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A TEST OF THE SERIAL INDEPENDENCE OF PRICE CHANGES IN SOYBEAN FUTURES†

The main empirical contribution of this article is to present evidence bearing on the hypothesis that price changes in soybean futures contracts are serially independent. The method of analysis used is capable of being applied to other sensitive price series. It consists of applying a mechanical trading rule to actual soybean futures price series, and to random rearrangements of such series.

In Part I of this article the results of a test of this hypothesis are described. The hypothesis itself is derived from some theories about the behavior of traders in futures markets. These theories and their relation to the evidence are discussed in Part II. Part III compares the results of this study with some important previous studies.[1]

Throughout this paper we shall have occasion to refer to price changes as being serially independent, or positively or negatively dependent, as positively or negatively correlated, or uncorrelated, or as exhibiting a trend. It will be helpful to discuss the meaning of these concepts at the outset.

Suppose that X_T and X_{T-N} are random variables whose values, x_T and x_{T-N}, are the change in the price of some commodity during the time interval indicated by the subscript. We can think of any two observed price changes during this interval as being a sample of one, from a bivariate joint probability mass function. If for a given N the bivariate probability mass function is identical for all values T, then by making observations for various values of T we can increase our sample size. The sample will be used to make inferences about characteristics of the probability mass function from which these observations have come. The concepts mentioned in the preceding paragraph are all possible characteristics of this probability mass function. To define their meaning, we assume that this probability mass function is known.

From the joint probability mass function we can compute marginal proba-

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† The author gratefully acknowledges the assistance of Myles Dryden, who helped develop and program the trading rules used in this chapter, John Rudan and other staff members of the Cornell Computing Center for splendid cooperation in providing the hours of machine time necessary, and Roger Gray and Holbrook Working for many helpful comments on earlier versions of the manuscript. This research was supported in part by funds supplied to the Graduate School of Business and Public Administration, Cornell University, by the Whitehall Foundation. Only the author is responsible for the defects that remain in spite of such generous help.

[1] The studies commented on deal mainly with commodity futures prices. A collection of studies dealing with similar problems in connection with stock market prices has recently been published (*5*).

bility density functions for X_T and X_{T-N}. The two random variables are said to be *independent* if and only if the value of the joint probability mass function for x_T and x_{T-N} is always equal to the product of the corresponding marginal probabilities. If the two random variables are not independent, we will say they are *dependent.*

From the joint probability mass function we can always compute mean or expected values of X_T given that X_{T-N} has some particular value. These are called the conditional expected values of X_T given x_{T-N}, and are usually written $E(X_T \mid X_{T-N} = x_{T-N})$. If the two random variables are independent, all of the conditional expected values of X_T will be identical. The converse is not necessarily true. If the two random variables are dependent we can compute a marginal expected value for X_T by multiplying each conditional expected value by the marginal probability of the corresponding value of X_{T-N}, and summing the products. The price difference during period T will be said to exhibit a positive (or negative) *trend* if the marginal expected value of X_T is positive (or negative). If the $E(X_T) = 0$ the variable will be said to have no trend.

If we multiply the expression $[x_T - E(X_T)][x_{T-N} - E(X_{T-N})]$ by the corresponding value of the joint probability mass function, and sum the product over all possible combinations of values of X_T and X_{T-N}, the resulting sum is called the covariance of X_T and X_{T-N}. If the covariance is positive (negative) the random variables are said to be *positively* (negatively) *correlated.* If the covariance is zero the variables are *uncorrelated.* If two variables are independent, it follows that they will be uncorrelated. However, the converse is not necessarily true.

If the expression given in the previous paragraph is multiplied by the conditional probability of X_T given some particular value of X_{T-N}, and the product summed over all values of X_T, the sum will be equal to $[E(X_T \mid X_{T-N} = x_{T-N}) - E(X_T)][x_{T-N} - E(X_{T-N})]$. If the sign of this expression is positive (negative) then X_T will be said to be *positively* (negatively) *dependent* on X_{T-N} when $X_{T-N} = x_{T-N}$. X_T may be positively dependent on X_{T-N} for certain values of the latter variable, and negatively dependent for other values. However, if X_T and X_{T-N} are independent, positive or negative dependence is impossible.

A random variable X_T may have a positive trend, and may be both positively and negatively dependent on X_{T-N}, but yet uncorrelated with X_{T-N}. It may be helpful to give an example.

Table 1 shows a hypothetical joint probability density function. The marginal

TABLE 1.—HYPOTHETICAL JOINT AND MARGINAL PROBABILITY MASS FUNCTIONS OF TWO DEPENDENT BUT UNCORRELATED RANDOM VARIABLES

	X_{T-N}					
	-1	0	1	2	3	P_{X_T}
X_T						
-1	0.10	0.00	0.00	0.10	0.00	0.20
0	0.00	0.00	0.05	0.10	0.05	0.20
1	0.05	0.00	0.10	0.00	0.05	0.20
2	0.05	0.10	0.05	0.00	0.00	0.20
3	0.00	0.10	0.00	0.00	0.10	0.20
$P_{X_{T-N}}$	0.20	0.20	0.20	0.20	0.20	...
$E(X_T \mid X_{T-N})$	0.25	2.50	1.00	-0.50	1.75	...

distributions of X_T and X_{T-N} are identical, and their common expected value is one, so the price differences have a positive trend. The reader may verify that the two variables are uncorrelated but dependent. Moreover, X_T is negatively dependent on X_{T-N} when the latter variable is one unit away from its marginal expected value in either direction. X_T is positively dependent on X_{T-N} when the latter variable deviates by two units from its marginal expected value in either direction.

I

Soybean price series used.—The data used were the daily maximum, minimum, and closing prices for the May soybean futures contract traded on the Chicago Board of Trade. The data were collected for consecutive days, ending with the day trading in the contract expired. Trading years are identified by the calendar year in which the contract expired. The basic time series run from 157 to 208 consecutive trading days. The length was determined arbitrarily by the ease of obtaining data. When possible, data were obtained from the *Annual Reports of the Chicago Board of Trade.* When these were not conveniently available *The Wall Street Journal* was used.

Because some of the trading rules used required information on prices for the previous ten days, the rules were applied beginning on the eleventh day for which data were available. Because we were not primarily interested in the special behavior often attributed to prices on the final days of a contract's life, trading was terminated by closing any open positions ten trading days before the contract expired. Thus the actual period available for trading varied from 137 to 188 consecutive trading days per year. The data used were for the ten consecutive contracts expiring May 1952 through May 1961, inclusive.

