



**AgEcon** SEARCH  
RESEARCH IN AGRICULTURAL & APPLIED ECONOMICS

*The World's Largest Open Access Agricultural & Applied Economics Digital Library*

**This document is discoverable and free to researchers across the globe due to the work of AgEcon Search.**

**Help ensure our sustainability.**

Give to AgEcon Search

AgEcon Search  
<http://ageconsearch.umn.edu>  
[aesearch@umn.edu](mailto:aesearch@umn.edu)

*Papers downloaded from **AgEcon Search** may be used for non-commercial purposes and personal study only. No other use, including posting to another Internet site, is permitted without permission from the copyright owner (not AgEcon Search), or as allowed under the provisions of Fair Use, U.S. Copyright Act, Title 17 U.S.C.*

## ESSAY 5

# TECHNICAL INDIVISIBILITIES AND THE DISTRIBUTION OF INCOME: A MIXED INTEGER PROGRAMMING MODEL OF PUNJAB AGRICULTURE

CARL H. GOTSCH AND SHAHID YUSUF

After a spate of recriminations about the distributive effects of the so-called Green Revolution, it has become clear to students of rural development that the root cause of social inequalities is the distribution of control over scarce agricultural resources and that the distributive effects of technology tend to be derived and secondary. Indeed, insofar as the seed-fertilizer-pesticide component of increased agricultural output is concerned, it is precisely this type of highly divisible technology that offers at least some measure of hope that small, medium, and large farmers will all benefit from increased productivity. The available evidence concerning diffusion and adoption practices in areas with reasonably hospitable agro-climatic environment, primarily those with adequate moisture, tends to bear out this hope. A number of studies indicate that though there may be a short lag in the time of adoption as the smaller farmers of the community observe the trials of their larger neighbors, and though the level of purchased inputs may be somewhat less on small farms as a result of short-term credit constraints and a higher degree of risk aversion, in general, where structural changes are fueled only by a highly divisible technology, the process of accumulation is unlikely to be seriously detrimental to the economic position of the smaller farmers.<sup>1</sup>

It should not be concluded from the preceding evidence, however, that there need be *no* concern about technologically created income disparities between large and small farms. Inevitably, the increased productivity of land and water resources has led to a demand for new—largely indivisible—mechanical inputs. Prominent among these have been tubewells, pumps, threshers, tractors, tillers, and engines and motors of all kinds. It is in this particular nexus that the potential for technologically related increases in income disparity is to be found. For although theoretical solutions that increase input divisibility exist in the form of hire service arrangements and the cooperative purchase and use of machinery, in practice these have frequently been slow to emerge. The result, particularly

<sup>1</sup> A portion of this literature is reviewed, complete with caveats, in C. H. Gotsch (3).

when government regulations have caused capital in general or specific capital intensive inputs to be undervalued, has been to set in motion a type of structural transformation that is contrary to the dictates of basic factor endowments.

The events of the past decade in the Pakistan Punjab illustrate with unusual clarity the argument made above. Many of these issues have been addressed in a general way in earlier studies, but no attempt has thus far been made to provide a quantitative assessment of just what the impact of the "lumpiness" associated with current equipment packages is on the distribution of income by farm size. When posed in the context of a linear programming framework, the question is relatively simple: What would the optimal solution be if some or all of the decision variables were required to take on discrete values?<sup>2</sup>

While the question is simple to ask, it is by no means simple to answer, at least from a computational point of view. The algorithm used in this paper involves a number of steps. First, a continuous solution to the problem is obtained in which decision variables are permitted to take on fractional values. That is, farmers may purchase fractional parts of tractors, tubewells, threshers, etc. If all the variables designated as "discrete" were by chance to take on integer values, the continuous solution would also be the optimal solution. When this is not the case, the algorithm places sequential restrictions on the feasible sets that forces each of the variables to take on integer values. The search for the optimal solution then consists of enumerating possible solutions in a directed tree with branches on which integer solutions are established. Every terminal node of the tree, from which no further branching is possible, provides either a feasible solution or a set of integer values that cannot be fitted into a feasible solution. In the latter case further branching is precluded.

The number of potential solutions (nodes) to programs with several integer constraints can be quite large, hence the need for a search procedure that produces convergence toward an optimal solution with a minimum number of iterations. Most computer programs use some version of the method described by A. H. Land and A. G. Doig for this purpose (4).

#### A MIXED INTEGER MODEL OF PUNJAB AGRICULTURE

The problem posed in the model's formulation is straightforward. Suppose that a farmer has a certain size of holding (to which a surface water allotment is attached) and the labor of his family. What is the optimal package of equipment that should be purchased?

The items from which he may choose—and the items that take on integer values in the model—are bullocks (and associated traditional implements), a Persian wheel, a tubewell, a thresher, a diesel engine, and a tractor (with implements). He is assumed to borrow money with his holding as collateral to effect the purchase. The model's structure and a description of the rationale for the various constraint sets is as follows.

<sup>2</sup> Readers interested in the literature on integer programming may wish to refer to H. M. Wiggartner (5). For a practical application to the problem of choosing farm machinery, see Dale Colyer and Francis Vogt (2).

