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MEASUREMENT OF A RANDOM PROCESS IN FUTURES PRICES*

This paper presents the results of application of a new method of time-series analysis, which appears to give the first really satisfactory answer to the question of whether commodity futures markets facilitate excessive daily price fluctuations. The method uses an entirely new test statistic, the index of continuity, rather than autocovariance as do virtually all other methods. In order to show why the new statistic is needed, I will first sketch the current status of the relevant area of time-series analysis, and demonstrate the failure of existing methods to solve the problem at hand.

RECENT TRENDS IN TIME-SERIES ANALYSIS

From the viewpoint of the economist engaged in research into economic relationships, progress in development of time-series analysis can be gauged by the kinds of "cycles" which can be detected. These may be true sine waves of fixed period and amplitude, or the apparent cycles which occur in series with purely random movements, or they may be intermediate types such as the disturbed cycles of autoregressive schemes. Usually the processes which generate the cycles embody or reveal interesting economic relationships, and the parameters of the processes can be given economic interpretation.

Much of the early work in time-series analysis was concerned with the decomposition of a series into trend, seasonal or other cyclic, and random components. Often the cycle so isolated was of indeterminate character. Among the first procedures which at least by implication addressed themselves to the question of the type of process which generates cycles was the periodogram. It tested for cycles of fixed period against the counter hypothesis of a purely random series. A systematic development of the theory of stationary stochastic processes, beginning in about 1930, permitted the formulation of a large sample theory for the harmonic, autoregressive, and moving average processes. The parameters of even high order processes can be estimated, although the methods are quite cumbersome and almost require the use of electronic computers. These three processes—harmonic, autoregressive, and moving average—are thought to correspond to many real processes, including many of those in operation in economic situations.

There appear to be many difficulties involved in applying the methods of

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analysis of stochastic processes to economic data, in spite of the seemingly satisfactory state of the theory. Since many economic series are short, the large-sample estimates are of doubtful validity. The process in operation in the series may not be stationary, and in fact may be intermittent or at times completely obscured by random factors. Most of the methods of parameter estimation depend fundamentally on the correlogram, that is, the graph of successive autocorrelation coefficients. Empirical correlograms have been shown to have highly autocorrelated terms, and to depend heavily on the particular sample drawn. Results of correlogram analysis of economic time series have been generally disappointing to date, being both inconclusive and of doubtful validity.[1] Usually only the first one or two terms of the correlogram have been used, and even these often have not differed significantly from zero. However, it must be granted that the possibilities have not been exhausted for successful application of correlogram methods.

Often in time-series analysis greater interest attaches to the *type* of process in operation than to the particular values of the parameters of a process once specified. The type of process is interesting to the scientist because it determines the nature of the mechanism in operation, and the mechanism may be basically unaltered under a wide range of parameter values. Furthermore, the type of process may be genuinely in doubt. Tests of significance, which in general help us choose between alternative parameter values, are in consequence not as important as are tests of goodness of fit, which choose between alternative specifications of process type. Sometimes more than one specification is acceptable on the basis of the test of goodness of fit. Procedures for choosing among the acceptable specifications can be devised, but one can never determine by purely statistical means that *the* true process has been found. All statistical investigation encounters this problem, of course, but it is of special importance here.

THE THEORY OF ANTICIPATORY PRICES

A time series, such as a commodity futures price series, can be analyzed in one of two ways. First, one may apply formally and naively the methods for estimating each of several processes, and choose the best fitting process as the one which is presumably in operation in the series. This will normally involve some *a posteriori* rationalization of the existence of such a process in the series. The test of goodness of fit will not ordinarily provide conclusive evidence that this is the true process, but quite possibly the rationale will be sufficiently plausible so that the results may be accepted.

The second approach, which seems preferable because of the lack of power of the tests of goodness of fit, is to determine the type of process which is implied by the structure of the market, or other institution being studied. Then only the order of the process and certain parameters need be estimated by statistical means. It is still necessary that the data fit the specification satisfactorily, but acceptance of the hypothesis rests primarily on its *a priori* probability, based on economic considerations.

