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Forage Response to Swine Effluent: A Cox Nonnested Test of Alternative Functional Forms Using a Fast Double Bootstrap

Seong C. Park, B. Wade Brorsen, Arthur L. Stoecker, and Jeffory A. Hattey

A Cox nonnested test is conducted using a fast double bootstrap (FDB) method to select among three competing functional forms (linear response plateau, quadratic, and Mitscherlich-Baule) to model forage yield response to nitrogen applied with swine effluent. The quadratic is rejected in favor of one of the other functional forms in all cases. The FDB *p* values differed slightly from the single bootstrap *p* values. Buffalograss was slightly more profitable than bermudagrass and has the ability to use almost as much nitrogen as bermudagrass.

Key Words: Cox nonnested test, fast double bootstrap, linear response plateau, Mitscherlich-Baule, nitrogen response, swine effluent

JEL Classifications: C12, Q12

The Oklahoma Panhandle is one of the leading swine producing regions in the United States (Lowitt, 2006). Confined swine production facilities in this region produce massive amounts of manure that are typically flushed into anaerobic lagoons to facilitate decomposition. Crop and grassland close to swine production facilities are then irrigated with the swine effluent.

Land application of animal waste has been widely adopted by livestock operators in this region to use nutrients in manure. The application cost of nitrogen in swine effluent is low for nearby land (Carreira, 2004) because the effluent is typically delivered to cropland through an existing irrigation system. Available land for manure application is an important limiting factor for livestock operations.

Improved forage systems in the Oklahoma Panhandle are a potential alternative to crops such as corn and wheat because forage grasses use less water and chemicals and have lower variable costs (Allen et al., 2005). The Panhandle region was originally grassland and is suited to cattle grazing. With better-adapted cultivars of grasses in an improved forage system, high per-acre yields and profits are possible (Krall and Schuman, 1996). An ongoing challenge for producers is providing an adequate supply of forage throughout the year (Gillen and Berg, 2005; Park et al., 2011) and so there is interest in expanding forage production in this region.

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Forage production using manure not only removes nutrients in the form of hay from land receiving manure, but also reduces the potential for environmental damages from runoff and water impairment (Sims and Wolf, 1994). Several field trials showed that anaerobic swine effluent and commercial fertilizer applied at comparable rates had similar effects on the cumulative dry matter yield and nutritive value of forages, which indicate that swine effluent is equally effective as commercial fertilizers in supplying nutrients to forages including bermudagrass and Johnson grass (Adeli and Varco, 2001; Brink et al., 2003). Producers benefit from applying swine effluent to forage crops as a result of reduced waste management costs and reduced need for chemical fertilizers, because swine lagoon effluent (SE) contains multiple essential crop nutrients (McAndrews et al., 2006).

When properly applied at rates based on plant nutrient requirement, SE can replace costly inorganic fertilizers with minimal environmental and odor concerns (Al-Kaisi and Waskom, 2002). Agronomic benefits of using SE includes the build-up of multiple essential crop nutrients, soil fertility, tilth, soil aeration, and increased presence of beneficial microorganisms (Sutton et al., 1986). Additionally, soil erosion resulting from wind can be reduced. Manure can substitute for commercial fertilizers by supplying the multiple essential crop nutrients (such as nitrogen, phosphorus, potassium, sulfur, calcium, magnesium, and micronutrients) contained in animal wastes. Swine effluent has sufficient phosphorus that the level of phosphorus is expected to increase over time (Carreira et al., 2006), but we do not address potential problems created by the build-up of phosphorus in the soil.

Moreover, recent shocks in energy and corn markets make animal manure an economically viable alternative to inorganic fertilizer (Park et al., 2010). Manure's economic feasibility is greatly affected by shipping cost because of its lower concentration of crop nutrients than inorganic fertilizers. Therefore, animal manure has been continuously applied on cropland nearby to concentrated animal feeding operations in excess of plant needs (Park et al., 2010). However, previous experimental work on nitrogen response has dealt almost entirely with commercial fertilizers and more often with grain than with forage. In evaluating application of animal wastes, there is a need to better understand yield response.

The importance of response functions to nutrients has long been of interest to agricultural economists (e.g. Heady and Pesek, 1954). The importance of the functional form of empirical production functions has been recognized in previous studies (Ackello-Oguto, Paris, and Williams, 1985; Frank, Beattie, and Embleton, 1990; Paris, 1992). Polynomial functional forms such as quadratic and square root have been widely used to model crop response to nutrients. However, polynomial specification of crop response to fertilizer nutrients often generates a high optimal level of fertilizer, which could lead to environmental damage from application of excess fertilizer if the functions overestimate the optimal level (Ackello-Oguto, Paris, and Williams, 1985).

The von Liebig function is an alternative to polynomial functions (Kaitibie et al., 2007). With von Liebig functional forms, a plateau exists in which crop yield does not increase with addition of nonlimiting production inputs (Ackello-Oguto, Paris, and Williams, 1985). Previous studies (Berck and Helfand, 1990; Paris, 1992; Paris and Knapp, 1989) found the von Liebig functional form usually fits better than other functional forms for grain yield. One problem that this previous literature has had to deal with is how best to select among nonlinear functional forms that are not nested.

In the case of linear models, nonested tests are well developed. For nonlinear models, a suggested nonnested test based on the Cox statistic is to calculate a bootstrap p value as the proportion of simulated log likelihood ratio statistics that are more extreme than the actual log likelihood ratio statistic (Goldman, 1993; Lee and Brorsen, 1997; McLachlan, 1987; Williams, 1970). Coulibaly and Brorsen (1999) and Kapetanios and Weeks (2003) find that such a single bootstrap of the likelihood ratio statistic has better size and power properties than asymptotic tests such as Pesaran and Pesaran (1993).

