Holbrook Working

MEASUREMENT OF CYCLES IN SPECULATIVE PRICES*

The term "speculative price" is here used to mean the price of any commodity or security arising from trading on an organized speculative market. As regards commodities, this definition restricts consideration to the prices of futures. Since spot (or "cash") prices of commodities follow prices of futures very closely, most conclusions reached with regard to futures prices of a commodity apply with only slight modification to the spot prices of the same commodity.

Speculative prices deserve critical statistical study from at least three standpoints. One is the practical point of view of the trader, who seeks knowledge of characteristics of price fluctuations in order to profit from the knowledge. Another is the point of view of the citizen, or of his representative in public office, who seeks to know whether the net effect of speculation is good or bad, or who seeks to maximize the net social advantage that may accrue from speculative markets. And a third is the point of view of the economic theorist who may study speculative prices as one of the best sources of empirical quantitative evidence on the behavior of aggregative economic expectations.

From whichever of these viewpoints we regard speculative prices, we need information from scientific statistical study. A good many people hold confident opinions on speculative price behavior, but the opinions on some basic questions are contradictory. For example, John Stuart Mill said that speculation tends to smooth out price fluctuations, and most economists since his time have held that opinion; but John Maynard Keynes declared that speculation tends to aggravate price fluctuations, and that view is now widely held by economists. Among people close to the speculative markets, there are many who believe firmly that "mechanical" systems of price forecasting are reasonably reliable, and there are many others who deny the validity of all such systems.

The statistician who seeks to resolve these differences of opinion by appeal to the facts should not expect to find his task easy. The obvious facts of speculative price movements have been on record for many years, and many people have

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studied them without settling the disputed issues. If we hope to do better, our hope must be placed either in bringing new collateral evidence to bear on the questions, or in finding new concepts and new techniques of analysis which will extract from the known facts meanings which have hitherto eluded us.

The present paper follows the latter course. It makes use of the concept of "disturbed cycles" advanced originally by G. Udny Yule in 1927 (*26*). The concept gives evidence of being one of that class of great ideas which appear now and then in science, and prove useful far beyond the range of their original application. Yule conceived the idea as an aid toward explaining characteristics of sunspot cycles which defied explanation under the concepts previously employed. A few years later Sir Gilbert Walker tried Yule's idea in dealing with cycles in barometric pressure. Herman Wold (*19*) took it up from a mathematical standpoint and showed, in 1938, that Yule's concept was very interestingly related to some other ideas of the behavior of time series. Sixteen years after the concept was advanced, M. G. Kendall published what I believe were the first applications of it in the field where it may find its greatest usefulness—that of economic fluctuations (*13*). The concept seems to provide means, for the first time, of dealing realistically and logically with the sort of cycles commonly encountered in economics and in biology.

Before we look farther into the subject of cycles in general, let us examine some of the characteristics of fluctuations in speculative prices, with which we seek to deal.

SOME CHARACTERISTICS OF WHEAT FUTURES PRICES

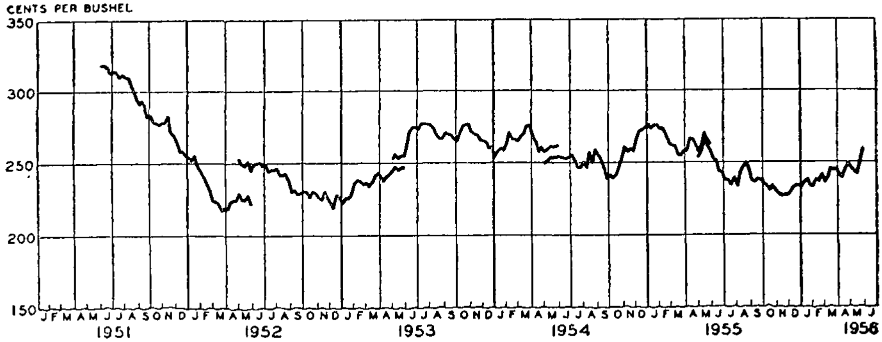
If a chart be drawn of the price of any commodity or security which is traded on an active speculative market, the observed price movements will be found to suggest the presence of irregular cycles. These evidences of cyclical behavior have been the subject of much study, a good deal of it spurred less by pure scientific curiosity than by the consideration that finding a pattern of true cyclical behavior in a speculative price could be comparable to the discovery of at least a small gold mine.