*Objective Function*

Maximize:

$$\sum_i r_i X_i - \sum_m (c_m(P)P_m + c_m(H)H_m + c_m(T)T_m + c_m(L)L_m + c_m(G)G_m) - \sum_j d_j Z_j, \quad (1)$$

where

- $r_i$  = annual net revenue of one acre of crop  $i$   
 $X_i$  = acres of crop  $i$   
 $c_m(P)$  = variable cost per hour of tubewell operation  
 $P_m$  = hours of tubewell operation in month  $m$   
 $c_m(H)$  = variable cost per hour of Persian wheel operation  
 $H_m$  = hours of Persian wheel operation in month  $m$   
 $c_m(T)$  = variable cost per hour of tractor operation  
 $T_m$  = hours of tractor operation in month  $m$   
 $c_m(L)$  = variable cost (wages) per hour for hired labor  
 $L_m$  = hours of hired labor in month  $m$   
 $c_m(G)$  = variable cost per hour of grain thresher operation  
 $G_m$  = hours of grain thresher operation in month  $m$   
 $d_j$  = fixed charge of equipment  $j$   
 $Z_j$  = number of units of equipment  $j$ .

Values for most of the parameters in the objective function involve either the calculation of net revenue (gross revenues minus variable costs) for the crop activities or the assignment of variable costs alone to the resource augmenting activities. However, the parameters associated with the variables representing the animal and mechanical technologies from which the farmer may choose ( $Z_j$ ) are measures of the fixed charge of purchasing and owning a capital asset. Since these values are crucial to the model's behavior, some comments on the method of calculating them are given below.

The first problem is to select a time period over which the interest and principal of a loan are assumed to be repaid. One possibility would be to base the calculation on a financial transaction that reflects institutional lending practices in the area. For example, in Pakistan, a prominent source of medium-term credit is the Agricultural Development Bank. Although its loans differ somewhat by equipment type, most are for three to five years with an interest charge of 7 to 9 percent.

Another approach would be to base the fixed charge calculation on the life of the asset. Farm management studies show much of the equipment to have a useful life at least double the repayment period allowed by most institutions. The choice of repayment period is an exceedingly important parameter in determining the ability of small farmers to invest in equipment and alternative assumptions about its character will be explored.

A second issue that arises in calculating fixed charges for a static, one-period model involves the appropriate assumption regarding the way in which the loan is amortized. For example, in Pakistan and elsewhere the usual commercial transaction involves equal payments on the principal in each time period and interest charged on the declining balance. The effect of this arrangement is to make the fixed payments at the beginning of the repayment period larger than

those required at the end. Alternatively, with a real-estate type of loan, the fixed charge may be calculated so that equal installments will retire the principal over the repayment schedule, with an increasing proportion of each installment credited against principal.

In order to make comparisons of alternative interest rates in the model simulations, it was assumed that equal payments would be made to pay for equipment purchases.

### Land Constraints

All input and output coefficients reflect units required per acre of crop activity. Hence the land coefficient,  $a_{im}$ , equals one for all time periods in which land is committed to a particular crop. (The empirical estimates of land use include the time required to prepare the land for sowing since one of the options is to leave the land fallow.)

$$\sum_i a_{im} X_i \leq S, \quad (2)$$

where  $a_{im}$  = input of land into crop  $i$  in month  $m$

$X_i$  = acres of crop  $i$

$S$  = total amount of land available (farm size) .

### Water Constraints

$$\sum_i b_{im} X_i - d_m P_m - e_m H_m \leq W_m, \quad (3)$$

$$P_m - \eta U_m \leq 0, \quad (4)$$

$$H_m - \epsilon V_m \leq 0, \quad (5)$$

where  $b_{im}$  = acre inches of water required by one acre of crop  $i$  in month  $m$

$X_i$  = acres of crop  $i$

$d_m$  = acre inches per hour of tubewell operation

$P_m$  = hours of tubewell operation

$e_m$  = acre inches per hour of Persian wheel operation

$H_m$  = hours of Persian wheel operation

$W_m$  = fixed canal water supply (acre-inches)

$U_m$  = hourly capacity of a single tubewell in month  $m$  (acre-inches)

$V_m$  = hourly capacity of a single Persian wheel in month  $m$

$\eta, \epsilon$  = number of tubewells and Persian wheels purchased by the farmer (restricted to integer values) .

Water requirements for the model have been determined from consumptive use estimates made for the Punjab by several reputable engineering firms.<sup>3</sup>

### Power Constraints

**Bullocks.**—Bullocks are the traditional source of power for agriculturalists in the Punjab and are likely to remain so among small farmers in the foreseeable future.<sup>4</sup>

<sup>3</sup> Since such estimates are generally based on the evapotranspiration requirements for maximum yield, "consumptive use" estimates tend to bias downward the benefits of tractor mechanization. It has been well established that there are substantial diminishing returns to water applications, i.e., yields decrease less than proportionally when less than the full consumptive-use requirements is supplied. As a result, most arid-zone farmers find it more profitable, at the margin, to extend rather than intensify their activities. This, in turn, puts a premium on having enough tractor power to increase the ratio of cropped to cultivated area.