Something close to the first approach is most often used in economic analyses.

[1] See Gerhard Tintner, *Econometrics* (Wiley, 1952) for a number of applications and discussions of the results.

A process of one of the well developed types is fitted after the most casual theorizing, just to see if it gives a good fit. It may be that more sophisticated and realistic theories do not ordinarily lead to stochastic processes of one of the standard types. But if this is true, a conscious process of simplification with an eye to retaining the essential features of the economic structure in the stochastic process would be more likely to yield valid results.

In this study the second course is pursued and an attempt is made to derive the stochastic processes implied by two theories of market price formation, and to subject the theories to empirical test by estimating the implied processes. The two theories are not mutually exclusive, but they do present opposing views about the probable existence and importance of excessive price fluctuations generated in futures markets. No attempt is made at complete expositions of either theory. Rather only those facets which touch upon the implied stochastic process and the central question of excessive fluctuation are developed at all fully.

The first theory is that of Professor Taussig (*1*). He held that the equilibrium price is only roughly determinate, and that the market price is free to fluctuate within a penumbra of uncertainty about the equilibrium price. At the edge of the penumbra there would be a reaction, or at least a check, presumably caused by equilibrating forces asserting themselves, returning the price toward equilibrium.[2] Since no one knows just where within the penumbra the price should be, it moves in this zone in response to the slightest impulse, which may be completely unrelated to market information. Furthermore, and this is perhaps Taussig's main point, movements away from equilibrium in the penumbra of uncertainty will not necessarily call forth an immediate return to equilibrium, but may well produce further movement in the same direction. Taussig assigns a big role to the market manipulator who, for example, may sell in hopes of inducing further sales and an artificially low price, at which he buys and awaits the return of price to equilibrium. The timing of the reversal of price movement varies, and sometimes the manipulator fails in his objective.

Taussig's theory unfortunately cannot be reformulated as a stationary stochastic process, at least without considerable simplification. A Markov chain with reflecting barriers corresponding to the edges of the penumbra might be an approximate model. The analogy is far from complete, however. For instance Taussig's theory apparently differentiates between price changes caused by manipulation and those caused by market information, and hence one cannot apply the same transition probabilities, which imply the same probable subsequent course of events, to the two situations. The probabilities to be assigned should perhaps depend on the path previously taken, which would make the process non-Markovian and make analysis much harder.

Even the correlogram implied by the theory is not clear. Presumably there would be numerous short runs of various lengths, many but not all of them followed by abrupt reactions. This might imply rather high negative correlation coefficients for several days' lag, for a series of price changes. Or if the runs are

[2] This phenomenon resembles in many respects the "support" and "resistance" levels of chart traders, although the support and resistance seems often to be attributed to more esoteric causes than equilibrating forces. If we grant that they are the same, we would conclude that price does not immediately bounce off the edge of the penumbra and continue toward the center, but rather remains near the edge for some time.

of several days' duration there may be positive correlations followed by negative ones. The correlogram, which completely describes the autoregressive and the moving average schemes, appears to tell little about this kind of process.

It would evidently be difficult or impossible to test Taussig's theory by standard statistical methods, because it fails to conform to any of the well developed stochastic processes. But maybe it is wrong anyway. Past prices have had to stand the test of market clearing and presumably cannot be grossly in error very long. The primary task of the traders in the market is thus reduced from the formidable one of estimating the correct price *level* to the much more modest one of gauging the price *changes* appropriate to changes in the demand and supply conditions. This they might well be able to do. So maybe the penumbra is not there.

Holbrook Working has developed a theory of anticipatory market prices which reflects quite a different view of market behavior (2). He argues that anticipatory prices depend on expectations regarding the future course of events affecting demand and supply of the commodity being traded. Traders base their expectations on market news, which in broad and highly organized commodity markets, he suggests, tends to be generally accurate, timely, and relevant.

