SHIFTING CULTIVATION ON THE FARM? DEGRADING FARM PRACTICES AND OPTIMAL LONG TERM LAND REHABILITATION

A preliminary investigation.

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ABSTRACT

If farming practices are degrading the land by enhancing salinisation and soil erosion, and the appropriate way to deal with the problem is by planting trees or regenerating native bush, farmers are faced with several questions. Assuming a rather homogenous area where cropping yields have been decreasing, when should a farmer stop cropping and start planting? Conversely, when should cropping be resumed? On a long term basis, what is the economically optimal pattern between cropping and land rehabilitation? How do differences in soil quality and related production and rehabilitation rates affect management outcomes? The basic question is first investigated in terms of optimal switching times, when switching incurs a cost, then optimality conditions are derived for the three-stage problem. Finally, the practical relevance of the study is assessed given the real situations farmers face. Namely, spatial synergies, such as alley-cropping, are likely to be preferred to temporal solutions. But the time dimension will never be absent from sustainable farm management.

Keywords: Sustainable agriculture; optimal control; land rehabilitation; salinity management; phase farming.

1 INTRODUCTION : PROBLEM DESCRIPTION

1.1 Phase farming: a typical example

- Generics

The problem addressed here is best introduced by the following example. Consider a farmer and a homogenous area of flat land dedicated to annual cropping. Over the years, cropping degrades the land in some way, thereby reducing yields. Although falling yields may be offset by increased inputs, this raises costs and ultimately reduces profits. At some stage, the land is in such poor condition that cropping must be stopped and, unless the land be abandoned, some form of land rehabilitation is initiated. This assumes no land use other than farming is profitable. During the rehabilitation phase, over the years, the land and its productive potential is gradually restored, but at the cost of forgone profits from not cropping. Thus, during rehabilitation, two opposing forces are at work. On the one hand, increasing land quality implies increasing future yields and profits, while, on the other hand, forgone profits from not cropping accumulate. At some point, when the land has recovered enough of its productive potential, it becomes preferable again to revert to cropping. And the cycle starts over again.

This description is not complete, however. At the time of the first switching, from cropping to rehabilitation, there are costs associated, typically in the form of an up-front investment. Likewise, another up-front investment is associated with the second switch, from rehabilitation back to cropping. At both times, switching from one activity to the other is not frictionless, it incurs a cost.

The problem then is to know what are the optimal switching times from one activity to the other such that some performance function of the farm is maximised over a given time horizon. For instance, this function could be the sum of discounted profits over the whole period, or the value of the land at terminal time. The problem may have a finite horizon, relating to the farmer's retirement, or an infinite one, relating to the sustainability of farming practices for future generations.

Choosing when to switch is not the only decision available. The type or intensity of rehabilitation, which may be more or less costly, is also a decision variable. Likewise, cropping intensity, or the type of cropping system, may affect the rate of land degradation over time.

- *Specifics*

Many real-world examples correspond to this type of problem. One of importance in Western Australia relates to soil salinity in the dryer wheatbelt areas. Clearing of the original vegetation, deep-rooted woody perennials, and replacement by short-rooted annual crops and pastures, have knocked the water cycle system off balance. Deep-rooted shrubs and trees tapped water down to some depth and their high level of evapotranspiration, due to the hot and dry climate, kept the level of saline groundwater low. With the trees and shrubs gone, the saline watertable rose closer to the surface, eventually in some places hitting the crops' root systems. Where this has happened, yields or profits per hectare have fallen (Williams, 1989; Hall & Hyberg, 1991; Ferdowsian et al., 1996; State of the Environment, 1996).

One solution CALM[1] and others are looking into for reversing the process is the planting of oil mallee trees (Bartle et al., 1996). This tree species has been viewed with some hope because not only should it reverse the salinisation process, it should also produce revenues to the farmer by yielding essential oils for the solvent industry. This is obtained by processing the leaves of the oil mallee.

In this case, the cost of switching from cropping to rehabilitation by trees mainly involves the planting of the trees and their protection in early years. With the reverse switch, a cost may be associated with removing the stumps, either mechanically or otherwise. However, if new technologies and new markets allow for the wood to fetch a high enough price, the cost associated with this second switch may be negative, meaning a positive net benefit.

Similar problems may involve restoration of land fertility in the form of organic matter content, soil structure (due to soil compaction, hard pans, waterlogging, and so forth) and water retention capacity. The idea here is different from the more classical remedies, where restoration of some aspect of land quality takes the form of inputs. Nutrient deficiency can be restored by fertiliser applications; water deficiencies, where

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possible, by irrigation; acidity, by liming. In the present case, restoration demands a time-consuming function excluding, partially or totally, the initial most profitable activity.

Obviously, however, this setting is neither general nor always appropriate.