Wagenmakers et al. (2004) and later Godfrey (2007) and Godfrey (2009, pp. 297–300), however, argue the single bootstrap does not consider the variability as a result of using estimated parameters,[1] which causes bias toward rejecting null hypotheses. Wagenmakers et al. (2004) propose a nonparametric bootstrap of the parametric bootstrap, but such an approach can be computer-intensive. Kapetanios and Weeks (2003) also consider a double bootstrap procedure to be more accurate than the ordinary bootstrap, but the double bootstrap is costly because second-level bootstrap samples are generated for each first-level bootstrap sample. We extend this previous literature by using the fast double bootstrap (Davidson and MacKinnon, 2001, 2007), which is an alternative to the double bootstrap that considers the extra variability from estimated parameters and is much less computationally intensive than the double bootstrap.

Kaitibie et al. (2007) conducted a single bootstrap Cox test of alternative functional forms in a study of the effects of forage stocking density on average daily gain. The fast double bootstrap approach that we used has correct asymptotic size whereas the single bootstrap approach used by Kaitibie et al. (2007) does not. We also compare the single bootstrap and fast double bootstrap *p* values to provide a measure of the importance of considering the additional noise from estimating parameters.

We use the fast double bootstrap to select among three competing functional forms in modeling dry matter yield of four grass species in response to applications of swine effluent. The three functional forms considered are: linear response plateau (LP), quadratic (QD), and Mitscherlich-Baule (MB). In the next section, the three functional forms are defined and formulas for finding optimal nitrogen rates are derived. Next, we explain single bootstrap, double bootstrap, and fast double bootstrap methods and how they can be used to conduct Cox nonnested tests. The data used are from a long-term forage experiment in the Oklahoma Panhandle. The effects of alternative functional forms on the optimal level of nitrogen (N) and on expected profit are determined. Swine farms sometimes want to apply large amounts of N and these functional forms can give very different predictions at very high levels so the issue of what functional form to use is of considerable practical relevance in evaluating swine farms.

Expected Profit Maximizing Level of Nitrogen Fertilizer

Consider a farmer who wants to maximize expected profit from growing forages using swine effluent as a N fertilizer. The farmer is assumed to know the production function and thus the farmer's decision problem regarding an optimal N level can be represented as

$$\begin{aligned} \max_{N}\ & E(\pi_t|N) = pE[y_t] - rN \\ & \text{s.t. } y_t = f(N),\ N \geq 0 \end{aligned} \tag{1}$$

where π_t is the farmer's net profit above N cost at time t, y_t is the forage yield, N is the level of applied N, r is the price of applied N fertilizer, and p is the price of forage. The problem is that a different optimal level is obtained when a different functional form is assumed. We now discuss the determination of the optimal level of N for each of the three functional forms considered.

Linear Response Plateau

Using a univariate linear response plateau[2] functional form, the response of forage yield to the N in the SE can be expressed as:

$$y_{it} = \min\{\alpha_0 + \alpha_1 N_{it}, \alpha_2\} + u_t + \varepsilon_{it} \tag{2}$$

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[1] Note that the theoretical argument for the inconsistency of the single bootstrap approach that is made in Godfrey and Santos Silva (2004) and Pesaran and Pesaran (1995) is incorrect and was corrected in Godfrey (2007).

[2] Previous research (Berck and Helfand, 1990; Kaitibie et al., 2003) has considered models in which the plateau is also stochastic. Because our plots are irrigated, the year-to-year variation is small in the experiments considered here. In addition, stochastic plateau models often have difficulty converging, which makes it impractical to estimate them with a double bootstrap.

where y_{it} is dry matter yield from the i^{th} plot in year t; N_{it} is the amount of N applied; α_0, α_1, and α_2 are coefficients to be estimated; α_2 is the plateau; and u_t represents the year random effect,[3] $u_t \sim N(0, \sigma_u^2)$ and $\varepsilon_{it} \sim N(0, \sigma_\varepsilon^2)$.

The function is continuous, but its derivatives do not exist with respect to N at the knot point where the response and the plateau are joined. The level of N (N*) that maximizes expected net profit (equation 1) can still be determined using marginal factor/input cost (MFC) and marginal expected product value (MVP). A nonstochastic linear response plateau function will show constant positive marginal product when $\alpha_0 + \alpha_1 N < \alpha_2$, and N should be applied until MVP = MFC. However, applying N above the level needed to reach the maximum yield, α_2, will generate zero marginal product. The optimal N (N*) for the linear response plateau would be either the N level to reach the plateau (N_p) or zero:

$$N^* = \begin{cases} N_p, & \text{if} \quad VMP > MFC \\ 0 & \text{otherwise} \end{cases} \tag{3}$$

Quadratic

A quadratic function is specified as

$$y_{it} = \beta_0 + \beta_1 NA_{it} + \beta_2 NA_{it}^2 + \omega_t + \pi_{it} \tag{4}$$

where β_0, β_1, and β_2 are coefficients; ω_t represents the random error term associated with year *t;* $\omega_t \sim N(0, \sigma_\omega^2)$ and $\pi_{it} \sim N(0, \sigma_\pi^2)$. The quadratic function is continuously twice differential. Therefore, the optimal N rate (N*) can be found by solving the first-order conditions for profit maximization. The resulting optimal level is

$$N^* = \frac{\beta_1 - r/p}{2\beta_2} \tag{5}$$

Mitscherlich-Baule

A MB function for forage yield response is

$$y_{it} = \gamma_0 - \gamma_1 \exp(-\gamma_2 NA_{it}) + \varphi_t + \varnothing_{it} \tag{6}$$

where γ_0, γ_1, and γ_2 are coefficients; φ_t represents the random error term associated with year t; $\varphi_t \sim N\left(0, \sigma_\varphi^2\right)$ and $\varnothing_{it} \sim N(0, \sigma_\varnothing^2)$. The optimal level of N is obtained by substituting Eq. (6) into Eq. (1) and then solving the optimization problem. The optimal level of N (N*) is obtained by solving the first-order condition for N, which gives

$$N^* = -\frac{1}{\gamma_2 \left[\ln\left(\frac{\frac{r}{p}}{\gamma_1 \gamma_2} \right) \right]} \tag{7}$$