In an ideal market, existing knowledge of market conditions would be reflected in current price, and any new information, showing altered conditions, would produce a price movement. Truly new information emerges randomly, and so price movements would tend to be random. Working's model of price movement in an ideal market is thus a random walk; that is, price is the cumulation of random movements. Equilibrating forces restrain the random walk, of course, but during the life of any one future the random movements predominate. These random movements, far from being blind stumbling around in a penumbra, would be movements to accurately estimated new equilibrium prices.

No actual market behaves exactly like an ideal market. Many commodity futures markets approach the ideal, and differ principally in that traders react with varying skill to varying sources of information, and so some of the response to price-making forces is delayed. The initial response is inadequate, and subsequent corrective response is in the same direction. The delayed response is a rather small fraction of the total, and is dispersed over a considerable number of days, in a highly variable pattern.

This completes the essential economic argument, but for mathematical reasons, to reduce the number of parameters to be estimated, it is further specified that the average pattern of dispersion is approximately rectangular; that is, a constant percentage of the delayed price response occurs on each day of the dispersion span. This is a gratuitous assumption, and as a matter of fact we will modify it shortly, but even major changes in the specified pattern do not seem to call for revision of the economic interpretation.

With this assumption, Working's theory of anticipatory prices leads directly to a moving average stochastic process of order equal to the length of dispersion span and with uniform weights. Price changes are a moving average of the price changes which would have occurred had the market been perfect. The process is not quite stationary, since the variance of price influences varies over time, but this is not likely to be a serious discrepancy.

For large dispersion span and low per cent dispersed, the weights of the moving average process are very low. A moving average stochastic process which generates a series is completely determined by the mean and variance of the series and by the correlogram equal in length to the order of the process. The very low weights, with resulting low values of correlation coefficients, coupled with the difficulties mentioned above, make estimation of the process from the empirical correlogram extremely difficult.

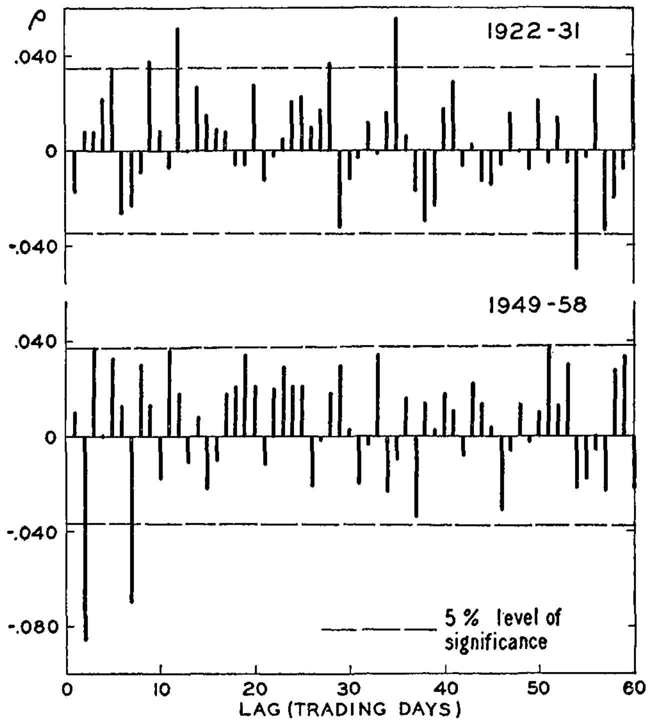
TABLE 1.—AUTOCORRELOGRAMS OF CLOSE-TO-CLOSE PRICE CHANGES, CORN FUTURES PRICES, CHICAGO, 1922–31 AND 1949–58

