Computing an Asymmetric Competitive Market Equilibrium

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Abstract. *Demand and supply are often asymmetric, that is, cross-price effects are not equal over all commodities Because of asymmetry, conventional surplus maximization formulations cannot be employed to compute a competitive market equilibrium This article compares alternative formulations under a system of equation, optimization, and iterative procedures for computation A general strategy for selecting an appropriate procedure is presented The iterative procedure is recommended for structural or complex nonlinear demand systems or for extremely large (size) problems The optimization procedure is suggested for large and medium (size) problems because of the availability of a computer solution package The system of equation formulation is suggested for modeling various types of economic behavior because of its flexibility*

Keywords. *Asymmetric demand and supply, market equilibrium*

For measuring social welfare as affected by farm policy, economists use social surplus or net social payoff, which is the sum of the consumer surplus and producer surplus (*28*) [1] The social surplus is the area below the demand function and above the supply function Samuelson (*28*) shows the equivalence of a maximization of social welfare problem to the general non-normative problem of market equilibrium among spatially separated markets as formulated by Enke (*6*) Takayama and Judge (*33*) reformulate Samuelson's model into a quadratic programming problem and suggest an efficient algorithm to compute competitive equilibrium Because of advances in computation, the maximization of the social surplus method has become a powerful tool in policy analysis

Many quadratic programming models and, in some situations, nonlinear programming models of higher order, with maximization of the social welfare as the objective function, have been applied to agricultural policy analysis CHAC (*5*), USMP (*12*), and CARD (*22*) are three large-scale nonlinear programming models that are used extensively Numerous other small-scale models are found in the economic literature

The welfare function used in these models is derived from a symmetric demand or supply function that satisfies the integrability condition This assumption of a symmetric demand and supply function implies that the cross-price effects are equal over all commodities That is, the effect of income on consumption is identical across all included commodities or is zero

The demand function is often not symmetric (*16*), therefore, it is not integrable [2] Under this situation, the welfare function cannot be formulated [3] Thus, formulation of the welfare maximization problem cannot be established [4] The convenient equivalence between the market equilibrium problem and the surplus maximization problem, therefore, is not available

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[1]Italicized numbers in parentheses refer to items in the References at the end of this article

[2]The integrability used here differs from that commonly used in consumption theory in which the demand functions are said to be integrable if they can be derived from a utility function (*17, 37*) Integrability here is concerned mainly with the mathematical relation from the demand functions to the welfare function It is implied that the order of integration of a sum of definite integrals is "path independent " A unique solution is found regardless of the path selected Mathematically, we can find a unique welfare function $W(p)$ from demand function $D_i(p)$ by $\delta W(P)/\delta P_i = D_i(P)$ if the following condition exists $\delta D_i(P)/\delta P_j = \delta D_j(P)/\delta P_i$ This condition implies that the demand functions are symmetric and all cross price effects are equal over all commodities

[3]Some asymmetric demand systems can be integrated Carey (*2*) showed that some asymmetric demand functions are "factor integrable " That is, they are integrable on being multiplied through by some nonzero factor so that the resulting functions become integrable However, there is no test that can be applied to discover whether it is possible to transform a set of nonintegrable functions into a set of integrable functions

[4]Many excellent articles show conditions of deriving the surplus function from observable ordinary (Marshallian) demand functions that are asymmetric Two representative articles are by Chipman and Moore (*3*) and by Hausman (*9*)

(*10*) Alternative methods of computing the competitive equilibrium solution should be employed We compare several methods here

Problem

An illustrative surplus model can be defined as problem 1a [5] Choose Q to maximize

$$Z = W(Q)$$

subject to

$$\begin{aligned} CG &\leq B \\ Q &\geq O \end{aligned} \tag{1}$$

where W is a surplus function that is the sum of W_i $(q_1, \ldots, q_n)$, Q is a $n \times 1$ vector with elements $q_1, \ldots, q_n$ G is a $m \times n$ technical input-output matrix, and B is a $m \times 1$ vector with elements $b_1, \ldots b_m$ representing available fixed resources By assuming that the inverse demand function $P_i = D_i(Q)$, $i=1, \ldots, n$, and the inverse supply function $P_i = S_i(Q)$, $i=1, \ldots, n$ are symmetric, we can express the surplus function as

$$W(Q) = \sum_i \int D_i(Q)dq_i - \sum_i \int S_i(Q)dq_i \tag{2}$$

where $\int$ means line integration

The supply function $S_i(Q)$ is frequently not used in practical application Instead, an activity model is used to represent the supply function A surplus model in such situations may be defined as *problem 1b* Maximize

$$Z = \sum_i \int D_i(Q)dq_i - C'X$$

$$\text{subject to } GX \leq B, \quad HX = Q, \quad Q \geq O, \tag{3}$$

where X and C are $k \times 1$ vectors of production activities and their corresponding costs, respectively, and H is an $m \times n$ transfer matrix relating production activties X to final output Q The formulation of problem 1a often appears in trade analysis, whereas the formulation of problem 1b appears mostly in production and resource allocation studies Here, we use primarily the formulation of problem 1a to develop alternative computational methods We include the formulation of problem 1b mainly for exposition

