Seasonal variability and a farmer's supply
response to protein premiums and discounts *

by

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ABSTRACT

This paper extends the analysis of the impact of a system of protein premiums and discounts to that on a farmer's planned production. Despite an unambiguously negative impact on expected profits of equally likely premiums and discounts, supply response to the introduction of such a system is shown to depend on the level of seasonal variability faced by the farmer.

In particular, farmers in regions which are more seasonally-unreliable are likely to feature a negative supply response, whereas those in regions which are more seasonally-reliable are likely to feature a positive supply response. Consequently, it is suggested that, overall, protein payments for what may have encouraged a shift of wheat-growing activity towards more seasonally-reliable areas.
INTRODUCTION

The Australian Wheat Board (AWB) has recently introduced a system of premiums and discounts for protein levels in wheat. With this system higher prices are paid if measured protein exceeds a specified level, while a price discount is applied if measured protein is below a specified level.

For farmers, the impact of this system on income from wheat-growing is complicated by the fact that the relationship between yield and protein depends on uncertain seasonal conditions. In particular, because yield and protein are jointly determined by uncertain season conditions through an inverse relationship (given available nitrogen), a farmer will find that, in the presence of protein payments, seasons of relatively high yield tend to coincide with relatively low protein content and therefore relatively low prices. As shown in Fraser (1996), this negative correlation between price and yield means that the introduction of a protein payments system centred on the protein level associated with a farmer’s initial level of expected yield decreases both the expected level and variability of income.¹

The aim of this paper is to extend the analysis of the impact of a protein payments system to that on a farmer’s planned production. At first glance it may be expected that, for a farmer concerned primarily with the level of expected profits from wheat growing, the negative impact of protein payments on this level would also mean a reduction in planned production. However, in a model of a risk neutral farmer making an optimal planned production decision, it is shown that the actual supply response may be positive or negative depending on the level of seasonal variability, and despite a uniformly negative impact on the level of expected profits. This result arises because the introduction of protein payments modifies the condition
determining optimal planned production in two conflicting ways. First, as recognised above, it introduces a negative effect through the negative correlation between price and yield. But second, by creating the opportunity for the farmer to reduce the probability of a discount and increase the probability of a premium through increased application of nitrogen (which is shared between yield and protein), the system also has a positive effect on the level of planned production. Moreover, the level of seasonal variability determines the relative strength of these two conflicting effects, with the negative effect increasing in magnitude relative to the positive effect with the level of seasonal variability. The potential therefore exists for the balance of these two effects at a lower level of seasonal variability to be reversed at a higher level.

The plan of the paper is as follows. Section 1 develops in detail the model outlined above, focussing in particular on the impact of the protein payments system on the first order condition for optimal planned production by a risk neutral farmer. Section 2 uses numerical analysis to illustrate the role of the level of seasonal variability in determining the direction of this impact. The paper ends with a brief conclusion.
SECTION 1: The Model

The model, based on that developed in Fraser (1996), specifies a farmer’s expected level (Ec(I)) of wheat income per hectare in the absence of protein payments as:

\[ \text{Ec}(I) = \bar{p} \bar{y}(N) \]  

(1)

where: \( \bar{p} \) = expected price per tonne  
\( \bar{y}(N) \) = expected yield per hectare given \( N \)  
\( N \) = level of available nitrogen.

Note that this specification assumes the farmer’s uncertain price and season are independent.

It is further assumed that yield (\( y \)) and protein (\( r \)) are jointly determined by uncertain seasonal conditions through an inverse relationship (given available nitrogen):  

\[ r = \frac{\gamma(N)}{y} \]  

(2)

where: \( \gamma(N) \) = function relating to soil type and available nitrogen (\( \gamma(N) > 0 \)),

and that yield uncertainty (and therefore protein uncertainty) has a multiplicative relationship with seasonal uncertainty (\( \theta \)):

\[ y = \theta \bar{y}(N) \]  

(3)

where: \( E(\theta) = 1 \).

