Modelling Natural Resource Policy as a Hierarchical Stackelberg Game

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Abstract

A unified approach to formulating policy is suggested for controlling water quality deterioration from agricultural production while reconciling the conflicting goals of primary interest groups in the farm policy process. This paper develops a Stackelberg game theoretic model of public policy formation, in an American setting, which simultaneously determines endogenous price supports and pollutant quotas, as well as the optimal permissible water contamination. The analysis distinguishes between the private and social opportunity costs of producing agricultural crops and using water as a repository for contaminants from agricultural sources. It is recognized that the social benefit of using the resource of water's assimilative capacity in agriculture is much below the private benefit to producers. Private and social benefits as well as optimal production and pollution solutions will vary as the relative weights which policymakers attach to different social constituents change. The method developed in this paper may be applicable to any policy process in which policymakers exercise indirect influence over industrial production decisions through economic instruments.

Introduction

While the importance of topsoil and the problem of topsoil loss have long been recognized, more than four decades of federal programs have done little to stay the erosion of American farmland (McConnell). One-third of U.S. cropland topsoil has been lost in the last 200 years (Walker), and sheet and rill erosion on U.S. cropland continues at a rate of 1.6 billion tons annually (USDA/SCS, 1990).

Increased public awareness of these issues continues to put pressure on policymakers to solve environmental problems while maintaining an abundant, inexpensive food supply. The federal government has responded to public pressure for remediation of this problem by launching several water quality management programs and policies at every level of government throughout the United States. However, several policy components bundled into the current farm legislation have conflicting effects. Helmers *et al.* (1992) stress that certain features of current farm programs result in a resource imbalance, such as intensive use of chemical fertilizers and associated water pollution. First, the high loan rates and support prices for certain crops tend to promote the intensive use of land and movement of environmentally sensitive land into production. Second, land retirement provisions such as the Acreage Reduction Program and the Conservation Reserve Program encourage farmers to set aside their marginal land and to achieve production goals through intensive use of their prime land. Thus, under the current regime of programs, the federal government is leasing land from farmers to remove it from production while promoting production by holding commodity prices artificially high.

Such policies seem to arise naturally out of a process in which different policy problems are examined and addressed individually. This piecemeal approach to policy formation inherently produces conflicts of purpose even within the same legislation. Some countries have attempted to aggregate certain types of legislation into larger, more consistent policy instruments with varying degrees of success. For example, New Zealand's Resource Management Act replaces several smaller pieces of environmental legislation with a single unified act governing the management and use of all natural resources in the country. However, unlike the United States, New Zealand is not attempting to provide industry price or income support to its agricultural sector. This is one of the major areas of conflict in U.S. agricultural legislation.

If the U.S. is to continue its policy of simultaneously supporting agricultural prices while instituting conflicting policies to protect the environment, an analytical system is needed which can consider policymaker's competing goals and balance them to optimize a policy's returns to society. This paper provides an analytical framework of public policy formation which determines simultaneously the endogenous price support and environmental policies as well as the optimal permissible sediment contamination to waterways. The endogenous policy formation is modeled as a hierarchical, Stackelberg game-theoretic decision process, by combining a dynamic sedimentation model with a market equilibrium model of price determination.

The Stackelberg model structure provides a formal model of bargaining processes among economic agents in a situation wherein a dominant player, such as a government, has the ability to enforce a strategy on other players by announcing a strategy before the others (Basar and Olsder, 1982). As the policymaker is constrained by the influence of interest groups in the policy process (Rausser and Freebairn, 1974), the policymaker's objective function is weighted to represent the welfare of interest groups – including the farmers group – through a suitable combination of policy instruments, such as price supports, a cap on total erosion permitted, and economic incentives or disincentives to polluters. The enforcement of such regulations will require producers to take one or more of the following actions: 1) change their production technology, 2) reduce production, and 3) adopt better management practices. These actions likely will affect the economic performance of production activities. Profit-maximizing individual producers will make their production decisions in response to prevailing output market conditions and water quality regulations. Policymakers will consider such production decisions as rational responses of farmers and will formulate price support and environmental policies endogenously.