Cox Nonnested Test

Two models are said to be nonnested when neither model cannot be expressed as a special case of the other. The nonnested hypotheses with H_0 as the null hypothesis and H_1 as the alternative can be written as

$$\begin{aligned} H_0 : Y &= f(X, \alpha, \sigma_u^2, \sigma_\varepsilon^2), \\ H_1 : Y &= g(X, \beta, \sigma_w^2, \sigma_\pi^2) \end{aligned} \tag{8}$$

where Y is a vector of observations of the dependent variable, X is a matrix of observations of independent variables, and α and β are parameter vectors under the null and alternative hypothesis.

The Cox test statistic is the difference between the log-likelihood ratio and the expected value of the log-likelihood ratio. The difficulty in calculating the Cox test is that this expected value cannot be obtained analytically for most nonnested tests and although the test statistic is asymptotically normal, the variance of the test statistic is not easily estimated. The Cox test

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[3] Weather and other effects associated with year are not controlled by the researcher and so the convention is to model year as a random effect. Grain yields are often nonnormally distributed across years (Harri et al., 2009), which would suggest either a stochastic plateau as in Tembo et al. (2008) or a nonnormally distributed random effect as in Bauer, Preacher, and Gil (2006). Although we know of no comparable work for forages, forages may also be nonnormally distributed. With irrigation, yields would be less affected by the nonlinear distribution of precipitation so any nonnormality would be lessened. Furthermore, with only 7 years of data, statistical tests have no power to reject the null hypothesis of normality.

statistic for testing the null hypothesis against the alternative hypothesis is

$$T_0 = L_{01} - E_0(L_{01}) \tag{9}$$

where L_{01} is the log-likelihood ratio, which is the difference in the estimated maximum log-likelihoods under the null and alternative hypotheses ($L_{01} = L_0(\hat{\alpha}) - L_1(\hat{\beta})$), $E_0(L_{01})$ is the expected value of the log-likelihood ratio under the null and alternative hypotheses, $\hat{\alpha}$ and $\hat{\beta}$ are the maximum likelihood parameter estimates of the null and alternative hypotheses, respectively, and T_0 is the Cox test statistic under the null hypothesis, which is asymptotically distributed with mean zero and variance φ_0^2 (Cox, 1962). The hypotheses are then reversed and the Cox statistic is calculated with H_1 as the null and H_0 as the alternative: $T_1 = L_{10} - E_1(L_{10})$.

Although the log-likelihood ratio is relatively easy to calculate, the difficulty is calculating the expected value of the log-likelihood ratio and its variance. Pesaran and Pesaran (1995) propose simulation procedures that estimate both the mean and the variance. Although their approach is asymptotically valid, it has been shown to have poor small sample properties (Coulibaly and Brorsen, 1999). An alternative is to use a single bootstrap and estimate only the mean (Coulibaly and Brorsen, 1999; Lee and Brorsen, 1997). The parametric single bootstrap generates Monte Carlo samples with the same number of observations as the original data using parameters estimated under the null hypothesis (Kaitibie et al., 2007; Williams, 1970). Because the test statistic being bootstrapped is not asymptotically pivotal,[4] the proof of consistency no longer holds, although Coulibaly and Brorsen (1999) show that such an approach has much better small sample properties than the Pesaran and Pesaran (1995) approach.

Bootstrap Tests

The model selection procedure uses the Cox test with a parametric bootstrap. We first explain the single bootstrap procedure. We next explain the double bootstrap and then finally the fast double bootstrap procedure that we use.

Ordinary (first level) Bootstrap

Let L_{01} denote a test statistic, and let $\hat{L}_{01}$ denote a realized value of L_{01}, which in our case is the actual log likelihood ratio from competing functions estimated with a particular sample of size N. When we consider a test that rejects when $\hat{L}_{01}$ is in the lower tail, the true *p* value based on $\hat{L}_{01}$ is $F(\hat{L}_{01})$, where $F(L_{01})$ is the cumulative distribution function of L_{01}.

With a parametric bootstrap using estimated parameters, the parameters estimated under the null hypothesis are used to generate B number of Monte Carlo samples, each of which is used to calculate a test statistic $L^*_{01,j}$ for j = 1, . . ., B. Then, $F(\hat{L}_{01})$ can be estimated by $\hat{F}^*_B(\hat{L}_{01})$, where $\hat{F}^*_B(L_{01})$ is the empirical distribution function of the $L^*_{01,j}$. The calculation of a *p* value with Cox nonnested test using an ordinary single bootstrap can be written as

$$\begin{aligned} p^*(\hat{L}_{01}) &= \hat{F}^*_B(\hat{L}_{01}) \\ &= \frac{1}{B+1}\left[\sum_{j=1}^{B} I\left(L^*_{01,j} < \hat{L}_{01}\right) + 1\right] \end{aligned} \tag{10}$$

where $p^*(\hat{L}_{01})$ is the bootstrap *p* value, $I(\cdot)$ is the indicator function that takes a value of one when the specified relationship is true, and $L^*_{01,j}$ which equals $L_0(\cdot) - L_1(\cdot)$ is the log-likelihood ratio for the j^{th} generated bootstrap sample. The null hypothesis that H_0 is true is rejected whenever $p^*(\hat{L}_{01}) < \alpha$ when the test level is α. The value one is added to both numerator and denominator to correct small sample problems. The problem with the single bootstrap is that it does not consider that estimated parameters are being used and so it could reject too often. Note that Monte Carlo studies (Coulibaly and Brorsen, 1999; Kapetanios and Weeks, 2003) show that the single bootstrap