1.2 Qualifications and relevance

Solutions to land degradation may be time-dependent or space-dependent, or both. The problem stated as above is time-dependent. An obvious alternative to dense tree plantations in alternation with cropping is combining both simultaneously into some agroforestry pattern, such as alley cropping. Another possibility is choosing strategic paddocks where trees should be planted on a permanent basis, allowing crops to thrive in adjacent paddocks. Many other configurations are possible, depending on local topography and hydrogeology.

What we are concerned here is therefore with a special case where switching from cropping to tree plantations and back over time would make sense. Such a system would produce a form of shifting cultivation on the farm, whereby, once the system has reached an equilibrium (or more rigorously, a steady state), an optimal proportion of the farm would be in trees and another in crops at any time. Such a scenario heavily depends, however, on at least three assumptions: the land is homogenous, the topography is flat, and relative prices and costs remain constant over time. This means that only vertical movements of saline groundwater are relevant and not horizontal movements. In reality, land is hilly, and groundwater moves from higher to lower areas. As we shall see, however, even with such simplifications, the problem is not an easy one to solve.

2. PROBLEM CHARACTERISATION

2.1 Previous investigations

This type of problem seems to have been first encountered in resource management in problems involving a switch to backstop technologies in extractive industries (Hoel, 1978; Dasgupta *et al.*, 1982). The problem investigated is that of

exercising some control over the time at which such technologies become profitable. In other contexts, problems of this kind have involved delivery lags associated with the acquisition of new capital goods, when some control over such lags is sought (Rossana, 1985). A general characterisation has been given by Tomiyama (1985) for two-stage optimal control processes. It was extended by Tomiyama and Rossana (1989) in the case where the performance function explicitly depends on the switching time. The introduction of non-zero switching costs was introduced by Amit (1986) in a key application that involved the switching from primary to secondary petroleum recovery. Kamien and Schwartz, in the second edition of their book (1991), point out that such problems are somewhat analogous to the existence of discrete jumps in the state variables of the system. An extension to multi-stage switching problems is carried out by Babad (1995), using multiprocess theory.

A slightly different version of this problem was tackled by Hertzler (1990). The main difference with our problem was the non-existence of a cost to the switching function (in the notation below, the ϕ function was always zero). The procedure developed was applied to the long term management of soil acidity using liming as a control (Hertzler, 1995). The problem was to know when to lime and when to stop liming, given dynamic interactions between topsoil and subsoil acidity. In this formulation, the problem was concave in time and general solutions could be identified. In our case, as will appear, this is no longer true. Also, Hertzler solves for the steady-state, assuming the system has got there. In our case, the world may have changed considerably before a steady-state is ever reached. The important aspect is the first stages of the transition.

Other work has tackled the economics of land rehabilitation and more particularly salinity abatement. Examples are Gomboso and Hertzler (1991) and Hertzler and Barton (1992). However, these approaches focused on the management of the rehabilitation phase itself. The switching problem was not considered. Similarly, Wang and Lindner (1990) examine the rehabilitation of degraded rangelands with stochastic effects on range re-growth, but again with no switching problem.

2.2 General formulation

- The two-stage problem

The basic formulation of the problem as described above is given by Amit (1986). It covers only the first two periods and involves only one switch. It can be written as follows:

$$(2.1) \quad J = \max \int_{t_0}^{t_1} f^1[t, x(t), u(t)]\, dt + \int_{t_1}^{t_2} f^2[t, x(t), u(t)]\, dt - \varphi\,[t, x(t_1), u(t_1)]$$

subject to

$$(2.2) \quad x'(t) = \begin{cases} g^1[t, x(t), u(t)] & \text{for} \quad t_0 \le t \le t_1 \\ g^2[t, x(t), u(t)] & \text{for} \quad t_1 \le t \le t_2 \end{cases}$$

$$x(t_0) = x_0 : t_1 , x(t_1), t_2 , x(t_2) \text{ free.}$$

In this formulation, J measures the net present value of the land. f^1 and f^2 are the two objective functions associated, in our example, respectively with profits from cropping and with revenues from mallee oil production. The function φ represents the cost of switching from cropping to rehabilitation, namely in terms of tree planting and initial protection. The switching time is t_1 and separates the first cropping period $[t_0, t_1]$ from the second rehabilitation period $[t_1, t_2]$. The function x(t) is the state variable and represents land quality or, in our example, salinity measured as the depth of a saline watertable. The function u(t) is the control variable. For the rehabilitation phase, it may represent tree density or number of trees per hectare.

The constraints are given by differential equations, called the equations of motion of the system, and describe the dynamics of land quality depending on what phase is active, cropping or rehabilitation. In our example, the first equation, given by the function g^1, would typically be negative and describe a decreasing saline watertable

depth. The second equatic would be positive and describe an increasing watertable depth as a function of time, the current level and tree density.