<sup>4</sup> When combined with constraint sets (2) and (3), the bullock constraint captures an interest-

$$\sum_i g_{im} X_i + H_m - \mu O_m \leq 0, \quad (6)$$

where  $g_{im}$  = bullock pair hours required by crop  $i$  in month  $m$

$X_i$  = acres of crop  $i$

$H_m$  = bullock pair hours used to operate the Persian wheel in month  $m$

$O_m$  = number of hours a pair of bullocks can work in month  $m$

$\mu$  = number of bullock pairs available (restricted to integer values).

*Stationary engine.*—A second source of power is the stationary engine. Currently, most are 20 to 25 h.p. cold-start, slow-speed diesel engines designed in the 1920s. Because of the asymmetry between the hours that tubewells, threshers, and the equipment powered by such engines can be operated, two constraints are necessary to describe the engine's role. Although tubewells and threshers compete for engine time during the daylight hours, only the tubewell can realistically be operated at night. Thus, in the following specification, the engine delivers two kinds of hours, daytime and nighttime.

$$G_m + P_m^1 - \varphi M_m^1 \leq 0, \quad (7)$$

$$P_m^2 - \varphi M_m^2 \leq 0, \quad (8)$$

where  $G_m$  = number of hours the thresher operates during month  $m$  (restricted to daytime)

$P_m^1$  = number of daytime hours pump operates in month  $m$

$P_m^2$  = number of nighttime hours pump operates in month  $m$

$M_m^1$  = daytime engine hours available from one engine during month  $m$

$M_m^2$  = nighttime engine hours available from one engine during month  $m$

$\varphi$  = number of engines available (restricted to integer values).

*Tractors.*—Tractor mechanization was introduced by adding another set of crop activities to the model whose tillage and harvesting requirements are described in terms of tractor hours. These exist side by side with the same crop activities carried out with bullocks and the other implements of the "traditional" package.

$$\sum_i h_{im} X_i - \lambda A_m \leq 0, \quad (9)$$

where  $h_{im}$  = tractor hours required by crop  $i$  in month  $m$

$X_i$  = acres of crop  $i$

$A_m$  = capacity of one tractor in month  $m$  (hours)

$\lambda$  = number of tractors purchased (restricted to integer values).

### Thresher Constraint

Only two wheat growing activities require a thresher but the acreage devoted to the crop is sufficiently large and the bullock power needed for alternative uses, e.g., preparing land for summer crops, is sufficiently important to warrant its inclusion in the equipment choice set. As noted earlier, its capacity is assumed to be limited by the number of suitable (daylight) hours available during the threshing season.

---

ing, traditional decision-making problem. Should the farmer use the bullocks to pump water with a Persian wheel for the maturation of the summer crop or should they be used to prepare the soils for the winter crop? This is particularly acute when part of the difficulty is to insure that there is enough summer fodder to feed the bullocks.

$$\sum_i k_{im} X_i - \alpha Q_m \leq 0, \quad (10)$$

where  $k_{im}$  = thresher time required for one acre of crop  $i$  in month  $m$

$X_i$  = acres of crop  $i$

$Q_m$  = capacity of one thresher in month  $m$  (hours)

$\alpha$  = number of threshers purchased and available (restricted to integer values) .

### Labor Constraints

Two kinds of labor, family and hired, are distinguished in the model.

$$\sum_i w_{im} X_i - L_m \leq F_m, \quad (11)$$

where  $w_{im}$  = labor hours required by crop  $i$  in month  $m$

$X_i$  = acres of crop  $i$

$L_m$  = hours of labor hired in month  $m$

$F_m$  = hours of family labor available in month  $m$  .

Specifying the amount of family labor available to the farm enterprise is always a difficult and arbitrary undertaking. Indeed, one characteristic of family-operated holdings is their ability to expand the short-term labor supply and thus to meet seasonal demands of planting, harvesting, etc. No attempt was made to adjust for a possible expansion of  $F_m$  in these critical periods and this should be borne in mind when assessing the model's results.

The amount of casual labor that can be hired is assumed to be unconstrained.

### Crop Constraints

*Fodder.*—Two constraints are necessary to insure that enough fodder is grown to feed bullocks when these are a part of the optimal equipment package.

$$X_b - 1.0\mu \geq 0, \quad (12)$$

where  $X_b$  = acres of winter fodder

1.0 = acres of winter fodder required to maintain one pair of bullocks

$\mu$  = number of bullocks pairs (restricted to integer values) .

$$X_f - 0.9\mu \geq 0, \quad (13)$$

where  $X_f$  = acres of summer fodder

0.9 = acres of summer fodder required to maintain one pair of bullocks

$\mu$  = number of bullocks pairs (restricted to integer values) .

*Sugarcane.*—A third, arbitrary, crop constraint is introduced to prevent the optimal cropping pattern from being dominated by sugarcane. Cane is an extremely high-value crop grown in the vicinity of sugarcane mills. Its transportation is expensive, however, and the market beyond the mills tends to be limited to subsistence uses. Several surveys suggest that the average devoted to the crop is rarely more than 10 percent of the cultivated area.

$$X_s \leq 0.1S, \quad (14)$$

where  $X_s$  = number of acres of sugar cane

$S$  = supply of land available for cropping, i.e., farm size .

