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IMPERFECT INFORMATION, CONSUMER THEORY, AND
ALLOCATIVE ERROR IN CONSUMPTION

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Traditional economic theory of the consumer assumes the existence of perfect information. However, in reality this assumption is rarely fulfilled. In this paper a model is presented which relaxes this assumption and explicitly introduces the possibility of imperfect information into the theory of consumer behavior. Specifically, the focus is on consumer decision making when the utility realized from a bundle of goods and services can be different from the utility anticipated during the budget allocation process.

It is assumed that a consumer allocates a fixed budget to a bundle of goods and services based on known prices, a set of information about the goods and services, and his or her perception of the utility to be derived from consumption. If the consumer's knowledge about the goods and services in the consumption bundle is inaccurate at the time of budget allocation, the utility actually realized from consumption will differ from the utility previously perceived. Allocative errors due to imperfect information can occur giving rise to losses in consumer welfare which can be used to infer the value of information.

The study of the economics of information was initiated by Stigler's seminal article in 1961 on the information search process. Several authors including Barron and Peterson; Gatswirth; Karni and Schwartz; Landsberger and Peled; and Pratt, Wise, and Zeckhauser expanded on Stigler's basic model in investigating the price information search process. Rothchild and Stiglitz identified the information problem as a form of risk and uncertainty and expanded the literature dealing with search costs. Nelson classified products into search or experience goods and broadened search to

encompass information about quality. Auld and Colantoni, Davis, and Swaminathan employed Lancaster's theory of consumer behavior to explore welfare losses due to less than perfect information. Hirshleifer provided a review of developments in information theory in a 1973 article.

Several studies indicate that consumers place a value on information by expressing a willingness to pay something extra for improved information. For example, Daly and Lenahan have studied this phenomena with respect to nutritional information and the price of food. The article by Peltzman evaluating drug regulations makes an important conceptual contribution to analyzing the welfare effects of imperfect information. Finally, Kotowitz and Mathewson allow for differences in perceived and realized characteristics of products, but do not conceptualize these differences in a utility framework.

This study is the first which integrates the economics of information and consumer utility theory. The possibility of imperfect information is introduced directly into the utility maximization process. The implications of misinformation on the resulting demand curves are then developed and consumer surplus is used to measure the value of information. The approach presented here has broad applicability and lays a foundation for further analysis.

Changes in consumer surplus occur because consumer demand for a product shifts when consumers discover the true nature of goods and services they have purchased. Demand for a given product may shift in either direction as the marginal rate of substitution between goods and services changes with improved information. Three utility states, a perceived, an optimum, and a realized, are delineated and their indirect analogues and expenditure func-

tions are defined in order to illustrate exact changes in consumer welfare. Corresponding demand curves are illustrated graphically. Suggestions for extensions and implications of the theoretical model are offered in the concluding remarks.

In the development of our basic model certain simplifying assumptions are made. The model is static examining two end-points of an implied learning process. The nature and rate of the learning process are ignored. The model also disregards consumers' attitudes toward uncertainty about the set of information they possess. Finally, the possibility that consumers may misconstrue accurate information is ignored. Some of the implications of relaxing these assumptions are explored later in the paper.

I. CONCEPTUAL FRAMEWORK

The utility perceived by the individual from consuming a vector of goods and services $X = (x_1, \dots, x_n) \in \bar{X}$ in R_+^n is denoted as:

$$(1) \quad U_p = U(X; M_0).$$

Perceived preferences on the set \bar{X} are assumed complete, reflexive, transitive, continuous, strictly convex and strongly monotonic. The parameters of (1) are given by the vector M_0 . This vector can be viewed as reflecting a consumer's preferences which depend upon his or her access to information about the goods and services in \bar{X} at the time of their acquisition. Hence, M_0 can be viewed as embodying a consumer's state of knowledge of the utility obtainable from $X \in \bar{X}$.