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[4] A statistic is asymptotically pivotal if its asymptotic distribution does not depend on unknown parameters. For example, if a statistic has an asymptotic Z, F, or chi-squared distribution, it would be asymptotically pivotal.

has good-sized properties despite its theoretical deficiencies. Thus, the double bootstrap correction is expected to make little practical difference[5] despite its superior asymptotic properties.

Double Bootstrap

The double bootstrap proposed by Beran (1988) can also be used to calculate *p* values. In particular, the double bootstrap can produce consistent *p* values when the parameters must be estimated (Kapetanios and Weeks, 2003).

The first step in the double bootstrap is to generate B_1 Monte Carlo samples, each of which is used to calculate a test statistic $L_{01,j}^{*}$ for $j = 1, \ldots, B_1$. Then, the ordinary *p* value, $p^{*}(\hat{L}_{01})$, is obtained as described in Eq. (10). The second-level bootstrap generates B_2 bootstrap samples using the estimated parameters from each first-level bootstrap sample (a total of B_2B_1 second-level bootstrap samples). Each of the B_2 bootstrap samples based on the jth first-level parameters are used to calculate likelihood ratio $L_{01,jl}^{**}$ for $l = 1, \ldots, B_2$. Then, we can define the second-level bootstrap p value for the j^th^ first-level bootstrap sample as

$$(11)\quad p_j^{**}(\hat{L}_{01}) = \frac{1}{B_2+1}\left[\sum_{j=1}^{B_2} I\left(L_{01,jl}^{**} < L_{01,j}^{*}\right) + 1\right]$$

where $p_j^{**}(\hat{L}_{01})$ is the *p* value for the statistic $L_{01,j}^{*}$ based on the empirical distribution function of the $L_{01,jl}^{**}$. Using $p_j^{**}(\hat{L}_{01})$, the double bootstrap *p* value can be written as

$$(12)\quad \begin{aligned} p^{**}(\hat{L}_{01}) &= \frac{1}{B_1+1} \\ &\quad \times\left[\sum_{j=1}^{B_1} I\left(p_j^{**}(\hat{L}_{01}) < p^{*}(\hat{L}_{01})\right) + 1\right] \end{aligned}$$

The *p* value, p^{**}, based on a Cox nonnested test, is the percentage of the second bootstrap *p* values that go beyond the first bootstrap *p* value (in our case lower than $p^{*}(\hat{L}_{01})$). Letson and McCullough (1998) use a Monte Carlo study to show that the double bootstrap has good-sized properties even when the test statistic being used is not asymptotically pivotal. However, the double bootstrap is costly in terms of computation because it requires generating a set of second-level bootstrap samples for each first-level bootstrap sample. This is especially true here because two of the three functional forms require nonlinear estimation.

Fast Double Bootstrap

Davidson and MacKinnon (2001) proposed the fast double bootstrap (FDB) to reduce the computational burden of the double bootstrap. FDB only needs one second-level bootstrap sample for each first-level bootstrap sample. For FDB to be asymptotically valid, the distribution of statistics ($L_{01,jl}^{**}$) from the second-level bootstrap samples must be asymptotically independent of the distribution of statistics ($L_{01,jl}^{*}$) from the first-level bootstrap samples. Davidson and MacKinnon (2007, p. 3262) argue that the asymptotic independence assumption is not very restrictive and will generally be true for extremum estimators where the null lies in the interior of the parameter space.

The *p* value based on a Cox nonnested test using the fast double bootstrap can be easily calculated:

$$(13)\quad \begin{aligned} p_F^{**}(\hat{L}_{01}) &= \frac{1}{B+1} \\ &\quad \times\left[\sum_{j=1}^{B} I(L_{01,j}^{*} < \hat{Q}_B^{**}(p^{*}(\hat{L}_{01})) + 1\right] \end{aligned}$$

where $p_F^{**}(\hat{L}_{01})$ is the FDB *p* value, $I(\cdot)$ is the indicator function that is one when the argument is true and zero when it is false, $\hat{Q}_B^{**}(p^{*}(\hat{L}_{01}))$ is the $p^{*}(\hat{L}_{01})$ quantile of $L_{01,j}^{**}$ which is a log-likelihood ratio for the j^th^ generated sample of the second-level bootstrap with the null and alternative hypotheses. Each level of the bootstrap uses 10,000 Monte Carlo samples with 112 observations in each.