The results of such studies continue to support the hopes of "chart traders," and to provide bases of operation for one type of market advisory service. But to many critical eyes, the results of these continuing and repeated studies seem to build ever higher the evidence that most of the apparent cycles in speculative prices are illusory. They are false cycles, like the "fool's gold" that sometimes misleads the amateur gold prospector.

As an illustration of false cycles, I present Chart 1. The data are wholly artificial; designation of years has been included only to carry out the illusion of an actual price record. If the chart were presented to an experienced student of wheat futures prices, without designation of years or of values on the price scale, and he were told that it was an historical record of actual wheat price movements and asked to identify the years represented, he could find that statement quite credible and start searching his memory to recall what years showed that series of price movements.

Except for the initial year, in which an apparently strong downward trend predominates, there is a marked suggestion of cyclicality in the fluctuations

CHART 1.—EXAMPLE OF UNPREDICTABLE PRICE MOVEMENT*



* Data from Holbrook Working, "Random-Difference Series for Use in the Analysis of Time Series," *Journal of the American Statistical Association*, March 1934, XXIX, Table 1 (beginning a little above the middle of column 4), modified by the addition of slightly different arbitrary constants to the five segments of the series which are separated by breaks.

shown in the chart. The "cycles," however, are purely illusory, and so is the "trend." Each continuous curve in the chart was obtained by starting at an arbitrary point and drawing the curve as a succession of purely random changes. The changes represented were derived from L. H. C. Tippett's table of random sampling numbers (1927).

Since the changes in the series are random, it follows that all of the changes lack any causal relation with previous or subsequent changes. And if there is no causal relation among groups of changes, there can be no real trends or cycles in the series.

The resemblance of the curves of Chart 1 to an actual speculative price series is a logical and necessary one. The principal characteristic of a speculative price is that its changes are unpredictable, or nearly so. If the changes in an actual price series are completely unpredictable from any knowledge of previous changes or values in the series, it follows that they are random changes, like those in the series of Chart 1.

There may be some tendency for people to regard randomness of change in a price series as irrational and undesirable. Randomness of price change is in fact a requirement for a perfect market in which current prices in the market should correspond to expected future prices. This may be seen most readily if we consider the consequences of assuming a contrary condition. Suppose that the price of May wheat today is $2.10, but that a sound basis exists for predicting a price increase. In that case, the price today ought not to be at $2.10, but at some higher figure which would truly reflect the known conditions. If, on the other hand, the current price of May wheat reflects the best possible appraisal of known conditions, it is still true that some change in price will undoubtedly occur tomorrow, but no basis exists for predicting what the change will be.

An interesting consequence follows from this reasoning. It is that if forecasters who attempt to predict changes in speculative prices have little success, their lack of success may reflect no discredit on their forecasting ability, but indicate that the market approaches the ideal of perfection.

Not all the evidence of cycles in speculative prices can be dismissed as illusory.

Security prices quite clearly reflect the business cycle, and though the mechanism of the business cycle is still a subject of dispute, the relation of security prices to it, I think, is one of the better understood aspects of business cycles.

Another type of cycle in speculative prices whose existence has been clearly demonstrated is that which I have elsewhere called "crop-scare and related cycles." Clear evidence of the existence of that type of cycle has been produced, so far as I know, only for wheat prices in the United States, but there is a good deal of reason to believe that the same general type of cycle probably occurs in prices of other speculative commodities and in prices of stocks. The name "crop-scare cycles" was not a very good one even to designate a kind of cycle regarded as possibly peculiar to wheat prices, and it is not at all applicable to any similar cycles which may exist in security prices. I propose, therefore, a new name, *episodic cycles*.

