geq 0$$

That is, we should like to know the quantity which maximizes surplus subject to the constraint that the firm's profit is nonnegative.

To solve this problem, note that $\partial S/\partial Q = A-b-BQ$ is positive for $Q < (A-b)/B$. Thus, for $Q < \hat{Q}$, surplus is an increasing function of output. Therefore, the solution can be determined by finding the largest value of Q less than $\hat{Q}$ for which profit is nonnegative. The solution, denoted $\tilde{Q}$, is the quantity for which the firm's average cost curve intersects the market demand curve from below. The solution is

$$\tilde{Q} = \frac{(A-b)}{2B} + \frac{[(A-b)^2 - 4Ba]^{1/2}}{2B}$$

The corresponding price is:

$$\tilde{P} = \frac{(A+b)}{2} - \frac{[A-b)^2 - 4Ba]^{1/2}}{2}$$

If $Q = \tilde{Q}$, total surplus (denoted $\tilde{S}$) is

$$\tilde{S} = \frac{(A-b)^2 + (A-b)\ [(A-b)^2 - 4Ba]^{1/2} - 2Ba}{4B}$$

Since $\partial S/\partial Q > 0$ for $Q < \hat{Q}$, and since $\hat{Q} > \tilde{Q} > Q_{CH}$, we know $\tilde{S} > S_{CH}$; Likewise, $\tilde{Q} > Q_{C-N}$ implies $\tilde{S} > S_{C-N}$.

By organizing the industry as a monopoly, and regulating its price and quantity to the appropriate levels ($\tilde{P}$, $\tilde{Q}$ respectively), society could improve the performance of the market relative to its performance as a Cournot or Chamberlin oligopoly. The "regulated monopoly" equilibrium is denoted $[\tilde{P}, \tilde{Q}]$. The regulated monopoly equilibrium maximizes surplus subject to the constraint that the monopolist covers its costs.

As an alternative to establishing a government-regulated monopoly in natural monopoly industries, these industries could be organized as cooperatives. If the supply side of the market was comprised of a single supply cooperative, the market equilibrium would be the cooperative's equilibrium, which is identical to the regulated monopoly equilibrium: $[P_c, Q_c] = [\tilde{P}, \tilde{Q}]$.

Although the equilibrium may be the same in either case, there may be certain advantages to organizing a natural monopoly as a cooperative rather than as a government-regulated privately-owned monopoly. In particular, government regulation does not always produce the theoretically optimal results; nor is regulation a free good. A regulatory agency is generally required to balance the opposing objectives of consumers and investors, to decide on the allowable rate of return, and to determine the optimal price and quantity. And, there is a danger that the regulatory agency will be "captured" by the firm it is supposed to regulate, leading to a nonoptimal result. In contrast, a cooperative's members are both its customers and its investors, so that conflicts between consumers and owners could be resolved internally, without external regulation. Members would be unlikely to press for a price in excess of $\tilde{P}$, because their resulting gain in producer surplus would be more than offset by a loss in consumer surplus.

Summarizing to this point, the total surplus attainable without subsidy in a natural monopoly equals the surplus that could theoretically be attained if the market were organized as a government regulated monopoly, or, as a supply cooperative; this surplus $\tilde{S}$ exceeds the surplus that would be attained in Cournot-Nash or Chamberlin equilibrium.

Other equilibria are possible if information is not free. Two that are of interest will now be illustrated.

Suppose there are two firms, a cooperative and a private (noncooperative) firm. Figure 3a illustrates an equilibrium which might be attained if information is not free. This figure shows the average cost curve (AC), the market demand curve (D), the private firm's demand curve (D_p), and the cooperative's demand curve (D_C). The cooperative is in an equilibrium [P_c, Q_c]. The market price P_m, which is the price charged by the private firm, is equal to P_c. Hence, farmers have no incentive to join or leave the cooperative. The private firm has a larger volume (Q_p), a lower average cost, and earns a profit. Such an equilibrium could persist only if farmers were ignorant of cost conditions. If farmers were aware that the market was a natural monopoly, they would presumably join the cooperative enabling it to capture economies of size and to reduce its price to $\tilde{P}$.

Figure 3b is identical to 3a, except that now it is assumed that there are two cooperatives, each serving half the market, and each in an equilibrium [P_c, Q_c]. A merger between the two cooperatives would enable economies of size to be captured. Such a merger might be prevented by lack of information regarding conditions of cost, or by unwillingness on the part of managers to jeopardize their positions by merging. Presumably, if members were aware of the situation, they would favor a merger. If members were in control of the cooperative, management could not prevent such a desirable merger.