Some characteristics of the data used are presented in Table 2. Prices at the beginning of the first trading period were only 21.5 cents per bushel less than at

TABLE 2.—SOME CHARACTERISTICS OF THE MAY SOYBEAN FUTURES CONTRACT PRICE SERIES USED TO TEST TRADING RULES*

(Cents per bushel, except as otherwise indicated)

Year	Length of time series[a] (*trading days*)	Closing prices: 10 days from Beginning	Closing prices: 10 days from End	Closing prices: Change	Within trading period: Maximum Price	Maximum Day	Minimum Price	Minimum Day
1952	192	289.500	296.500	+ 7.000	309.750	50	281.500	166
1953	157	299.500	301.000	+ 1.500	311.000	41	280.000	91
1954	157	270.250	393.500	+123.250	422.000	141	268.750	11
1955	161	282.250	252.250	− 30.000	299.000	24	247.250	134
1956	161	236.750	321.500	+ 84.750	340.000	148	234.000	40
1957	161	248.250	240.375	− 7.875	269.500	43	238.000	142
1958	204	250.500	226.625	− 23.875	251.750	13	222.625	148
1959	208	233.250	229.625	− 3.625	233.500	28	217.375	110
1960	203	222.000	213.875	− 8.125	233.500	70	211.000	185
1961	201	225.750	311.000	+ 85.250	334.500	174	220.500	76
Total	...	...	...	+228.250	...	...	...	...

* See text, pp. 119–20, for description and sources of data.
[a] Exceeds "trading period" by 10 days at the beginning and 10 days at the end.

the end of the last trading period. Thus the price trend measured in this way is less than 0.5 per cent per year. However, since the trading periods are not consecutive, this is not a very useful measure of trend.

Table 2 also shows the change in the closing prices from the beginning to the end of the trading period used each year. The part of the year arbitrarily chosen for testing exhibits a strong upward trend. A trader who followed a rule of buying futures at the beginning of each year's trading period, and selling them at the end of the year's trading period, would have earned an impressive total of over $2 per bushel over the ten-year period. In a futures market, the alternatives of taking a long position or a short position are equally relevant. A trader who systematically took short positions at the beginning of each trading period and held them until the end of the trading period would have suffered the impressive loss of over $2 per bushel.

Description of system-trading rules.—The mechanical trading rules used in this study were designed to reproduce the profits or losses that could actually have been achieved by an avocational trader following a rigid trading pattern. The rules require only information that would be available to anyone having access to an evening newspaper that carries the high, low, and closing prices of the Chicago May soybean futures contract each trading day. A decision is made after the end of trading on a given day, but is not executed until the following trading day.

One problem is that no public record is available of the time sequence of intraday price movements.[2] When a decision is made to buy (or sell) one contract of soybean futures during the next trading day, at what price should we assume this contract was executed? The procedure actually used was to define a random price variable such that any value between the high and low price on the day the trade was executed was equally likely. That is, the assumed probability distribution was the discrete counterpart of a uniform distribution. A discrete distribution is needed since prices are quoted and trades made only in even multiples of $\frac{1}{8}$ cent. To insure maximum comparability between the results of the various rules tested, the price that happened to be chosen on any given day was used for all rules. Thus if two rules both gave a buy signal on a given day, profits from both would be recorded as if the contracts were both executed at the same price on the following day.

Let $Q = Q(T, N)$ be a variable defined as follows:

$$Q = \frac{1}{N} [C(T) - C(T - N)]$$

where T and N are positive integers, and C(T) is the closing price of the May soybean futures contract on T^{th} trading day for which data were available for that trading year. Thus Q is the average daily increase (decrease) in closing prices during the previous N days.[3]

In general, the rules testing positive serial dependence can be described as follows:

[2] The Chicago Board of Trade does maintain a record of all intra-day prices issued over its wire service. But this record is not open for general inspection. (Information based on a letter to the author from the Chicago office of the Commodity Exchange Authority, dated February 26, 1963.)

[3] Q could also be defined as the first difference of an N period moving average.

If $Q > K$ take (or hold) a net long position of one contract on the next trading day.
If $Q < -K$ take (or hold) a net short position of one contract on the next day.
If $K \geqq Q \geqq -K$ make no trades on the next trading day.

For a given K the rules can be uniquely defined by N, the length of time over which the change in the closing price is calculated. The rules tested involve $N = 1, 2, 3, 5$, and 10.

Only two possible market positions are allowed. These are: long one contract or short one contract. A position with no open contracts is possible only from the first decision day in a given year until the first long or short position is assumed. Once an open position is assumed, it is maintained continually until the end of the trading year, shifting from long to short and vice versa, depending on the values of Q and K.

Two families of rules were used with different definitions of K. In the first family, K was a constant, k; i.e., Q was required to exceed one cent, two cents, or some other arbitrary value as shown in Table 3. In the second, $K = \alpha \, \text{Max} [PR(T), PR(T-1), PR(T-2), \ldots PR(T-9)]$. PR(T) was the range of prices observed on day T. In this case K is some fraction of the largest of the last ten daily price ranges. The idea behind this variation was that the size of the daily trading range might be an indication of the inherent variability of price changes. If price variability changed from year to year, this definition of K might be helpful in defining a significant variation in Q.

The rules described above have the characteristic that if the underlying distribution of price changes were serially independent with a mean value of zero, the before-commission profits generated by the rules would have an expected value of zero. With price changes having no trend, but positive serial dependence, the rules would tend to generate positive profits. The effects of nonzero trends are discussed later.

Results of rules testing positive dependence.—The results obtained from applying 20 examples of the first family of rules to May soybean futures prices for trading years 1952 through 1961 inclusive are given in Table 3. Table 3B gives the profits or losses after commissions from each rule each trading year. Table 3A gives various pertinent characteristics of the results of using the rules. Table 4 provides similar information for 20 examples of the second family of rules. In reference to Tables 3 and 4, a move is said to be completed when an open position is closed out because the trading period has come to an end, or because a long position is closed to take a short position, or a short position is closed to take a long position.

In order to compare the profitability of the various trading rules with other investments that might be available to a potential speculator, it sometimes will be helpful to calculate profits as the return on an initial investment. For this purpose we assume the trader starts with an initial investment equal to \$1 per bushel, or \$5,000, assuming trading is done consistently in units of one 5,000-bushel contract.[4]

[4] Although no broker would request a margin as high as \$1 per bushel to trade in soybeans, a trader might need more than the minimum margin to be able to follow any of these rules, if, for example, he had a series of trades resulting in losses at the start of his trading.

