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MEASUREMENT OF RELATIVE POVERTY

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1. INTRODUCTION

Although poverty is of considerable economic and political importance, its measurement is not a standard procedure [12]. Even basic conceptual issues relating to poverty measurement continue to evade consensus. This paper is concerned with measuring poverty of a group relative to that of some population to which the group belongs. While it is possible to use almost any poverty index as a basis for the measure of "relative poverty"[1] proposed below, some have more desirable properties than others. Three poverty indices are used to demonstrate an approach to measuring relative poverty. It will be shown that one of these has preferable characteristics.

The measure of relative poverty proposed here, which we will call the relative poverty ratio, R, has the following construction:

$$R_k = \frac{\text{the proportional contribution by group } k \text{ to population poverty}}{\text{the proportional contribution by group } k \text{ to population size}} \quad (1)$$

We can interpret R_k as follows. If R_k is less than/equal to/greater than unity, then the incidence of poverty in group k is less than/equal to/greater than the incidence of poverty in the population as a whole. The incidence of poverty in group k is R_k times that in the population as a whole. Compared with standard poverty indices such a measure conveys more information and is easier to understand, especially by those unfamiliar with the complexities of poverty and inequality measurement. Moreover, because economic policy makers are frequently more concerned with poverty within specific groups which constitute the population of interest, than with poverty for the population as a whole, R_k should be as useful as standard poverty indices, maybe even more so.

The denominator of (1) causes no problems but the numerator implies the use of some poverty measure. The numerator, (call it ρ_k), is given by

(2) $\rho_k = \frac{\text{contribution by group k to the population poverty index value}}{\text{the poverty index value for the whole population}}$

The problem is to select a measure of poverty so that R satisfies some reasonable axioms and properties. For example, if the chosen poverty index is the head count ratio (H = the number of poor as a proportion of the total population) then $\rho_k = m_k/m$ and $R_k = nm_k/mn_k = H_k/H$, where m and n are the number of poor and the size of the population respectively and the k subscript indicates the corresponding values for group k. Because $\partial R_k/\partial H_k > 0$, R_k will exhibit the directional properties of H_k. But the head count ratio, H, has few desirable features; indeed it has several undesirable ones. In order to find an acceptable poverty index we shall review the desirable properties of a poverty index in section 2, and evaluate several previously proposed indices accordingly in section 3. In section 4 the derivation of R will be presented, followed by an illustration of its use in section 5.

2. AXIOMS OF POVERTY MEASUREMENT

A. K. Sen [11] and Nanak Kakwani [5] have proposed the following axioms (Sen proposed axioms M and T while Kakwani proposed axioms MS and TS) for poverty measurement:[2]

AXIOM M (monotonicity): *If $(\Delta P)_d$ is the change in the poverty measure when the income of a poor changes by d, other things equal, then $(\Delta P)_d > 0$ for $d < 0$.*

AXIOM T (transfer): *If $(\Delta P)_{y,y'}$ is the change in the poverty measure when there is a unit transfer of income from a poor with income y to someone with income y', other things equal, then $(\Delta P)_{y,y'} > 0$ for $y < y'$.*

AXIOM MS (monotonicity sensitivity): *If $(\Delta P)_y$ represents the increase* *in the poverty measure due to a small reduction in income of a poor with income y, other things equal, then* $(\Delta P)_y > (\Delta P)_{y'}$ *for* $y < y'$.

Transfer sensitivity (TS): (there are two contrasting forms of the TS axiom)

AXIOM TS1: *If* $(\Delta P)_{i,i+k}$ *is the increase in the poverty measure due to a unit transfer from the ith poor with income* y_i, *to the (i+k)th poor, k being a positive integer, other things equal, then* $(\Delta P)_{i,i+k} > (\Delta P)_{j,j+k}$ *for* $y_i < y_j$.