The system of protein payments is specified as:

\[ \bar{P}_H = \bar{p} + x \text{ if } \theta < \frac{\gamma}{r_H \bar{y}} \]  
\[ \bar{P}_M = \bar{p} \text{ if } \frac{\gamma}{r_H \bar{y}} \leq \theta \leq \frac{\gamma}{r_L \bar{y}} \]  
\[ \bar{P}_L = \bar{p} - x \text{ if } \theta > \frac{\gamma}{r_L \bar{y}} \]  

(4)

where: \( r_L \) = critical low protein level
\[ r_H = \text{critical high protein level} \]
\[ \bar{p}_H = \text{expected price with protein premium} \]
\[ \bar{p}_L = \text{expected price with protein discount} \]
\[ x = \text{size of discount/premium}. \]

Finally, the critical protein levels are set symmetrically in relation to the protein level associated with the initial level of expected yield:

\[ r_H - \gamma / \bar{y} = \gamma / \bar{y} - r_L. \tag{5} \]

On this basis, the expected level \( E_1(1) \) of income in the presence of protein payments is given by

\[ E_1(1) = \bar{p}_H \int_{0}^{\gamma / \bar{y}} \theta \bar{y} f(\theta) d\theta + \bar{p} \int_{\gamma / \bar{y}}^{\bar{y}} \theta \bar{y} f(\theta) d\theta + \bar{p}_L \int_{\gamma / \bar{y}}^{\infty} \theta \bar{y} f(\theta) d\theta. \tag{6} \]

Substituting (4) and simplifying gives:

\[ E_1(1) = \bar{p} \bar{y} + x\bar{y}(w_1 - w_3) \tag{7} \]

where

\[ w_1 = \int_{0}^{\gamma / \bar{y}} \theta f(\theta) d\theta \]
\[ w_2 = \int_{\gamma / \bar{y}}^{\infty} \theta f(\theta) d\theta. \]

Since for \( \theta \) symmetrically distributed:

\[ w_3 > w_1, \tag{8} \]

the second term on the right-hand-side of (7) is negative so that for given N:

\[ E_1(1) < \mathcal{Z}_0(1). \tag{9} \]
Consider now the impact of the introduction of the protein payments system on the optimal level of planned production. In the absence of the system expected profit (E_o(\pi)) per hectare is given by:

$$E_o(\pi) = E_o(I) - c N - F$$

where:

- $c$ = cost per unit of nitrogen
- $F$ = fixed costs per hectare.

Consequently, optimal planned production is given by:

$$\frac{\partial E_o(N)}{\partial N} = c.$$  \hspace{1cm} (10)

Whereas in the presence of protein payments expected profit (E_1(\pi)) per hectare is given by:

$$E_1(\pi) = E_o(I) + \bar{y} x (w_1 - w_3) - cN - F$$  \hspace{1cm} (11)

so that optimal planned production is given by:

$$\left(\bar{p} + x(w_1 - w_3)\right) \frac{\partial \bar{y}(N)}{\partial N}$$

$$+ \bar{y} x \left(f(\gamma / \gamma_H \bar{y}) + f(\gamma / \gamma_L \bar{y})\right) = c  \hspace{1cm} (13)$$

where:

- $f(\gamma / \gamma_H \bar{y})$ = value of the probability density function of $\theta$ at $\gamma / \gamma_H \bar{y}$
- $f(\gamma / \gamma_L \bar{y})$ = value of the probability density function of $\theta$ at $\gamma / \gamma_L \bar{y}$.

Based on (8), the first term in the left-hand-side of (13) is smaller than the left-hand-side of (11). This is the manifestation of the negative impact of protein payments on expected income at the level of the marginal expected income from increased planned production. However, the second term on the left-hand-side of (13) is positive and represents the opportunity both to decrease the likelihood of a discount and to increase the likelihood of a premium that follows
from increasing planned production by increasing nitrogen and the associated sharing of this nitrogen between yield and protein. Consequently, a comparison of (11) and (13) shows that the overall impact of the introduction of protein payments on the optimal level of planned production as algebraically ambiguous. Nevertheless, it can be seen from (13) that the relative strength of these conflicting effects depends on the level of seasonal variability. In particular, a greater level of seasonal variability can be expected to increase the magnitude of \( w_3 \) relative to \( w_1 \), thereby increasing the magnitude of the negative effect on \( \bar{y} \) in (13). Moreover, a greater level of seasonal variability typically reduces the value of the probability density function at a given point, thereby reducing the magnitude of the positive effect on \( \bar{y} \) in (13). Consequently, the potential exists for two farmers, who differ only in terms of their respective levels of seasonal variability, to have opposite supply responses to the introduction of a protein payments system.