*Capital Constraint*

As indicated earlier, the purchase of equipment is assumed to be influenced by both the price of capital and by its availability, i.e., by credit rationing. The constraint introduced to reflect this latter consideration is as follows:

$$\sum_j n_j Z_j \leq C, \quad (15)$$

where  $n_j$  = total cost per unit of equipment  $j$

$Z_j$  = number of units of equipment  $j$

$C$  = maximum amount of credit that can be obtained.

In normal lending practice, there is not a one-to-one correlation between the available collateral and the value of  $C$ . Rural bankers in advanced countries tend to develop a good sense of the intelligence, reliability, and managerial capacity of their credit applicants and bend the collateral criteria in both directions. The same is true of village money-lenders in traditional societies. However, the newly organized government lending institutions in developing countries rarely have this type of intimate contact with their clients and the size of the loan is very often related rather mechanically to the size of the holding. This procedure has been followed in this exercise, the figure of 1/2 the current market value of the farm being chosen as the appropriate value for  $C$ .

## RESULTS OF MODEL SIMULATIONS

The simulations carried out with the model were designed to shed light on a number of issues. A first, and perhaps major point of interest, was the comparison of continuous and mixed integer solutions to ascertain the effects on net revenue, equipment packages, cropping patterns, and cropping intensities, of requiring the farmer to purchase his implements in integer-valued amounts. Such a comparison provides a direct measure of the distributive impact of technological indivisibilities on income. Second, the model was used to investigate the impact of different tractor subsidies on the pattern of mechanization. As indicated in the introduction, the government's mechanization policy made tractors available at real costs below those paid by farmers in capital-abundant countries. Although these subsidies were abolished with the currency reform, they may again become a live policy issue as Pakistan begins to manufacture its own tractors. Third, a sensitivity analysis was carried out to determine the impact of variations in the interest rate on the model's behavior. Fourth, an alternative repayment scheme was examined to investigate the effect on different farm sizes of the financial terms of equipment loans.

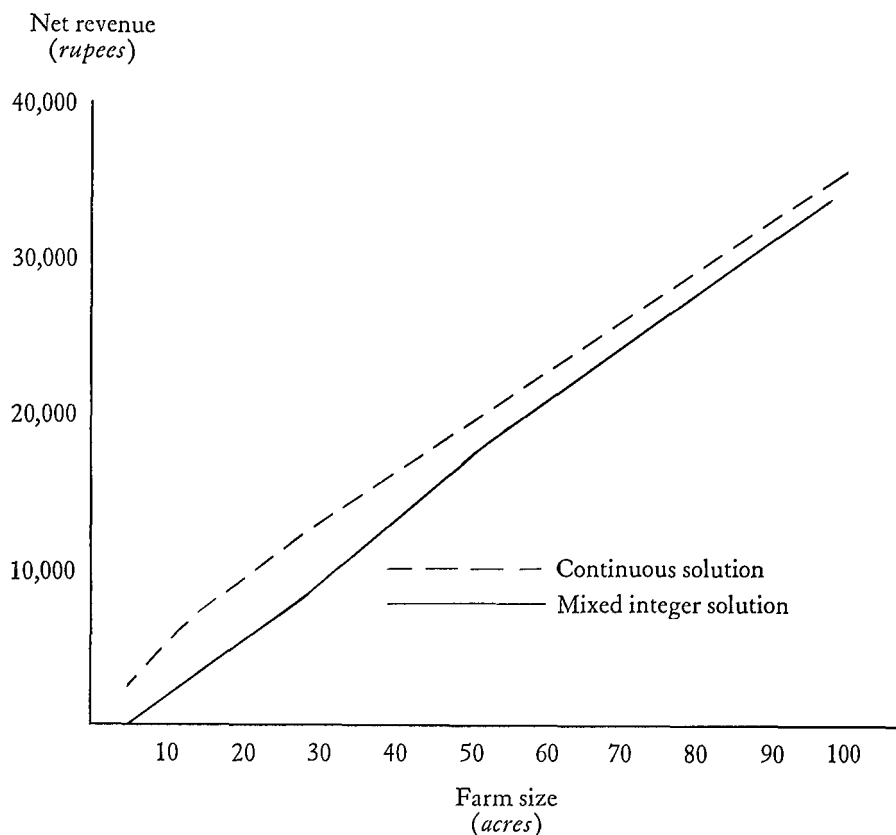
*Basic Solution*

Most of the data used to construct the basic input-output relationships in the model were collected in the Punjab in 1969/70. All commodity prices also reflect that year.<sup>6</sup> Financial repayment schedules are consistent with current lending practices. The interest on borrowed capital was set at 10 percent and tractor prices reflect the costs to farmers in 1970, i.e., costs that were approximately 60 percent of world market values.

<sup>6</sup> We are indebted to Bashir Ahmed for use of the basic tableau developed in 1.



CHART 5.1.—NET REVENUE FROM THE BASIC SOLUTION  
FOR DIFFERENT FARM SIZES



*Net revenue.*—As would be expected, the integer constraints affect severely the net returns of small farms (Chart 5.1). In fact, no feasible solution exists for the 5-acre holding when the technology choices are confined to integer values. The continuous solution for the same holding indicates, on the other hand, that if the various implements were available as a flow of services, the same farmer could obtain net returns of Rs. 3,140. Indeed, his total revenue is greater than that of a 10-acre farm where technology is subject to integer constraints.