AXIOM TS2: *If* $(\Delta P)_{x/x+h}$ *is the increase in the poverty measure due to a unit transfer of income from a poor with an income of x to a poor with an income of (x+h), h > 0, other things equal, then* $(\Delta P)_{x/x+h} > (\Delta P)_{y/y+h}$ *for* $x < y$.

In passing it should be noted that Kakwani's contention [5, p.438] that axioms M and MS necessarily imply axiom T is incorrect. Axiom MS is concerned with reductions in the incomes of poor, while axiom T involves an increase in the income of the recipient which could cause him to cross the poverty line. This is not possible when a poor suffers an income reduction. It is this possibility which causes Sen's poverty index to violate axiom T, contrary to Clark [3, p.517], while it satisfies both axioms M and MS [12, p.302]. Kakwani [6] revised his axiom T in accord with Sen's revision [12]. In this paper we will stay with the original, more demanding version of the transfer axiom given in [11].

While TS1 and TS2 appear similar, they are equivalent only if income levels of the poor are proportional to their income positions. In general, TS1 is applicable when "relative deprivation" of those in poverty is perceived as based on their ordinal income ranking. Axiom TS2 is applicable when relative deprivation is perceived as based on income differences. The prime motivation for the TS axioms is the perception that a given transfer from a poor to someone with more income should cause a greater increase in the poverty measure, the lower the income of the "donor". A problem lies in the characterization of the transfer. Axiom TS1 measures the transfer distance as the difference in the ordinal ranks of the "donor" and "recipient" (call this ordinal measurement), while axiom TS2 measures the transfer distance by the income difference between the two (call this income measurement). Transfer sensitivity based on ordinal measurement seems a little "peculiar" [1, p.868] because its effect is dependent upon the relative densities of the income distribution at various income levels. Transfer sensitivity based on income measurement seems more in accord with common sense.[3]

Finally, Noriyuki Takayama [13] and Blackorby and Donaldson [2] proposed that the following three properties are desirable in a poverty index. Given a poverty index $P = P(H,v,Y_p)$, where H is the proportion of the population which is poor, v is the mean income of the poor, and Y_p is a measure of the income inequality among the poor, then:

T1: $\partial P/\partial Y_p > 0$. T2: $\partial P/\partial v < 0$.[4] T3: $\partial P/\partial H > 0$.

Takayama actually specified the Gini coefficient of the poor (G_p) as the measure of income inequality but this is as a little too restrictive. Here we allow the possibility of other measures of income inequality, such as the coefficient of variation of the income distribution of the poor (V_p).

3. INDICES OF POVERTY

Crude, formerly popular, measures of poverty [3, p.515 and 12, p.294] include the head count ratio (H), the mean poverty gap (M), and the poverty gap ratio (I). They are defined as

(3) $H = m/n$

(4) $$M = \sum_{i=1}^{m} (z-y_i)/m = z - v$$

(5) $$I = M/z = 1 - v/z$$

where: n is the size of the population; m is the number of poor; y_i is the income of the i^{th} agent such that $y_i \leq y_{i+1}$ for all i; z is the poverty income line or poverty threshold (which is assumed to be the same for all agents in the population); and v is the mean income of the poor.

The head count ratio (H) violates all of the Sen and Kakwani axioms and satisfies only property T3. The mean poverty gap (M) and the poverty gap ratio (I) satisfy axioms M and property T2 only. It is clear that these simple measures of poverty have multiple deficiencies.[5]

Of the more recently proposed measures of poverty, those suggested by Sen [11][6] and Takayama [13] are arguably the best known. Both involve a rank order weighting (ROW) scheme applied to a set of income gaps. The Sen index (S) and the Takayama index (T) are

(6) $$S = \frac{2}{(m+1)nz} \sum_{i=1}^{m} g_i \ (m+1-i)$$

(7) $$= H \{ I + (1-I) G_p \}$$

where g_i is the poverty gap of the i^{th} poor such that $g_i \geq g_{i+1}$ for all i=1,m-1; ($g_i = z-y_i$ provided all have the same poverty line.)[7]
G_p is the Gini coefficient of the income distribution of the poor.