If one is to derive the surplus function equation, the demand function $D_i(Q)$ and the supply function $S_i(Q)$ must meet integrability conditions We consider the following two sets of linear demand functions:[6]

$$Q_{n\times1} = v_{n\times1} + V_{n\times n}P_{n\times1} \tag{4}$$
$$P_{n\times1} = d_{n\times1} + D_{n\times n}Q_{n\times1} \tag{5}$$

where Q is a vector of quantities of commodities demanded, P is a vector of the price of commodities, and v, d, D, and V are parameters of the direct demand function (equation 4) or the inverse demand function (equation 5) [7]

We also assume that the supply function can be expressed as a marginal cost function MC

$$MC_{n\times1} = s_{n\times1} + S_{n\times n}Q_{n\times1} \tag{6}$$

where s and S are parameters of supply functions If D and S are symmetric matrices, the objective function in equation 1 can be expressed as

$$W(Q) = A'Q + 1/2\, Q'EQ \tag{7}$$

where $A' = (d-s)'$ and $E = (D-S)$ Note that by substituting equation 7 for the objective function in equation 1, we have a quadratic programming problem that can be solved readily by a nonlinear programming solver (such as Minos, 5 0) (*29*) or by the use of grid linearization available in the IBM-MPSX370 system

The condition for a symmetric demand function can be stated as

$$\frac{\delta^2 W(Q)}{\delta q_i \delta q_j} = \frac{\delta D_i(Q)}{\delta q_j} = \frac{\delta D_j(Q)}{\delta q_i} = \frac{\delta^2 W(Q)}{\delta q_j \delta q_i} \tag{8}$$

A similar condition can be expressed for symmetric supply functions The condition (equation 8) is called the integrability condition When it is violated, the surplus function (equation 2) cannot be formulated as equation 7 Thus, we need alternative methods to derive a competitive equilibrium solution from sets of asymmetric demand and supply functions

We review and discuss three categories of alternative formulations that one can use to find the competitive equilibrium solution for asymmetric demand and supply functions. (1) a system of equations, (2) optimization, and (3) iterative procedures

[5]To simplify the discussion, we use only less-than or equal-to constraints Additional equality and greater than constraints can be included, if desired

[6]For simplicity, we use linear demand and supply functions and assume that demand and supply substitution matrices are definite

[7]Note that the set of equations 4 and 5 can be estimated independently of each other, or one can be derived from the other, if the inverse of V or D exists

System of Equations

Problem 1a can be formulated as a system of equations from which the competitive equilibrium can be solved The Lagrangian method and the complementary formulation are presented

Lagrangian Method

The Lagrangian method formulates a surplus maximization problem by a system of equations and obtains the competitive solution by solving this system Express the surplus maximization problem 1a

Maximize

$$Z = \sum_i W_i(q_i)$$

$$\text{subject to } \sum_i g_{ij} q_i \leq b_j \text{ and } q_i \geq 0 \qquad (9)$$

The Lagrangian function of this maximization problem is expressed by

$$L = \sum_i W_i(q_i) + \sum_j \mu_j(\sum_i g_{ij} q_j + t_j - b_j) + \sum_i \alpha_i(q_i - y_i^2) + \sum_j \Theta_j(t_j - u_j^2) \qquad (10)$$

The first-order conditions of the Lagrangian function gives the following system of equations (see *24* for derivation) as problem 2

$$W'_i(q_i) + \sum_j \mu_j g_{ij} + \alpha_i = 0 \quad \text{for } i = 1, \ldots, n \qquad (11)$$

$$\sum_i g_{ij} q_i + t_j - b_j = 0 \quad \text{for } j = 1, \ldots, m \qquad (12)$$

$$\alpha_i q_i = 0 \quad i = 1, \ldots, n \quad \text{and} \qquad (13)$$

$$\mu_j t_j = 0 \quad j = 1, \ldots, m \qquad (14)$$

where μ_j, α_i and Θ_j are Lagrangian multipliers, and where t_j, y_i, and u_i are slack variables Thus, solving problem 1a is equivalent to solving the system of equations in problem 2