Let,

$$(2) \quad U = U(X; M)$$

denote the utility function of the same consumer who now possesses complete information concerning the products in \bar{X} . The consumer's preferences are now reflected by the parameter vector M . The fundamental properties of preferences stated above are assumed to remain unchanged. Thus, (1) and (2) differ only in that the elements of M can differ from the corresponding elements of M_0 . If the consumer possessed perfect information then $M = M_0$.

The consumer with perfect information chooses vector X^* to maximize (2) subject to:

$$(3) \quad Y - P'X = 0$$

where P is a vector of known prices and Y is spendable income. Let,

$$(2') \quad U^* = U(X^*; M)$$

denote the result.

With incomplete or inaccurate information, the consumer chooses vector X^0 to maximize (1) subject to (3) and obtains

$$(1') \quad U_p^0 = (X^0; M_0).$$

However, upon consuming X^0 , the consumer realizes utility based on the "true" function (2) with parameter vector M .^{1/} Hence, realized utility is given by:

$$(4) \quad U^\theta = U(X^0; M).$$

Thus, (1) and (2) yield three states of utility, optimum (U^*), perceived (U_p^0) and realized (U^θ).^{2/} If $M \neq M_0$ for any element, say m_i , then

$x_1^0 \begin{matrix} > \\ < \end{matrix} x_1^*$. The vector X^0 is only a feasible solution to the constrained maximization of (2) since, by construction, X^* is an optimal solution. Hence,

$$(5) \quad U^\theta = U(X^0; M) \leq U(X^*; M) = U^*$$

The inequality between realized U^θ and the optimal state U^* suggests a measure for the value of information. The loss in welfare, $U^* - U^\theta$, can be viewed as the maximum value of information, in utility terms, yielding perfect knowledge of M .

Perceived utility (U_p^0) may be greater or less than realized utility (U^θ) depending on the values of M_0 relative to M , i.e.,

$$(6) \quad U_p^0 = U(X^0; M_0) \begin{matrix} > \\ < \end{matrix} U(X^0; M) = U^\theta.$$

However, since M and M_0 reflect mutually exclusive states of knowledge which cannot exist simultaneously for the consumer, no a priori comparison between perceived utility (U_p^0) and optimum utility (U^*) can be made.

A depiction of these results appears in Figure 1 where the broken curve represents the consumer's perceived indifference curve between goods x_1 and x_2 and the unbroken curve represents the consumer's indifference curve corresponding to (2). Point A depicts result (1'), point B depicts the maximum obtainable utility with complete information of x_1 , x_2 and corresponds to (2'). The indifference curve U^θ is the level of utility realized from the choice x_1^0 , x_2^0 and corresponds to (4). Notice that the utility realized at point A is less than the maximum utility obtainable with complete information, hence condition (5). The choice X^0 as opposed to the choice X^* can be viewed as an allocative error in the dispensing of spendable income, Y .

In other words, the maximum value of additional information leading to perfect knowledge of the utility attainable from the goods x_1 and x_2 (a situation reflected by the convergence of M_0 to M and a change in the consumer's choice from X^0 to X^*) is the increase in the consumer's utility from U^0 to U^* .

In order to treat losses in consumer welfare numerically, it is useful to derive demand functions corresponding to the perceived utility state (1) and the optimum state (2). This is illustrated in figure 2 for the case where complete information about x_1 results in a decrease in its demand. As above, the broken curves refer to the perceived utility function (1) while the unbroken curves refer to (2). In the next section, we pursue an analytical treatment of these respective demand functions and their interrelationships.

II. WELFARE LOSS AND THE VALUE OF INFORMATION

Having established the basic conceptual model, the next task is to provide a measure of losses in consumer welfare which occur when the consumer chooses X^0 and then discovers that X^* is preferred. Our approach is to develop this measure through the indirect utility and expenditure function relationships.