(8) $$T = \frac{2}{u^* n^2} \sum_{i=1}^{n} (u^*-y_i^*) \ (n+1-i)$$

(9) $$= H \{ (1-\psi) I + \psi G_p \}$$

where $\psi = H v/u^*$; $y_i^* = \text{Min} [y_i , z]$ i=1,n; and u^* = mean of y_i^*.
y^* is called the censored income distribution. It can be seen that T is the Gini coefficient[8] of the censored income distribution. The essential

difference between the ROW schemes of S and T is that S implies that relative deprivation is defined in terms of the ordinal income ranking among the poor only, while T defines relative deprivation in terms of the ordinal income ranking within the entire population.

While S is generally well behaved, it is possible for it to violate axiom T.[9] More specifically, when an income transfer causes the recipient to cross the poverty threshold the value of S may fall so violating axiom T [12 and 14]. Moreover, S is equally sensitive to all equal income transfers when the transfer distance is ordinally measured.[10] Therefore S will violate TS1. If the transfer distance is income measured, then S will be most sensitive to equal income transfers between poor where the income distribution is most dense. Income distributions of the poor will tend to be most dense at the top, and so S will tend to violate TS2. S satisfies axioms M, MS and the Takayama properties [3, p.517].

T is also well behaved in most circumstances but it is possible for T to violate both axioms M and T.[11] According to Kakwani such violations occur "only in unusual situations" [7, p.525].[12] T has the same characteristics as S in relation to the TS axioms. Moreover, T fails to satisfy property T3, although Takayama assures us that "larger H gives us larger T in almost all cases" [13, p.757].

The last poverty index which we will consider here was first proposed by Foster et al [4]. We will call it the deficit squared poverty measure, D. It is given by

$$D = \frac{1}{n z^2} \sum_{i=1}^{m} g_i^2 \tag{10}$$

$$= H \left(I^2 + (1-I)^2 V_p^2 \right) \tag{11}$$

where V_p is coefficient of variation of the income distribution of the poor. D is closely related to Clark's poverty measure[13] [3], which is

(12) $$C_\alpha = (H/z) \left(\sum_{i=1}^{m} g_i^{\alpha}/m \right)^{1/\alpha}$$

Letting $\alpha = 2$

(13)[14] $$C_2 = (H\ D)^{1/2}$$

and so

(14) $$D = H^{-1}\ C_2^{2}$$

D satisfies axioms M and T [4] as well as axiom MS. It fails TS2 [4], because, like the coefficient of variation of which it is a function, it is equally sensitive to equal income transfers at all income levels if distance is income measured [8, p.28 and 1, p.868]. Because income distributions of the poor tend to be most dense at the top, D will tend to satisfy axiom TS1. This is because axiom TS1 measures the transfer distance by ordinal rank differences which imply smaller income differences where the income distribution is most dense. Using equation (11) it can be shown that D also satisfies the Takayama properties where, in the case of T1, income inequality among the poor is measured by the coefficient of variation.

Although numerous other indices of poverty have been proposed [12] indices S, T and D are sufficient for this discussion. None of these three indices require arbitrary choice of parameters (as do measures proposed by Kakwani [5], Clark [3], and Foster [4]); from equations (7), (9), and (11) we can see that they are similar to each other in structure (even though S and T use a ROW scheme while D implies an income difference weighting system); and they are characterized by varying technical problems, with D appearing to be in the least trouble. Table 1 summarizes the properties of the poverty indices reviewed above as well as showing their limiting values.