This situation is illustrated numerically in the next section.
SECTION 2: Numerical Analysis

In order to undertake a numerical analysis of the model developed in the previous section the yield response function is assumed to take the Mitscherlich form:

\[
\bar{y} = m(1 - e^{-bn})
\]  

(14)

where:  
\( m \) = maximum yield  
\( d \) = axis parameter  
\( b \) = curvature parameter.

In addition, the functional relationship between protein, yield and nitrogen is specified as:

\[
r = \frac{\gamma N}{\theta \bar{y}}
\]  

(15)

This form satisfies the requirement of the model that, for given seasonal conditions (\( \theta \)), additional nitrogen is shared between yield and protein. Finally, it is assumed that the probability density function of seasonal conditions can be represented by the normal distribution. On this basis:

\[
w_1 = F(\gamma N / r_H \bar{y}) \left( 1 - \frac{\sigma_\theta Z(\gamma N / r_H \bar{y})}{F(\gamma N / r_H \bar{y})} \right)
\]  

(16)

\[
w_3 = (1 - F(\gamma N / r_L \bar{y})) \left( 1 + \frac{\sigma_\theta Z(\gamma N / r_L \bar{y})}{(1 - F(\gamma N / r_H \bar{y}))} \right)
\]  

(17)

where:  
\( Z(\gamma N / r_H \bar{y}) \) = ordinate of the standard normal distribution at the value of \( \theta \) corresponding to the high critical protein level

\( Z(\gamma N / r_L \bar{y}) \) = ordinate of the standard normal distribution at the value of \( \theta \) corresponding to the low critical protein level.
\[ F(\gamma_n / r_H \bar{y}) = \text{cumulative probability of } \theta \text{ being less than } \gamma N / r_H \bar{y} \]

\[ F(\gamma_n / r_L \bar{y}) = \text{cumulative probability of } \theta \text{ being less than } \gamma N / r_L \bar{y} \]

\[ \sigma_0 = \text{standard deviation of } \theta. \]

Note that this distributional assumption is consistent with the requirement of the model that \( \theta \) be symmetrically distributed.

Turning to the parameter values for the numerical analysis, base case assumptions are as follows:

\[
\begin{align*}
m &= 110 \\
d &= 80 \\
b &= 0.35 \\
c &= 700 \\
p &= 200.
\end{align*}
\]

In the absence of protein payments these assumptions result in the following initial optimal values:

\[
\begin{align*}
\bar{y}_o &= 100.00 \quad (N_o = 19.37) \\
E_o(\pi) &= 6440.25.
\end{align*}
\]

The base case specification of the protein payments system is as follows:

\[
\begin{align*}
\gamma &= 0.516 \quad (\gamma N_o / \bar{y}_o = 0.1) \\
r_H &= 0.105
\end{align*}
\]
\begin{align*}
  n_l &= 0.095 \\
  x &= 10 \quad \left( \bar{p}_H = 210; \bar{p}_L = 190 \right).
\end{align*}

Table 1 contains details of the optimal values for expected yield and profits following the introduction of such a protein payments system for three levels of seasonal variability.\textsuperscript{9} This table confirms the result presented in equation (9) that such a symmetrically-positioned protein payments system would reduce expected profits regardless of the level of seasonal variability. However, it also supports the suggestion made in relation to equation (13) that the potential exists for the optimal supply response of two farmers who differ only in terms of their respective levels of seasonal variability to have opposite supply responses to the introduction of a protein payments system. In particular, an increase in the level of seasonal variability increases the relative strength of the negative effect of protein payments both on expected profit and on marginal expected profit. Table 1 shows that for \( \sigma_0 = 0.6 \) this negative effect outweighs the positive effect relating to the opportunity both to increase the likelihood of a premium and to decrease the likelihood of a discount which follows from increasing nitrogen. Consequently, a farmer with this level of seasonal variability responds to the introduction of the protein payments system by reducing planned production, whereas farmers with the lower levels of seasonal variability in Table 1 would show a positive supply response.\textsuperscript{10}
CONCLUSION

This paper has extended the analysis of the impact of a protein payments system to that on a farmer's planned production. Because such a system has been shown to have a negative effect on expected profits even if positioned symmetrically in terms of the likelihood of a premium or discount, it could reasonably be inferred that this impact would feature a negative supply response.