Basic Market Equilibrium Framework

Following the simple market equilibrium framework developed by Helmers, *et al.* (1991) -- although for slightly a different purpose -- the impacts of restricting erosion rates on crop supplies and market prices are illustrated in figure 1. The analysis assumes that farmers attempt to maximize profits from crop production using technologies that generate variable erosion rates with a fixed land area and other inputs. Assuming an input inelasticity of production, the resulting marginal cost (lagged supply response) curve (S_1) is upward sloping. For simplicity, the supply function is assumed to be linear. Farmers are faced with a given downward sloping market demand curve D.

Given a support price G_t, higher than market price, equilibrium output is determined at the intersection of G_t and the supply curve S_1 at Q_t. The intersection of market demand D and the equilibrium output Q_t yields an equilibrium market price of P_t. If producers agree, voluntarily or by regulatory pressure, to restrict erosion arising from production such that the total supply curve shifts backward to S_2, the equilibrium market price rises to P_t^*. Suppose that farmers are responsive not only to support prices but also to market prices. The cost-related price increase in period t will have a positive recursive impact on the producers acreage (output) response in period $t+1$, which is reflected by a forward shift in the supply curve S_3. This new supply curve will yield an equilibrium output of Q_{t+1}^*, reducing the market price to P_{t+1}^*. This oscillation in market prices is represented by a converging cob-web in figure 1, assuming that both input and lagged price elasticities of supply are less than one. In the long run, prices could diverge if the supply is more elastic with respect to erosion control costs and market price. It is evident from this illustration that a change in the average level of erosion produced may have impacts on market prices and output, given no concurrent technological change. Also, note that for any given support price, the government payments also fluctuate as market prices change.

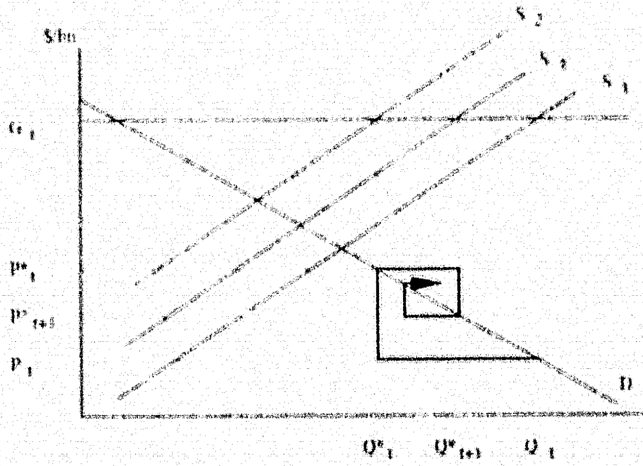

Figure 1. Erosion reduction impacts on equilibrium output and price of agricultural commodities under price support program.

This analogy of market equilibrium can be extended to analyze the impacts of commodity policies such as support price and various acreage (supply) control programs. The government has several alternatives available to reduce the production of certain crops in order to reduce the environmental externalities of agricultural production. But policymakers' alternatives are limited by pressure from agricultural interest groups. Policymakers want to combine these policy instruments in such a way that the interests of farmers, water quality advocates, and taxpayers (in terms of costs of commodity programs) are balanced according to the relative importance policymakers place on the concerns of the various constituents. When multiple policy instruments are applied simultaneously to achieve more than one goal, the dynamics of the market mechanism become extremely complicated to visualize. The model developed in the next section attempts to express both regulatory and economic choices exercised by the government and farmers in a single decision-making framework, combined with a physical water quality model of sedimentation

Hierarchical Stackelberg Game-Theoretic Model for Pollution Control

The concept of a hierarchical decision-making problem was first introduced by von Stackelberg and was later applied to dynamic differential game problems by others (Basar and Olsder, 1982). The application of this tool to environmental problems, however, has been rare (see Koyama, 1990, for an exception). In the Stackelberg water quality control model developed in this section, the producers' (followers') dynamic problem first would be developed and all necessary conditions be derived. These conditions then would be included as constraints in the development of the policymaker's (leader's) dynamic problem in which the leader tries to balance the income from and environmental costs of pollution-generating production activities as well as program costs.

Noncooperative interaction between the leader and the followers is assumed. The model explicitly assumes cooperative behavior is not possible and characterizes the interactive open-loop strategies of policymaker and the farmers. As Negri (1989) points out, even noncooperative behavior can result in a socially efficient decision strategy if antagonistic behavior triggers threat strategies. Such threat strategies are quite possible in a hierarchical decision-making problem like this one, wherein the leader of the game (policymaker) possesses statutory power to enforce desired behavioral action by followers. It also is possible that a noncooperative game played repeatedly could evolve into cooperative game behaviour, which generally is accepted as a Pareto efficient equilibrium solution.