To better explain Eq. (13), consider an example for bermudagrass and the null hypothesis

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[5] In discussing the single bootstrap approach, Godfrey (2009, p. 300) says "results reported in Godfrey and Santos Silva (2004) include examples of important departures from the desired significance level." However, Godfrey and Santos Silva (2004) actually find incorrect size as a result of using too few (399) bootstrap samples.

of the linear plateau vs. the alternative of Mitscherlich-Baule. B is 10,000 since there are 10,000 bootstrap samples. For FDB, there are 10,000 first-level bootstrap samples and 10,000 second-level bootstrap samples. The first-level bootstrap *p* value $(p^*(\hat{L}_{01}))$ in this example is calculated as 0.0123 (122 of the 10,000 first-level likelihood ratios are less than the actual likelihood). The 10,000 likelihood ratios from the second-level bootstrap samples are then ordered from smallest to largest. The term $\hat{Q}_B^{**}(p^*(\hat{L}_{01}))$ is any number between the 123rd and 124th smallest likelihood ratios from the second-level bootstrap. We use the midpoint of the interval between the 123rd and 124th smallest likelihood ratios, which in this example is –5.0859. Thirteen of the 10,000 first-level likelihood ratios are less than –5.0859, which gives a *p* value of 0.0014. So using a significance level of 0.05, the linear plateau model would be rejected in favor of Mitscherlich-Baule for bermudagrass.

Data

The forage dry matter yields used to estimate the models are from a forage experiment conducted at the Oklahoma Panhandle Research and Extension Center near Goodwell, OK (latitude 36°35′ N, longitude 101°37′ W, and elevation 992 m) from 1999–2005. Mean annual precipitation and temperature at the station are 435 mm and 13.2°C, respectively. Four grass species included in this experiment are bermudagrass, buffalograss, orchardgrass, and wheatgrass. All plots were established and fully irrigated under a center-pivot irrigation system. The four forage grasses were grown on a Gruver soil series (fine, mixed, superactive, mesic Aridic Paleustoll) on 0–2% slopes and harvested in June, July, August, and September.

In the first year of the experiment, each plot was randomly assigned to one of the strategies using a randomized complete block design. Because residual effects (e.g. nutrient carry-over) were expected to have a significant effect, each strategy was maintained on the same plot throughout all 7 years of the experiment. The experiment had four replications and four N treatment levels so the 7 years provided 112 observations for each species. Swine effluent was obtained from a local anaerobic single-stage lagoon near the research station, the same type of effluent available to producers. Swine effluent was applied at equivalent N rates of 50 and 150 lbs N per acre after the first monthly cutting in June. The 450 lbs N per-acre rate was split into two applications; the first application came after the first cutting in June and the second just after the second cutting in July. The actual rate of N varied from the target rate of N as a result of uncertainty about the nutrient content of the SE (Table 1). An errors-in-variables problem would be created if we used analysis of variance, but we do not have such a problem because we use a regression with the actual rate of N as the explanatory variable. The levels of phosphorus and potassium as well as other micronutrients were tested at the beginning of the experiment and were sufficient to meet plant requirements so none of these nutrients were added. Soil tests in later years demonstrated no need for additional nutrients. Phosphorus increased during the experiment as expected, especially on plots with high levels of SE. The high levels of phosphorus are not expected to affect yields.

All plots were fully irrigated under a center-pivot irrigation system following standard practices used by producers in the region. The SE was field-applied through the center-pivot system as part of the June and July irrigation water applications. The data are presented in Figure 1. It is difficult to discern which functional form can best fit the data from looking at the plots.

Different market prices of dry matter yields for each of four grasses were adopted. First,

Table 1. Actual Amount of Nitrogen Applied (N lb/ac) by Swine Effluent

	Targeted N Amount		
Statistic	50	150	450
	Observed N Amount		
Mean	48.14	144.71	433.86
Standard Error	10.89	32.88	99.10
p	0.67	0.69	0.68

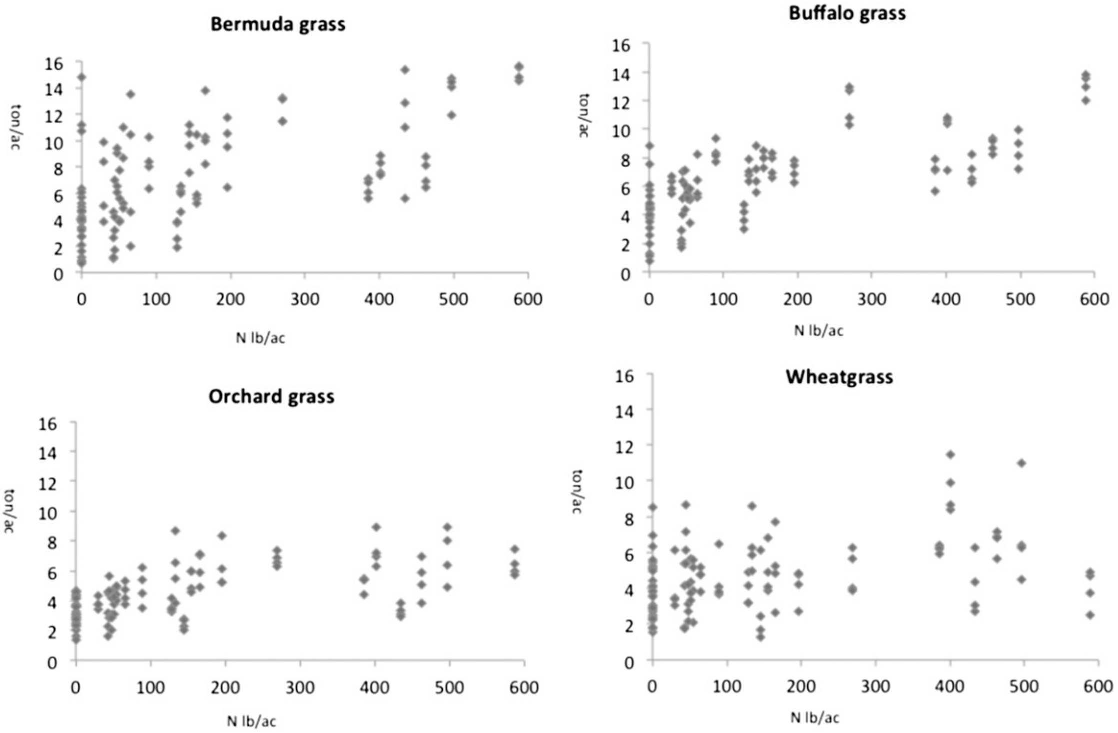

Figure 1. Dry Matter of Four Grasses (ton/ac) to the Amount of Nitrogen Actually Applied Using Swine Effluent (N lb/ac)