However, using a model developed in section 1 it was shown that the introduction of a protein payments system has two conflicting effects on the optimal level of planned production. The first is negative and follows from the impact on expected profits. But the second is positive and relates to the opportunity the farmer has both to reduce the probability of a discount and increase the probability of a premium through increased application of nitrogen which is shared between yield and protein. Moreover, as illustrated by the numerical analysis in section 2, the relative strength of these effects can be reversed by changes in a farmer's level of seasonal variability. In particular, the greater is this level the stronger is the negative effect on planned production.

Consequently, it is suggested that the introduction of protein payments is more likely to have reduced planned production in areas of greater seasonal variability and increased planned production in areas of lesser seasonal variability. In so doing the AWB's protein payments system can be seen to have encouraged overall a shift of wheat-growing activity towards more seasonally-reliable regions of the wheatbelt.
REFERENCES


The implications for expected income of an asymmetrically-positioned system are quite straightforward. In particular, protein payments centred above the initial protein/expected yield level increase the likelihood of a discount and therefore have a stronger negative impact on expected profit. The reverse applies for a system centred below the initial level.

Although the stabilising impact of protein payments on the variability of income can be expected to have a positive impact on the planned production of a risk averse farmer, this feature of the supply response to protein payments is considered to be of a second order of importance compared with the expected profit impact. Therefore, in order to simplify the analysis, further consideration of its role is omitted.

This specification is consistent with preliminary scientific evidence. See Robinson (1995) for details.

Note that it is not statistically precise to refer to $\gamma / \bar{y}$ as the expected protein level ($\bar{r}$) because $r$ is an hyperbolic function of $\gamma$.

Note that this derivative assumes an increase in available nitrogen is "shared" between yield and protein for a given value of $\theta$. This assumption seems consistent with scientific evidence and is represented algebraically by:

\[ \frac{\partial \bar{r}(\gamma / \bar{y})}{\partial \theta} > 0. \]

See Paris (1992) for details of empirical support for this functional form.

See footnote 5 for further details.

See Fraser (1988) for this derivation.

Note that this range seems consistent with existing estimates of wheat yield variability around Australia. See Anderson, Dillon, Hazell, Cowie and Wan (1988).

Further numerical analysis shows that this pattern of results is not sensitive to the size either of the critical protein bandwidth or of the protein premium/discount. In each case a variation in size affects the magnitude of the two terms on the left-hand-side of (13) similarly. However, an asymmetrical positioning of the critical protein levels will either increase or decrease the relative strength of the negative effect on marginal expected profit. Consequently, if a premium is considerably more likely than a discount, then even a farmer with $\sigma = 0.6$ may exhibit a positive supply response. While if a discount is considerably more likely than a premium, then even a farmer with $\sigma = 0.4$ may exhibit a negative supply response.
Table 1

Results of the Impact of Introducing a Protein Payments System on Optimal Expected Yield and Profits

<table>
<thead>
<tr>
<th></th>
<th>( \bar{Y} )</th>
<th>( E(\pi) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>No protein payments</td>
<td>100.00</td>
<td>6440.25</td>
</tr>
<tr>
<td>( \sigma_0 = 0.2 )</td>
<td>100.49</td>
<td>6301.84</td>
</tr>
<tr>
<td>( \sigma_0 = 0.4 )</td>
<td>100.13</td>
<td>6130.26</td>
</tr>
<tr>
<td>( \sigma_0 = 0.6 )</td>
<td>99.95</td>
<td>5966.44</td>
</tr>
</tbody>
</table>