Lag	1922–31	1949–58	Lag	1922–31	1949–58
1	−.017	.012	31	−.003	−.020
2	.008	−.086[a]	32	.012	−.004
3	.008	.037[b]	33	−.001	.034[c]
4	.022	−.000	34	.016	−.024
5	.035[b]	.033[c]	35	.056[a]	−.010
6	−.026	.013	36	.006	.016
7	−.023	−.070[a]	37	−.017	−.034[c]
8	−.009	.030[c]	38	−.030[c]	.014
9	.038[b]	.013	39	−.023	.003
10	.009	−.018	40	.018	.018
11	−.007	.036[c]	41	.029[c]	.011
12	.052[a]	.018	42	−.006	−.008
13	−.000	−.011	43	.003	.002
14	.027[c]	.008	44	−.013	.014
15	.015	−.022	45	−.014	.004
16	.009	−.010	46	−.006	−.031[c]
17	.008	.018	47	.016	−.006
18	−.006	.021	48	−.000	.014
19	−.006	.034[c]	49	−.008	−.002
20	.028[c]	.021	50	.021	.010
21	−.012	−.012	51	−.005	.037[b]
22	−.002	.020	52	.014	.013
23	.005	.029	53	−.005	.030[c]
24	.021	.021	54	−.050[a]	−.022
25	.023	.021	55	−.003	−.018
26	.010	−.021	56	.032[c]	−.006
27	.017	−.001	57	−.034[c]	−.023
28	.037[b]	.018	58	−.020	.028
29	−.032[c]	.030[c]	59	−.008	.034[c]
30	−.012	.003	60	.032[c]	−.022

[a] Significant at 1 per cent level.
[b] Significant at 5 per cent level.
[c] Nearly significant.

The correlograms of the series of changes in closing prices of corn futures on the Chicago Board of Trade for two ten-year periods, 1922–31 and 1949–58, up to lag 60, are shown in Table 1 and Chart 1. Several of the coefficients of correlation are significantly different from zero, but no clear pattern emerges. For instance, significant values seem to be as likely at high orders of lag as at low. Also, high positive coefficients frequently occur adjacent to high negative values. More-

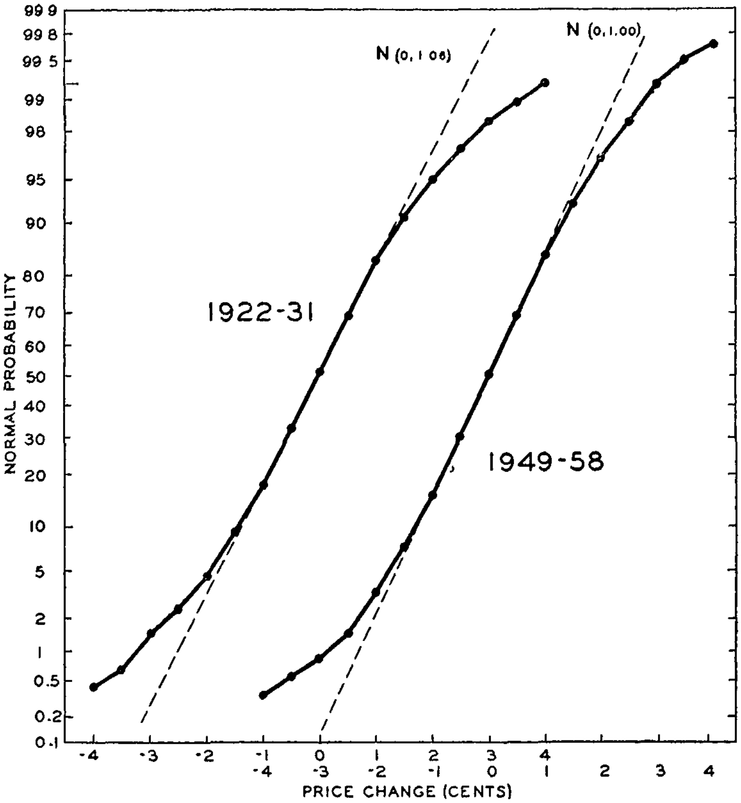
CHART 1.—Autocorrelograms of Close-to-Close Price Changes, Corn Futures Prices, Chicago, 1922–31 and 1949–58



over, there appears to be no correspondence between the correlograms for the two periods. The general appearance of the correlograms virtually rules out the possibility of a low-order process of any sort, and strongly suggests that the series is indeed a random walk. Because the correlogram method lacks power to detect moving average processes with very low weights, we should not accept this negative conclusion on the basis of this evidence.