In an alternative comparison, Table 5.1 presents both the continuous and integer solutions in terms of (1) net revenue per acre, and (2) the divergence between the two as a percentage of the integer solution. The severe distributive effects of forcing small farmers to make discrete input choices in the presence of technological indivisibilities are apparent from the fact that net revenue from the continuous solutions are roughly double those obtained from the integer results.

As farm size increases, of course, the divergence between the two solutions decreases, both because of diminishing returns to land in the continuous solution and because of increasing returns in the integer case. In the 75- to 100-acre range, the difference in the model results is only 4 percent.

*Equipment packages.*—Table 5.2 shows the optimal equipment packages for

TABLE 5.1.—NET REVENUE PER ACRE BY FARM SIZE,  
BASIC SOLUTION  
(Rupees)

	Farm size (acres)							
	5	10	15	25	35	50	75	100
Net revenue per acre (continuous solution)	628	548	498	444	417	395	373	362
New revenue per acre (mixed integer solution)	0	211	261	305	355	355	354	351
Difference as a percentage of the mixed integer solution		160	91	46	17	11	5	3

TABLE 5.2.—OPTIMAL EQUIPMENT PACKAGES BY SIZE OF FARM

	Farm size (acres)							
	5	10	15	25	35	50	75	100
Continuous solution								
Bullocks	.17	.33	.50	.83	1.17	1.67	1.71	1.40
Persian wheel	.00	.00	.00	.00	.00	.00	.00	.00
Stationary engine	.03	.07	.10	.17	.24	.35	.52	.66
Tractor	.12	.24	.37	.61	.85	1.22	1.96	2.75
Tubewell	.03	.07	.10	.17	.24	.35	.52	.66
Thresher	.03	.07	.10	.17	.24	.35	.50	.66
Mixed integer solution								
Bullocks	0	2	2	0	1	2	1	2
Persian wheel	0	1	0	0	0	0	0	0
Stationary engine	0	0	1	1	1	1	1	1
Tractor	0	0	0	1	1	1	2	3
Tubewell	0	0	1	1	1	1	1	1
Thresher	0	0	1	0	0	0	1	0

various farm sizes under continuous and integer assumptions. In the continuous case, the use of tractor power and tubewell water through hire services is attractive to the very small farmer since it permits him (1) to devote land to high valued crops such as cotton, wheat, and sugarcane rather than bullock fodder, and (2) to raise his cropping intensity from the traditional 110 percent to full double-cropping.

Imposition of the integer constraints on the model creates an infeasible solution in the 5-acre case. This can be interpreted as a statement that a really small farmer cannot meet the fixed charges required to purchase the animals and equipment needed to till his land. What happens practically in these cases is that the farm operator spends a good portion of his time as an agricultural laborer, using the proceeds from this labor to hire tractor services. Or, he may agree to work personally for a larger farmer in exchange for the use of the larger farmer's animals.

Adding integer constraints to the 10-acre model limits equipment purchases of the 10-acre holding to bullocks and a Persian wheel. The result, as Table 5.3

TABLE 5.3A.—OPTIMAL CROPPING PATTERNS AND CROPPING INTENSITIES BY SIZE OF FARM  
(Basic solution)

	Farm Size							
	5 acres				10 acres			
	Continuous solution		Integer solution		Continuous solution		Integer solution	
	(acreage)	(percent)	(acreage)	(percent)	(acreage)	(percent)	(acreage)	(percent)
Winter crops								
Wheat	4.34	44.1			8.67	44.1	4.71	33.6
Barley								
Oilseeds								
Food legumes							.56	4.0
Fodder	.17	1.7			.33	1.7	2.00	14.3
Sugarcane	.50	5.1			1.00	5.1	1.00	7.1
Summer crops								
Rice								
Cotton	3.44	35.0			6.90	35.0	1.01	7.2
Maize	.72	7.3			1.44	7.3	1.72	12.3
Fodder	.17	1.7			.33	1.1	2.00	14.3
Sugarcane	.50	5.1			1.00	5.1	1.00	7.1
Total	9.84	100.0			19.67	100.0	14.00	100.0
Cropping intensity		197				197		140

TABLE 5.3B.—OPTIMAL CROPPING PATTERNS AND CROPPING INTENSITIES BY SIZE OF FARM  
(Basic solution)

	Farm Size							
	15 acres				25 acres			
	Continuous solution		Integer solution		Continuous solution		Integer solution	
	(acreage)	(percent)	(acreage)	(percent)	(acreage)	(percent)	(acreage)	(percent)
Winter crops								
Wheat	13.01	44.1	11.50	47.9	21.66	44.1	22.50	45.0
Barley								
Oilseeds								
Food legumes								
Fodder	.50	1.7	2.00	8.3	.83	1.7		
Sugarcane	1.50	5.1	1.50	6.3	2.50	5.1	2.50	5.0
Summer crops								
Rice								
Cotton	10.34	35.0	1.75	7.3	17.23	35.1	22.50	45.0
Maize	2.17	7.4	3.75	15.6	3.60	7.3		
Fodder	.50	1.7	2.00	8.3	.83	1.7		
Sugarcane	1.50	5.1	1.50	6.3	2.50	5.1	2.50	5.0
Total	29.52	100.0	24.00	100.0	49.15	100.0	50.00	100.0
Cropping intensity		197		160		197		200

shows, is a substantial decline in cropping intensity (197 to 140 percent) and a shift in the cropping pattern to provide fodder for bullocks.