TABLE 3.—RESULTS OF SOYBEAN RULES TESTING POSITIVE DEPENDENCE, WITH ACTION SIGNALS BASED ON ABSOLUTE SIZE OF CHANGE IN THE MOVING AVERAGE

A. SUMMARY

Action signals		Total profits or losses[a] (*cents per bu.*)				After commission number of years[b]		Commission (*cents per bushel*)	Total number of moves
Length of moving average (*days*, N)	Change required (*cents per bu.*, k)	1952–61 After commission	1952–61 Before commission	After commission, in year with Best profits	After commission, in year with Worst losses	With profits	With losses		
1	1.0	476*	338*	1	112*	1	9	138	384
1	2.0	176*	112*	18	108*	3	7	64	177
1	3.0	59*	29*	100	66*	4	5	30	83
1	4.0	146	163	91	19*	7	2	17	46
2	1.0	12*	52	104	40*	4	6	64	178
2	2.0	3	28	83	46*	4	5	25	69
2	3.0	71	84	93	44*	5	3	13	37
2	4.0	36	44	51	19*	3	4	8	21
3	1.0	112	154	142	27*	4	5	42	116
3	2.0	75	89	108	50*	7	3	14	40
3	3.0	67	73	61	42*	4	3	6	18
3	4.0	20	23	41	16*	1	2	3	9
5	0.5	9	57	52	53*	3	7	48	133
5	1.0	110	131	72	54*	7	2	21	57
5	1.5	102	112	43	20*	6	2	10	28
5	2.0	149	155	77	22*	6	1	6	18
10	0.5	25	47	65	50*	5	5	22	61
10	1.0	197	203	100	14*	7	1	6	17
10	1.5	124	127	113	45*	3	2	3	7
10	2.0	58	60	54	29*	2	1	2	5

B. ANNUAL PROFITS AND LOSSES[a] AFTER COMMISSION
(*Cents per bushel*)

Action signals N	k	1952	1953	1954	1955	1956	1957	1958	1959	1960	1961	Total
1	1.0	86*	60*	42*	110*	1*	29*	16*	1	21*	112*	476*
1	2.0	41*	15*	18	48*	1*	11	5*	4*	17	108*	176*
1	3.0	35*	24*	100	8	39*	3	14*	0	8	66*	59*
1	4.0	4	19*	91	17*	25	12	20	0	1	29	146
2	1.0	17*	37*	104	2	46	26*	25*	21*	2	40*	12*
2	2.0	46*	31*	83	7	10*	12	18	0	2*	28*	3
2	3.0	20*	44*	93	3*	19	6	18	0	2	0	71
2	4.0	9*	19*	28	16*	51	13	0	0	0	12*	36
3	1.0	22*	5	142	0	53	27*	5	6*	21*	17*	112
3	2.0	11*	4	108	14	4*	1	11	0	2	50*	75
3	3.0	19*	19*	45	42*	61	13	0	0	0	28	67
3	4.0	0	0	16*	0	5*	0	0	0	0	41	20
5	0.5	18*	11*	46	53*	52	14*	13*	4*	3*	27	9
5	1.0	26*	54*	61	5	15	12	17	0	8	72	110
5	1.5	20*	12	43	15*	35	9	0	0	3	35	102
5	2.0	22*	2	77	16	35	14	0	0	0	27	149
10	0.5	50*	10	55	37*	65	31*	10*	4*	7	20	25
10	1.0	14*	1	100	34	35	16	0	0	1	24	197
10	1.5	10*	0	113	45*	37	0	0	0	0	29	124
10	2.0	0	0	54	0	33	0	0	0	0	29*	58
Sum		462*	299*	1,303	300*	502	5*	6	38*	4	130*	581
Average per year		23*	15*	65	15*	25	0	0	2*	0*	6*	29

[a] Losses indicated by asterisks. Profits and losses rounded to the nearest cent.
[b] Total does not always equal 10 because of years in which profits or losses were less than half a cent per bushel, or in which no trading occurred.

TABLE 4.—RESULTS OF SOYBEAN RULES TESTING POSITIVE DEPENDENCE, WITH ACTION SIGNALS BASED ON RELATION OF CHANGE IN MOVING AVERAGE TO LARGEST PAST TEN DAILY PRICE RANGES

A. SUMMARY

Action signals		Total profits or losses[a] (*cents per bu.*)							
		1952–61		After commission, in year with		After commission number of years[b]		Com-mission (*cents per bushel*)	Total number of moves
Length of moving average (*days*, N)	Change (*fraction of daily range*, α)	After commission	Before commission	Best profits	Worst losses	With profits	With losses		
1	0.2	460*	264*	26	105*	1	9	196	544
1	0.5	114*	41*	15	58*	4	5	73	204
1	0.7	128	161	90	32*	6	4	33	93
1	1.0	153	161	114	22*	4	5	8	23
2	0.2	7*	88	129	44*	4	6	95	265
2	0.5	162	184	110	27*	5	5	22	62
2	0.7	93	101	110	44*	5	4	8	23
2	1.0	14*	13*	12	26*	1	1	1	2
3	0.2	110	172	104	27*	3	6	62	173
3	0.5	173	183	119	30*	5	5	10	27
3	0.7	107	110	81	11*	5	3	3	9
3	1.0	0	0	0	0	0	0	0	0
5	0.2	134	167	101	28*	5	5	33	92
5	0.5	57	60	80	28*	4	4	3	8
5	0.7	0	0	0	0	0	0	0	0
5	1.0	0	0	0	0	0	0	0	0
10	0.05	125	161	58	32*	7	3	36	99
10	0.10	108	136	84	43*	5	5	28	77
10	0.15	181	199	122	26*	7	3	18	50
10	0.20	227	237	122	33*	7	3	10	29