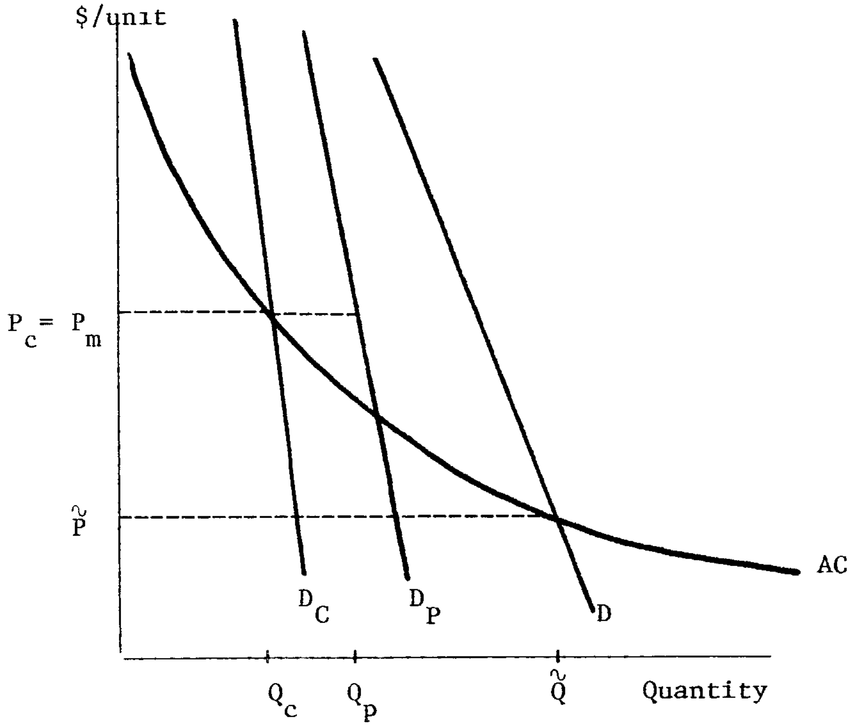

Figure 3a.

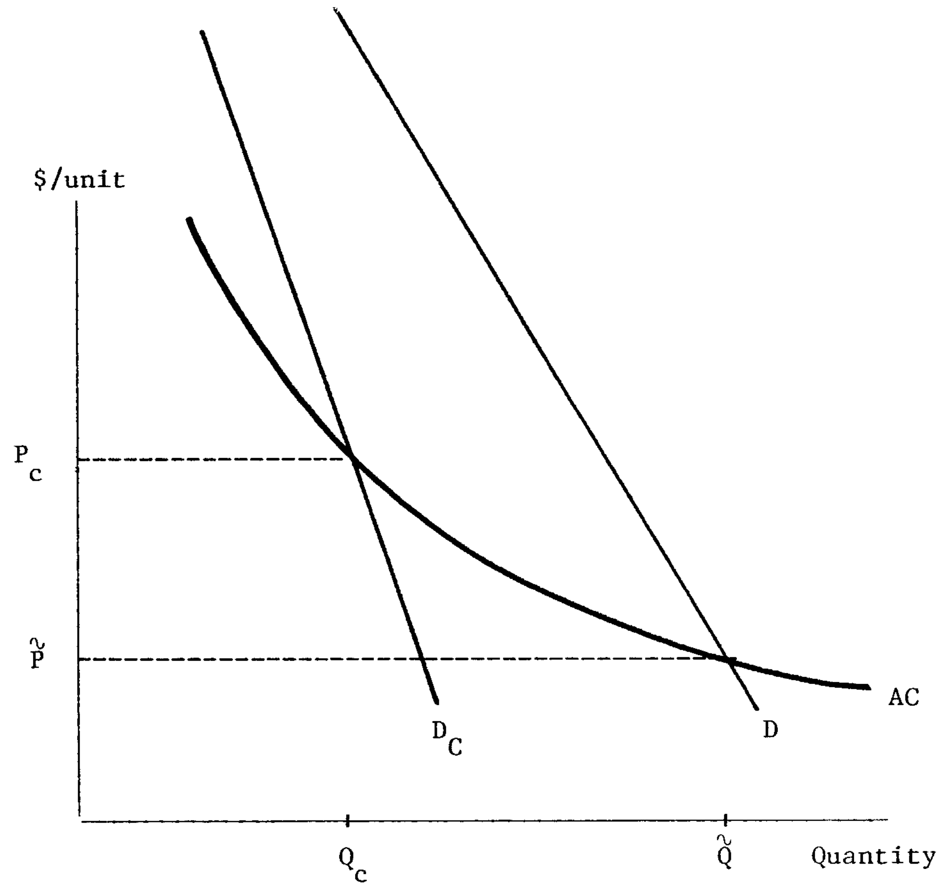

Figure 3b.