buffalograss is more valuable than bermudagrass because it is higher in protein and second, cool-season grasses such as orchardgrass and wheatgrass have a much higher price as a result of seasonality in forage markets. In the winter, forage supplies dwindle when less is growing and winter forages fetch a premium because they are much higher in protein than the summer forages. The prices of forage for two cool-season grasses such as orchardgrass and wheatgrass were determined as the cost of beef weight gain divided by the amount of forage required by a grazing animal (Kaitibie et al., 2003). For example, assume that a grazing steer requires 10 pounds (dry matter) of standing forage for 1 pound of weight gain according to previous studies (Ishrat, Epplin, and Krenzer, 2003; Krenzer et al., 1996) and that the cost per pound of weight gain is 0.45 (Tumusiime et al., 2011), then, the price per pound of forage is $0.45/10 = $0.045. In terms of warm-season grasses, hay prices from the most recent Oklahoma Hay Report were used assuming that buffalograss is premium-quality grass hay and bermudagrass is only good-quality grass hay. The predicted prices of 1 ton of dry matter yields are $52.50, $62.50, $90.00, and $90.00 for bermudagrass, buffalograss, orchardgrass, and wheatgrass, respectively. The application cost of N in the SE is assumed to be $0.41 per pound (Park et al., 2011).

Results

The models are estimated using the NLMIXED procedure in SAS (SAS Institute Inc., 2003). The NLMIXED procedure is convenient because it can estimate nonlinear models that have random effects. The estimation procedure is restricted maximum likelihood, which maximizes an approximation to the likelihood integrated over the random effects to estimate nonlinear mixed models (SAS Institute Inc., 2003). One of the common problems associated with nonlinear optimization is obtaining convergence. To address this issue, first, we used scaling so that the diagonal elements of the Hessian were of similar magnitude. Second, initial estimates were obtained using the first-order approximation (method = FIRO in SAS). These estimates were then used as starting values using the trust-region algorithm (TRUREG). If convergence was not obtained (which is rare), the simulated observation was not used.

Estimated dry matter response functions to the amount of N actually applied using SE are shown in Figure 2 for each of the four grasses. The estimated functions are similar for low levels of N but differ substantially at the highest levels of N. The LP tends to show the largest marginal product of N. As a result, the LP also shows less ability for the forage to use very high levels of N than do the other two functional forms.

The parameter estimates, as well as estimates of the variances, for the three competing functional forms are reported in Table 2. Most of the parameters are statistically significant. The exceptions are the squared terms in the quadratic model for bermudagrass and wheatgrass, two terms (γ_1, γ_2) in the MB parameters for wheatgrass and the variance of year random effect for the linear plateau model. The LP model has the highest likelihood (lowest –2 log likelihood) for two of the forages and the MB has the lowest for the other two forages.

The likelihoods indicate the preferred models because the models all have the same number of parameters and the dependent variable is not transformed. However, the Cox nonnested test is needed to determine if the differences in log likelihood values are statistically significant. The Cox test *p* values are presented in Table 3.

The quadratic functional form is dominated by the other two functional forms. The quadratic is significantly worse in all cases except