Let the indirect utility function corresponding to the constrained maximization of perceived utility (1'), be denoted as:

$$U_p^0 = V(P, Y; M_0)$$

and let the indirect utility function corresponding to (2') be denoted as:

$$U^* = V(P, Y; M).$$

If (1) represents the consumer's beliefs, then the perceived Marshallian demand functions corresponding to (1') are:

$$(1'') \quad x_i^0 = - \frac{\partial V(P, Y; M_0)}{\partial P_i} / \frac{\partial V(P, Y; M_0)}{\partial Y} = V_i(P, Y; M_0), \text{ for all } i,$$

which can differ from the Marshallian demand functions corresponding to (2'):

$$(2'') \quad x_i^* = - \frac{\partial V(P, Y; M)}{\partial P_i} / \frac{\partial V(P, Y; M)}{\partial Y} = V_i(P, Y; M), \text{ for all } i.$$

These functions correspond to those depicted in Figure 2. If X^0 are data, then it is the perceived functions which are observable where M_0 underlies the parameters we frequently attempt to empirically estimate. If information and experience cause consumer beliefs to change, then the perceived demand functions are not structural in an econometric sense.

The expenditure function corresponding to (1) can be defined as:

$$(7) \quad G(P, U_p^0; M_0) = \min_X P'X, \text{ s.t. } U_p^0 = U(X; M_0),$$

and similarly for the case of (2):

$$(8) \quad G(P, U^*; M) = \min_X P'X, \text{ s.t. } U^* = U(X; M),$$

where U_p^0 and U^* are values defined previously. In terms of Figure 1, the value of (7) corresponds to point A while the value of (8) corresponds to point B. The next step is to show the relationship between the indirect utility and expenditure functions and realized utility, U^θ .

The realized indirect utility function can be obtained by substituting (1'') into (2) for all i which yields a function containing both parameter vectors M_0 and M . Similarly, the relationship between expenditure and realized utility (U^θ) can be obtained by expressing (4) as a function of (1'') and then, upon substituting this result into the budget constraint (3), obtaining the realized expenditure function:^{3/}

$$(9) \quad G(P, U^\theta; M_0, M).$$

It may be useful to note that, in reference to figure 1, (9) corresponds to point A at utility level U^θ . Hence, it is not surprising that (9) contains parameters from both (1) and (2). Since $U^\theta \leq U^*$, then

$$(10) \quad C.V. = G(P, U; M_0, M) - G(P, U; M) \geq 0,$$

that is, the minimal expenditure to obtain utility U with perfect knowledge of M is less than or equal to the expenditure incurred to obtain the same level of utility when M is not known exactly. Hence, (10) is a measure of welfare loss in monetary terms. As M_0 converges to M , (10) approaches zero. Another way to phrase this same concept is to interpret (10) as the amount of money, the compensatory variation (C.V.), a consumer would need to be paid to remain at $U = U^\theta$ after obtaining perfect knowledge of M .

The problems of using compensating and equivalent variation or consumer surplus as exact measures of consumer welfare are relatively well known. Chipman and Moore (1979) and others have shown that constant marginal utility of income is both a necessary and sufficient condition for compensating and equivalent variation and consumer surplus to be equivalent and precise measures of changes in consumer welfare.^{4/} For purposes of this section,

we assume constant marginal utility of income, i.e. we assume that the denominator of Roy's identity in (2'') for all i are constant.^{5/} This assumption simplifies the following exposition of consumer surplus measures of welfare for the problem considered in this paper.

Consumer surplus with perfect knowledge of M is, for any i -th good, defined as:

$$CS_i^* = \int_0^{x_i^*} V_i^{-1}(X, Y; M) dx_i - p_i x_i^* \approx \int_{p_i^b}^{p_i} V_i(P, Y; M) dp_i$$

where V_i is the Marshallian demand function (2''), V_i^{-1} is its price inverse and p_i^b is some price of x_i for which $x_i \approx 0$. This familiar concept is illustrated in figure 3.a. If $M_0 \neq M$, then it is possible for any good to be underconsumed and any other good overconsumed relative to the optimal choice, x_j^* , x_s^* , $j \neq s$, since the optimization of utility requires fulfillment of the income constraint. The case of underconsumption is illustrated in Figure 3.b. where x_j^o is given by (1'') for $j=i$ at price p_j .^{6/} Consumer surplus realized (CS_j^θ) from the choice x_j^o is:

$$CS_j^\theta = \int_0^{x_j^o} V_j^{-1}(X, Y; M) dx_j - p_j x_j^o \approx \int_{\hat{p}_j}^{p_j^b} V_j(P, Y; M) dp_j + (\hat{p}_j - p_j) x_j^o.$$