Indicative of the value of supplementary water is the appearance of a tubewell in the optimal equipment package of a 15-acre farm. As the continuous solution shows, this is in part the result of the joint profitability of sharing an engine between the well and a mechanical threshing machine. It also shows that if the capacity to produce water could be freely purchased, only approximately 1/10 of a tubewell would be demanded. Application of the yes-no decision constraint, however, dictates that a complete well be installed.

Under the assumption that tractors are sold at the subsidized prices prevailing in the late 1960s, the model results suggest that a profit-maximizing farmer operating 25 acres should purchase a tractor. However, despite the fact that the land would be double-cropped, a tractor on a farm of this size would tend to have considerable excess capacity. As a result, the need for a thresher to insure timely harvesting is decreased to the point where it is dropped from the optimal equipment package.

Changes in equipment packages as farm size increases beyond 25 acres are related primarily to shifts in draft power availability. Some bullocks continue to be used at peak periods, the number increasing as the capacity of a tractor unit is reached and decreasing after the purchase of a new unit. Except for the 25-acre farm, none of the larger units is entirely without bullocks, a result that is consistent with empirical observations made by several researchers who have conducted field studies of mechanized operations in the Punjab (1).

### *Removal of Tractor Subsidies*

One of the most hotly debated policy issues involving mechanical inputs has been the pricing of tractors. As the description of the assumptions underlying the basic solution indicated, tractors were entered at prices actually paid by farmers prior to the economic reforms of May 1971. At that time, the rupee was drastically devalued and the cost of tractors to the farmer increased significantly. (Credit continued to be supplied at a subsidized rate and hence the new price still does not reflect fully the opportunity cost of capital to the economy.)

Table 5.4 outlines the nature of the changes that occurred in the optimal solution when the new tractor prices were inserted in the matrix. First, and most obvious, the use of tractors declines drastically in the continuous case and the tractor no longer appears in any of the optimal solutions under the integer case. Instead, bullocks become the sole source of draft power. In the continuous solution the use of threshers increases significantly when compared with the subsidized tractor case. In the absence of tractors, the ability to break animal-power constraints increases in value. The level of tubewell services purchased declines, because without mechanical power it is difficult to practice double-cropping. This in turn leads to a decrease in the demand for supplementary irrigation water.

Second, the increase in the price of tractors decreases the difference between the net revenues of continuous and mixed integer solutions on small farms. Instead of 160 percent in the case of pre-1970 tractor prices, the divergence is only 120 percent for 10-acre farms. For 15-acre farms, it is 68 percent instead of 90 percent.

TABLE 5.4.—NET REVENUE AND OPTIMAL EQUIPMENT PACKAGES  
(Tractors at world prices)

	Farm size (acres)							
	5	10	15	25	35	50	75	100
Equipment package								
Continuous solution								
Bullocks	.47	1.23	1.58	1.97	2.89	3.86	6.19	8.26
Persian wheel								
Stationary engine	.10	.18	.20	.48	.66	.98	1.42	1.89
Tractor	.04	.06	.05	.02		.06		
Tubewell	.03	.06	.09	.10	.15	.21	.32	.43
Thresher	.10	.18	.20	.48	.66	.98	1.42	1.89
Mixed integer solution								
Bullocks		2.00	2.00	3.00	4.00	5.00	7.00	9.00
Persian wheel		1.00						
Stationary engine			1.00	1.00	1.00	1.00	1.00	2.00
Tractor								
Tubewell			1.00	1.00	1.00	1.00	1.00	1.00
Thresher			1.00	1.00	1.00	1.00	1.00	2.00
Net revenue (Rs./acre)								
Continuous solution	547	467	420	371	342	319	297	284
Mixed integer solution	0	211	261	302	307	298	284	276
Difference as a percent of mixed integer result		120	61	23	11	7	5	3

The decrease in divergence between the continuous and integer solutions as a result of increased tractor prices flows from the decline in net revenue per acre in the continuous case. A comparison of the two solutions shows that in the integer case, there is no change in net revenues; the tractor does not enter into either the "with" or "without" subsidy solution. However, the increased prices cut fractional purchases drastically and reduce net revenues in the continuous solution by approximately 15 percent.

Comparison of the with and without subsidy solutions also shows, in the integer case, a decline in the difference between the net revenues per acre on small holdings (10 to 15 acres) and those obtained on medium sized (35 to 50 acres) and large holdings (75 to 100 acres) at higher tractor prices. For example, under the assumption that tractors are subsidized, the difference in net revenues per acre between small- and medium-sized farms is Rs. 131 per acre and between small and large, Rs. 128 per acre. When tractor prices are set to reflect world market prices, the differences are reduced to Rs. 78 per acre and Rs. 56 per acre, respectively. The result is again due to the fact that, in the integer case, small farmers are isolated from changes in tractor prices; a tractor is not a part of their optimal equipment package in either solution. Medium and large farmers, on the other hand, experience a considerable decline in revenues at higher prices.