B. ANNUAL PROFITS AND LOSSES[a] AFTER COMMISSION
(*Cents per bushel*)

Action signals N	α	1952	1953	1954	1955	1956	1957	1958	1959	1960	1961	Total
1	0.2	73*	70*	26	105*	2*	50*	19*	21*	69*	77*	460*
1	0.5	36*	1	15	36*	21	0	4	5*	20*	58*	114*
1	0.7	32*	18*	90	10*	53	23	26*	4	5	39	128
1	1.0	0	8*	114	26*	48	12	22*	17*	11*	63	153
2	0.2	34*	43*	129	14*	30	38*	17	0	10*	44*	7*
2	0.5	27*	8*	110	16*	56	2*	8*	4	5	48	162
2	0.7	19*	44*	110	0	61	13	6*	11*	22*	11	93
2	1.0	0	0	12	26*	0	0	0*	0	0	0	14*
3	0.2	28*	25*	104	21*	62	27*	3*	0	12*	60	110
3	0.5	17*	24*	119	15	61	30*	3*	2	26*	76	173
3	0.7	2*	0	12	17	8*	0	11	11*	7	81	107
3	1.0	0*	0	0	0	0	0	0	0	0	0	0
5	0.2	14*	8*	101	21*	52	3	28*	4	3*	48	134
5	0.5	5*	2*	12	28*	8*	2	0	0	6	80	57
5	0.7	0	0	0	0	0	0	0	0	0	0	0
5	1.0	0	0	0	0	0	0	0	0	0	0	0
10	0.05	32*	2	54	3	58	10*	10*	4	0	56	125
10	0.10	44*	3	84	13*	65	13*	21*	3	1*	45	108
10	0.15	15*	4	122	22*	78	26*	4	8	3	25	181
10	0.20	7*	11	122	33*	68	32*	6	14	5	73	227
Sum		385*	229*	1,336	336*	695	175*	104*	22*	143*	526	1,163
Average per year		19*	11*	67	17*	35	9*	5*	1*	7*	26	58

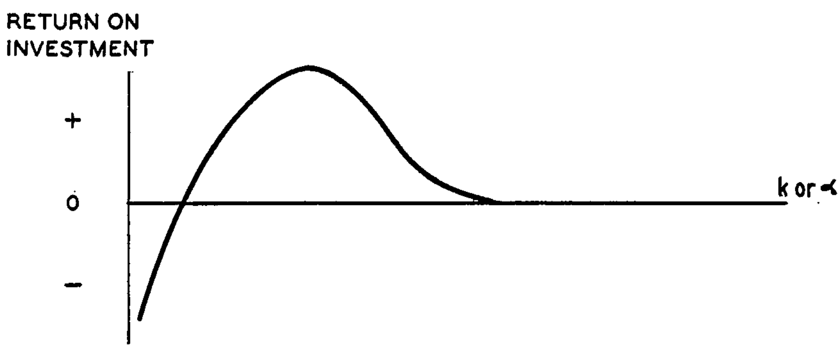
[a] Losses indicated by asterisks. Profits and losses rounded to the nearest cent.
[b] Total does not always equal 10 because of years in which profits or losses were less than half a cent per bushel, or in which no trading occurred.

Looking at profits after commissions, we find that a large number of the trading rules lead to significant positive returns. Of the 20 rules in Table 3, eight return 7.5 per cent per year or more, and eight others yield positive returns of less than 7.5 per cent. Of the 20 rules in Table 4, twelve yield returns of greater than 7.5 per cent, one yields a positive return of less than that amount, and two result in no trading at any time. Two rules in each of the tables yield losses greater than 7.5 per cent per year. We shall consider these in detail later.

For any given length of moving average, a fairly regular pattern of gains or losses results. The pattern is illustrated in Chart 1. If k (or α) is set very low, large losses result. As this parameter is increased, the losses first decline, and then profits result reaching a peak of 15 or 20 per cent per year. If the parameter is further increased, the profits decline and finally approach zero when k or α is so great that no trading results.

The parameters k and α determine how large a price change must be in order to be considered significant. For small values of these parameters, open positions are switched from long to short and back quite frequently. Consequently commission expenses are large. For sufficiently large values of the parameters, little or no trading is done.

CHART 1.—PATTERN OF GAINS OR LOSSES RESULTING FROM APPLICATION OF TRADING RULES



If we look more closely at the rules that are at least moderately profitable—say, more than 7.5 per cent per year—we find another interesting pattern. Most of the individual years generate losses, or very small gains. But a few years produce very high profits. Few of the very profitable rules would look very profitable if the year in which they made their best profit were excluded. Among generally profitable rules, the profits are much more concentrated in a few years than are the losses. For example, if we take the rule for which $N = 1$ and $k = 4$, we find profits were about 91 cents per bushel in 1954, 25 cents per bushel in 1956, and 29 cents in 1961. For both families of rules, two of these years have the highest total profits. Referring back to Table 2, we note that only in 1954, 1956, and 1961 was the within-the-trading-period price range greater than $1 per bushel. Also, in all three of these years, prices rose from the beginning to the end of the trading period by a much larger amount than in any other years.

Certain gross features of these observations suggest that the price changes may not be the result of serially independent price changes whose mean value was zero. In three of the ten periods, the price range was greater than $1 per bushel. In each case where a large price range is observed, the direction of change is upward.

Both the relative frequency and the direction of the large price changes observed could be explained by assuming that under certain circumstances the distribution of price changes during this time of year tends to have a positive expected value. The fruitfulness of this approach depends on whether by using some objective criteria one could identify in advance the circumstances under which positive trends are likely to occur. A search for such criteria is beyond the scope of this study. However, we can analyze the results of the trend-trader rules on the assumption that such criteria could be found, and ask whether there is any other evidence of positive serial dependence.

If one believes that the price trend will be upward in certain periods, the alternative of taking a position on the short side of the market is irrelevant. In this case one should compare the results that could be obtained by following the trading rule with the results of a consistent long position. In none of the three years in which there is an apparent strong upward trend are the profits resulting from the average of the 40 trading rules as great as could have been obtained from a consistent long position. Thus if one interprets the strong upward trend in three years as being due to the existence of a positive expected value in those three years, the results lend no support to the hypothesis of positive serial dependence. If anything, the consistent losses suggest the possibility of negative serial dependence. This possibility will be considered in detail below.

On the other hand, if one rejects the assumption that it is possible to identify in advance certain years in which there will be a positive price trend, then both the apparent price trends and the profits that result from applying the trading rules to the entire price series must be interpreted as evidence supporting the existence of positive serial dependence in the data.

The evidence examined so far suggests two possible conclusions, but does not enable us to distinguish between them: either the price changes exhibit positive serial dependence or their expected value is positive during some periods but not during others.

Rules testing negative dependence.—The trading rules that ostensibly generate large losses also deserve careful consideration. These rules are characterized by the fact that only small price changes are required to initiate a change from long to short, or vice versa. Consequently, many trades take place. The two most "unprofitable" rules generate average "losses" before commissions in excess of 25 cents per bushel per year.

A trading rule generating before-commission losses can always be modified to produce profits equal to the previous losses. The rule under which the gains or losses were calculated was: if $Q > K$ take (or hold) a long position the next day. To convert losses to profits, modify it to read: if $Q > K$ take (or hold) a short position the next day. Similarly modify the rule so that if $Q < -K$ a long position is taken (or held). Substituting long for short and short for long in the original statement of the trading rules converts before-commission gains into losses, and losses into gains. In this modified version, the rule is: if prices drop, buy; if they rise, sell. With these modified rules, profits would suggest the possibility of negative dependence.