This value is illustrated by the sum of the area in the upper triangle and rectangle of figure 3.b, or equivalently, by $CS_j^* - W_j$. It follows from (5) that $CS_j^* - CS_j^\theta \geq 0$. Hence, the maximum welfare gain from exact knowledge of M with respect to a single good x_j , is, in value terms,

$$W_j = CS_j^* - CS_j^\theta.$$

Consumer surplus realized when $x_s^o \geq x_s^*$ at price p_s is illustrated in figure 3c. In this case,

$$CS_s^\theta = \int_0^{x_s^o} V_s^{-1}(X, Y; M) dx_s - p_s x_s^o \approx - \int_{p_s}^{p_s^b} V_s(P, Y; M) dp_s - (p_s - \bar{p}_s) x_s^o$$

which is equivalent to the triangle CS_s^* (consumer surplus with perfect knowledge of M) less the right hand triangle W_s .^{7/} The welfare loss from the overconsumption of x_s is given by:

$$W_s = CS_s^* - CS_s^\theta$$

The implications of this welfare analysis is that a single error in the consumer's choice of a good can cause, through the budget constraint, nonoptimal choices of other goods and services. Hence, it follows that the total value of consumer welfare gain from exact knowledge of M is the summation of the gains $\sum_i^n W_i$ over all goods and services in \bar{X} .^{8/} The larger the budget share of the good for which the consumer's knowledge of M is incomplete, the greater can be the error induced in the choice of other goods. The estimation of a single W_i is, therefore, a lower bound to the total gain from exact knowledge of M.

III. CONCLUDING REMARKS

Several extensions and applications of this framework are worth noting. For example, the basic concept can be applied to the Lancaster model. In Lancaster's approach, utility is a function of the characteristics of goods and services. Let $BX = Z$ denote the true relationship between a vector of goods X and a vector of characteristics Z and let $B_o X = Z_p$ denote the corresponding perceived relationship. Then equations (1) and (2) are specified in terms of Z_p and Z respectively. In this case, (1) is $U_p = (X; M_o, B_o)$ and (2) is $U = U(X; M, B)$. The interpretation is that consumers face two

sources of error, imperfect information of the characteristics embodied in X and imperfect information as to the utility obtainable from the characteristics. For example, access to improved nutrition information could have two effects: (a) The original perception of the nutrient content (characteristics) of foods might be reevaluated causing different foods to be consumed and, (b) the individual might readjust his or her preferences between nutrition and flavor, based on a better understanding of the utility to be realized from a healthier diet.

Under conditions of imperfect information, a consumer's perceived utility will likely be subject to uncertainty. Since M is unknown before the choice X^0 is made, M_0 can be viewed as the consumer's subjective estimate of M . Thus, in a Bernoullian context, the consumer can be viewed as maximizing the mathematical expectation of equation (1) subject to (3). If the axioms of orderability, transitivity, continuity and independence are valid, then

$$U(X^*;M) - E[U(X^0;M_0)] \geq 0.$$

The consumer who is uncertain of her or his subjective estimate of M_0 may experience a corresponding disutility of uncertainty, depending on his or her attitude towards risk. Under these circumstances, consumer welfare can be enhanced not only through the convergence of M_0 toward M , the perceived and true utility parameters, but also by decreasing consumers' uncertainty as to the values of M .

The convergence of M_0 toward M implies a learning process. This paper has not sought to explain this process.^{9/} The rate of the learning process might be explained by the individual's cognitive ability, which

is expected to be correlated with some socio-economic factors such as education. A fully developed theory of learning might yield more rigorous justifications for the role of these factors in consumer behavior than our current fairly ad hoc rationales. A theory of learning may also lead to a plausible econometric specification of (1''), which incorporates cognitive and information variables to explain the changes in M_0 over individuals and/or over time. In an economy typified by rapidly evolving technological and marketing modifications the observed demand relationships (1'') will not be truly structural. The learning process may give rise to structural shifts in the demand functions.