The Producer Model

It is assumed that farmers have no incentive to consider the water pollution externality of sedimentation in their crop production decisions. Their objective is to maximize net revenue (market return plus government payment less production cost) from agricultural production, subject to pollution restrictions and support price levels imposed by the policymaker. Producers, who are followers in this hierarchical decision game, are assumed to lack the ability to influence their leader's decision once it is made. Alternatively, they attempt to make optimal decisions regarding input use, production practices and total acreage under different crops in response to the government decision variable.

In order to keep the analysis manageable, two crop commodities are considered in this model. As indicated earlier, endogenous market prices have a key role in both farmers' and policymaker's decision making. The dynamics of the two commodity market is modeled as a recursive system of output (or crop acreage) and price equations (Waugh, 1962). Adopting a simple partial adjustment model of acreage response (Nerlove, 1958), farmers' policy-responsive planting decisions are assumed to take the following form:

$$(1) \qquad A_{it} = a_{i0} + a_{i1} P_{1t-1} + a_{i2} G_{1t} - a_{i3} P_{2t-1} - a_{i4} G_{2t} + a_{i5} A_{it-1} \quad i = 1,2$$

where A_{it}, P_{it} and G_{it} are planted acres, market price (\$/lb) and government price (\$/lb), respectively, of crop i in period t, and a_{it} are model parameters. Each crop's acreage depends on its own market and support prices, lagged crop acres, and cross market and support prices.

The model sacrifices some realism in the case of inverse price equations in order to keep the analysis tractable. The model assumes the market equilibrium commodity prices are influenced only by the current year's production, ignoring the effects of residual stocks from previous production periods. The *ceteris paribus* price equations for two commodities are given by:

$$(2) \qquad P_{it} = b_{i0} - b_1 h_i y_i(\varepsilon_i) A_{it} \quad i = 1,2$$

where h_i is the ratio of harvested acres to planted acres; $y_i(\varepsilon_i)$ is the crop yield (lbs/ac) and is a function of the erosion control technology used ε_i. Positive but diminishing marginal returns to high erosion technologies are assumed; therefore $y_i'(\varepsilon_i) > 0$ and $y_i'(\varepsilon_i) < 0$. The product $h_i y_i(\varepsilon_i) A_{it}$ gives the total crop production in period t.

By substituting (2) into (1) and simplifying, the following equations of acreage movement can be obtained.

(3) $$A_{it} - A_{it-1} = c_{i0} + c_{i1}A_{it-1} + c_{i2}G_{it} + c_{i3}A_{jt-1} + c_{i4}G_{jt} = f_{it}(A_{it-1}, A_{jt-1}, G_{it}, G_{jt})$$

where $i = 1,2$ and $i \neq j$.

Let R_{it} represent periodic unit net crop returns ($/ac) which is measured as the sum of market returns and support payments less per acre production costs. Formally,

(4) $$R_{it} = h_i P_{it}(\varepsilon_{it}, A_{it}) y_i(\varepsilon_{it}) + \omega_i D_{it}[G_{it}, P_{it}(\varepsilon_{it}, A_{it})]\Phi_{it} - C_{it}(\varepsilon_{it}).$$

The first term on the right hand side (RHS) of (4) is gross market return per planted acre ($/ac), adjusted by the harvested-to-planted acreage ratio, which yields the actual per acre market return. The second term represents the per acre government payment which is measured as a product of rate of deficiency payment $D_{it}[G_{it}, P_{it}(\varepsilon_{it}, A_{it})]$ ($/lb) and the constant term program yield ω_i (lb/ac), adjusted by the ratio of acreage planted under the government program to total crop acres planted Φ_{it}. Thus, the second term, when multiplied by the total planted acres A_{it}, gives the total deficiency payments made to the program participants. D_{it}, by definition, meets the following restriction:

(5) $$D_{it}[G_{it}, P_{it}(\varepsilon_{it}, A_{it})] = G_{it} - P_{it}(\varepsilon_{it}, A_{it}) \quad \text{if} \quad G_{it} > P_{it}$$

$$= 0 \quad \text{if} \quad G_{it} \leq P_{it}$$

The term $C_{it}(e_{it})$ represents cost of production ($/ac) and is assumed to be the sum of erosion control dependent unit variable costs and unit fixed costs of all other production inputs. A linear inverse relationship is assumed between production cost and erosion rates.