Figure 3. Alternative suboptimal equilibria. In 3a, the cooperative is in equilibrium with output Q_c and price P_c, and a private firm is in equilibrium with output Q_p and price P_m. The private firm earns a profit but the cooperative does not. In 3b, each of two cooperatives is in an equilibrium with output Q_c and price P_c. In either situation, farmers would be better off if only one cooperative would produce at output level $\tilde{Q}$ and price $\tilde{P}$.

3. Increasing Then Decreasing Returns to Size

The final situation analyzed in this paper is one in which there are ITDRS. The cost function is (14) and the parameters of the model are as follows: $A = 4$, $B = a = b = d = 1$, $c = -1$, $M = 2$. The market is a "natural oligopoly" in the sense that the equilibrium number of firms exceeds 1 but is nevertheless small. In this market,

$$[P_{C-N}, Q_{C-N}] = [2.31, 1.69]$$

$$[P_{CH}, Q_{CH}] = [2.56, 1.44]$$

$$[\hat{P}, \hat{Q}] = [2.0, 2.0]$$

and $S_{C-N} = 1.945$, $S_{CH} = 1.784$, and $MAX(S) = 2.0$

Consider the effects of replacing only one of the noncooperative firms with a supply cooperative. The cooperative would, by previous assumption, restrict membership so that the demand curve it faced from its members intersected its average cost curve at the point of minimum average cost. A demand curve which has this property is $P = 4 - 2Q_c$. By adding two such demand curves horizontally, the market demand curve is obtained. The noncooperative firm would be a monopolist with respect to the remaining portion of the market; its demand equation would be $P = A - 2Q_p$. In equilibrium, $[P_c, Q_c] = [2.0, 1.0]$. The noncooperative firm's profit maximizing equilibrium is $[P, Q_p] = [3.27, .721]$. The resulting surplus is $S = 1.892$. Whereas the resulting surplus exceeds the surplus in Chamberlin equilibrium, it is less than the surplus in Cournot-Nash equilibrium. Therefore, if the market was originally in a Cournot-Nash equilibrium, surplus would be reduced by replacing one of the Cournot duopolists with a cooperative.

However, if <u>both</u> private firms were replaced by supply cooperatives, each would reach an equilibrium $[P_c, Q_c] = [2.0, 1.0]$. The resulting surplus would then be a maximum.

By construction, it was possible in this example to have two cooperatives of optimal size at a market equilibrium. Under other sets of parameters, the demand curves of the optimally-sized cooperatives might not add up to the market demand. For farmers who are not members of optimally sized cooperatives, this leaves several alternatives, including:

(1) form a cooperative of suboptimal size; or (2) bargain to obtain input Q at its marginal cost from optimally sized cooperatives. Alternative (2) involves cooperatives treating members and nonmembers differently.

This completes our discussion of the impacts of supply cooperatives on the performance of farm input markets. The presence of the right number of agricultural supply cooperatives can improve market performance under some circumstances. In nearly every circumstance, the performance of the market may be improved by the introduction of one or more agricultural cooperatives.

CONCLUSIONS

It is hoped that this paper will provide a basis for further analysis of the impacts of cooperatives on the performance of farm input markets. Useful extension of this paper would include: allowing for heterogeneous products; allowing for different cost functions among suppliers, and consideration of multiproduct or multienterprise cooperatives' impacts on market performance. Further analysis could reinforce or contradict our conclusion that cooperatives can improve the performance of farm input markets.

A major limitation of this analysis is that it is static, rather than dynamic. It is not possible to infer from this analysis what impacts supply cooperatives would have on market performance in a dynamic context.

The model ignores uncertainty, and this is another important limitation. Presumably, uncertainty will influence members' behavior vis a vis a cooperatives, as well as the cooperative's equilibrium. As of this time, only one author has introduced uncertainty into a model of an agricultural cooperative:[9/] not even he considers the effects of risk aversion on members' behavior. This important area needs further research.

Our analysis sidesteps the issue of control of cooperatives. This issue is, of course, vital. As the Farmer Cooperative Service points out, cooperatives are in principle democratically controlled by their members

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9/ Royer [17]; Royer assumed members were risk neutral.

[20, p.4]. In practice, however, members may relinquish or lose control to hired professional managers.[10/] Management may then pursue its goals (e.g., organizational growth, higher salaries, more perquisites) at the expense of members. By ignoring the issue of control, we have foreclosed a potentially interesting inquiry into the appropriate incentives and constraints to be imposed on cooperatives' managers. This is another area which merits additional research.