TABLE 1

Summary of the properties of the poverty indices discussed

Property	H	I	S	T	D
			Index		
Value if there are no poor.	0	0	0	0	0
Value if all income is monopolized. (assuming large n)	1	1	1	1	1
Value if all poor have same income.	H	I	HI	$H\psi I$	HI^2
Axiom M: monotonicity.	vw	s	s	s/v	s
Axiom T: transfer.	v	v	v	s/v	s
Axiom MS: monotonicity sensitivity	vw	vw	s	s	s
Axiom TS1: transfer sensitivity.	vw	vw	vw	vw	us
Axiom TS2: transfer sensitivity.	vw	vw	uv	uv	vw
Property T1: $\partial\_/\partial Y_p>0$	vw	vw	s	s	s
Property T2: $\partial\_/\partial v<0$	vw	s	s	s	s
Property T3: $\partial\_/\partial H>0$	s	vw	s	s/v	s

Key: s=satisfies;

v=violates (when the transfer recipient crosses the poverty line);

s/v=usually satisfies but may violate in "unusual cases";

vw=violates weakly (value does not change);

us=usually satisfies, if the income distribution is most dense at top.

uv=usually violates, if the income distribution is most dense at top.

4. MEASUREMENT OF RELATIVE POVERTY

We will begin with using poverty index D because table 2 shows it to be the most satisfactory of those considered. When poverty is measured by index D, the contribution by group k to the level of poverty in the population [4, p.764], C_k, is

$$(12) \qquad C_k^{(D)} = \frac{1}{n z^2} \sum_{\substack{i \in k \\ i \leq m}} g_i^2$$

$$(13) \qquad = \frac{n_k}{n} D_k$$

where D_k is the level of poverty within group k as measured by index D. The (D) superscript on C_k indicates that the D index is being used to measure poverty. If there are L disjoint, collectively exhaustive groups in the population [4, p.764], then

$$(14) \qquad D = \sum_{k=1}^{L} \frac{n_k}{n} D_k$$

That is, the value of the population D index is the simple weighted average of the D index values of any disjoint collectively exhaustive set of subpopulations, where the weights are the respective subpopulation proportions.[15] In other words, when the D index is used, poverty of the total population is related to the poverty in its constituent groups in a very simple way. Equation (14) describes an attractive property of D which neither S nor T exhibit.

From (13) and (2)

$$(15) \qquad \rho_k^{(D)} = \frac{n_k D_k}{n D}$$

and so from (1) and (15)

$$(16) \qquad R_k^{(D)} = D_k/D$$

That is, the relative poverty ratio for group k, when D is used as the measure of poverty, is the ratio of the poverty level in group k to the population poverty level. $R_k^{(D)}$ will be positive with a value greater/less than 1.0 indicating that group k has an incidence of poverty greater/less than the total population.

From (14) and (16) we get

$$(17) \qquad \frac{\partial R_k^{(D)}}{\partial D_k} = (1 - \rho_k^{(D)}) / D > 0$$

$R_k^{(D)}$ will satisfy the same axioms and exhibit the same properties as D_k. In other words, $R_k^{(D)}$ satisfies axioms M, S, MS and most likely TS1, and it exhibits the three Takayama properties. Moreover D_k and $R_k^{(D)}$ will give the same poverty ranking of any set of disjoint groups within a given population. It is not possible to make similar statements about $R_k^{(S)}$ and $R_k^{(T)}$. No expressions for $R_k^{(S)}$ and $R_k^{(T)}$ corresponding to equation (17) exist. When S and T are used to measure poverty, the expressions for R_k are not as simple as (16). They are

$$(18) \qquad R_k^{(S)} = \frac{n \sum_{\substack{i \in k \\ i \leq m}} g_i \, (m+1-i)}{n_k \sum_{i=1}^{m} g_i \, (m+1-i)}$$

and

$$(19) \qquad R_k^{(T)} = \frac{n \sum_{\substack{i \in k \\ i \leq m}} (u^*-y_i^*) \, (n+1-i)}{n_k \sum_{i=1}^{m} (u^*-y_i^*) \, (n+1-i)}$$

The ROW schemes used by S and T complicate the numerators in (18) and (19) respectively. As a consequence $R_k^{(S)}$ and $R_k^{(T)}$ cannot be expressed in terms of the respective group index values (ie. S_k and T_k). This means that no stable relationship exists, such as (14), between the population S (or T) value and the S_k (or T_k) values of its constituent groups.