The aggregate producers' problem is to maximize the present value of the stream of net returns over the interval $0 \leq t \leq T$. That is,

(6) $$\text{Max } J' = \sum_{t=1}^{T} \rho^t \left[R_{1t}(A_{1t}, G_{1t}, \varepsilon_{1t})A_{1t} + R_{2t}(A_{2t}, G_{2t}, \varepsilon_{2t})A_{2t}\right]$$

subject to equations of acreage movement (3), given initial acreage levels $A_{10} = A_1^0$ and $A_{20} = A_2^0$, and total erosion restriction

(7) $\varepsilon_{1t}A_{1t} + \varepsilon_{2t}A_{2t} \le E_t$,

where E_t is the policymaker's restriction on the maximum erosion in the region. The inequality in (7) provides flexibility to producers in terms of optimally adjusting erosion rates ε_{it} (producers' control variables) among two model crops in order to meet the total erosion production restriction. The ρ in (6) represents a discount factor measured as $(1+\delta)^{-1}$, where δ is real annual discount rate.

In classical supply response literature, acreage is generally considered a producer decision variable. In this model, acreage is considered a dynamic state variable in the control-theoretic sense, and this variable is driven directly by endogenous lagged market prices and current period government prices given by (3). The lagged market price is determined by lagged output as governed by (2); and the latter is determined by the producers' lagged control variable ε_{it-1}. Thus, this recursive interaction makes the model unique in that the periodic state variable of the model is influenced intertemporally by the previous period's decision value but not the current period's value.

The above producers' problem can be solved by setting up a discrete-time *current value* Hamiltonian,

(8) $H_t^c(A_{1t}, A_{2t}, \varepsilon_{1t}, \varepsilon_{2t}, \lambda_{1t+1}, \lambda_{2t+1}, \mu_t; G_{1t}, G_{2t}, E_t)$

$= R_{1t}(A_{1t}, G_{1t}, \varepsilon_{1t})A_{1t} + R_{2t}(A_{2t}, G_{2t}, \varepsilon_{2t})A_{2t} + \lambda_{1t}f_{1t}$

$+ \lambda_{2t}f_{2t} + \mu_t(\varepsilon_{1t}A_{1t} + \varepsilon_{2t}A_{2t} - E_t)$

where λ_{it} is the costate variable measuring the marginal private benefit ($/ac) from planting an additional acre of land to crop i, and μ_t is the multiplier function measuring the marginal costs to agriculturists' from reducing the allowable erosion production limit by an extra unit.

The Hamiltonian function (8) can be solved by applying the Pontryagin Maximum Principles, which yield the following necessary conditions:

(9) $R_{it}'(\varepsilon_{it})A_{it} + \mu_t A_{it} = 0 \quad i = 1,2$

$$(10)\quad \rho\lambda_{it+1} - \lambda_t = -\left[R_{it} + R'_{it}(A_{it})A_{it} + \rho\lambda_{1t+1}f'_{1t+1}(A_{it}) + \rho\lambda_{2t+1}f'_{2t+1}(A_{it}) + \mu_t\varepsilon_{it}\right],\quad i = 1,2$$

$$(11)\quad A_{it} - A_{it-1} = f_{it}(A_{1t-1}, A_{2t-1}, G_{it}, G_{jt}),\qquad A_{i0} = A_i^0,\quad i = 1,2$$

and inequality constraints:

$$(12)\quad \mu_t \geq 0;\qquad \mu_t(\varepsilon_{1t}A_{1t} + \varepsilon_{2t}A_{2t} - E_t) = 0$$

The functions A_{it} and ε_{it}, when solving the above optimal system, represent the optimal state and control decision responses of followers to a given set of values of the leader's decision variables G_{it} and E_t.