The erratic patterns in the correlograms may arise in part from the nature of the distributions of price changes, shown in Chart 2. The distribution for each 10-year period has mean near zero, and is symmetrical and very nearly normally distributed for the central 80 per cent of the data, but there is an excessive number of extreme values. Also, some of these are quite extreme, being 8 or 9 standard deviations from the mean. If these extreme values occur randomly the estimates of correlation coefficients are consistent, but the confidence intervals shown are somewhat too narrow. Indeed, the cross-product of two of the higher extreme values would be of the same order of magnitude as the autocovariances found in the study. Examination of the pattern of occurrence of all price changes in excess of three standard deviations from zero, and of the cross-products of these changes, indicated that they have no special autocorrelation properties, and it is assumed that the data are essentially homogeneous and that underestimation of the variance of the autocorrelation coefficients is probably the only problem arising from presence in the data of an excessive number of extreme values.

CHART 2.—Distribution of Close-to-Close Price Changes, Chicago Corn Futures, 1922–31 and 1949–58, Compared to Normal Distribution



THE INDEX OF CONTINUITY

Holbrook Working developed the test of the index of continuity specifically to analyze anticipatory prices. The index is based on the ratio of the range of a series on an interval to the sum of the ranges of the subintervals within the interval. The subintervals are of equal length and non-overlapping. The series of values of index of continuity, designated H, for one-day subintervals and interval lengths of 2, 4, 8, . . . , 256 days, trace a curve which forms the basis for estimation procedures. Positive values of H occur when the range of the series tends to exceed the expected range of a random-walk series over intervals of a given length. Positive values of H thus imply movement of the series predominantly in one direction; in other words, relatively long, but perhaps interrupted, runs and positive autocorrelation. Negative values of H imply an excessive number of reversals in the series, so that the range tends to be less than that of a random-walk series. Autocorrelations between members of a series at any lag within the interval length tend to affect the index of continuity for that interval length similarly. Hence the index of continuity test relaxes the assumption of rigid lag between initial and corrective movements, and substitutes the more realistic assumption that the correction occurs sometime within a given interval of time.

Since the index of continuity is based on the range, i.e., the difference between maximum and minimum, it is rather difficult to treat analytically. With the assistance of two associates,[8] Working has determined the approximate distribution of H for a random series, thus providing a test of the hypothesis that a series is random. Some of the basic mathematical work on the test remains to be done, but enough has been done to make the test usable.

The parameters of a "rectangular-tailed" dispersion pattern, and hence the weights of the particular type of moving average stochastic process postulated in Working's theory, have been estimated by Monte Carlo methods, dispersing a random-walk series in ways corresponding to several moving average processes (*3*). Such dispersions give rise to a family of curves of H-values which can be approximated by a family of logistic curves depending on the parameters of dispersion. In this way the parameters of dispersion can be estimated from the empirical H-curves for one-day subintervals.

Empirical H-curves for the two ten-year periods of corn prices, and the logistic curves fitted to them, are tabulated below, and are shown in Chart 3A:

Interval length	1922–31 Period		1949–58 Period	
	Empirical	Fitted	Empirical	Fitted
2	−.006	.001	.012	.001
4	−.022	.002	.008	.003
8	−.033	.006	−.007	.009
16	−.032	.017	−.003	.023
32	.005	.038	.030	.051
64	.048	.074	.082	.100
128	.133	.112	.181[a]	.164
256	.148	.139	.253[b]	.220