#### *Variation of the Interest Rate*

The sensitivity of the model to variations in the interest rate depends to a great extent, of course, on the farm size being considered. Table 5.5 shows the results of

TABLE 5.5.—NET REVENUE AND OPTIMAL EQUIPMENT PACKAGES UNDER  
ALTERNATIVE INTEREST RATES  
(25-acre farm)

	Interest rate				
	5	10	15	25	35
Equipment package					
Continuous solution					
Bullocks	.83	.83	.83	.83	1.08
Persian wheel					
Stationary engine	.17	.17	.17	.14	.14
Tractor	.61	.61	.61	.51	.39
Tubewell	.17	.17	.17	.14	.14
Thresher	.17	.17	.17	.14	.14
Mixed integer solution					
Bullocks			3.00	3.00	1.00
Persian wheel					
Stationary engine	1.00	1.00	1.00	1.00	
Tractor	1.00	1.00			
Tubewell	1.00	1.00	1.00	1.00	
Thresher			1.00	1.00	
Net revenue (Rs./acre)					
Continuous solution	11,616	11,093	10,549	9,496	8,507
Mixed integer solution	8,554	7,617	6,997	5,833	4,821
Difference as a percentage of the mixed integer solution	36	46	51	63	76

parametrically varying the assumed cost of capital from 5 percent to 35 percent when holding size is set at 25 acres. (Tractor prices have again been set at the domestic price prevailing before the monetary reforms of 1971.)

As Table 5.5 indicates, the crucial change occurs between 10 percent and 15 percent. When the former is assumed, the optimal equipment package continues to include a tractor; with the latter, the source of power becomes bullocks with a thresher added to break the power bottlenecks that occur because of the overlap of harvesting and planting operations in the spring.

The results of varying the interest rate also underline the difference in the sensitivity of the continuous and mixed integer equipment packages to the cost of capital. This can be seen in the comparison of the 10 percent and 15 percent solutions which show no change in the continuous solution while a substantial alteration has occurred in the mixed integer package. A similar phenomenon occurs in the comparison of the 25 percent and 35 percent cases where modest changes in the continuous case can be contrasted with an integer solution that borders on infeasibility.<sup>6</sup>

Comparison of the equipment packages leads naturally to an examination of the impact of interest rates on net revenues. In the continuous case, the decline is substantial but not surprising over the 5 to 35 percent range, i.e., 27 percent. In

<sup>6</sup> The first integer solution for 35 percent produced an equipment package similar to that shown for 25 percent. The net revenue was Rs. 4,589 compared with the Rs. 4,821 associated with optimal package involving only one pair of bullocks and assorted traditional implements.

the integer solution, however, the effect is significantly greater. From Rs. 8,500 at 5 percent, net revenue decreases to Rs. 4,800, at 35 percent, a decline of 43 percent.

The distributive effects of interactions between the price of capital and technological indivisibilities flow in a straightforward way from the preceding observations. To illustrate this point, Table 5.5 also shows the percentage difference between the continuous and integer solutions at various interest rates. At 5 percent it is modest: 36 percent. However, at 15 percent interest, the difference has increased to 51 percent and at 35 percent interest, the net revenue from the same 25-acre farm under the continuous technology assumption is approximately 75 percent greater than that generated by the integer solution. When seen in this context, i.e., in a situation where the technology needed to increase productivity is indivisible and relatively capital-intensive, the fact that small farmers are frequently forced to pay higher rates of interest in the capital market than large farmers obviously compounds the already serious distributive consequences of yes-no decisions about implement purchases.

### *Alternative Repayment Schedules*

The discussion in the section describing the specification of the model's fixed charge parameters indicated that the conditions of the repayment were an important element in determining the extent to which technological indivisibilities worsened the distribution of net revenue by farm size. Previous solutions have assumed that these were imposed by local financial institutions and did not necessarily reflect the actual life of the machine. Table 5.6 shows the impact on net revenues and optimal equipment packages when the repayment schedule is changed to reflect a fixed charge that would pay off interest and principal over the time that the machine (or animals) could actually generate funds for loan repayment. The other assumptions of the basic solution, i.e., tractor prices at their pre-1970 domestic level, a 10 percent interest rate, and so on, remain as before.<sup>7</sup>

Liberalizing medium-term credit policies by extending the repayment schedule to more closely reflect the life of the asset clearly has a significant effect on the extent to which small farmers are able to avoid the "lumpiness" problem. Comparison with Table 5.3, for example, shows that the profit-maximizing farmer with 10 acres would now purchase a tubewell and motor instead of investing in a Persian wheel. The result would be an increase in net revenue per acre of approximately Rs. 135 or 50 percent and a reduction of the difference between the continuous and integer solutions from 160 percent to 80 percent. Similar changes, although of a lesser magnitude, are to be found in comparison of the two solutions on a 15-acre farm.

### CONCLUSIONS

The policy conclusions that the modeling exercise suggests are several. In a given agricultural situation, the wider the range of capacity available in the relevant technologies, the less the distributive impact of mechanical inputs will be. Judging from the implements available in the Indian Punjab, where farms tend

<sup>7</sup> Note that stretching out the repayment period is not equivalent to altering the interest rate on loans since the lifetimes of various implements vary considerably. These differences are usually not captured fully in loan agreements.