The Policymaker Problem

For the purpose of this analysis, only sedimentation of surface water from sheet and rill erosion is considered, although the problem can be extended to other types of surface water contamination, such as nutrient loadings. This consideration is based on the fact that sediment is the largest polluter of ponds, streams, rivers and reservoirs in the United [illegible] Resources Council, 1978). Sediment trapped in ditches and lakes reduces water [illegible]acity and increases the likelihood of flooding. It increases dredging costs of rivers and harbours, silts reservoirs, damages wildlife habitats and diminishes recreational enjoyment of water resources. The annual offsite cost of erosion in the United States is estimated to be US$6.2 billion (Clark et al., 1985).

Denote S_t as stock of sediment [tons]. The physical dynamics of sedimentation are governed by the following difference equation:

$$(13)\qquad S_t - S_{t-1} = \alpha\left(\varepsilon_{it-1}A_{it-1} + \varepsilon_{jt-1}A_{jt-1}\right) - \beta S_{t-1},$$

where ε_{it-1} is the erosion rate (tons/acre) resulting from production of crop i, A_{it-1} is the acreage in production of crop i, α is the sediment growth coefficient (the proportion of erosion that enters the waterway as sediment), and β is the sediment decay coefficient (the proportion of erosion stock which is carried away through natural action of water). The term $S_t - S_{t-1}$ measures the periodic net growth in sediment stock.

Numerous hydrological and agronomic factors, including soil type and profile, land surface conditions (crop residues) and tillage practices affect the both the erosion of topsoil and the transport of eroded soil into waterways as sediment. These factors vary widely across a given region and between regions. Consideration of all the location-specific hydrological and agronomic factors makes the model less tractable and therefore is ignored. Thus, the constant values of α and β across the region imply that the stock of sediment is unaffected by, and has no value for, the spatial variation in the hydrological characteristics which have potential to influence the erosion and sediment transport processes.

The policymaker has multiple objectives of maximizing net agricultural returns, minimising environmental damage from erosion, and minimising the burden on taxpayers of agricultural subsidies. These objectives compete in that a contribution to one objective always costs the reduced attainment of another. To accommodate society's shifting attitudes, the policymaker's objectives utilize a weighting system, indicating the relative importance to society of the policymaker's competing goals.

The level of net agricultural return is the same as in the producers' problem. Estimating the environmental costs of sediment is a difficult task but has been attempted in several past studies (Clark et al., 1986; Alexander and English, 1996). Unit sediment damage costs ($/ton) are assumed to increase at an increasing rate, represented by the quadratic function:

(14) $$C_t^S(S_t) = (0.5\Theta S_t^2),$$

where Θ is the unit cost function parameter. The component of policymaker objectives which reflect taxpayer interests is represented simply by the amount of deficiency payments made to program participants.

The present value of the stream of net social benefits to the society over interval $0 \le t \le T$ is

(15) $$\text{Max } J^g = \sum_{t=1}^{T} \rho^t \left[a\left(\sum_{i=1}^{2} R_{it} A_{it}\right) - b\left(0.5\Theta S_t^2\right) - c\left(\sum_{i=1}^{2} \omega_t \Phi_{it} D_{it} A_{it}\right)\right],$$

where $a, b,$ and c are the weights attributed to the agricultural revenue, environmental cost, and program cost components of policymaker objectives, respectively, such that $a+b+c=1$ to maintain the convexity of the objective function. A value of $a=b=c=0.33$ indicates that the policymaker assigns equal weight or value to all the interest groups.

Since agricultural revenue is one of the goals in the policymaker's decision process, the policymaker must consider – in order to achieve a Pareto optimal solution – the expected rational producer reaction to his announced policy strategy as an integral part of the policy design. Therefore, the policymaker's problem is to select policy functions G_{it} and E_t which maximise (15) subject to (13), a given initial condition $S_0 = S^0$, and all the necessary conditions of the producers' problem (9) to (12). Note that the producers' costate variables become state variables in the policymaker's problem.