First, we note that $W'_j(q_i)$, which is a partial derivative of $W_i(q_i)$ with respect to q_i, in equation 11 is a marginal net return function, which is the difference between demand price (p_i) and supply price (marginal cost, mc_i)

$$W_i'(q_i) = p_i - mc_i \qquad (15)$$

Thus, if p_i and mc_i are available regardless of whether they are in a nonlinear or asymmetric structure, we can formulate equation 11 as

$$P_i(q_1, \ldots, q_n) - MC_i(q_1, \ldots, q_n) + \sum \mu_j g_{ij} + \alpha_i = 0 \qquad (16)$$

$$i = 1, \ldots, n$$

By solving the system of equations 12, 13, 14, and 16 we can find the exact competitive equilibrium solution (q_i^*, p_i^*)

Second, to solve the system of equations is to solve n + m equations 12 and 16 with 2n + 2m unknowns (n q's, n α's, m μ's, and m t's) However, n+m of these variables have a zero value from equations 13 and 14 Thus, equations 12, 13, 14, and 16 provide 2^{n+m} sets of n+m linear equations in n+m variables, and one of these sets will yield the solution The computation can get out of hand very quickly, even for problems with a modest number of variables Nevertheless, for a smaller number of variables (fewer than five) this procedure is a useful tool, especially when a computer package for the solution is not available

Complementarity Formulation

Takayama and Uri (*35*) illustrated that quadratic programming (QP) models are a subset of linear complementary programming (LCP) models, and they suggested the use of LCP formulation when the coefficient matrix of the demand or supply function is asymmetric For a certain class of LCP models, the principal pivoting method or the Lemke method (*19*) leads to a solution We now construct a LCP problem using Kuhn-Tucker conditions for a competitive equilibrium solution of the surplus model, with linear asymmetric demand and supply functions

Given the demand and the supply equations 5 and 6, the Kuhn-Tucker condition for problem 2 can be expressed as

$$(d + DQ) - (s + SQ) + G'\mu \leq 0 \qquad (17)$$
(marginal revenue ≤ marginal cost)

$$GQ - B \leq 0 \qquad (18)$$
(resource use ≤ resource available)

By using α and t as vectors of slack variables, we can formulate an LCP problem as problem 3

$$\begin{bmatrix} \alpha \\ t \end{bmatrix} = \begin{bmatrix} d-s \\ B \end{bmatrix} + \begin{bmatrix} D-S & G' \\ G & 0 \end{bmatrix} \begin{bmatrix} Q \\ \mu \end{bmatrix}$$

$$\begin{bmatrix} \alpha \\ t \end{bmatrix} \begin{bmatrix} Q \\ \mu \end{bmatrix} = 0, \begin{bmatrix} \alpha \\ t \end{bmatrix} \geq 0, \begin{bmatrix} Q \\ \mu \end{bmatrix} \geq 0 \qquad (19)$$

We can solve the LCP equation by using Lemke's algorithem described by Zangwill and Garcia (*38*) By including both G and G′ matrices, the LCP formulation increases the size of the problem to be solved This method may not be an efficient tool for solving a large-scale problem

Because the matrix in the LCP formulation is generally sparse, efficient computation methods to exploit the sparsity have been developed (*19, 21, 36*) Rutherford (*27*) has reported on the application of the LCP formulation and algorithm to a large empirical model of the Norwegian economy Furthermore, because market behaviors can be formulated as problems of complementarity, researchers have made considerable progress in developing an efficient algorithm (*18, 23*)

Optimization Formulation

We describe three alternatives for building an optimization model equivalent to problem 1a the average, the Plessner-Heady, and the minimum rent methods

Average Method

Carey (*2*) has suggested the average (AV) method, if the off-diagonal elements of matrices D and S are similar or the off-diagonal elements are relatively small in comparison with the diagonal elements That is, income effects are similar or small for all goods The AV method involves averaging the off-diagonal elements (d_{ij}, d_{ji} for all $i \neq j$), that is

$$(d_{ij} + d_{ji})/2 = \bar{d}_{ij} \text{ and } (s_{ij} + s_{ji})/2 = \bar{s}_{ij} \tag{20}$$

and entering the average values of $\bar{d}_{ij}$ and $\bar{s}_{ij}$ in the off-diagonal positions d_{ij} and s_{ij} of matrices D and S to form the new matrices $\bar{D}$ and $\bar{S}$

The newly constructed symmetric matrices $\bar{D}$ and $\bar{S}$ then replace D and S in the welfare function (equation 7) The problem becomes a quadratic programming problem and can be solved This method alters the marginal cost and price relationship, so the Kuhn-Tucker first-order conditions of the problem are no longer valid Thus, the solution from this method can only roughly approximate the competitive equilibrium solution