This problem seeks to determine the value of t_1 (when to switch) and the value of u(t), say tree density, such that J be maximised over the entire period spanning from initial t_0 to final t_2. A general procedure does exist for defining optimality conditions, although in practice solutions come by through the use of numerical algorithms (Wong *et al.*, 1985; Teo & Jennings, 1991; Stewart, 1992).

- *The multi-stage problem*

Our problem as described in the introduction is a generalisation of Amit's (1986) and Tomiyama's (1985) problems, in that it involves not two, but many, possibly an infinite number of stages. In light of Babad's (1995) work however, it remains a two-process (or two-phase) problem, with cropping and rehabilitation being the two processes.

For clarity, it may be noted that different periodisations are involved in this type of problem. *Phases* or *processes* may designate the type of different activities, such as cropping and rehabilitation. A third phase or process could be imagined in sequence to the other two. *Cycles* are the number of full sets of phases over time, e.g. the cropping-rehabilitation cycle[2]. *Stages* define the number of times a phase or process is considered. A three-stage, two-phase problem would, for instance, look at the following sequence of cropping and rehabilitation, of the form C-R-C. Finally, *periods* count time units, such as years. Each phase usually has a different number of time periods. Outside steady-state conditions, each stage also has a non-constant number of time periods. Thus, our problem is potentially a two-phase, multi-stage, multi-period problem.

Its general formulation must allow for the alternation of the two phases, each of which would typically have different durations (the cropping phases would usually be of different lengths compared to the rehabilitation phases). Furthermore, the duration of each cropping (resp. each rehabilitation) stage would differ, until a steady-state is reached, if ever. The problem can be generalised as follows:

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[2]) Note that *rotations* are another sort of cycle, specific to a given phase. For instance, a fixed sequence of crops during the cropping phase.

$$(2.3)\quad J = \max \sum_{k=0}^{n} \Big\{ \int_{t_{2k}}^{t_{2k+1}} f^1[t, x(t), u(t)]\, dt + \int_{t_{2k+1}}^{t_{2k+2}} f^2[t, x(t), u(t)]\, dt$$

$$- \varphi_{2k}\ [t, x(t_{2k+1}), u(t_{2k+1})]$$

$$- \varphi_{2k+1}\ [t, x(t_{2k+2}), u(t_{2k+2})]\ \Big\}$$

subject to

$$(2.4)\quad x'(t) = \begin{cases} g^1[t, x(t), u(t)] & \text{for} \quad t_{2k} \le t \le t_{2k+1} \\ g^2[t, x(t), u(t)] & \text{for} \quad t_{2k+1} \le t \le t_{2k+2} \end{cases} \qquad \text{and } k = (0, 1, \ldots, n)$$

$$x(t_0) = x_0\,; \qquad t_{2k+1}\,,\ x(t_{2k+1}),\ t_{2k+2}\,,\ x(t_{2k+2}) \text{ free,}$$

where, this time, both switches appear explicitly and recurrently. The optimality conditions for this generalised problem are analogous to the two-stage one, but, of course, more involved.

Using these general guidelines, optimality conditions may be derived, starting with the two-stage specification.

3. DERIVING OPTIMALITY CONDITIONS

3.1 The basic method, two-stage problem

For the basic two-stage problem, the general procedure is to first form the Hamiltonians H_1 and H_2 of the system (Amit, 1986; Kamien & Schwartz, 1991):

$$(3.1)\quad \begin{aligned} H_1 &\equiv f_1 + \lambda_1 g_1 \quad \text{for} \quad t_0 \le t \le t_1 \qquad \text{where } \lambda_1 = \lambda_1(t) \\ H_2 &\equiv f_2 + \lambda_2 g_2 \quad \text{for} \quad t_1 \le t \le t_2 \qquad \text{where } \lambda_2 = \lambda_2(t) \end{aligned}$$

the interpretation of which comes in terms of dynamic annual profits, meaning the current profits affected by all discounted future losses and gains. λ_1 and λ_2, functions of time, are Lagrange multiplier functions (or the costate variables) and measure the

marginal value of land quality (the state variable), given f and g. There are two standard optimal control conditions for optimality, namely:

$$(3.2)\quad \begin{array}{llll} H_{u,1} = 0 & \text{and} \quad \lambda'_1 = -H_{x,1} & \text{for} & t_0 \le t \le t_1 \\ H_{u,2} = 0 & \text{and} \quad \lambda'_2 = -H_{x,2} & \text{for} & t_1 \le t \le t_2 \end{array}$$

plus two conditions specific to this type of problem:

$$(3.3)\quad H_1(t_1) - \phi_t(t_1) = H_2(t_1) \quad \text{if} \quad t_0 < t < t_2 \ \text{ and } \ t_0 \neq t_1 \ \text{ and } \ t_0 \neq t_2$$