With the introduction of learning, a distinction must be made between information, which is external to the consumer, and knowledge, which is internal to the consumer and consists of processed information. Mental processing errors introduce the possibility of inaccurate knowledge even when the information available is accurate.

Two prevailing sources of information in our society which influence consumers' evaluation of goods and services are advertising by private enterprise and regulations and education promulgated by the government. By stressing and perhaps exaggerating the beneficial aspects of a product, advertising may be viewed as an attempt to create a situation wherein the perceived utility prior to consumption is greater than or at least equal to the utility realized, $U_p^0 \geq U^\theta$. Under the influence of advertising it is possible that actual consumption exceeds the optimum level, as Kotowitz and Mathewson point out. Government regulatory and educational programs tend to focus on those areas where consumption might exceed the optimum level because of imperfect information. The consumer welfare framework presented above suggests how the gain or loss from advertising and government programs can be measured.

In this paper losses in consumer welfare due to imperfect information were examined by relaxing the perfect information assumption of the neo-classical theory of consumer behavior. Measures of welfare losses were derived through analyzing changes in consumer surplus which result from allocative error, i.e. purchasing the wrong mix of goods and services prior to obtaining perfect information. The value of perfect information was hypothesized to be equal to welfare losses incurred in its absence. Furthermore, it was shown that utility realized from the consumption of goods and services acquired with less than perfect knowledge can never be greater than utility received from products selected with perfect knowledge. Among the important implications of this theory is that observed demand functions may not be structural and cannot be expected to hold across different states of knowledge.

The assumption of perfect knowledge excessively restricts the ability of the traditional theory to explain consumer behavior. If the consumer prefers x_1 to x_2 , she or he cannot then prefer x_2 to x_1 within the axioms of rational behavior. By introducing imperfect information, behavior previously cast as irrational can now be explained in the context of the theory.

As Green states in Consumer Theory:

We suggest, without having anything original to offer, that the theory could be improved if the information available to consumers and their interactions were taken into account (p. 29).

The bibliography on the economics of information has grown quite long. However, revision of the fundamental theory of consumer behavior to encompass the possibility of imperfect information has not previously been carried out. This study attempts to partially fill that gap.