Other methods, such as imposed integrability as a condition in estimating the demand function, are used by Pressman (*26*) and by Littlechild and Rousseau (*20*) The imposition of symmetric conditions in estimating the systems of the demand function is also popular in economics to reduce the number of parameters to be estimated

Plessner-Heady Method

Analysts have extensively used Plessner and Heady's (PH) primal-dual formulation (*25*) to find the competitive equilibrium solution This formulation does not require using the surplus function, which cannot be formulated under the asymmetric condition. Their objective function is formulated instead as maximization of the difference between net return and imputed costs of fixed, binding resources The constraints include both the primal and the dual formulation of an optimization model

The PH formulation equivalent of problem 1a can be expressed as problem 4 (see *22* for detail)

Maximize

$$Z = P'(v+VP) - Q'(s+SQ) - B'\mu$$

subject to

$$\begin{aligned} &v + VP - Q \le 0 \\ &\text{(supply} \ge \text{demand)}, \\ &-B + GQ \le 0 \\ &\text{(resource use} \le \text{resource available) and} \\ &P - (s+SQ) - G'\mu \le 0 \\ &\text{(marginal revenue} \le \text{marginal cost)} \end{aligned} \tag{21}$$

When the competitive equilibrium is reached, the value of the objective function becomes zero. Takayama and Judge (*32, 34*) also use the primal-dual method for spatial market equilibrium problems.

The PH formulation includes primal and dual components and thereby increases the size of the model. For large-scale problems, the primal-dual formulation becomes expensive to solve For a medium-scale problem, however, this formulation is a practical tool and has been applied extensively (Stoeker (*31*) and Bhide (*1*) are two typical applications)

Minimum Marginal Rent Method

The minimum marginal rent (MR) method is derived from the PH method Given a PH formulation as described in problem 4 and assuming $Q > 0$ (this assumption is valid, especially if Q is referred to as aggregate national production of major commodities), we have

$$P - (s + SQ) - G'\mu = 0 \tag{22}$$

Further, because demand is equal to supply when a competitive equilibrium solution is reached (in the situation of limited resource, the market price > marginal cost, but demand = supply), we have

$$v + VP - Q = 0 \tag{23}$$

Substituting

$$B'\mu = Q'G'\mu \tag{24}$$

in the objective function of equation 21 we have

$$Z = P'(v + VP) - Q'(s + SQ) - Q'G'\mu \tag{25}$$

By combining equations 22, 23, and 25 and by assigning $R = G'\mu$, we can formulate the MR problem as problem 5

Maximize

$$Z = P'(v + VP) - Q'(s + SQ) - Q'R$$

subject to $v + VP - Q = 0, \quad -B + GQ \leq 0,$

$$P - (s + SQ) - R = 0, \tag{26}$$

Problem 5 is a nonlinear programming problem If we further use βR to replace the term $Q'R$ in the objective function and we assign an arbitrarily large constant number for β, which should be greater than the Q^* of optimal solution, we will have a nonlinear separable programming formulation that can be solved by a linear programming technique We use a large constant value for β the objective function to minimize rents while maximizing the returns. This method has been applied in land use study *(15)*

Because the MR formulation does not include the production technology data matrix G, its size can be much smaller than the PH formulation However, it is difficult to apply this method to the activity model (problem 1b) in which the supply function is not explicitly formulated

Iterative Procedure

The iterative procedure is a computational method of solving a market equilibrium model To use the procedure, one divides the surplus model into demand and supply submodels An iterative procedure is then used to interact between these two submodels until the process converges To illustrate the procedure, we reformulate problem 1b as problem 6 [8]

[8] We use problem 1b, instead of problem 1a, because the iterative procedure is applied to the activity model in most practical applications

Demand submodel $\bar{Q} = v + VP$

Supply submodel — choose X to minimize

$$Z = C'X$$

subject to $HX = Q, \quad GX \leq B, X \geq 0$ (27)

Equilibrium condition —$P = MC$

where MC is an optimal dual-variable vector (shadow price vector) corresponding to the demand balance constraint $HX = Q$ We can start an iterative procedure by assigning an initial value for P in the demand submodel and by obtaining Q, which is then used as the right-side value of the demand balance constraint in the supply submodel We then solve the supply component for the value of MC If $P = MC$, we reach the equilibrium solution Otherwise, we use MC as P in the demand submodel for the next iteration