$$(3.4)\quad \lambda_1(t_1) + \phi_x(t_1) = \lambda^+_2(t_1)$$

with the usual terminal conditions written as:

$$(3.5)\quad H_2(t_2) = 0, \qquad \lambda_1(t_2) = 0 \ \text{ or } \ \lambda_2(t_2) = 0$$

where the notation H_u means the partial derivative of H relative to u, or

$H_u = \partial H / \partial u$, and

$\lambda' = d\lambda / dt$,

$\phi_x = \partial\phi / \partial x$

and the notations $\lambda^-(t_1)$ and $\lambda^+(t_1)$ mean the value of $\lambda(t)$ just to the left and just to the right of t_1 respectively, knowing that in general they are not equal (whence the analogy to problems with jumps in their state variables).

Condition (3.3) states that if there is a time t_1 at which H_1 less the marginal cost of switching from g^1 to g_2 equals H_2, then it is optimal to switch at t_1. Condition (3.4) then adds that if such a switch occurs, then the marginal value of the state variable (land quality) evaluated according to f_1 and g_1, plus the marginal cost with respect to land quality, must equal the marginal value of land quality evaluated according to f_2 and g_2.

3.2 Application to the three-stage cropping - rehabilitation problem

A general solution procedure to our problem is yet to be obtained. Here, we restrict ourselves to an initial investigation, exploring optimality conditions in the first three stages. It may be realised that the corresponding time period may span well over 100 years, meaning that the characteristics of further cycles will depend dramatically on uncertainties in future technologies and prices.

The profit function during the cropping phase may be written in the following general form:

(3.6) $\pi(X,t,u,X_0,t_0)\, e^{r(t-t_0)}$

while the revenue function during the rehabilitation phase may be written as

(3.7) $R(X,t,u,X_0,t_0)\, e^{r(t-t_0)}$

where X is the index for land quality, namely salinity or watertable depth, u is some control variable, like cropping intensity or tree density, X_0 and t_0 are initial land quality and initial time respectively, and r is the discount rate.

The general formulation for the 3-stage problem then is:

$$
\begin{aligned}
(3.8)\quad J = \text{Max} & \int_{t_0}^{t_1} \pi_1(X,t,u,X_0,t_0)\, e^{r(t-t_0)}\, dt \\
& + \int_{t_1}^{t_2} R_1(X,t,u,X_1,t_1)\, e^{r(t-t_0)}\, dt \\
& + \int_{t_2}^{t_3} \pi_2(X,t,u,X_2,t_2)\, e^{r(t-t_0)}\, dt \\
& - \phi_1(X_1, t_1, u)\, e^{r(t-t_0)} \\
& - \phi_2(X_2, t_2, u)\, e^{r(t-t_0)}
\end{aligned}
$$

subject to the constraints

$$X' = \begin{cases} f_1(X, u, t) & t_0 \le t \le t_1 \\ f_2(X, u, t) & t_1 \le t \le t_2 \\ f_3(X, u, t) & t_2 \le t \le t_3 \end{cases}$$

X_0, t_0 given

$X(t_1)$, t_1, $X(t_2)$, t_2, $X(t_3)$, t_3 free

$X(t) \ge 0$ for all t

The Hamiltonians of the system are given in each stage by:

- in stage 1, cropping phase:

$$H_1(t) = \pi_1(X,t,u,X_0,t_0)\, e^{-r(t-t_0)} + \lambda_1(t)\, f_1(X, u, t) \tag{3.9a}$$

- in stage 2, rehabilitation phase:

$$H_2(t) = R_1(X,t,u,X_1,t_1)\, e^{-r(t-t_0)} + \lambda_2(t)\, f_2(X, u, t) \tag{3.9b}$$

- in stage 3, again cropping phase:

$$H_3(t) = \pi_2(X,t,u,X_2,t_2)\, e^{-r(t-t_0)} + \lambda_3(t)\, f_3(X, u, t) \tag{3.9c}$$

The procedure involves "solving backwards". Assuming t_2 to be found already, we determine the optimal solution for phase 3, by setting the partial derivatives of the Hamiltonians (H), the Lagrangians (L) and the time derivatives of the costate variables (λ) equal to zero.:

(3.10) $$\frac{\partial H_3}{\partial u} = \frac{\partial \pi_2}{\partial u}(\,.\,)\, e^{-r(t-t_0)} + \lambda_3(t)\, \frac{\partial f_3}{\partial u}(X, u, t) \quad \text{where } (\,.\,) = (X, u, t, X_i, t_i) \;\; i = [1, 2, 3]$$

$$\lambda'_3(t) = -\frac{\partial \pi_2}{\partial x}(\,.\,)\, e^{-r(t-t_0)} - \lambda_3(t)\, \frac{\partial f_3}{\partial x}$$

$$\lambda_3(t) = 0$$

$$\frac{\partial L_3}{\partial x} = \frac{\partial H_3}{\partial x} + \mu_3 = 0$$

$$\frac{\partial H_2}{\partial u} = \frac{\partial R_1}{\partial u}(\,.\,)\, e^{-r(t-t_0)} + \lambda_2(t)\, \frac{\partial f_2}{\partial u}(X, u, t)$$

$$\lambda'_2(t) = -\frac{\partial R_2}{\partial x}(\,.\,)\, e^{-r(t-t_0)} - \lambda_2(t)\,\frac{\partial f_2}{\partial x}(X, u, t)$$

$$\frac{\partial L_2}{\partial x} = \frac{\partial H_2}{\partial x} + \mu_2 = 0$$

$$\frac{\partial H_1}{\partial u} = \frac{\partial \pi_1}{\partial u}(\,.\,)\, e^{-r(t-t_0)} + \lambda_1(t)\,\frac{\partial f_1}{\partial u}(X, u, t)$$

$$\lambda'_2(t) = -\frac{\partial \pi_2}{\partial x}(\,.\,)\, e^{-r(t-t_0)} - \lambda_1(t)\,\frac{\partial f_1}{\partial x}(X, u, t)$$

$$\frac{\partial L_1}{\partial x} = \frac{\partial H_1}{\partial x} + \mu_1 = 0$$

The Lagrangian multipliers μ_1, μ_2, μ_3 are determined by the fact that the water level is not allowed to be negative (that is, rise above the surface). We therefore need to incorporate the Lagrangians. We also have:

(3.11) $\mu_1.X(t) = 0$ for $t_0 \le t \le t_1$

$\mu_2.X(t) = 0$ for $t_1 \le t \le t_2$

$\mu_3.X(t) = 0$ for $t_2 \le t \le t_3$

We could also introduce an extra constraint of the type $X(t) \ge X(t_0)$ for the start of the third stage (second cropping phase).

The matching conditions at t_2 are given by:

(3.12) $$H_2(t_2) - \frac{\partial \varphi_2}{\partial x}(X_2, t_2, u)\, e^{-r(t-t_0)} + r\,\varphi_2(X_2, t_2, u)\, e^{-r(t-t_0)} = H_3(t_2)$$

$$\lambda_2(t_2^-) + \frac{\partial \varphi_2}{\partial x}(X_2, t_2, u_2)\, e^{-r(t-t_0)} = \lambda_3(t_2^+)$$

If there is no such t_2, then the switch from rehabilitation back to cropping does not take place. This could happen, for instance, if revenues from trees were high enough.

The matching conditions at t_1 are given by:

(3.13) $H_1(t_1) - \frac{\partial \varphi_1}{\partial x}(X_1, t_1, u)\, e^{-r(t_1 - t_0)} + r\,\varphi_1(X_1, t_1, u)\, e^{-r(t_1 - t_0)} = H_2(t_1)$

$$\lambda_1(t_1) + \frac{\partial \varphi_1}{\partial x}(X_1, t_1, u_1)\, e^{-r(t_1 - t_0)} = \lambda_2(t_1^+)$$

If there is no such t_1, then the switch from cropping to rehabilitation does not take place, and the farmer simply goes on cropping. This could happen, for instance, if cropping was very profitable and if trees yielded no revenue and produced slow land rehabilitation rates.

4. EXPLORING A SPECIFIC PROBLEM FORMULATION

4.1 Specifying the functions; information needs

Considering first the cropping phase, it appears reasonable to define a profit function $\pi(.)$ related to crop yields, prices, and costs, typically:

(4.1) $\pi(.) = [p.Y(X_t) - C]\, e^{-rt}$ where r is the discount rate of future earnings.

X_t measures the depth of saline watertable and Y is crop yield. This formulation would assume price and cost invariance, where in reality one should write $p = p(t)$ and $C = C(t)$, possibly stochastic.

As a function of soil salinity, that is, as an inverse function of watertable depth, crop yields may be expected to follow a falling logistic curve, of the type:

(4.2) $Y(X_t) = \frac{Y_{max}}{(1 + ve^{-wX})}$ with $Y(X_0) = Y_0$ given

where Y_{max} is the maximum average crop yield corresponding to zero salinity, w the speed at which yield falls with increasing salinity and v a scaling parameter. The reason for a logistic is simple enough. Before the saline watertable hits the zone where the root system of the crop is sensitive, decreases in watertable depth do not much affect yields. When the water level reaches this zone, things start happening very quickly,

until, in the last zone, crop yields are already close to zero so that any further increases in water levels hardly have any further effect. As will appear later logistic functions may lead to difficulties in integration, and may have to be replaced by simpler, even if less appropriate, functions, such as an adjusted polynomial.