The current value Hamiltonian for this problem is written as

(16) $$H_t^g(A_{1t}, A_{2t}, \varepsilon_{1t}, \varepsilon_{2t}, \lambda_{1t+1}, \lambda_{2t+1}, \mu_t; G_{1t}, G_{2t}, E_t)$$

$$= a\left[\sum_{i=1}^{2} R_{it} A_{it}\right] - b\left[0.5\Theta S_t^2\right] - c\left[\sum_{i=1}^{2} \omega_t \Phi_{it} D_{it} A_{it}\right]$$

$$+ \varsigma_{1t} f_{1t} + \varsigma_{2t} f_{2t} + \varsigma_{st} f_{st} + \xi_{1t} F_{1t} + \xi_{2t} F_{2t}$$

$$+ \upsilon_{1t}[R_{1t}'(\varepsilon_{1t})A_{1t} + \mu_t A_{1t}] + \upsilon_{2t}[R_{2t}'(\varepsilon_{2t})A_{2t} + \mu_t A_{2t}]$$

$$+ \upsilon_{3t}\mu_t + \upsilon_{4t}\mu_t(\varepsilon_{1t} A_{1t} + \varepsilon_{2t} A_{2t} - E_t)$$

where $F_{it} = \lambda_{it} - \lambda_{it+1}, \lambda_{iT} = 0$ is the modified form of producers' adjoint equation (10), ς_{it} is the costate variable measuring the marginal present net social benefit (\$/ac) from allowing an additional acre of land to be planted to the given crop; ς_{st} is the costate variable measuring the marginal social cost from allowing an extra unit of sediment to contaminate the water; ξ_{it} is the costate variable associated with producers' costate variable λ_{it}; υ_{1t} and υ_{2t} are the shadow prices associated with producer optimality equations governing erosion rates, and measure the marginal social costs to the policymaker as a result of producers deviation from optimal behavior by allowing an additional unit of sediment to be produced; and υ_{it} and υ_{it} are the multipliers associated with the producers' optimality conditions and constraint qualifications.

Following Basar and Olsder (1982) and after some simplification, maximization of (16) requires that the following Pontryagin necessary conditions be met:

Optimality Conditions:

(17) $aR'_{it}(G_{it})A_{it} + \varsigma_{1t}f'_{1t}(G_{it}) + \varsigma_{2t}f'_{2t}(G_{it}) = c\omega_t\Phi_t A_{it} D'_{it}(G_{it}), \quad i = 1,2$

(18) $aR'_{it}(\varepsilon_{it})A_{it} + \rho\varsigma_{it+1}f'_{it+1}(\varepsilon_{it}) + \upsilon_{it}R''_{it}(\varepsilon_{it})A_{it} = c\omega_t\Phi_t A_{it} D'_{it}(\varepsilon_{it}), \quad i = 1,2$

Policymaker's Adjoint Equations:

(19)

$$\rho\varsigma_{it+1} - \varsigma_{it} = -\{a[R_{it} + R'_{it}(A_{it})A_{it}] - c\omega_t\Phi_t[D_{it}(A_{it}) + D'_{it}(A_{it})A_{it}] + \rho\varsigma_{1t+1}f'_{1t+1}(A_{it})$$

$$+ \rho\varsigma_{2t+1}f'_{2t+1}(A_{it}) + \rho\varsigma_{3t+1}f'_{3t+1}(A_{it}) + \upsilon_{it}[R'_{it}(\varepsilon_{it}) + \mu_t]\}, \quad \varsigma_{iT} = 0, \; i = 1,2$$

(20) $\rho\varsigma_{3t+1} - \varsigma_{3t} = b\Theta S_t - \rho\varsigma_{3t+1}f'_{3t+1}(S_t), \quad \varsigma_{3T} = 0$

Policymaker's Other Static Constraints:

(21) $R'_{it}(\varepsilon_{it})A_{it} + \mu_t A_{it} = 0, \quad i = 1,2$

(22) $\varepsilon_{1t}A_{1t} + \varepsilon_{2t}A_{2t} = E_t$

(23) $\upsilon_{1t}A_{1t} + \upsilon_{2t}A_{2t} = 0$

Policy Maker's State Equations:

(24) $S_t - S_{t-1} = \alpha(\varepsilon_{1t-1}A_{1t-1} + \varepsilon_{2t-1}A_{2t-1}) - \beta S_{t-1}, \quad S_0 = S^0$

$$(25) \qquad A_{it} - A_{it-1} = f_{it}(A_{1t-1}, A_{2t-1}, G_{it}, G_{jt}), \qquad A_{i0} = A_i^0,\ i = 1,2$$

$$(26) \qquad \lambda_{it} - \lambda_{it+1} = F_{it}(A_{it}, e_{it}, G_{it}, \lambda_{1t+1}, \lambda_{2t+1}, \mu_t), \qquad \lambda_{iT} = 0,\ i = 1,2$$