5. MEASUREMENT OF RELATIVE POVERTY IN MINNESOTA

The indices of poverty and relative poverty ratios discussed above have been applied to 1979 income distribution data for Minnesota families grouped on a racial and geographical basis.[16] The results are presented in table 2. In reading table 2 it should be noted that the central city groups are subsets of the nonrural groups. Column 1 will not add correctly unless this is taken into account. Also the small numbers of racial minority farm families made separate analyses of these groups meaningless.

In reading table 2 recall that a large R value means a high incidence of poverty. From the table, we can see that minority poverty in Minnesota is ubiquitous, but in central city areas the situation deserves special attention. Also from the table it can be seen that the incidence of "white" family poverty is greatest among rural farm families. In this case care is appropriate because farm incomes tend to be more volatile than nonfarm incomes and so farm incomes can be abnormally high or low in any given year. However the year in question, 1979, was not an abnormally good or bad year for farming in Minnesota. In addition, analyses for other years have yielded similar results to those presented in table 2.

It is not the purpose here to analyse the structure of poverty in the state of Minnesota. The intention of presenting table 2 is to demonstrate how the relative poverty ratio values, R_k, convey more information about relative poverty than the basic index values. The R_k values are more effective than the standard S, T or D values, in drawing attention to those groups with a poverty problem. R_k values in excess of 2.0 should attract attention while R_k values above 3.0 should indicate the need for considerable concern.

TABLE 2

Poverty measures for Minnesota families : grouped by race & geography

Groups	% of total population	Poverty Indices S	T	D	Relative Poverty Ratios $R^{(S)}$	$R^{(T)}$	$R^{(D)}$
All families	100.0	.051	.039	.023	1.000	1.000	1.000
White families							
All	97.4	.048	.037	.022	0.949	0.955	0.949
Rural farm	8.0	.105	.081	.051	2.236	2.083	2.244
Rural nonfarm	26.1	.064	.048	.028	1.235	1.217	1.234
Nonrural	63.3	.035	.026	.015	0.668	0.704	0.667
Central city	17.8	.045	.034	.020	0.870	0.887	0.869
Black families							
All	1.11	.137	.105	.068	2.968	2.727	2.979
Rural	0.01	.098	.075	.045	1.995	1.889	1.996
Nonrural	1.09	.137	.106	.068	2.978	2.736	2.990
Central city	0.92	.153	.116	.076	3.319	3.038	3.332
American Indian							
All	0.70	.151	.109	.069	3.024	2.814	3.027
Rural	0.27	.133	.096	.060	2.615	2.465	2.615
Nonrural	0.43	.163	.117	.075	3.299	3.055	3.303
Central city	0.26	.202	.142	.094	4.137	3.799	4.144
Asian & Pacific Isldrs.							
All	0.54	.129	.101	.065	2.858	2.621	2.872
Rural	0.03	.169	.131	.088	3.848	3.484	3.869
Nonrural	0.51	.126	.099	.064	2.804	2.573	2.817
Central city	0.22	.193	.144	.098	4.263	3.863	4.283

Data source: U.S. Census Reports: 1980. Minnesota

A surprising feature of table 2 is the similarity between the values of $R^{(S)}$ and $R^{(D)}$ for corresponding groups. This similarity has been noted for all "real world" income distributions analysed to date. However it is not difficult to construct income distributions where dissimilarity of $R^{(S)}$ and $R^{(D)}$ is observed. Also from table 2 we see that the $R^{(T)}$ values are of the same order of magnitude as the other R values for corresponding groups, but slightly more conservative.