Because true episodic cycles in wheat prices appear as isolated occurrences in a series which seems to be composed mostly of false cycles, it is difficult to establish a clear distinction between the true and the false cycles. There exist no well-established statistical tests of significance which are very useful to prove the validity of any distinction that may be drawn at the time a study is made. In these circumstances, it is particularly important to test conclusions by applying them to a price record which is quite independent of the one from which the conclusions were drawn. A good test of that sort is now possible because the original conclusions regarding episodic cycles in wheat prices were published in 1931, and a good deal of the subsequent period provides a record of prices formed under conditions reasonably comparable with those of the study period.

The primary criterion for identification of the beginning of an episodic cycle was originally expressed as "an increase of 14 cents or more in the deflated weekly average prices (deflated to 1913 price level) within a period of 5 weeks or less" (*20*, p. 19). On both practical and theoretical grounds it now seems to me that the criterion should be expressed in terms of an extreme range of prices within a specified interval of time, rather than in terms of weekly averages.[1] In those terms the criterion would be a price advance of 17 cents (of 1913 purchasing power) or more from the low price of any one day to the high price of any day less than 6 weeks later. If we express the criterion in terms of 1949 price level, the price advance to be specified is 38 cents.

Characteristics of price behavior following such a price increase were found to vary somewhat according to certain identified circumstances. Subsequent experience has verified the original findings in all respects except one, and it will be convenient to state the observed characteristics and to consider the more recent evidence simultaneously.

Actual occurrence of episodic cycles in wheat prices since 1930 has been limited to about the first half of the period which has elapsed since then. From 1942 to the summer of 1946, price controls were in effect, and governmental control of wheat exports subsequently has apparently given the price movements new characteristics. Prices since 1946 have not been stabilized, however; in early 1947 the weekly averages for the May future rose from $1.97 in the last week of Janu-

[1] This change in formulation makes no change in results from application of the criterion, either before or after 1930.

ary to \$2.62 in the second week of March; and later in 1947 the price of the next year's May future rose from \$2.10 to \$3.00 between the last week of June and the final week of November—an advance that was followed after an interval of two months by an abrupt decline to an average of \$2.36 for the third week of February 1948. But these fluctuations, depending heavily on administrative decision, do not fit the pattern of free market behavior.

The 1930s saw two upward price movements which meet the formal specifications for the upward phase of an episodic cycle, but which I leave out of account because their causation was clearly quite different from that of quantitatively similar movements previously observed.[2] One of these movements was that during the month following devaluation of the dollar in 1933, and the other was that during a few days following the outbreak of war in 1939. With these left out of account, the troubled decade of the 1930s produced a normal amount of experience with price advances such as had earlier been found to initiate episodic cycles.

The hypothesis which was not clearly borne out by experience during the 1930s was that the declining phase of a cycle initiated by news of crop damage tended to be more gradual if it occurred late in the year. Specifically it had seemed that declines tended to be rapid from cycle culminations in the month of May, slower if the culmination was in June or July, and slower yet, and perhaps considerably delayed, if the culmination was in late summer or autumn. None of the cycles of the 1930s happened to culminate in May, and the only one which showed a distinctly slow declining phase may have done so for a reason not related to its occurrence fairly late in the harvest season.