Economic Interpretation of Optimality System

The essential feature of the above optimization procedure is that the solution to conditions (17) to (26) will simultaneously achieve policymaker and producers maximization goals. Unlike the producers' optimal solutions, the policymaker's optimal solutions explicitly consider water quality in terms of sedimentation and the associated damage potential. Condition (17) requires the policymaker set the target price of each crop such that the weighted marginal program costs with respect to target price equal the sum of the weighted marginal return to producers and the marginal net social benefits accruing from acreage changes resulting from a unit change in the target price. Similarly, condition (18) requires that policymaker's choice of erosion control technology be established such that the weighted marginal program costs with respect to the erosion rate equal the sum of the weighted marginal return to producers and the marginal social benefits of allowing water quality to be contaminated by an incremental increase in the erosion rate, net of marginal costs to the policymaker resulting from producers' deviation from their rational strategy.

The policymaker's adjoint equations (19) for acreage costate (shadow value) variables further demonstrate the policymaker's consideration of various groups' interests in deciding the optimal steady state solutions of acreage and acreage costate variables. Equation (19) ensures that the time rate of change in the acreage costate variables is balanced with the sum of the weighted marginal returns to the producers, the weighted marginal savings to the taxpayers, the marginal future opportunity costs of two crop acreage variables and water quality and a marginal "premium" on producers' rational response -- all resulting from an incremental change in the given crop acreage. Comparing (19) with the producers' adjoint equations in (10) demonstrates the distinction between the social and private shadow values of crop land. Although (10) and (19) are difficult to solve, one can intuitively find that the social benefit of planting an acre to each crop is much less than the private benefit. Equation (19) accounts for the dollar value of future direct environmental damage costs [$\rho\varsigma_{1t+1}f'_{1t+1}(A_{it})$] and the indirect damage to the environment through future acreage growth resulting from the current period acreage increase [$\rho\varsigma_{1t+1}f'_{1t+1}(A_{it}) + \rho\varsigma_{2t+1}f'_{2t+1}(A_{it})$], in addition to marginal benefits to producers.

The adjoint variable ς_{it} represents the implicit value of water resources, measuring the social opportunity costs of using water as a sink for sediment generated from agricultural sources. Equation (20) mandates that the time rate of change in this opportunity cost (value of capital depreciation) be balanced with the weighted marginal direct costs of sediment abatement ($b\Theta S_t$) minus the marginal change in the future value of sediment stock [$\rho\varsigma_{3t+1}f'_{3t+1}(S_t)$] as a result of incremental increase in erosion.

Equations (21) and (22), which represent the producers' optimality and erosion production constraints, become the part of the policymaker's solution process. These conditions reinforce the notion that the producers reveal their profit-maximizing strategies to policy instruments. Then the policymaker responds by enforcing the socially best possible policy-mix. This hierarchical decision process assumes away any possibility that farmers will consider strategic interaction by counter-reacting to the policymaker's strategy (McBryde, 1993). Condition (23) has a special meaning in that the two cro acres valued at their social costs occurring as a result of producers deviating from the rational strategy must be equal and opposite. In other words, the social costs incurred by the policymaker as a result of producers deviating from their optimal solution of one crop acreage must be equally offset from the social gains through an adjustment in the other crop acre. Equations (24) through (26) are the original state equations for the policymaker's problem.

The model assumes that the societal weights attached to each of the constituents in the policy process are fixed during the finite planning horizon of the problem. If these weights change during the planning horizon, social opportunity costs of agricultural productions and water quality will change. Accordingly, the optimal paths of crop acreage, sediment stocks and government policy variables will have to be reexamined by resolving the model, taking into account the appropriate state and costate boundary conditions prevailing at that time.

Numerical Application

The model developed in the study is currently being applied to corn and soybean crops. A computer algorithm in the Maple mathematical programming language is developed for solving the system of optimality equations. Preliminary runs of the model with ad hoc model parameter estimates show that the model converges to feasible solutions, but is particularly sensitive to the functional form of the production cost equations. The latter issue is being addressed and valid empirical data currently are being developed to apply the model to the Midwestern agricultural region.