Figure 1

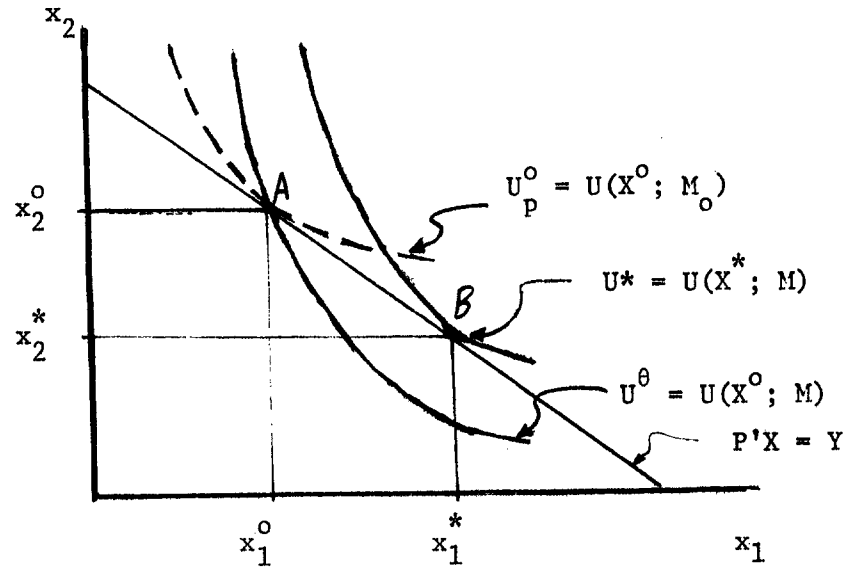


Figure 2

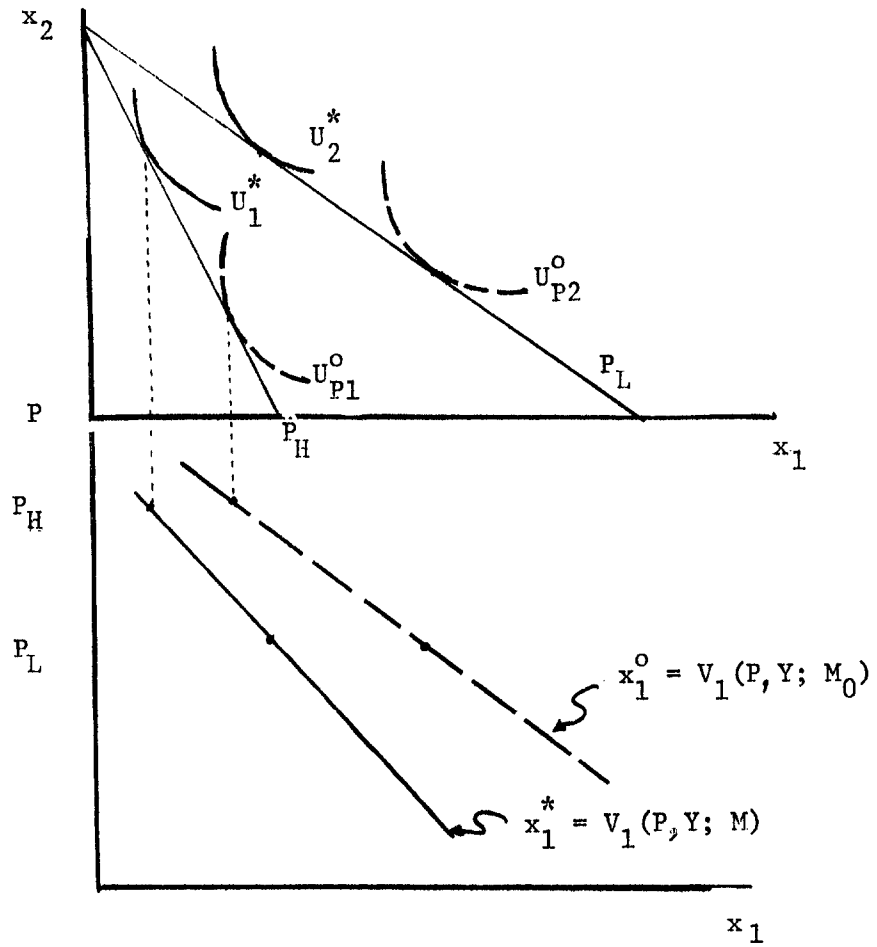
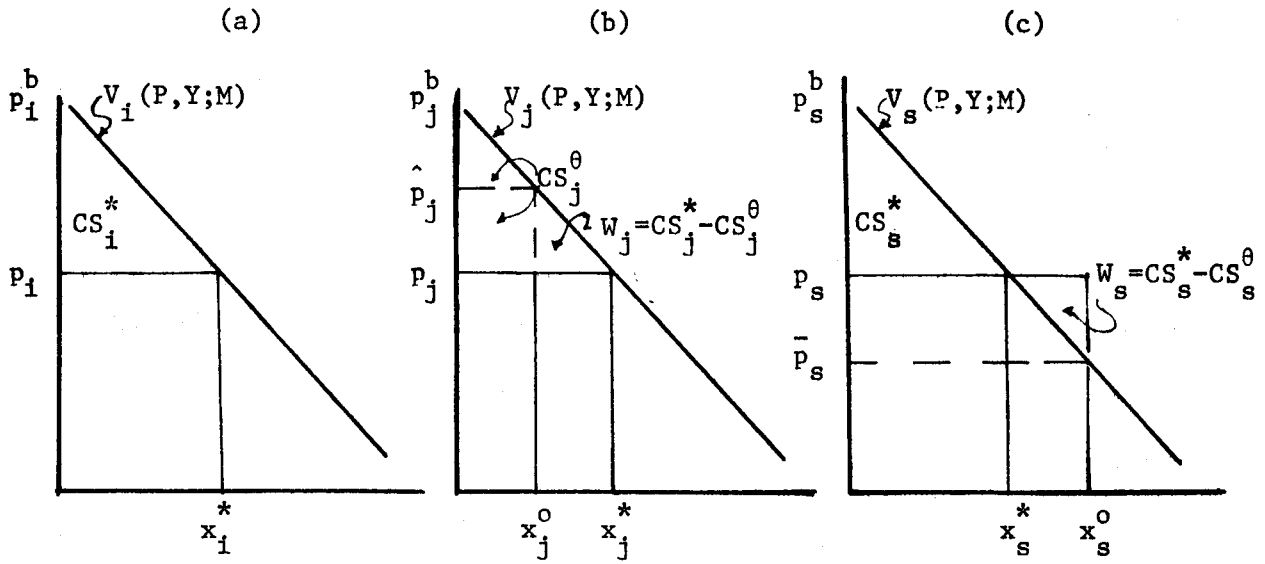