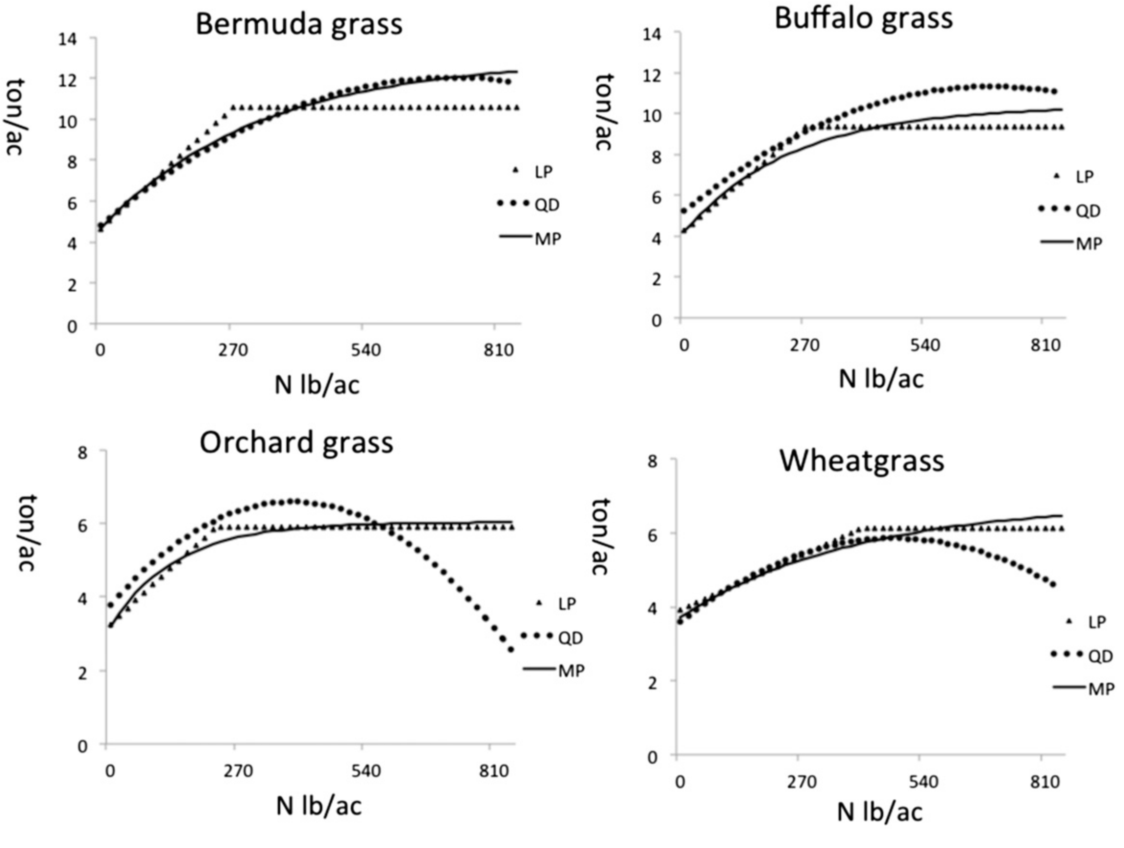

Figure 2. Estimated Dry Matter Response Function of Four Grasses (ton/ac) to the Amount of Nitrogen Actually Applied Using Swine Effluent (N lb/ac)

Table 2. Dry Matter Yield (ton/ac) Response of Four Grasses to Nitrogen Applied (N lb/ac) for Three Functional Forms (Linear Plateau, Quadratic, and Mitscherlich-Baule Models)

	Parameter Estimates (SE)			
Functional Form/Parameters	Bermuda	Buffalo	Orchard	Wheatgrass
Linear response plateau (LP)				
α_0	4.6439* (0.0561)	4.2764* (0.0296)	3.2589* (0.3654)	3.9031* (0.0257)
α_1	0.0220* (0.0003)	0.0188* (0.0001)	0.01193* (0.0017)	0.0055* (0.00004)
α_2	10.5780* (0.1244)	9.3513* (0.0645)	5.9192* (0.3792)	6.1149* (0.0399)
σ^2_ε	5.1007* (0.7040)	1.6894* (0.2330)	0.9335* (0.1287)	2.0460* (0.2823)
σ^2_u	4.6789 (2.6715)	1.6503 (0.9360)	0.7733 (0.4446)	1.2258 (0.7289)
–2 log likelihood	519.6	396.3	328.6	414.5
N rates of the plateau yield (lb/ac)	269.46	269.46	223.08	401.09
Quadratic (QD)				
β_0	4.8132* (0.3672)	5.2489* (0.2389)	3.7847* (0.1649)	3.5992* (0.2676))
β_1	0.0201* (0.0044)	0.0178* (0.0026)	0.0146* (0.0020)	0.0095** (0.0029)
β_2	−0.000014 (0.000009)	−0.000013** (0.000005)	−0.000019* (0.000004)	−0.00001 (0.000006)
σ^2_π	4.8764* (0.6642)	1.6741* (0.2265)	1.0198* (0.1363)	2.0771* (0.2823)
σ^2_ω	7.3395* (1.5410)	2.8586* (0.5592)	4.3090* (0.9483)	1.9344* (0.5700)
–2 log likelihood	516.7	398.3	335.7	416.2
Mitscherlich-Baule (MB)				
γ_0	12.9312* (2.2002)	10.3879* (0.7386)	6.0308* (0.2692)	6.8100* (1.5533)
γ_1	8.2848* (2.0293)	6.1627* (0.6864)	2.8426* (0.2826)	3.0980 (1.4407)
γ_2	0.0031** (0.0016)	0.0041** (0.0011)	0.0072* (0.0017)	0.0026 (0.0021)
σ^2_ϕ	4.8202* (0.6568)	1.6563* (0.2226)	0.9374* (0.1273)	2.1061* (0.2849)
σ^2_φ	3.9321* (0.7759)	1.8865* (0.3698)	0.5286* (0.1162)	1.2053** (0.0205)
–2 log likelihood	514.9	392.9	330.1	417.1

Note: * and ** are significant at one and 5% confidence levels, respectively.

SE, standard error.

Table 3. Probability Values from Cox Parametric Bootstrap Test–Single Bootstrap and Fast Double Bootstrap (FDB)

Forage	Method	Ho:LP Ha:QD	Ho:QD Ha:LP	Ho:LP Ha:MB	Ho:MB Ha:LP	Ho:QD Ha:MB	Ho:MB Ha:QD
Bermudagrass	Single	0.0441	0.9923	0.0123	0.9488	0.0448	0.8363
	FDB	0.0253	0.9998	0.0014	0.9983	0.0253	0.9995
Buffalograss	Single	0.6057	0.0948	0.0493	0.6303	0.0005	0.9778
	FDB	0.6903	0.0943	0.0427	0.9380	0.0001	1.0000
Orchardgrass	Single	0.9281	0.0081	0.3537	0.1506	0.0147	0.7825
	FDB	1.0000	0.0068	0.3220	0.1152	0.0145	0.9999
Wheatgrass	Single	0.9387	0.0595	0.9612	0.0377	0.7600	0.1187
	FDB	0.9954	0.0614	0.9963	0.0204	0.9358	0.0276

Note: Because the numbers in the table are *p* values, values less than 0.05 are considered significant.

with bermudagrass and the linear plateau. The MB is preferred for the two summer forages (with bermudagrass the difference is significant, but with buffalograss, it is not), but the LP is preferred for the two winter forages (for orchardgrass the difference between MB and the LP is not statistically significant).

Bermudagrass shows more response to N than the other forages with the production functions being close to linear. Some past research has shown bermudagrass response to N at even higher levels of N than the levels considered here (Redfearn et al., 2010).