Finally, we note that cooperatives have been analyzed primarily in the context of partial equilibrium analysis. It may be possible to view cooperatives as general equilibrium phenomena. For example, in the terminology of game theory, a cooperative could be viewed as a "coalition," and the formation of cooperatives could propel the economy towards an equilibrium belonging to the "core."[11/] It could be that with non-convex production sets (and consequently, economies of scale), formation of cooperatives is preferable to independent action by large numbers of small firms. The role of cooperatives in a market economy might best be understood in this context.

10/ See Trifon [19, p.235]; Henning and Laubis [13, p.40]; Vitaliano [22, p.37]; Peregaux [16, p. 119-21]; and Helmberger & Hoos [11, p.288].

11/ Formally, a coalition is a subset of players in a game, and the core is the set of imputations not blocked by any coalition. See Malinvaud [14] for further discussion.

References

1. Bar, Josef, "A Mathematical Model of a Village Cooperative Based on the Decomposition Principle of Linear Programming", AJAE, Vol. 57, No. 2 (May 1975), pp. 353-57.

2. Bishop, Robert L., "Duopoly: Collusion or Warfare?", AER, Vol. 50, No. 5 (December 1960), pp. 933-61.

3. Clark, Eugene, "Farmer Cooperatives and Economic Welfare", JFE, Vol. 34, No. 1 (February 1952), pp. 35-51.

4. Clark, Eugene, "Farmer Cooperatives and Economic Welfare - Rejoinder", JFE, Vol. 34, No. 3 (August 1952), pp. 404-07.

5. Chamberlin, Edward, The Theory of Monopolistic Competition, Harvard University Press; Cambridge: 1933.

6. Currie, J. M., J. A. Martin and A. Schmitz, "The Concept of Economic Surplus and Its Use in Economic Analysis", Economic Journal, Vol. 81, (December 1971), pp. 741-99.

7. Dewey, Donald, "Industrial Concentration and the Rate of Profit: Some Neglected Theory", JLE, Vol. 19, No. 1 (April 1976), pp. 67-78.

8. Fellner, William, Competition Among the Few, August M. Kelley; New York: 1960.

9. Gislason, Conrad, "Cooperatives and Economic Welfare", JFE, Vol. 34, No. 4 (November 1952), pp. 555-63.

10. Helmberger, Peter G., "Cooperative Enterprise as a Structural Dimension of Farm Markets", JFE, Vol. 46, No. 3 (August 1964), pp. 603-17.

11. Helmberger, Peter and Sidney Hoos, "Cooperative Enterprise and Organization Theory", JFE, Vol. 44, No. 2 (May 1962) pp. 275-90.

12. Henderson, James and Richard E. Quandt, Microeconomic Theory: A Mathematical Approach, Second Edition, McGraw-Hill Book Co., New York: 1971.

13. Henning, G. F. and R. E. Laubis, Financial Structure of Agricultural Business Organizations, Research Bulletin 880, Ohio Agricultural Experiment Station, Wooster, Ohio: April 1961, 54 pp.

14. Malinvaud, E., Lectures on Microeconomic Theory, North Holland Publishing Co., Amsterdam: 1972.

15. Nourse, Edwin G., "The Economic Principles of Cooperation", in Abrahamsen and Scroggs, eds., Agricultural Cooperation, University of Minnesota Press: 1957, pp. 161-83.

16. Perregaux, E. A., "The Future of Farm Cooperatives in the United States", JFE, Vol. 29, No. 1, February 1947, pp. 115-127.

17. Royer, Jeffrey, "A General Nonlinear Programming Model of a Producers Cooperative in the Short Run", Unpublished doctoral dissertation, Iowa State University: 1978, 167 pp.

18. Schaars, Marvin A., "Basic Principles of Cooperatives: Their Growth and Development", in Abrahamsen and Scroggs, eds., Agricultural Cooperation, University of Minnesota Press: 1957, pp. 183-203.

19. Trifon, Raphael, "The Economics of Cooperative Ventures - Further Comments", Journal of Farm Economics, Vol. 43, No. 2 (May 1961). p. 215-35.

20. U.S. Department of Agriculture, Farmer Cooperative Service, Legal Phases of Farmer Cooperatives, FCS Information 100, May 1976.

21. Varian, Hal R., Microeconomic Analysis, W. W. Norton and Company, New York: 1978.

22. Vitaliano, Peter, "The Theory of Cooperative Enterprise - Its Development and Present Status", in Agricultural Cooperatives and the Public Interest, NCRR Publication 256, Bruce W. Marion, ed., September 1978, pp. 21-42.