Although not shown in table 2, it is a simple matter to obtain the proportion of total poverty attributable to any group by multiplying the the corresponding population proportion (column 1) by one of the relative poverty ratio values (in one of the last three columns). For example, rural farm families, which constituted 8.0% of all families in Minnesota in 1979, contributed 17.9% (= 8.0 x 2.244) of family poverty in Minnesota in that year, when D is used as the index of poverty. The corresponding figures are 17.9% and 16.7% using the S and D indices respectively.

While the relative poverty ratio values in table 2 highlight the relative intensity of poverty among the groups considered, it is necessary to remember that intensity does not reflect aggregate absolute levels. Gold is more dense than water but a gallon of water is heavier than a wedding ring. Therefore while the incidence of poverty is greater among nonwhite families than among white families, 92.4% of poverty in Minnesota in 1979 (= 97.4 x .949) occurred within the white population. Obviously the white population should be further disaggregated to determine where/if white poverty is concentrated.

In addition, tracking changes in R for specific groups over time can be instructive. For example it has been found that even though relative poverty among white farmers, as measured by R, has been increasing over time, the "absolute" poverty of this group, as measured by S,T or D has been declining. But the relative poverty measures, R_k, and the "absolute" poverty levels, S,T, and D, have been increasing for all minority racial groups.

An attractive property of R is that it is not particularly sensitive to the choice of the poverty threshold whereas basic poverty indices such as S, T and D are sensitive. Very little change was noted in the R values in table 2 when the poverty threshold was raised and lowered by over 10%.

6. CONCLUSION

When there is interest in the levels of poverty within and between groups which belong to some larger population, the relative poverty ratio, R, provides more understandable, and perhaps more meaningful, information than the standard poverty indices. The relative poverty ratio based on the poverty index D, $R^{(D)}$, appears to be the most acceptable of the possibilities considered here. $R^{(D)}$ satisfies the same axioms and properties as D poverty index, while D itself is able to withstand comparison with the better known poverty indices proposed by Sen, S, and Takayama, T. Moreover, $R^{(D)}$ is much easier to compute than either $R^{(S)}$ or $R^{(T)}$. Theory and practice both support $R^{(D)}$ as a useful measure of relative poverty.

FOOTNOTES

1. In this paper the term "relative poverty" is not used in the same sense as by Sen [11] when he discusses relative deprivation as a basis for a system ordinal rank weights, or by Blackorby and Donaldson [2].

2. The axioms in the text are not exactly as stated by Sen [11] and Kakwani [5], but their meaning is preserved. The original statements of the Sen and Kakwani axioms are given in appendix A.

3. The Gini coefficient (G) is equally sensitive to equal income transfers at all income levels if the transfer distance is measured by rank order. Because it uses rank order weights, G will be most sensitive to a given income transfer over a specified income interval where the income distribution is most dense [1]. The coefficient of variation (V) is equally sensitive to an equal income transfers when distance is measured by the income difference of the "recipient" and "donor" [1]. V will be least sensitive to a given income transfer over a given ordinal distance where the income distribution is most dense.

4. Takayama's property T2 is equivalent to the requirement that, given H and Y_p, $\partial P/\partial I > 0$, where $I = 1 - v/z$.

5. See Sen [12, p.294] and Clark et al [3] for more discussion of the deficiencies of H, M and I.

6. In [9] and [10] the Sen index is proposed in a slightly more "primitive" form.

7. By assuming that all agents have the same poverty line considerable complication irrelevant to the current discussion is avoided. Different

poverty thresholds may be justified, indeed, may be necessary, if individuals live in significantly different economic environments (as in reality they do). Some of the relevant questions are raised, although they are not adequately addressed, in Sen [12]. It is suggested that if more than one poverty threshold is applied, then rank orders should be determined on the basis of the poverty gaps, not income levels. The literature has been far from transparent on this issue. The procedure suggested by Blackorby and Donaldson to surmount this "minor" problem [2, p.1060] appears to be in conflict with Sen's discussion of this "complex" issue [12, p.292].