A difficult characteristic of this model is that some of the dynamic variables move in time from known initial conditions to unknown terminal conditions, while others move from unknown initial conditions to known terminal conditions. Because of this, not only must the model provide solutions over multiple time periods, the entire system must be solved repeatedly to determine the fitting initial conditions in the optimal solution.

Limitations of the Study

The follower's opportunity of responding to the leader's strategy is omitted in the hierarchical decision game. In farm commodity programs, it is possible that farmers may not participate in government programs at all and produce only for the market. U.S. farm programs are voluntary, and the government has no control over farmer participation. Under this situation, target price instruments to affect erosion may not impact water quality to the degree expected.

Also, in order to ensure producers are compliant and do not exceed their permitted erosion quotas, policymakers must expend resources for enforcement activities. Such enforcement could take the form of random audits and fines levied for noncompliance. The cost of enforcement would become a component of the policymaker's objective function, while the expected penalty for noncompliance would be subtracted from the producers' objective function. The current model does not include these costs but can be modified to incorporate them, given the availability of proper data.

Finally, the sedimentation subcomponent of the model is based on simplistic assumptions. The hydrological parameters are assumed to be deterministic and temporally and spatially invariant. Sedimentation is a time-based process, and corrective regulations which do not account for the time-delayed environmental impacts could lead to a socially suboptimal erosion level.

Conclusions

The United States government has responded to the need for control of erosion from agricultural nonpoint sources. However, several policy components of the current farm program have conflicting effects. While policy instruments such as price supports tend to promote the intensive use of land, certain other provisions of the farm program attempt to remove environmentally sensitive land from production. Such policies seem to arise naturally out of a process in which different policy problems are examined and addressed individually, and such a piecemeal approach to policy formation inherently produces conflicts of purpose even within the same legislation. Little attention has been paid to developing a unified approach to formulating agricultural policy which considers a broader view of related and conflicting policy issues.

This paper develops a Stackelberg game-theoretic framework of public policy formation which simultaneously determines the endogenous price support and erosion production quota, as well as the optimal permissible water contamination (sediment stock). The model is based on the premise that profit-maximizing individual producers make production decisions in response to prevailing output market conditions and water quality regulations. Crop market dynamics are represented by a recursive system of output and crop price equations. Policymakers may consider producers' production decisions to be rational responses and will formulate price supports and ceilings on erosion output, in order to maximize a weighted social objective function representing the welfare of the primary interest groups, which include agriculturists, water users, and taxpayers.

The analysis distinguishes between the private and social opportunity costs of producing model crops and using waterways as a dump site for erosion from agricultural sources. It is intuitively recognized that the social benefit of erosion-producing activities in agriculture is much below the private benefit to producers. In light of this analysis, the configuration of private and social benefits as well as optimal production and pollution paths will vary as the weights which policymakers attach to different social constituents change.

While this model is developed in a setting of agricultural price supports versus water quality, the hierarchical framework of the model could apply to any case of interaction between government and industry in which the government must balance competing objectives in selecting a level of regulation while the industry has no *a priori* interest in the object of regulation outside of the regulation itself.

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Appendix
Key to Variable Definitions (excluding Figure 1 discussion)

A_{it} = planted acres in commodity i in time t
P_{it} = market price of commodity i in time t
G_{it} = government target price of commodity i in time t
h_i = ratio of harvest acres to planted acres of commodity i
y_i = yield per acre of commodity i
e_i = per acre erosion rate for commodity i
R_{it} = net returns per acre of commodity i in time t
ω_i = official program yield for commodity i
D_{it} = deficiency payment for commodity i in time t
Φ_{it} = government participation rate for commodity i in time t
C_{it} = unit production cost of commodity i in time t
ρ = discount factor
E_t = maximum allowed erosion in time t
S_t = sediment stock in time t
α = sediment growth coefficient
β = sediment decay coefficient
Θ = unit sediment damage cost
a, b, c = societal objective weights
λ_{it} = costate variable -- marginal private benefit of planting in crop i in time t
μ_t = multiplier function -- marginal cost to producer in time t of reducing erosion limit by one unit
ς_{it} = costate variable -- marginal net social benefit of planting in crop i in time t
ς_{1t} = costate variable -- marginal social cost of sedimentation
ξ_{it} = policymakers' costate variable associated with producers' costate variable, $\hat{\lambda}_{it}$
ν_t = marginal social cost of sediment production above optimal level