One can use iterative procedures to deal with noncomputable surplus functions in two ways The first way approximates the value of the surplus at each iteration and locates the equilibrium through the iterative process The second way uses the assumed market adjustment process or search technique in iteration until the market price in a demand submodel is equal to the supply price in the supply submodel This method thereby avoids direct formulation of the surplus function

A typical example of the first method is the iterative procedure described in the Project Independence Evaluation (PIES) algorithm *(11)* At each iteration, the algorithm diagonalizes the asymmetric demand matrix to approximate the surplus function Another example of the first method is described by Carey (2) A typical example of the second method is the use of the tatonnement procedure which adjusts the price in response to the excess demand *(7, 8, 30)* Another example is the use of search algorithms, such as fixed-point, Jacobian, Gauss-Seidal, and gradient methods (see *3* for a discussion of these methods) One major problem with the tatonnement procedure is uncertainty in convergence of the iterative process The major disadvantages with the search method are inefficiency (fixed point), uncertainty in convergence (Jacobian and Gauss-Seidal), and difficulty in the approximation of derivation (gradient) in each iteration Huang *(13)* discusses convergence conditions for some of these procedures Another problem is in the application of these methods to find the equilibrium of problem 5 when resources are limited Huang and Heady *(14)* suggest an iterative procedure to locate the equilibrium by using dual variable information obtained in the solution at each iteration Extension of this method to large-scale models has not yet been developed

Suggestions

Table 1 summarizes the alternative formulations discussed in three categories (1) systems of equations, (2) optimization models, and (3) iterative procedures. We compare these three formulations in general terms We use the LG method in the first group, the MR method in the second group, and the PIES procedure in the third group as their respective group representative to solve the following examples

$$v = \begin{bmatrix} 100 \\ 80 \end{bmatrix} V = \begin{bmatrix} -0\ 4 & 0\ 2 \\ 0\ 15 & -0\ 25 \end{bmatrix}$$

$$s = \begin{bmatrix} 20 \\ 16 \end{bmatrix} \quad G = \begin{bmatrix} 0\ 5 & 0\ 9 \\ 0\ 7 & 0\ 5 \end{bmatrix} \text{and } B = \begin{bmatrix} 70 \\ 50 \end{bmatrix}$$

In this example, S is a null matrix and H is an identity matrix The equilibrium solution for this problem is

$$q_1^* = 26\ 32 \quad p_1^* = 311\ 26 \quad \mu_1^* = 55\ 39$$
$$q_2^* = 63\ 16 \quad p_2^* = 254\ 11 \quad \mu_2^* = 376\ 52$$

To solve this problem by the LG method, we use 16 (2^4) possible sets of simultaneous equations One set yields the equilibrium solution, the rest of the solutions are either infeasible or suboptimal

Table 1—Problem size and solution method

Type of formulation	Row	Column	Slack variables	Total variables	Solution method
			Method		
1 System of equations					
LG	2n+2m	n+m		2n+2m	Solve at most 2^{n+m} sets of simultaneous systems of equations
LCP	2n+m	n+m		2n+m	Lemke's algorithm by solving at most n+m set of systems of equations
2 Optimization models[1]					
AV	m+1	n	m	n+m	Separable programming or gradient method[1]
PH	2n+m+1	2n+m	2n+m	4n+2m	Separable programming or gradient method
MR	2n+m+1	3n	2n+m	5n+m+1	Separable programming or gradient method
3 Iterative procedures					
PIES	m+1	n	m	n+m	Each iteration solves an LP model
Tatonnement and search	m+1	n	m	m+n	Each iteration solves an LP model

[1]AV, PH, MR, and PIES will have additional rows and columns for quadratic terms if the separable programming technique is used

Source. (29)

In using the MR method, we assign $\beta = 100{,}000$ and obtain $r_1^* = 343.16$ and $r_2^* = 341.89$ Using $r_i = \sum_{j=1}^{2} q_{ij}\, \mu_i$, $i=$ and 2, we compute the value for μ_1^* and μ_2^*.[9] The PIES method takes three iterations to converge to the optimal solution At each iteration, we formulate a diagonalized demand system $q_k = D_k(P_k)$ by substituting the initial value (or the i^{th} value computed in the previous iteration) P_i, $i=k$, into the demand $D_k(P_1, .., P_k, P_n)$ and then express the inverse demand matrix as $P_k = D_k(q_k)$ Using P_k, we formulate a surplus function This surplus function and the resource constraints are used to set up a quadratic programming problem that is then solved by a separable programming technique A new set of values of P_i, $i=i, \;,n$, is obtained from the inverse demand function[10] This set of P_i values is used as the initial values for the next iteration