Table 5 gives the results of counter-trend trading using the two most profitable counter-trend rules. Both rules have higher profits on a before-commission basis than any rules previously considered. Furthermore, the profits are rather evenly distributed over time. Eliminating even the most profitable year would not importantly affect our evaluation of these modified rules.

When brokerage commissions are taken into account, the total profits are significantly reduced but not eliminated. Commissions were calculated at the rate of 0.36 cent per bushel ($18 per contract) for each round-turn transaction. This is the rate nonmember traders would pay. However, a trader who was a member of the Board of Trade would be subject to a much lower commission rate, as well as to certain fixed charges, such as membership dues, that do not depend on his volume of trading.

If the modified trading rules were applied to data generated in such a way that successive price changes were known to be serially independent, the rule would still occasionally generate positive profits by chance. Thus the conclusion that the soybean futures price changes exhibit negative serial dependence rests on a judgment that profits as large as those actually observed would have been very unlikely if the price changes were not negatively dependent. To test this judgment we have artificially constructed some data having the observed trend, but no serial dependence. The most profitable modified trading rule was applied to this data.

The procedure for deriving serially independent data with the same trend was as follows. Suppose that there are TT consecutive trading days for which data are available in a given trading year. Then there are TT — 1 daily price differences. The last ten do not affect any of the trading rules and are believed to be subject to special influences, so they were eliminated. The remaining TT — 11

TABLE 5.—ANNUAL PROFITS OR LOSSES, BEFORE AND AFTER COMMISSION, OF TWO MODIFIED TRADING RULES TESTING NEGATIVE DEPENDENCE

(Cents per bushel, except as otherwise indicated)

Trading year	$N=1$ $\alpha=0.2$			$N=1$ $k=1$		
		Profits or losses[a]			Profits or losses[a]	
	Number of moves	Before commission	After commission	Number of moves	Before commission	After commission
1952	64	50	27	59	64	43
1953	49	52	34	39	46	32
1954	44	42*	58*	52	23	4
1955	54	86	66	54	91	71
1956	38	11*	25*	28	10*	20*
1957	53	31	12	35	16	4
1958	62	3*	25*	32	4	7*
1959	62	1*	23*	13	5*	10*
1960	65	46	22	23	13	5
1961	53	58	39	49	94	76
Total	544	266	69	384	336	198

[a] Losses indicated by asterisks. Profits and losses rounded to the nearest cent.

TABLE 6.—RESULTS OF APPLYING THE MODIFIED TRADING RULE FOR N = 1 AND k = 1 TO SIX RANDOM REARRANGEMENTS OF DATA FOR 1953–56: ANNUAL PROFITS OR LOSSES AFTER COMMISSIONS

(Cents per bushel)

1953	1954	1955	1956
+64.645	+41.435	− 4.675	−29.630
+11.460	−22.845	− 7.015	−35.115
− 0.945	−39.270	−19.920	−38.160
− 7.100	−58.985	−27.170	−40.330
−22.305	−75.000	−52.825	−54.010
−26.805	−92.845	−65.695	−71.535

price differences were rearranged in a random order.[5] By so rearranging the data we guarantee that there is no systematic serial dependence. The profitability of the trading rule used on the rearranged prices can then be compared with the profits from the same rule on the actual prices to estimate if the latter could have occurred by chance from independent price differences.

From among the modified trading rules, the one for N = 1 and k = 1.0 was selected. Table 6 gives the after-commission profits that result from applying this rule to six random rearrangements of the data for each of the four years 1953 through 1956. The results are arranged in order of descending profitability. When applied to the actual data for these four years, the rule yielded profits of 88.095 cents per bushel. There are 6^4, or 1,296, separate combinations of four-year profit totals that can be constructed from the results in Table 6. Each of these would be equally likely if the price changes during these four years were serially independent. The largest of the profits that can be constructed from this table is 71.775 cents per bushel, which is less than the profits actually observed.

We conclude that the evidence supports the hypothesis that price changes in the May soybean futures exhibit statistically significant negative serial dependence.

II

The evidence presented above seeks to test a hypothesis which has both practical and theoretical significance.

From a purely practical viewpoint, the serial independence of price changes bears on the question of whether certain styles of speculative trading, such as those commonly referred to as chart trading, are potentially advantageous. From the point of view of economic theory, the independence of successive price changes has a bearing on the efficiency of the market process because in a theoretically ideal commodity market all price changes would be serially independent.

[5] The trading results also depend on the daily price range. Let CL(T), MX(T), and MN(T) be the closing price, the maximum price, and the minimum price on day T. Then CL(T) − CL(T − 1) is the amount of the $(T-1)^{th}$ daily price difference in the original ordering. Suppose that after rearranging, the price difference of [CL(T) − CL(T − 1)] becomes the J^{th} price difference. Let CL*(J + 1), MX*(J + 1), and MN*(J + 1) be the closing price, the maximum price, and the minimum prices on day J + 1 after the rearrangement. The data were rearranged so that the following condition would hold:

a) $MX(T) - CL(T) = MX^*(J+1) - CL^*(J+1)$
b) $MN(T) - CL(T) = MN^*(J+1) - CL^*(J+1)$

It is well known that many futures market traders use mechanical trading systems to help make their trading decisions. By a mechanical trading system we mean one in which buy-and-sell signals are determined solely on the basis of past behavior of the market in respect to prices, volume of trading, size of open interest, etc. The profitability of any such mechanical trading system, insofar as it is not simply the result of price trends, depends on the existence of some sort of serial dependence of price changes.[6] Thus a mechanical trading rule is a means of testing for serial dependence. The moving average rule used in the previous section was selected as a test statistic partly on this basis.

Even if price changes are serially independent with no trend, they may occasionally produce patterns that resemble those which chart traders believe have predictive significance (*14*, p. 21). Price series generated under these assumptions have the technical name of simple random walks.[7]

Evidence about the profit-potential of a mechanical trading rule also can be analyzed to provide a judgment about the speed and efficiency of a speculative market response to new information affecting supply or demand.

To understand this, it is useful to imagine a hypothetical futures market where all actual and potential traders are immediately and simultaneously aware of any new information pertaining to the price of futures contracts. In such a market during periods when no new information becomes available, there would be little trading and few price changes.[8] As new information became available, the traders would analyze it. If they felt a change in their market positions was desirable, they would change their positions as rapidly as possible. There would thus be a short-lived flurry of trading during which prices would adjust to whatever new level was thought justified on the basis of the new information.