8. Equation (8) is only one of several forms for the Gini coefficient which appear in the literature. See [8], [1] and [13] for others.

9. Sen subsequently modified his transfer axiom [12, p.302] to make it less demanding because his measure violated axiom T as it was stated in [11, p.219]. The modification to axiom T consisted of the following added qualification: "unless the number below the poverty line is strictly reduced by the transfer". In the present discussion we reject the proposition that "(it) is arguable whether a poverty measure should not show increased poverty whenever some income is transferred from a poorer to a richer person no matter whether this makes the richer person cease to be regarded as poor because of his crossing the poverty line" [12, p.302]. We contend that it should.

10. From equation (7) S is an increasing function of G_p, the Gini coefficient of the income distribution of the poor. The Gini coefficient gives equal weight to unit income transfers which are equidistant in rank order terms. Therefore, other things equal, so does S.

11. Violation of axiom T in this case is somewhat contrived because a transfer from a poor to a nonpoor will fail axiom T if the index violates axiom M. In other words, index T satisfies axiom T in all but the cases of those transfers from poor to nonpoor where it violates axiom M [3, p.521].

12. Unfortunately such unusual situations do occur unexpectedly from time to time.

13. Foster et al [4] proposed a general class of poverty indices of the form $P_\alpha=(1/n)\sum_{i=1}^{m}(g_i/z)^\alpha$ which similar to the Clark index [3].

14. From equations (11) and (13) we can obtain:

$C_2 = [\ H^2\ (\ I^2 + (1-I)^2\ V_p^2\ ]^{1/2}$

which provides some insights into the structure of Clark's measure [3] in the case where $\alpha=2$.

15. From equation (14) $\partial D/\partial D_k = n_k/n > 0$, and so D satisfies the subgroup monotonicity axiom proposed by Foster et al [4] which can be stated as: *If* (ΔP_k) *is the change in the poverty measure of subgroup k which occurs when there is some change to the income(s) of a member(s) of that group, with* n_k *unchanged, and if* (ΔP) *is the corresponding change in the population poverty measure, other things equal, then* $(\Delta P_k) > 0$ *implies* $(\Delta P) > 0$.

Foster et al [4, p.763] claim that both the Sen and Takayama indices violate their subgroup monotonicity axiom.

16. The results presented and discussed in section 5 are taken from a much larger study which analysed the incidence of farm/rural/urban poverty in Minnesota, 1949-79. The various assumptions made in the analysis are given in the report of this larger study, which is in preparation.

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APPENDIX A

The Sen and Kakwani Axioms of Poverty Measurement

Sen [10, p.219] and Kakwani [4, pp.438-439 and 5, pp.17-18] stated their axioms as follows:

Axiom M (monotonicity): Given other things, a reduction in the income of a person below the poverty line must increase the poverty measure.

Axiom T (transfer): Given other things, a pure transfer of income from a person below the poverty line to anyone who is richer must increase the poverty measure.

Kakwani assumes: y_i is the income of the ith agent, $y_i < y_{i+1}$ for all i

Axiom MS (monotonicity sensitivity): If $(\Delta P)_i$ represents the increase in the poverty measure due to a small reduction in the income of the ith poor, then $(\Delta P)_i > (\Delta P)_j$ for $j>i$.

Axiom TS1 (transfer sensitivity I): For any positive integer ρ and any pair of poor individuals i and j, if $j>i$, then $(\Delta P)_{i,i+\rho} > (\Delta P)_{j,j+\rho}$ where $(\Delta P)_{i,i+\rho}$ is the increase in the poverty measure due to a transfer of income from the ith poor to the $(i+\rho)$th poor.

Axiom TS2 (transfer sensitivity II): If a transfer of income takes place from the ith poor with income x_i to a poor with income (x_i+h), then for a given $h>0$ the magnitude of the increase in poverty measure decreases as i increases."

It is suggested that the meaning of these axioms is preserved in the restated axioms, TS1 and TS2, in the text.