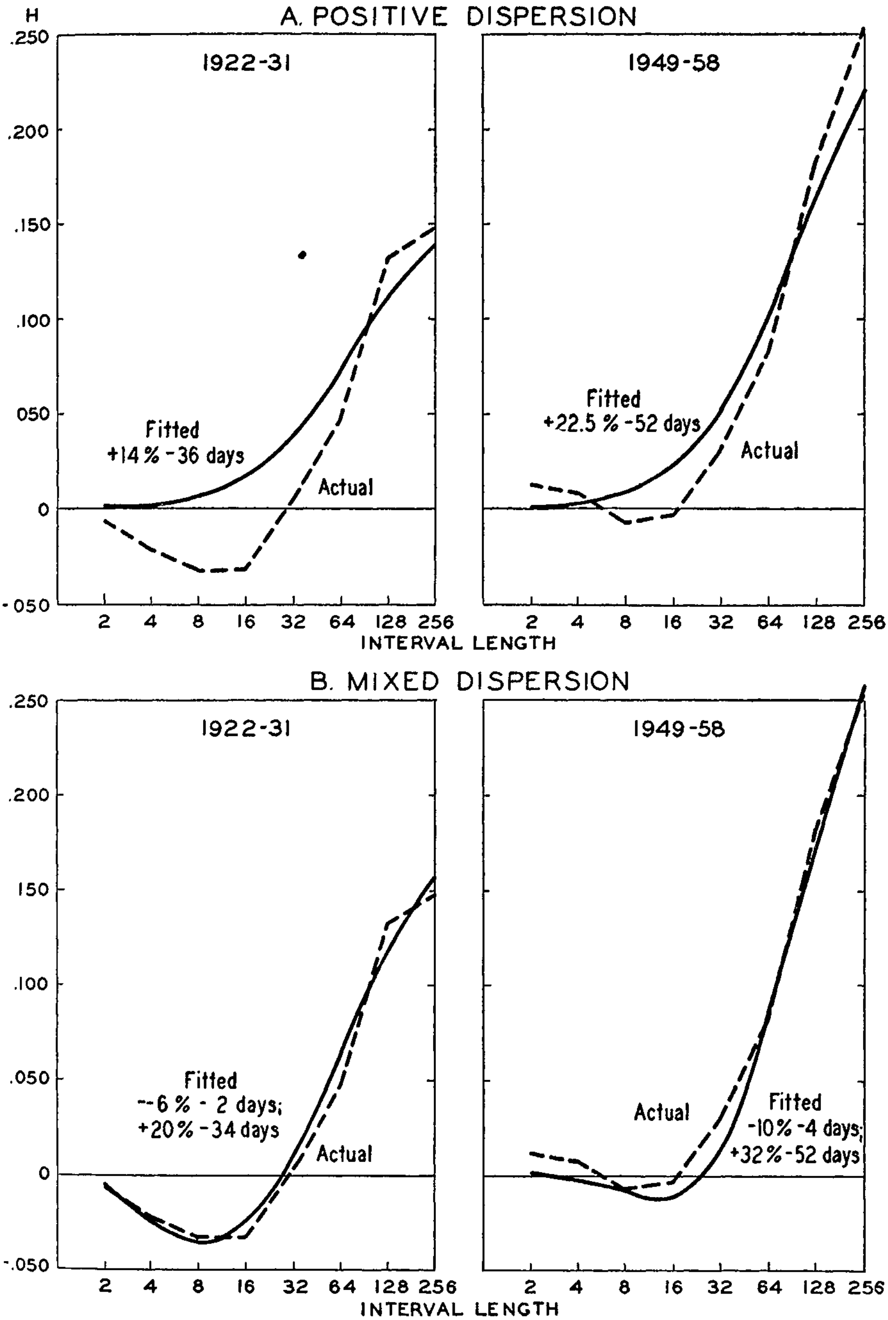
[a] Significant at 5 per cent level.
[b] Significant at 1 per cent level.

These fitted curves correspond to dispersion parameter estimates of 14 per cent dispersed over 36 days for the 1922–31 period, and 22.5 per cent dispersed over 52 days for the 1949–58 period.

The fit is fairly good. The general character of the empirical curve conforms rather well to the fitted theoretical curve. The negative H-values at low interval lengths are definitely anomalous, however. A much better fit is achieved if it is assumed that the weights of the lower portion of the dispersion span are negative. This might result from profit-taking by quick-turn traders, or if some group of traders tends to regard most price movements as somewhat excessive. This is compatible with the rest of the theory of anticipatory prices, even though it does introduce an element of trading that is not immediately motivated by market information. The modification results in a moving average process with four parameters; a negative per cent dispersed over a short span, and a positive per cent dispersed over a longer span. The curve of H-values resulting from this mixed dispersion pattern is approximately the sum of the curves which result from the two dispersions taken separately. The parameter estimates for the mixed dispersion patterns for the two periods are: —6 per cent dispersed over 2 days

[8] Phillipe Berthet and Claude S. Brinegar.