Figure 3



1/ We are abstracting from the case where a consumer realizes indirect effects at some distant point in time such as from the carcinogenic properties of tobacco or obesity from the consumption of high caloric foods. Furthermore, those consumers who remain ignorant or, who, after obtaining and evaluating more information about the characteristics of goods and services, in \bar{X} , still prefer X^0 suffer no loss in welfare since in this case $M = M_0$ and $X^* = X^0$.

2/ We ignore here the problems of converging M_0 to M and the time dimension and welfare loss this might entail. In other words, while a single realization or experience in consuming X^0 yields (4), this single experience may not be sufficient for the consumer to accurately perceive M and, therefore, possess complete and accurate knowledge of the utility to be derived from future consumption bundles in \bar{X} . We also ignore here the consumer's attitudes toward the uncertainty of not knowing M exactly when the choice X^0 is made.

3/ For the case of a two good Cobb-Douglas utility function, the realized expenditure function is:

$$G(P, U^\theta; M_0, M) = (m_{01} + m_{02}) \left(\frac{m_1}{m_{01}} \frac{m_2}{m_{02}} \right)^{-\frac{1}{r}} U^{\theta r} (P_1^{m_1} P_2^{m_2})^{\frac{1}{r}}$$

and realized indirect utility function is:

$$U_t^\theta = V(P, Y; M_0, M) = (m_{01} + m_{02})^{-\frac{1}{r}} \left(\frac{m_1}{m_{01}} \frac{m_2}{m_{02}} \right) P_1^{-m_1} P_2^{-m_2} Y^{\frac{1}{r}}$$

where $r = m_1 + m_2$ and where m_{01} , m_{02} are the perceived and m_1 , m_2 the "true" parameters associated with goods x_1 and x_2 respectively.

4/ Willig [1973] also shows that compensating and equivalent variations are the relevant surplus concepts for cost-benefit and welfare analysis and that either can be closely approximated by the consumer surplus areas of the Marshallian demand curve.

5/ This is a strong assumption. Three conditions under which it holds are outlined by Samuelson. One of the three that would also be useful to assume is that all income elasticities are unitary, a result obtained by assuming homoethetic preferences. The practical implication of these assumptions is that the Marshallian and the Hicksian demand curves converge and measures of changes in consumer surplus are identical to measures of compensating variation.

6/ For purposes of visual clarity, the perceived demand functions (1'') are not drawn through the coordinates p_j, x_j^0 and p_s, x_s^0 in figure 3(b) and (c) respectively.

7/ This is precisely the measure of welfare loss employed by Peltzman.

8/ This aspect of the gain is ignored by Peltzman and Kotowitz and Mathewson.

9/ Some interesting recent work in this area has been published by Cross.

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