In this hypothetical market, a trader might be successful because he was better at analyzing and interpreting the information simultaneously available to all. For example, his special talents might enable him to make a better assessment of the consequences of a heavy rainstorm in Minnesota on the supply of soybeans. It should not be possible, however, for a trader to be successful merely by analyzing past price movements, since they would tell him nothing about the new infor-

[6] H. S. Houthakker appears to have misinterpreted some of his results by not considering the possibility that price changes, although independent, might have a nonzero expected value. Houthakker compared the results of a consistent long or short position in wheat and corn futures between arbitrary dates for a long period of years with the results of a similar position that could be closed by a stop-loss order at an earlier date. If price changes were independent and had a zero expected value, the addition of the stop-loss order should not change the expected profit. In fact, Houthakker finds: "In every future, whether long or short, it is possible to do better by using some stop percentage than by using none" (*8*, p. 167). All of the futures contracts Houthakker uses have a definite price trend during the period when trading was possible. Since this is the case, a stop-loss rule would lead to a change in expected profits even if successive price changes were serially independent. To test independence, we need to know whether the observed change in profit with a stop-loss rule is greater than would be observed in an independent series with the same trend.

[7] The difference between random variation and a random walk is vividly illustrated in the following quotation: "A familiar illustration of random variation is the variation in the number of spots that turn up when a pair of 'true' dice is thrown after being thoroughly shaken each time. By proceeding from that illustration one might illustrate random walk by drawing on a chart a line that moves forward one space for each throw of the pair of dice, and up or down by a number of units equal to the number of spots minus seven (seven being the expected number of spots on each throw)" (*17*, p. 446).

[8] Some trading might be initiated by speculators whose price forecasts had not changed but whose ability to absorb the financial risks of a futures market position had been altered by changes in their personal situations such as might result from sickness, or from changes in the results of other independent business interests. Trading might also be initiated by hedgers.

mation that will become available to the market. System trading could not be profitable in this market, in the long run, except perhaps by taking advantage of price trends.

Compare this ideal market with a second market with two categories of traders. The first consists of traders who learn about new information relatively early. We call this group the "insiders." A second category consists of traders who only hear about new information *after* the insiders have heard about it. We call this group the "outsiders."[9] If new information became available in such a market, we would expect a double market response. The first would occur as the insiders learned of the new information and acted on it. The second would occur later when the outsiders learned of the new information.

The direction of the first price response could be predicted unambiguously. It would depend solely on the insiders' interpretation of the new information. If the new information were bullish, the insiders would buy and thus raise prices; if the new information were bearish, the insiders would sell and thus depress prices.

The direction of the second market response cannot be predicted unambiguously. Suppose that the new information is bullish. Three alternatives are possible.

One possibility is that prices would rise twice—once when the insiders heard the news, and again when the outsiders heard the news. A second possibility is that when outsiders learned of the news and began to buy, there would be a flurry of trading activity, but no change in price. A third possibility is that prices might decline as outsiders come into the market. Presumably in the latter case, the first price rise would be greater than the subsequent decline.

The first possibility might occur if the delayed demand from the outsiders were only partly supplied by sales from insiders. The second possibility might result if the insiders were willing to sell the entire quantity demanded by the outsiders, so that no appreciable rise in price would be necessary to supply the outsiders' requirements. The third possibility is likely if insiders tried to sell a greater quantity than was demanded by the outsiders, so that the buying activity from the outsiders would be accompanied by a decline in prices.[10]

In the first and second cases, prices would always move in the right direction. But in the second case the movements would be more prompt. In the third case, there would be destabilizing price movements; prices would first rise too much, and then be corrected by a fall.

The second case can be described as the most efficient. Prices react exactly as

[9] It would be more accurate to refer to a continuum of traders arranged in order according to how quickly they became aware of new information. The device of referring to two categories is introduced merely to simplify the exposition. There would be no essential change in our conclusions if we introduced a more complicated classification of traders along these lines.

The term "insider" as used here implies only that these traders learn of new information relatively early compared to other traders. There is no implication that access to this category is in any way restricted, or that the information is not public. For example, we could consider that an insider is anyone who hears of new information within two hours of the time it is carried on a ticker tape.

[10] The discussion in the text avoids the question of which traders sell when the insiders buy in response to bullish news. This need create no difficulties. The sellers might be any group of traders other than the insiders who are willing to sell on a price rise. For example, the sellers might be outsiders who were long and are willing to take profits on an apparently random price rise. By hypothesis, the outsiders have not yet heard the "news" that would explain the rise. Alternatively, if cash prices are somewhat more sluggish than futures prices, the sellers might be short hedgers for whom the futures price rise means a better-than-usual relationship between cash and futures prices.

in the hypothetical market where all traders are always equally well informed. The cases where all are equally well informed, and where insiders perfectly predict subsequent outsiders' behavior, could not be distinguished by observing price movements only.[11] In either case, there would be only a limited possibility of profits for traders using mechanical systems. Price movements would be as unpredictable as the new information that causes them. Even if insiders do not always perfectly anticipate outsiders, our conclusion is the same provided insiders are as likely to underestimate as to overestimate the outsiders' response to news.

However, if there is a systematic tendency for a price rise to be followed by a subsequent further rise, or by a subsequent fall, then a market might also contain system traders who could, in principle, earn long-run profits, even in the absence of price trends. Thus evidence that a trading system generates positive profits that are not simply the result of following a trend also constitutes evidence of market imperfections. It is from this point of view that system trading acquires its fundamental significance in this study.

It may seem strange to present a model of a commodity futures market that distinguishes various groups of traders but does not include hedgers among the groups explicitly considered. The rationale for this approach is that this model is designed primarily to investigate certain limited features of price behavior in a commodity futures market. Although hedgers are not considered explicitly, the model can be interpreted as implicitly including them. Suppose a hedger changes his open position in the futures market on the basis of some information available to him before it is available to other traders. Such a hedger would be, at that moment, an insider. The relevant outsiders might be professional traders on the floor of the exchange. Alternatively the hedger might alter his open position in the futures market on the basis of information that had been available to other traders before it became available to him. At that moment such a hedger would be an outsider.

To give some examples, suppose the hedger to be an elevator operator:

(A) The farmers from whom the elevator operator purchases the commodity in question (and whom we assume are not futures traders) offer him a larger quantity of the commodity than he or anyone else had anticipated. The elevator operator makes futures sales to offset at least some of his purchases. In this situation, the hedger is an insider.