The equation of motion during the cropping phase may plausibly be reduced to a constant rate, independent of time, such that

(4.3) $X'(t) = \delta < 0$ for $t_0 \leq t \leq t_1$

meaning that, under cropping, irrespective of cropping intensity or rotation types, land degrades at a characteristic rate, δ (in meters/year), arguably site specific.

In contrast, under rehabilitation with block tree plantations, the rate at which the watertable drops should depend both on the current level and on the control variable, tree density. The relationship is of the logistic type, thus:

(4.4) $X'(t) = \rho X_t(1 - X_t/m).D_t$ for $t_1 \leq t \leq t_2$

where ρ is the intrinsic rate of watertable fall per unit time, m is the maximum distance of watertable to the surface as a result of tree pumping, and D_t is tree density, in trees/ha. Tree density D_t is taken to affect reduction in watertable levels linearly. This is true only within reasonable bounds, namely before tree competition for water sets in. References for the rehabilitation phase are still few. Work like that of Schofield (1990) and Bell et al. (1990) is hard to come by.

The revenue function during rehabilitation will depend on the average revenue per tree (R) in terms, for instance, of leaf material produced for mallee oil, on the growth function of the trees G(t), and on tree density D_t. This yields the expression:

(4.5) $Rev(t) = R.G(t).D_t.e^{-rt}$

Again, in reality, we would have R = R(t), possibly stochastic. G(t) is the harvestable growth function of the trees as a function of time. The fact that G(t) does not appear as a control variable means that the tree species has been pre-specified on other grounds, presumably agronomic. A more general formulation would also seek to optimise the

tree species as a function of some characteristic, but this involves further complications. G(t) may be given by a spline function such as:

$$(4.6)\quad G(t) = \underline{G}\,(1 - e^{g(t-\tau)}) \quad \text{if } t > \tau$$
$$= 0 \quad \text{otherwise.}$$
$$G(t_0) = G_0 \text{ given}$$

where $\underline{G}$ is the maximum canopy mass, or leaf production per year, and g is the growth rate of the canopy as a function of time, and τ is the time lag before which the leaves yield no revenue. Note that although factors like salt concentration in the watertable affect tree growth and rehabilitation efficiency, they are site specific and can be considered as given.

Finally, the cost of switching from cropping to rehabilitation may be considered a linear function of the number of trees, where to a fixed cost per ha C_0 is added a unit cost C_1 of planting and protection per tree, such that $\phi = C_0 + C_1.D_t$.

4.2 Formulating a specific two-stage rehabilitation problem

With these assumptions, the two-stage problem becomes, with $t_0 = 0$:

$$(4.7)\quad J = \max \int_{t_0}^{t_1} [p.\frac{Y_{max}}{(1+re^{-wX})} - C]\, e^{-rt} dt + \int_{t_1}^{t_2} R.\underline{G}\,(1 - e^{-g(t-\tau)}).D_t\, e^{-rt}\, dt - [C_0 + C_1.D_t]\, e^{-rt_1}$$

subject to the constraints

$$X'(t) = \begin{cases} \delta < 0 & \text{for} \quad t_0 \le t \le t_1 \\ pX_t(1 - X_t/m).D_t & \text{for} \quad t_1 \le t \le t_2 \end{cases}$$

$X(t_0) = X_0$; and t_1 , $X(t_1)$, t_2 , $X(t_2)$ free

There is only one control in the rehabilitation phase, namely tree density, and no control in the cropping phase. This is because once cropping is chosen, cropping practices do not affect much the dynamics of watertable movements.

Quite obviously, the operational value of such a model hinges on the validity of the functional forms specified in equations (4.2) to (4.6). Much work needs yet to be done to reduce the uncertainty surrounding them, and to lend credibility to numeric simulations.

4.3 Problem formulation for more than two stages

The equivalent problem for the general multi-stage case is, again with $t_0 = 0$:

$$(4.8) \quad J = \max \sum_{k=0}^{n} \Big\{ \int_{t_{2k}}^{t_{2k+1}} \Big[p \frac{Y_{max}}{(1+ve^{-uX})} - C\Big] e^{-rt}\, dt + \int_{t_{2k+1}}^{t_{2k+2}} R.\underline{G}\,(1 - e^{-g(t-\tau)}).D_t\, e^{-rt}\, dt$$
$$- [C_0 + C_1.D_{t,2k+1}]\; e^{-rt_{2k+1}}$$
$$- [K_0 + (K_1 - V)\, D_{t,2k+1}]\, e^{-rt_{2k+2}} \Big\}$$

subject to the constraints

$$x'(t) = \begin{cases} \delta < 0 & \text{for} \quad t_{2k} \le t \le t_{2k+1} \\ \rho X_t(1 - X_t/m).D_t & \text{for} \quad t_{2k+1} \le t \le t_{2k+2} \end{cases}$$