CHART 3.—Comparison of Empirical and Fitted Curves of Index of Continuity



and +20 per cent dispersed over 34 days, for 1922–31, and −10 per cent dispersed over 6 days and +32 per cent dispersed over 52 days, for 1949–58. The actual and fitted curves are tabulated below and shown in Chart 3B:

Interval length	1922–31 Period		1949–58 Period	
	Empirical	Fitted	Empirical	Fitted
2	−.006	−.005	.012	.001
4	−.022	−.023	.008	−.002
8	−.033	−.036	−.007	−.007
16	−.032	−.024	−.003	−.011
32	.005	.010	.030	.015
64	.048	.063	.082	.082
128	.133	.118	.181[a]	.172
256	.148	.156	.253[b]	.256

[a] Significant at 5 per cent level.
[b] Significant at 1 per cent level.

The degrees of freedom are reduced, but the improvement in fit seems to be sufficiently pronounced to warrant the introduction of the more complex pattern of dispersion.

The two periods yield strikingly similar patterns of continuity, and may be combined to yield a single estimate of −8 per cent dispersed over 4 days and +27 per cent dispersed over 45 days. This implies that 81 per cent of the price effect of demand and supply influences occurs on a single day, presumably the day the influences occur and/or the day they become known to the majority of traders in the market. There is then some reaction away from this price movement, even though the initial movement had been insufficient to bring the price to equilibrium. Finally corrective movements occur, resulting in total movement which is just appropriate. If this is a true picture of the operation of the corn futures market, then it approaches very closely the ideal.

We do not yet have a mathematically determined test of goodness of fit, whereby we can evaluate the fit achieved. Other specifications of process might yield an even better fit than is obtained here, though this does not seem very likely. But the most plausible alternative hypothesis—the Taussig theory—can be rejected simply on the basis of the general character of the curve of indexes of continuity.

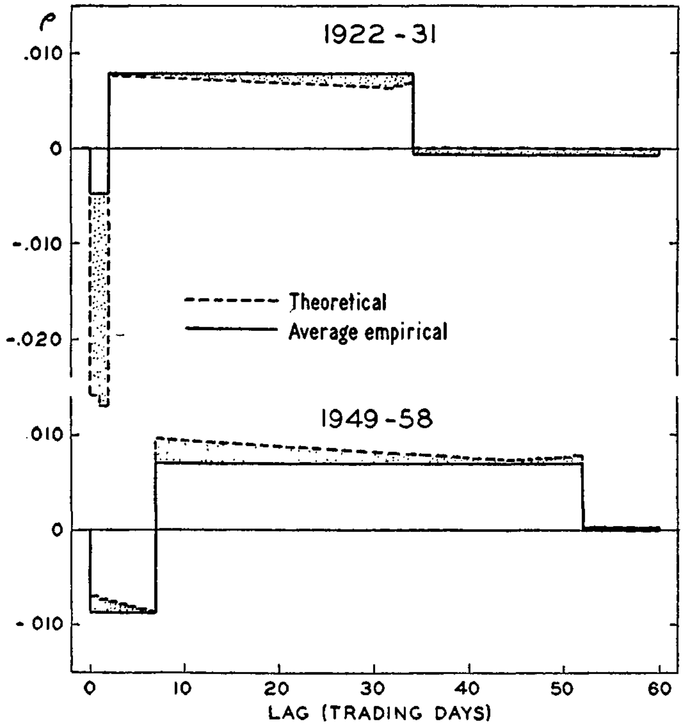
If the dominant source of price movement were the bounding of price up and down between the edges of a penumbra, the index of continuity should have high positive values at low interval lengths and high negative values at longer interval lengths. If most movements result from over-excited response to minute stimuli and what might be called panicky reactions to previous movements, whether or not consciously manipulated, the index of continuity would have high negative values at *all* interval lengths. Movements of the type Taussig speaks of may occur, occasionally, and the empirical content of the theory consists in saying that these are the dominant sources of movement, or that they occur often enough to have adverse effects on traders and on the resource allocation function of the market. This is disproved by the high *positive* values of the index of continuity at long interval lengths.