(B) The elevator operator sells some of the commodity to an exporter. At the same time he adjusts his own position in the futures market. The market had previously been aware of the unfilled export order. In terms of his relation to the news about the export order, the elevator operator is an outsider.

Since nothing in the data we have analyzed enables us to classify hedgers as being usually insiders or usually outsiders, we have not mentioned them ex-

[11] The two cases might be distinguished by relating price movements and volume of trading. If all traders are equally well informed, a new piece of information leads to only one flurry of trading activity. If some learn of the new information later than others, there might tend to be two flurries of trading activity, even if only the first is associated with a significant price movement.

Terms such as "predict" and "anticipate" can be interpreted metaphorically, if desired. It is not essential to the argument that insiders consciously try to estimate what other traders will do. The argument depends solely on the quantities that insiders are willing to sell at the going price compared to the amount outsiders would like to buy at that price.

plicitly. However, the term "trader" is used, rather than "speculator," to indicate that hedgers are to be included.

It may be of interest to compare the two models presented above with a somewhat similar model of price formation developed by Holbrook Working.[12] The basic concepts of primary price changes originating from changes in the information available to traders and of lags in traders' awareness of new information as causing price changes to be gradual rather than instantaneous are borrowed from Working (*16*, p. 195).

Working's model is presented as realistic, with only minor exceptions (*16*, p. 192). Our first model is potentially unrealistic in that it explicitly assumes all traders are simultaneously aware of all new information. One of the main purposes of the article is to test this hypothesis. Taken literally, the hypothesis is untenable. We do not intend it should be taken in this way. In fact, the smallest time interval considered in our empirical tests is one day. If nearly all traders are, as Working describes them in presenting his model, "persons of rather exceptional trading ability and judgment, emotionally stable, with a large fund of pertinent knowledge, skilled in using their knowledge, and they give all of their working time and energy to the business of trading and keeping appropriately informed" (*16*, pp. 193–94), then it seems not unrealistic to assume that nearly all of the price effects of new information would take place within the trading day on which the new information became available to traders. A trading day, for this purpose, should be defined as the period of time from the close of trading on one calendar day to the close of trading on the next calendar day in which trading takes place.

Our first model also differs from Working's in that it does not assume different traders tend to concentrate on obtaining different kinds of information. This assumption is not logically necessary to Working's conclusions, as was pointed out by Robert Weidenhammer in his discussion of Working's paper. In addition, in another study that attempted to determine whether such specialization occurs, we were unable to find any evidence for it, at least among avocational traders (*12*).

Since Working's model assumes some lag in response to information, he also allows for a small group of inept traders who can do little else than take what advantage they can from the slight gradualness of price movements that are a result of the lag in information. This inept group is presumably what we refer to as system traders. If our first model were appropriate, there would be no interday lags in price adjustments to new information, and no role for system traders who did not follow the market extremely closely. The study referred to above also contains data suggesting that few avocational system traders have access to intra-day information on price changes (*12*).

Our second group of models is an attempt to develop some of the theoretical possibilities that are open to us if we relax the assumption that new information is simultaneously available to all traders. The initial response to new information may either be in the wrong direction or, if in the right direction, it may be just

[12] Working has published his ideas in various versions. The comparison presented in the text is with the version presented in *16*. This version seems to be Working's most complete and authoritative statement of his model. Other statements of his ideas may be found in *15*, *17*, and *18*.

appropriate, too large, or too small (cf. *15,* pp. 1431–32). The three versions of the model provide explanations for the last three possibilities.

Admittedly other explanations also may be possible. In particular, neither the models presented here nor Working's model takes explicit account of the possible consequences for price behavior of the quantity of open contracts or of the volume of trading. A tendency for intra-day price changes to be negatively correlated has previously been reported by Working (*19,* p. 124). This type of negative correlation provides the basis for a specialized style of trading known as scalping, which serves the function of increasing the liquidity of a commodity market. In commenting on an earlier version of this paper, Working has suggested that the negative dependence of inter-day price changes reported here may reflect essentially the same forces that give rise to negative correlation among successive intra-day price changes. Exploring this possibility would require restating the present models so that traders' desire for liquidity was considered explicitly. For example, a trader desiring to close out a long position who placed a sell order "at the market" might be presumed to place a higher premium on liquidity than if he had made his order conditional on obtaining at least a certain price.

On purely statistical grounds it must be accepted that a system-trading rule could lead to positive profits even if successive price changes were statistically independent, provided their expected value was not zero.[13] One might ask: "On what economic interpretation could the assumption that trends exist be consistent with the assumption of an ideal market?"

One answer is this: "If a trader seeks to maximize expected monetary profits, and the expected value of the change in futures prices is always zero, then his expected profit is also zero; so he has no incentive to take a market position." This interpretation, advanced by John Maynard Keynes and others, is that a nonzero expected price change is required to induce speculators to assume risks, and is offered by hedgers as a payment for transferring risk. If hedgers had net short positions in the market, speculators would be net long. One would expect a positive expected price change if this theory were correct. Our soybean data cover a period of the year when we would expect hedgers to have net short positions in the futures market.[14]

Another explanation for the existence of price trends, advanced recently by Roger Gray (*6* and *7*), and supported by Working (*17*), is based on the market balance concept.[15] The basic idea is that a trend in futures prices exists in those

[13] One possibility not considered in the text is that in the true distribution of daily price changes, the mathematical expectation of the outcomes is undefined. This is logically possible. We leave a consideration of this possibility to others who may be better qualified to cope with it. Nevertheless, it might be mentioned that several observers have noted that large price changes occur relatively more frequently with speculative price series than would be true if these series followed a normal distribution (see *8* and *10,* p. 318). The class of probability distributions having this characteristic includes those distributions whose means are undefined.

[14] For a fuller discussion of this hypothesis and some relevant evidence from other commodity markets, see *2, 3, 6, 7, 9,* and *13.*

[15] It is interesting that Working, in describing the concepts of reliability of anticipatory prices and of market balance, finds no contradiction between the two. Working presents two interpretations of the concept of reliably anticipatory prices. On the one hand, he implies that what is meant is that prices are "mainly appropriate responses" to new information. On the other, he suggests "we need to consider reliability of expectations and what ought to be expected in the light of available information" (*18,* p. 447). If the first meaning is accepted, then prices may be reliably anticipatory even if there is a risk premium or a lack of market balance. If the second interpretation is adopted, the lack of market balance or the existence of a risk premium implies that prices are not reliably anticipatory.

markets and at those times when there is an insufficient volume of speculation to balance the hedging that takes place. "The significant requirement for balance is enough participation by speculators to balance the hedging. The thriving markets are those with adequate speculation to serve hedging needs economically" (*6*, p. 312).