$k = (0, 1, \ldots, n) \in N$

$X(t_0) = X_0$ given; $\quad t_{2k+1}$, $X(t_{2k+1})$, t_{2k+2}, $X(t_{2k+2})$ all free

where the new variables K_0, K_1 and V refer to the second switch, from rehabilitation back to cropping. K_0 is a fixed per hectare cost related to land preparation and K_1 a variable per tree cost, typically related to de-stumping. V is the stock per unit sale value of the trees on removal for cropping. It may be that $V > K_1 + K_0$ and so the cost of switching from rehabilitation back to cropping may be negative, meaning a net positive benefit per hectare. The effects of such a switching benefit on, for instance, the length of cropping phases is not straightforward. The model would need to be solved.

5. CONCLUDING COMMENTS

The problem : multistage two-phase optimal control with switching costs

The type of problem examined in this paper is a difficult one. No solution has been attempted as yet. Rather, a rigorous formulation and a characterisation of optimality conditions have been sought as a preliminary step. It departs from standard optimal control problems in several ways. It considers more than one stage, with a switch from one stage to the other. The profit function as of the second stage explicitly depends on the timing of the switch. Costs are associated with switches, analogous to discrete jumps in the state variables. Phases have different durations and so do stages of each phase, at least until a steady state is reached, if ever. Due to the multi-periodic nature of each phase (several years), the transition to a steady state becomes a more important problem than the characterisation of a steady-state.

And yet, difficult as it is, this problem is a highly simplified one. Namely, prices and costs have been considered constant, as have revenues from rehabilitation, when they should be variable and stochastic. System dynamics, particularly of watertable movements, and the equation for tree growth are also simplified. No interactions have been considered, and no uncertainties assumed in effects on crop yields or rehabilitation functions.

The approach : phase farming for sustainable agriculture

Quite likely, phase farming is not and will not be a general practice. Many other strategies are available that take advantage of spatial, rather than temporal, synergies. Alley-farming, hedge rows, and site specific strategies taking advantage of topography, hydrology and exposure to winds are examples. However, it is unlikely that the time element will ever be totally absent. Many traditional systems, such as nomadism, transhumance, slash and burn and shifting cultivation, were based on cyclical and recurrent patterns. Nevertheless, technological innovations and a structurally changing economy make very long term strategies somewhat irrelevant. It is not clear how this conundrum may be solved. Possibly, limited phase farming, such as ensuring a profitable return to stage three, as considered in this paper, may be sufficient. A

strategy like minimising the cost of any future switch may be an option preserving reversibility and ensuring flexibility. Switch-cost minimisation strategies, subject to uncertain benefits over time, is as yet an unexplored problem.

The future : the economics of sustainable farming management

The time for philosophical considerations on the sustainability of agriculture, and of the economy in general, is likely to draw to its end. Although they have been useful in highlighting the needs for research and attracting the attention of researchers and funding bodies, it is time to take issue at a more operational level. From the quickly developing literature on the subject, and from the exploratory considerations in this paper, several conclusions may be drawn for the future, regarding research of practical value in this field.

Firstly, behind the general idea of sustainability, which acts as a rallying emblem or battle banner rather than as an operational concept (Pannell and Schilizzi, 1996), very different and often extremely complex management issues are involved. More often than not, these issues include long time horizons, time management, uncertainties and stochastic processes, irreversibilities, and highly nonlinear, typically logistic dynamics, all at once. If problems are to be tackled in a tractable manner, they have to be broken up into sub-problems, the cost of which, however, means restricted applicability leading to a burgeoning of specific applications. This paper illustrates the matter.

Secondly, the mathematics involved are becoming so elaborate that more collaboration between economists and mathematicians will be necessary than has been the case in the past. It is simply inefficient for economists to invest the time and energy in mastering the increasingly varied fields of advanced mathematics needed to address sustainability problems. Indeed, those mathematicians who have devoted most of their time and energy to the appropriate methods, and who may be suffering from increased budget cuts, are in rising need of applications to their solution procedures and algorithms. Economists would benefit from such division of labour and collaboration all the more so that interactions with the natural science disciplines, such as ecology, hydrology and soil science, will be increasingly needed. More so than in the

past, there will be trade-offs between convenient, conventional simplifications and relevance to real-world management problems.

Thirdly, general qualitative results, such as those using implicit functional forms only defined by their mathematical properties, will become increasingly irrelevant, as the outcomes of not only the magnitude, but the direction of changes become parameter and number specific. For instance, optimal management rules valid for salinity abatement may not be valid for abatement of soil erosion; those valid for saline watertable abatement in monotonously sloping land may not be valid in undulating regions; or those valid for wind erosion abatement may not be valid for water erosion abatement, even if the general optimising functional always relates to farm profits. As dynamics are more seriously considered, management strategies become process-specific.