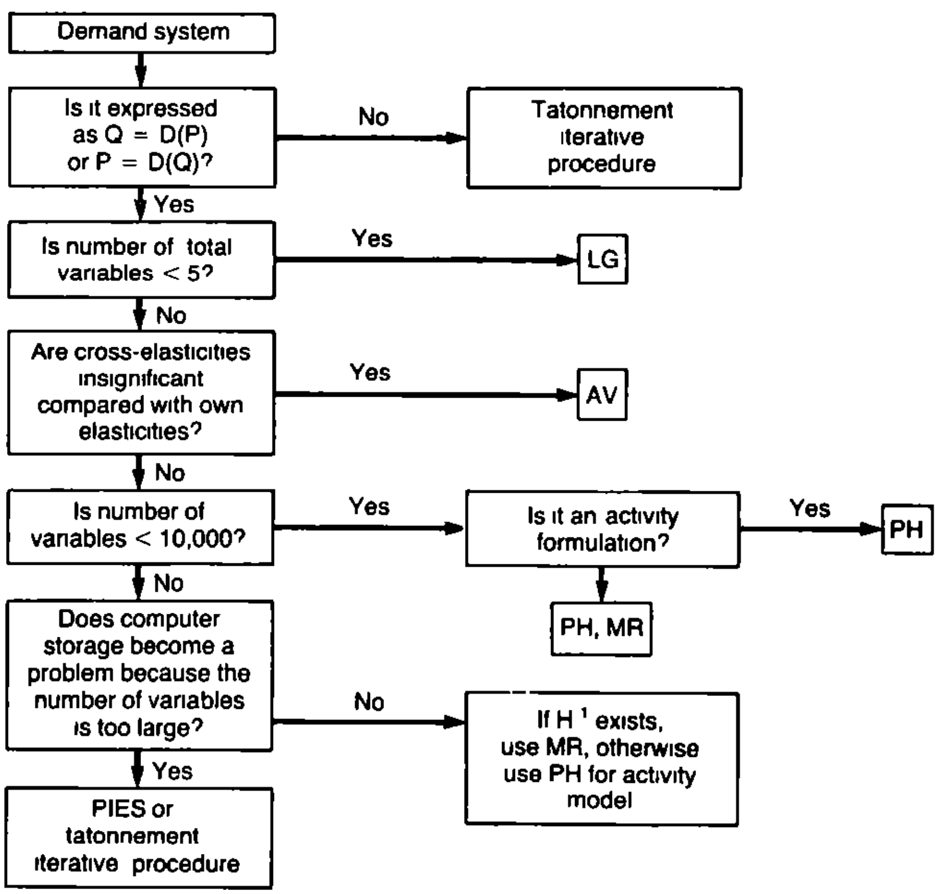
[9]In this case, the G is a square matrix If not, computing unique μ_1 and μ_2 from r_1 and r_2 values may not be possible

[10]For problem 6, the prices are the shadow prices of the demand balance constraints $HX = Q$

The LG method gives the exact competitive equilibrium solution. The MR method and PIES procedure (and other methods we have mentioned here) only approximate the exact solution In this example, the difference between the exact and the approximate solution is insignificant In general applications, the difference is probably due to precision of computation rather than to the method used

Selection of an optimal method of solving a given problem is decided by the three factors. (1) economic interpretation of the formulation and solution process, (2) availability of a computational package for each method, and (3) size of the problem. The justification of the optimization model rests on the meaningfulness of the objective function, whereas for the system of equations, its justification rests only on the set of conditions stipulated because equilibrium does correctly reflect market operation The iterative procedure provides a dynamic process of market adjustment toward the equilibrium

Strategy for selecting optimal method of deriving competitive equilibrium from an asymmetric demand system



Selection is frequently determined by the accessibility of a computer package The computer software for the first group (system of equations) and the third group (iterative procedure) is generally not rich, but the software for the second group is available commercially Solution techniques for solving a large-scale programming model have recently become more advanced. Thus, optimization (programming) formulation is widely used in applied research

From a computational viewpoint, the optimization model and the system of equations formulation are equivalent Carey (*2*) showed that the objective function of the PH method is redundant By dropping the objective function, the PH method becomes the LCP method and, therefore, can be handled within the scope of existing LCP algorithms Furthermore, from a practical viewpoint, the LCP is a natural way to formulate a model to reflect specific market behaviors For these reasons, considerable research efforts are currently underway at various universities to develop an efficient algorithm for solving the LCP models