The differences between the *p* values with the single bootstrap and the fast double bootstrap are mostly small as expected. Using a 5% significance level, in only one case does the conclusion about significance change. This is an important finding because it indicates that past empirical work that used a single bootstrap likely reached the correct conclusion. The fast double bootstrap values tend to be more extreme than the single bootstrap *p* values, but otherwise there is no clear pattern.

Optimal Nitrogen Application Rates

The expected profit maximizing N application rates were determined to compare the economic implications of the three competing models (LP, QD, and MB). Expected profit, economically optimal grass yield, and economically optimal N rates for both models are presented in Table 4.

Buffalograss has the highest expected profit regardless of the functional form used. It is the only one of the four forages that is native to the region. Buffalograss has yields nearly as high as bermudagrass and has higher quality forage. The expected profits do not differ widely across forages, so relatively small changes in prices could change the optimal forage.

The quadratic function has the highest recommended optimal N level for three of the four forages. The quadratic is well known to yield higher recommended levels of N.

For SE, in some cases, the price is effectively zero. Swine producers are simply looking for a place to apply the effluent. Table 5 shows the expected yield maximizing nitrogen rates (zero price of N). Optimal N rates did not change with the LP model. However, with the quadratic model, there are big changes in optimal N rates, 716.79 lbs per acre for bermudagrass, 660.26 lbs per acre for buffalograss,[6] 392.60 lbs per acre for orchardgrass, and 464.31 lbs per acre for wheatgrass, respectively. Finally, with the MB models, unlimited N rates maximize dry matter yields as a result of the functional specification. Compared with the quadratic model, however, maximum dry matter yields attainable with huge fertilizer application in the MB models are just slightly higher

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[6] The buffalograss model is nearly linear, which produces the ridiculously high estimate of yield maximizing level of nitrogen. This result is not realistic as a result of the projection being outside the range of the data.

Table 4. Expected Profit Maximizing Nitrogen Rates

	LP	QD	MB
Bermudagrass			
Expected profit ($/ac)	444.86	394.26	**390.86**[a]
Economically optimal dry matter yield (ton/ac)	10.58	10.93	**10.44**
Economically optimal nitrogen rate (N lb/ac)	269.46	438.44	**383.36**
Buffalograss			
Expected profit ($/ac)	473.97	474.97	**415.26**
Economically optimal dry matter yield (ton/ac)	9.35	10.34	**8.80**
Economically optimal nitrogen rate (N lb/ac)	269.46	417.29	**328.35**
Orchardgrass			
Expected profit ($/ac)	**441.04**	463.21	400.20
Economically optimal dry matter yield (ton/ac)	**5.92**	6.38	5.40
Economically optimal nitrogen rate (N lb/ac)	**223.08**	270.36	208.72
Wheatgrass			
Expected profit ($/ac)	**385.90**	377.42	364.97
Economically optimal dry matter yield (ton/ac)	**6.11**	5.29	5.05
Economically optimal nitrogen rate (N lb/ac)	**401.09**	241.25	218.83

Note: The profits calculated here are net returns since the only cost deducted is the cost of the nitrogen applied.

[a] Optimal solutions for the best functional form were highlighted in bold.

LP, linear response plateau; QD, quadratic; MB, Mitscherlich-Baule.

than those for bermudagrass and wheatgrass and even lower than those for buffalograss and orchardgrass. Moreover, overapplying animal manure close to where the manure is produced as a result of large manure shipping costs will result in environmental problems (e.g. eutrophication). The nitrogen–phosphorus ratio in manure is less than the crop requirements so that the amount of phosphorous applied with manure is typically higher than the annual removal by plants. Phosphorous accumulation in soil over time is commonly found when the difference in nitrogen–phosphorus ratio between crop and manure is great (Park et al., 2010).

Summary and Conclusion

This article uses the fast double bootstrap to conduct a Cox nonnested test. Results with the fast double bootstrap confirm results with the ordinary bootstrap, which suggests that using estimated parameters has small effects. The single bootstrap assumes less variability in the statistic than is actually the case. The fast double bootstrap *p* values were slightly larger than the single bootstrap *p* values when the null hypothesis cannot be rejected, whereas the *p* values from FDB were slightly smaller than those with the ordinary bootstrap when the null hypothesis can be rejected.

The test is used to select among three competing response functions, LP, QD, and MB, for

Table 5. Expected Yield Maximizing Nitrogen Rates

	LP	QD	MB
Bermudagrass			
Dry matter yield (ton/ac)	10.58	12.02	12.93
Optimal nitrogen rate (N lb/ac)	269.46	716.79	—
Buffalograss			
Dry matter yield (ton/ac)	9.35	11.13	10.39
Optimal nitrogen rate (N lb/ac)	269.46	660.26	—
Orchardgrass			
Dry matter yield (ton/ac)	5.92	6.65	6.03
Optimal nitrogen rate (N lb/ac)	223.08	392.60	—
Wheatgrass			
Dry matter yield (ton/ac)	6.11	5.80	6.81
Optimal nitrogen rate (N lb/ac)	401.09	464.31	—

Note: The yield maximizing nitrogen rates with MB are almost infinite.

LP, linear response plateau; QD, quadratic; MB, Mitscherlich-Baule.

dry matter yield of four forage grasses from an experiment in the Oklahoma Panhandle. The quadratic is rejected in favor of one of the other functional forms in all cases.

The highest optimal N rates are found with the quadratic model except for wheatgrass. Buffalograss showed the highest expected returns and wheatgrass had the lowest. All four forages were able to use relatively high levels of N, which is important for producers looking for crops that can make use of large quantities of SE.

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