REFERENCES

Amit R. (1986). Petroleum reservoir exploitation: switching from primary to secondary recovery. *Operations Research*, 34(4): 354-49.

Australian Government, Department of the Environment, Sport and Territories (1996). Australia State of the Environment 1996. (Chap. 6 and 10).

Babad H.R. (1995). An infinite-horizon multistage dynamic optimization problem. *Journal of Optimization Theory and Applications*, 86(3): 529-52.

Bartle J.R., Campbell C., and White G. (1996). Can trees reverse land degradation? Paper to the Australian Forest Growers Conference, Mt. Gambier, SA.

Bell R.W., Schofield N.J., Loh I.C., and Ban M.A. (1990). Groundwater response to reforestation in the Darling Range of Western Australia. *Journal of Hydrology*, 119: 179-200.

Chisholm A.H. (1992). Australian agriculture: a sustainability story. *Australian Journal of Agricultural Economics*, 36(1): 1-29.

Dasgupta P., Gilbert R.J., and Stiglitz J.E. (1982). Invention and innovation under alternative market structures: the case of natural resources. Review of Economic Studies, 49: 567-82.

Ferdowsian R., George R.J., Lewis F., McFarlane D.J., Short R., and Speed R. (1996). The extent of dryland salinity in Western Australia. In *Proceedings, Fourth National Conference on the Productive Use and Rehabilitation of Saline Lands.* Promoco Conventions, WA.

Gomboso J. and Hertzler G. (1991). A hydrological-economic modelling approach to dryland salinity in Western Australia. Agricultural and Resource Economics Discussion paper 1/91, The University of Western Australia, Nedlands, WA 6907.

Hall N. and Hyberg B. (1991). Effects of land degradation on farm output. Paper to the 35th Annual Conference of the Australian Agricultural Economics Society, University of New England, Armidale, NSW, February.

Hertzler G. (1990). Dynamically optimal adoption of farming practices which degrade or renew the land. Paper to the 34th Annual Conference of the Australian Agricultural Economics Society, University of Queensland, St. Lucia, Brisbane, 13-15 Feb.

Hertzler G. and Barton J. (1992). Dynamic model of dryland salinity abatement. . Agricultural Economics Discussion Paper 4/92, The University of Western Australia.

Hertzler G. and Tierney W. (1995). Dynamically optimal management of topsoil and subsoil acidity. Agricultural and Resource Economics, The University of Western Australia, Nedlands, WA 6907.

Hoel M. (1978). Resource extraction when a future substitute has an uncertain cost. *Review of Economic Studies*, 45: 637-44.

Kamien M.I. and Schwartz N.L. (1991). *Dynamic Optimization: The Calculus of Variations and Optimal Control in Economics and Management*. North Holland. (Chap. 18)

Pannell D.J. and Schilizzi S. (1996). Sustainable agriculture: a question of eternity, ecology, ethics, economics or expedience? Paper to the 41st Annual Conference of the Australian Agricultural and Resource Economics Society, Gold Coast, Queensland, Australia, 21-24 January.

Rossana R.J. (1985). Delivery lags and buffer stocks in the theory of investment by the firm. . *Journal of Economic Dynamics and Control*, 9: 153-93.

Schofield N.J. (1990). Determining reforestation area and distribution for salinity control. *Hydrological Sciences Journal*, 35(6): 1-9.

Stewart D.E. (1992). A numerical algorithm for optimal control problems with switching costs. *Journal of the Australian Math. Society*, Series B, 34: 212-28.

Teo K.L. and Jennings L.S. (1991). Optimal control with a cost on changing control. *Journal of Optimization Theory and Applications*, 68(2): 335-57.

Tomiyama K. (1985). Two-stage optimal control problems and optimality conditions. *Journal of Economic Dynamics and Control*, 9: 317-37.

Tomiyama K. and Rossana R.J. (1989). Two-stage optimal control problems with an explicit switch point dependence. *Journal of Economic Dynamics and Control*, 13: 319-37.

Wang and Lindner R.K. (1990). Rehabilitation of degraded rangeland under optimal management decisions. Agricultural Economics Discussion Paper 7/90, The University of Western Australia, Nedlands WA 6907.

Wong K.H., Clements D.J., and Teo K.L. (1985). Optimal control computation for nonlinear time-lag systems. *Journal of Optimization Theory and Applications*, 47: 91-107.

Williams J. (1989). Land degradation : evidence that current Australian farming practice is not sustainable. In: Hampson R.J. (ed.), *Proceedings of the 16th National Conference of the Australian Farm Management Society*, Queensland Agricultural College, Gatton.