The iterative procedures are market simulation solution algorithms They provide a computational alternative even when the other two approaches are available Because of the iterative process, the procedures are relatively expensive in terms of computational time They are often used in situations where the demand system cannot be stated as $P = f(Q)$ or $Q = f(P)$, and the programming and the LCP formulations are, therefore, impossible When the demand system is expressed in a structural form or by a system of computer language, iterative procedures are especially useful There are many ways of developing an iterative procedure for solving a particular problem There are many examples of using the tatonnement procedure, for which convergence has not been proven theoretically, several researchers have reported fast convergence (*4*) Finally, when dealing with an activity model as defined by problem 1b, one can choose between the PH and the MR method, depending on the relation $HX = Q$ in equation 3 The MR method is applicable only if H is a square and invertible matrix. If so, we obtain $X = H^{-1}Q$ and substitute it into the objective function in problem 1b In reality, H is most likely to be a rectangular matrix Under this case, the PH method should be used Because the PH method includes the primal and dual components, size may become too large for prevailing computer computation in a large-scale modeling problem If so, one would need to design an iterative procedure The figure suggests a general strategy for selecting a proper method

References

(1) Bhide, S "A Programming Model Incorporating Linear Demand Function for Grains and Vegetable Oils An Analysis of United States Agriculture in 1985." Ph D dissertation, Iowa State Univ , 1980

(2) Carey, M "Integrability and Mathematical Programming Models A Survey and a Parametric Approach," *Econometrica,* Vol 45, 1977, pp 1957-76

(3) Chipman, J S , and J C Moore "Compensating Variation, Consumer Surplus, and Welfare," *American Economic Review,* Vol 70, 1986, pp 933-49

(4) Dervis, K , J de Melo, and S Robinson *General Equilibrium Models for Development Policy* Cambridge Cambridge Univ Press, 1982, pp 491-96

(5) Duloy, J H , and R D Norton "CHAC, A Programming Model of Mexican Agriculture," *Multi-level Planning Case Studies in Mexico* (ed L Goreux and A Manne) Amsterdam North-Holland Publishing Co , 1973, pp 291-37

(6) Enke, Stephen. "Equilibrium Among Spatially Separated Markets Solution by Electric Analogue," *Econometrica,* Vol 19, 1951, pp 40-47

(7) Ginsburgh, U H , and J L Waelbroeck *Activity Analysis and General Equilibrium Modeling* Amsterdam North-Holland Publishing Co , 1981

(8) Harrison, M J "Software Tools for Combining Linear Programming with Econometric Models," *Computer and Mathematical Programming* NBC Special Publication 505 U S Dept of Commerce, 1976

(9) Hausman, J A "Exact Consumer's Surplus and Deadweight Loss," *The American Economic Review,* Vol 71, No 4, 1981, pp 662-76

(10) Hogan, W W "Energy Policy Models for Project Independence," *Computer and Operations Research,* Vol 2, 1975, pp 251-71

(11) ________ "Project Independence Evaluation System Structure and Algorithms," *Proceedings of the American Mathematical Society Short Course on Mathematical Aspects of Production*

and Dıstrıbutıon of Energy, San Antonıo, TX, 1976

(12) House, Robert "USMP Regıonal Agrıcultural Programmıng Model Theoretıcal and Data Descrıptıon " Internal dıstrıbutıon manuscrıpt U S Dept of Agr , Econ Res Serv , 1982

(13) Huang, W Y "Potentıal of Some Iteratıve Procedures for Solvıng an Equılıbrıum Hybrıd Model " Unpublıshed manuscrıpt, U S Dept of Agr , Econ Res Serv , Natl Res Econ Dıv , 1985

(14) __________, and E O Heady "Usıng Iteratıve Procedures to Solve Partıal Equılıbrıum Hybrıd Models," *Sımulatıon and Modelıng, Proceedıngs Eıght IASTED Internatıonal Symposıum* (ed M H Hamza), 1983, pp 71-76

(15) Huang, W Y , and H C Hogg "Estımatıng Land Use Patterns· A Separable Programmıng Approach," *Agrıcultural Economıcs Research,* Vol 28, No 1, Jan 1976, pp 22-33

(16) Hurwıcz, L , and H Uzawa "On the Problem of Integrabılıty of Demand Functıons," *Preferences, Utılıty and Demand, a Mınnesota Symposıum* (ed J S Chıpman, L Hurwıcz, Marcel K Rıchter, and Hugo F Sonnenscheın) New York Harcourt, Brace, and Co , 1971, p 114

(17) Johnson, S R , Z A Hassan, and R D Green *Demand Systems Estımatıon* Ames Iowa State Unıv Press, 1984, pp 35-37

(18) Jones, P C , R Saıgal, and M Schneıder "Computıng Non-Lınear Network Equılıbrıa," *Mathematıcal Programmıng,* Amsterdam North-Holland Publıshıng Co , Vol 31, pp 57-66