The risk premium concept implies that price trends are a normal characteristic of all futures markets. The trends are necessary to attract speculators and enable them to earn an adequate return on their somewhat risky investment. By denying that a price trend is normal or necessary, the market balance concept apparently leaves us free to interpret a price trend as evidence of a kind of market imperfection due to insufficient mobility by speculators, or to barriers against their entry.

To summarize: the existence of a trend in futures prices would be interpreted, by those who accept the risk premium concept, as simply the mechanism by which speculators earn a normal return; the same fact would be interpreted as indicative of insufficient mobility or of barriers to entry by those who accept the market balance concept. In either case, the existence of a trend implies that the *level* of futures prices is not reliably anticipatory. However, the existence of a trend in prices, for whatever reason, does not preclude the possibilty that *changes* in futures prices might be reliably anticipatory in the sense that the changes are mainly appropriate responses to changes in information.

III

There is a long history of attempts to test the serial independence of various time series data. A comprehensive treatment of this literature would be beyond the scope of the present article. The results of some recent studies, into which context the present evidence must be fitted, can be indicated briefly.

An article by S. S. Alexander (*1*) summarizes and evaluates much of the earlier work, and presents some original evidence. Referring to earlier studies, Alexander concludes: "The many statistical studies which have found speculative prices to resemble a random walk have dealt with changes over uniform periods of time" (*1*, p. 23). Alexander believes that changes in industrial security prices, at least, are not independent. His conclusion is based on a trading rule similar to those used in this paper in that they do not involve a fixed time period. His rule is: if the market goes up X per cent, go long and stay long until it moves down X per cent, at which time sell and go short until it moves up X per cent. Based on American industrial stock price averages from 1897 to 1959, he concludes that this rule yields positive results for X ranging from 5 to 30 per cent. Thus he finds a tendency for price changes to be followed by subsequent price changes in the same direction. He gives no evidence for values of X below 5 per cent, and no evidence of the statistical reliability of his conclusions.

Alexander assumes that his speculator can switch from long to short, or vice versa, when the price change is exactly X per cent. B. Mandelbrot has criticized this aspect of Alexander's results (*11*, pp. 417–18). If speculative prices actually follow a stable Paretian rather than Gaussian distribution function, as Mandelbrot believes, then a speculator could not actually switch at exactly the prices Alexander assumes, and Alexander's calculated profits for the speculator are systematically higher than could actually be obtained. We do not know how much of a bias would be introduced by this defect in Alexander's test procedure. However, the same criticism would not apply to the results presented in this article, since our assumed purchases and sales take place after the price change that signals them, and at a price between the observed high and low prices on the subsequent day.

A study by Arnold Larson presents some evidence on the serial independence of price changes in Chicago corn futures prices covering the periods 1922–32 and 1949–58 (*10*). His test procedure, an index of continuity developed by Working, makes it possible to compare a price change with subsequent changes over an interval without assuming that subsequent changes are dependent with any fixed time lag. But the initial change can be compared with subsequent changes over intervals of different lengths. The two sets of data considered yield similar patterns. Combining them, Larson finds price movements dispersed through time as follows. Eighty-one per cent of the movement occurs on the initial day. This is followed by a four-day period during which there is a tendency for prices to move in the opposite direction; but the magnitude of the movement is only eight per cent of the total price change. Following this, there is a 45-day period during which there is a tendency for prices to move in the direction of the original movement by an amount equal to 27 per cent of the total movement.

Larson's results for corn futures indicate that there is some evidence for both positive and negative serial dependence of price movements. However, since according to his measure "81 per cent of the price effect of demand and supply influence occurs on a single day, presumably the day the influences occur and/or the day they become known to the majority of traders in the market," Larson concludes: "If this is a true picture of the operations of the corn futures market, then it approaches the ideal very closely" (*10*, p. 323). Our conclusion is that soybean futures prices exhibit negative serial dependence that is both statistically and practically significant. Our criterion of practical significance is the possibility of profits by using a mechanical trading rule. Larson's criterion of practical significance is the relative size of the negative movement compared with the original positive movement. It will require further research to determine whether the differences between Larson's results and ours are due to differences in the data, differences in the statistical test procedures, or differences in the criteria of practical significance.

A study by Paul H. Cootner (*4*) presents a theory which would lead one to expect negative correlations between price changes over short time intervals and positive correlations over longer time intervals. He also presents some empirical evidence based on the weekly price changes of individual American common stocks. Cootner's theory is similar to ours in that it presupposes two distinct groups of market participants. His grouping is based on differences in the ability of the market participants to form an estimate of the future price level; ours is based on differences in the speed with which the market participants respond to new information. The two bases of classifying market participants are not mutually exclusive. Either one, by itself, leads to similar but not identical implications as to how price changes should behave. Neither the empirical tests presented in this paper nor in Cootner's study are sufficient to distinguish between the two theories. However, Cootner's results are inconsistent with the hypothesis that successive price changes are statistically independent, provided it is assumed that the price changes have a Gaussian distribution. If the price changes have a stable Paretian distribution, then some of Cootner's results are consistent with the hypothesis of statistically independent price changes (*11,* p. 410).

Mention can be made of an attempt to test our hypothesis on data from still another futures market. Cootner kindly supplied the author with punched cards containing rubber futures prices. A modified version of the system-trading rules testing negative dependence was applied to the July rubber futures contracts maturing in calendar years 1948–50 and 1953–60. Since Cootner's price data contained only closing prices, it was necessary to assume that all trades took place at the closing prices. The trading rules for $N = 1$ and various values of k exhibited a pattern similar to that shown in Chart 1. However, for values of k down to about 133 per cent of the normal round-turn commission, the rules did not yield positive profits. If the rubber futures prices exhibit negative serial dependence, the tendencies are not large enough to be taken into account by traders who must pay standard brokerage commissions.

Working has reported a tendency for positive serial dependence (with lags of more than one day) in prices for Chicago corn, wheat, and rye futures (*15,* p. 1433), and negative serial dependence in intra-day futures prices (*18,* p. 448; *19,* p. 124).

The present state of knowledge might be summarized as follows. In some markets, evidence of positive serial dependence or of negative serial dependence has been uncovered. In the case of rubber futures a search for negative serial dependence apparently failed. Clearly additional research will be necessary before we can generalize about the conditions under which nonindependence of price changes can be expected.

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