TABLE 5.6.—NET REVENUE AND OPTIMAL EQUIPMENT PACKAGE ASSUMING A REPAYMENT PERIOD  
EQUAL TO THE LIFE OF THE EQUIPMENT

	Farm size ( <i>acres</i> )							
	5	10	15	25	35	50	75	100
Equipment package								
Continuous solution								
Bullocks	.17	.33	.50	.83	1.17	1.67	.67	
Persian wheel								
Stationary engine	.04	.07	.12	.17	.24	.35	.57	.77
Tractor	.12	.27	.49	.66	.93	1.33	2.33	3.27
Tubewell	.04	.07	.08	.17	.24	.35	.57	.77
Thresher	.04	.03	.12	.06	.09	.13	.05	
Mixed integer solution								
Bullocks		2.00	2.00		1.00	2.00	2.00	2.00
Persian wheel								
Stationary engine		1.00	1.00	1.00	1.00	1.00	1.00	1.00
Tractor				1.00	1.00	1.00	2.00	3.00
Tubewell		1.00	1.00	1.00	1.00	1.00	1.00	1.00
Thresher			1.00			1.00	1.00	
Net revenue ( <i>Rs./acre</i> )								
Continuous solution	237	631	573	526	499	478	461	452
Mixed integer solution	0	348	392	448	465	442	444	443
Difference relative to the mixed integer solution		81	46	17	7	7	4	2

to be smaller, Pakistan's agricultural engineers should be able to develop smaller engines, pumps, and threshers than are currently available. There will, of course, tend to be some decline in the effectiveness of money spent on machinery; it costs little to increase the capacity of machines. But benefits obtained by a more rapid diffusion of small-scale units would undoubtedly outweigh the savings generated by a higher level of machine output per monetary unit invested.

A second point that flows from the model results has to do with the distributive effects of pricing capital when the technology in which it is embodied contains significant indivisibilities. It will be recalled that increasing tractor prices to more nearly reflect the opportunity cost of capital reduces the difference in net revenues per acre between small and medium and between small and large farms significantly. It follows from this that more appropriate factor pricing would produce not only gains in social efficiency, but less disparity in the distribution of income as well.

Moreover, it might even be useful to go a step further and, in the interests of technology diffusion, consider subsidies on small sizes of pumps, engines, and threshers. Because such small units are of relatively little use to the larger farmers (the cost of increasing capacity through the purchase of larger units is relatively low), subsidizing them would be one of the few programs that, in the Pakistani context, would result in the benefits of government policies being captured by the target groups.

Simulations relating to the impact of conditions in the rural capital market on the optimal equipment were also carried out. The results predictably showed that, among other things, the lower the interest rate, the less the divergence between the continuous and integer solutions. The mechanism in this case is the obvious one in which lower fixed charges make the purchase of "lumpy" units by small farms economically feasible. However, before a low interest-rate policy is recommended within a given economic and political environment, strong evidence would be necessary establishing the case that the smaller farmers would indeed be the recipient of the loans. Otherwise one could expect the worst solution that the model produced, i.e., low prices for tractors, the most indivisible item of technology considered.

Improving the terms of lending in any other way, e.g., extending the repayment period, would have problems similar to those associated with lowering the interest rate. However, one persistent practice found in many developing countries that is highly discriminatory in its distributive effects is stringent capital rationing for the purchase of durable goods. For years, Pakistani cultivators who owned less than 25 acres could not obtain funds to purchase a tubewell. This, despite the fact that the collateral represented by their land holdings was many times that required to guarantee the costs of the well. The reasoning of officials who were responsible for the regulations was that a tubewell obviously had the capacity to irrigate two or three times that much land and thus the 25-acre holding was clearly "too small." The issue of the profitability of the investment—and the justification for considerable excess capacity—was not given due consideration.

Finally, it should again be emphasized that the continuous and mixed integer solutions to the problem of technological choice represent two extremes: black and white. Reality, as usual, is a shade of grey. In most countries, including Paki-

stan where agriculture is organized around private holdings, markets involving the sale of water and the hiring of threshing and tractor services have emerged. Indeed, in some cases, viable cooperatives have been formed whose basic glue is a highly profitable—but indivisible—piece of equipment. Almost inevitably, however, this process has been accompanied by considerable delay, delay in which the larger again became a bit stronger relative to the smaller. In the long run, of course, this is the mechanism by which the process of structural transformation of capitalist agriculture in developing countries has proceeded. The issue that is being raised with increasing frequency is whether this is a viable paradigm of agricultural development for the Third World.

#### CITATIONS

1 Bashir Ahmed, "Farm Mechanization and Agricultural Development: A Case Study of the Pakistan Punjab" (unpub. Ph.D. diss. Michigan State Univ., 1972).

2 Dale Colyer and Francis Vogt, "Farm Machinery Decisions: Using Mixed Integer Programming," *Research Bulletin No. 931* (Agr. Exp. Sta., Univ. Missouri, Columbia, 1967).

3 C. H. Gotsch, "Economics Institutions and the Generation of Employment in Rural Areas" in *Employment in Developing Nations*, ed. by E. O. Edwards (New York, 1974).

4 A. H. Land and A. G. Doig, "An Automatic Method of Solving Discrete Programming Problems," *Econometrica*, 28, 1960.

5 H. Martin Wingartner, *Mathematical Programming and the Analysis of Capital Budgeting Problems* (Chicago, 1967).