(19) Lemke, C E "Bımatrıx Equılıbrıum Poınts and Mathematıcal Programmıng," *Management Scıence,* Vol 11, 1965, pp 681-89

(20) Lıttlechıld, S C , and J J Rousseau "Prıcıng Polıcy of a U S Telephone Company," *Journal of Publıc Economıcs,* Vol 4, 1975, pp 35-36

(21) Mathıesen, L "Computatıonal Experıence ın Solvıng Equılıbrıum Models by a Sequence of Lınear Complementarıty Problems " Technıcal Report SOL 83-1 Stanford Unıv , Systems Optımızatıon Laboratory, 1983

(22) Meıster, A D , C C. Chen, and E O Heady *Quadratıc Programmıng Models Applıed to Agrıcultural Polıcıes* Ames Iowa State Unıv Press, 1978

(23) Pang, J S , and D Chan "Iteratıve Methods for Varıatıonal and Complementarıty Problems," *Mathematıcal Programmıng,* Amsterdam North-Holland Publıshıng Co , Vol 24, 1982, pp 282-313

(24) Pfaffenberger, R C , and D A Walker *Mathematıcal Programmıng for Economıcs and Busıness* Ames Iowa State Unıv Press, 1976

(25) Plessner, Y , and E O Heady "Competıtıve Equılıbrıum Solutıon wıth Quadratıc Programmıng," *Metroeconometrıca,* Vol 17, 1965, p 117

(26) Pressman, I "A Mathematıcal Formulatıon of the Peak-Load Problem," *The Bell Journal of Economıcs and Management Scıence,* Vol 1, 1970, pp 304-26

(27) Rutherford, T , "Computıng General Equılıbrıum ın a Complementarıty Format, NORGE " M A thesıs, Stanford Unıv , Dept of Operatıons Research, Dec 1982

(28) Samuelson, P. "Spatıal Prıce Equılıbrıum and Lınear Programmıng," *Amerıcan Economıc Revıew,* Vol 42, 1952, pp 283-303

(29) Saunders, M A "Technıcal Report SOL 71-31," *Mınos Systems Manual* Stanford Unıv , Dept of Operatıons Research, 1977

(30) Schatzer, R J "Tatonnement Modelıng wıth Lınear Programmıng· Demand and Supply for Some Unıted States Crops ın 2000 " Ph D dıssertatıon, Iowa State Unıv , Dept of Economıcs, 1982

(31) Stoeker, A L , "A Quadratıc Programmıng Model of U S Agrıculture ın 1980 Theory and Applıcatıon " Ph D dıssertatıon, Iowa State Unıv , 1974

(32) Takayama, T , and G G Judge "Alternatıve Spatıal Equılıbrıum Models," *Journal of Regıonal Scıence,* Vol 10, 1970, p 1-12

(33) __________ "Equılıbrıum Among Spatıally Separated Markets A Reformulatıon," *Econometrıca,* Vol 32, 1964, pp 510-24

(34) __________ *Spatıal and Temporal Prıce and Allocatıon Models* Amsterdam North-Holland Publıshıng Co , 1971

(35) Takayama, T , and N Urı "A Note on Spatıal and Temporal Prıce and Allocatıon Modelıng Quadratıc Programmıng or Lınear Complemen-

tarity Programming?" *Regional Science and Urban Economics,* Vol 13, 1983, pp 445-70

(36) Tomlin, J "Robust Implementation of Lemke's Method for the Linear Complementarity Problem" Technical Report SOL 76-24 Stanford Univ, System Optimization Laboratory, 1976

(37) Varian, H R *Microeconomic Analysis* New York W W Norton and Company, Inc, 1978, pp 100-01

(38) Zangwill, W I, and C B Garcia *Pathways to Solutions, Fixed Points, and Equilibria* Englewood Cliffs, NJ Prentice-Hall, Inc, 1981

Doubt

If you are convinced that we have adequate procedures for measuring price elasticities of consumer demand, you will not develop a better method If you believe that existing theory of cooperative behavior is adequate, you will not develop a better theory If you believe that all existing theories, models, and measures are adequate, you will not develop anything better If you believe all significant questions have been properly asked, you will never ask a new important question. If you do not doubt something, you will have nothing to research, if you doubt nothing, you are not justified in doing research because you are doing unneeded work Some people doubt only what they have been taught to doubt It can prove fruitful to doubt what no one else has doubted You will never solve a problem that you are unaware of "Necessity is the mother of invention" is an old saying My reading of history leads me to believe that dissatisfaction also has been the mother of many inventions

George Ladd
Imagination in Research, 1987

(See review, p 32)