The test of continuity seems to demonstrate the existence of a moving average stochastic process of the type postulated by Working's theory of anticipatory prices. At the same time it disproves the theory of Taussig and other critics of futures markets. Correlogram analysis failed to do either of these things. We cannot on this account dismiss the correlogram test completely, however. In order that we may accept the moving average process, it is necessary to show that the empirical correlogram is not inconsistent with the theoretical correlogram which is implied by such a process.

Theoretical correlograms for the moving average processes estimated by the index of continuity method were computed for the two 10-year periods. (The negative dispersion span for 1949–58 was increased from 6 days to 7 days in order to include the second large negative correlation coefficient.) The empirical correlation coefficients over the spans so determined were then averaged and the empirical and theoretical correlograms compared. The results are shown in Chart 4. There is one major discrepancy, for the negative dispersion span for the 1922–31 period. Other than that, the agreement is close. This analysis lends further support to the hypothesis that the moving average is the true type of process in operation.

Perhaps an estimating procedure based on the smoothed correlogram could be constructed. Moving average processes of simple types, such as the rectangular, have associated with them correlograms which are also rather simple and reflect the pattern of the weights of the process. Autoregressive processes have correlo-

CHART 4.—Comparison of the Theoretical Correlograms Associated with the Estimated Dispersion Patterns and the Empirical Correlograms Averaged over the Dispersion Spans



grams which are damped oscillatory. The empirical correlogram may have a pattern of one of these types, hidden by random elements which smoothing would eliminate. It is surprising to find virtually no discussion of correlogram smoothing in the literature of time-series analysis, in view of the importance of smoothing in this field. There are hazards in the use of the smoothed correlogram, and there may be insurmountable obstacles, but it is an intriguing possibility.

SUMMARY AND CONCLUSIONS

Analysis based on the index of continuity has demonstrated the existence of a high-order, low-weight moving average stochastic process generating price changes, as envisioned by Working's theory of anticipatory prices. At the same time it has denied the existence of the excessive fluctuation supposed to occur by many critics of futures markets. Autocovariance analysis appeared unable to do either.

These statistical results can be given the economic interpretation that market price changes are closely tied to market news, which tends to be a true reflection of changing demand and supply conditions. But this interpretation rests on the assumption that price can not wander away from equilibrium and remain away for long. For if it could, price movements could be oriented about misinformation or pure guesses, provided only that the misinformation or guesses be random. Taussig and Working both assume that equilibrating forces soon assert themselves, but other critics of futures trading may disagree, and to them much of the argument of this paper may appear tautological. Whatever interpretation one places on the moving average process, it is not to be taken as the only imperfection possible in futures markets.

The potential scope of application of the index of continuity cannot be accurately predicted at this time. It may be that few markets are as near the ideal as are the corn and wheat futures markets, and that any further departure from the ideal introduces factors which will have to be detected and analyzed by other means. On the other hand, the index may prove to be a versatile tool for the analysis of time series from many fields of study, and may be capable of detecting some processes other than the moving average.

CITATIONS

1 F. W. Taussig, "Is Market Price Determinate?" *Quarterly Journal of Economics,* May 1921, pp. 394–411.

2 Holbrook Working, "New Ideas and Methods for Price Research," *Journal of Farm Economics,* Dec. 1956, pp. 1427–36, and "A Theory of Anticipatory Prices," *American Economic Review,* May 1958, pp. 188–99.

3 Arnold B. Larson, "Evidence on the Temporal Dispersion of Price Effects of New Market Information" (diss. Stanford University, Calif., 1960).