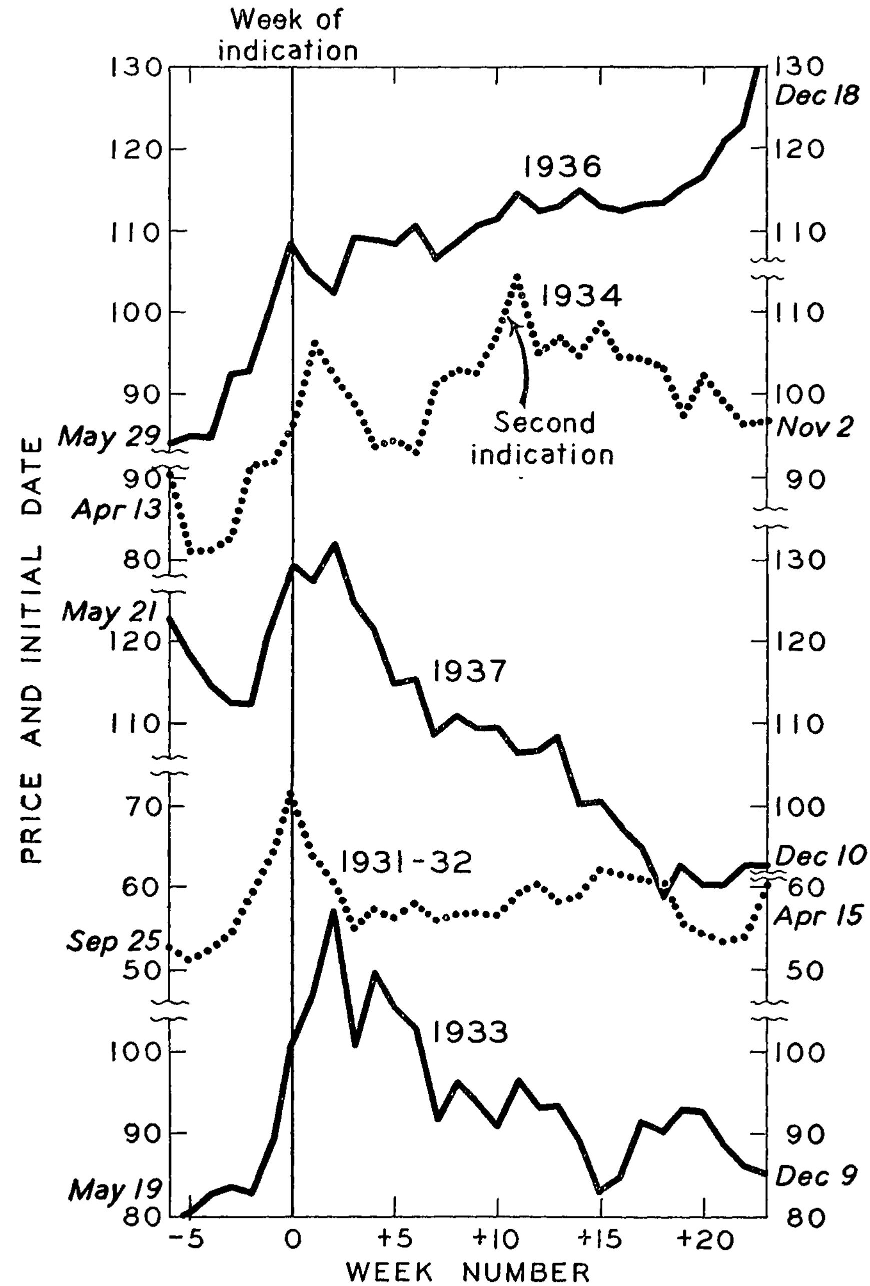
On the other hand, previous experience had indicated that price advances which met the general criteria for episodic cycles, but which culminated in the month of August, were rather likely *not* to be followed by price decline, and this seemingly peculiar exception to the general rule found confirmation in the 1930s. Such a price advance occurred in 1936, and raised the question whether in that instance subsequent decline was to be expected, as had sometimes occurred, or whether decline should not be expected. Contemporary study of the circumstances led to the opinion that in this instance, decline should *not* be expected (*8*, p. 23; *21*, p. 11), and the formal indication of the beginning of an episodic cycle was therefore rejected. The outcome confirmed this judgment.

The record of price movements in this particularly interesting, exceptional case is shown as the top curve in Chart 2. Records for other years follow in the order in which they will be discussed. It should be emphasized that the curves in Chart 2 are based on closing price quotations for one day only in each week. The cyclical appearance of the lower four curves in the chart could be considerably enhanced by plotting weekly averages, as was done in the initial study which revealed these cycles. The use of averages, however, gives an artificial smoothing, and it seems desirable here to avoid such smoothing lest the cyclical appearance be thought to depend on it.

In 1934 the criteria for an upward phase of an episodic cycle were satisfied twice: first near the end of May, and again in early August. Such superposed

[2] This consideration argues only that there was ground to doubt whether the price advances in question could safely be accepted as initial phases of episodic cycles; economic considerations which cannot be explained clearly at this point permitted arguing, at the time these price advances occurred, that they ought *not* to be regarded as initial phases of episodic cycles. They proved in fact not to be.

Chart 2.—Wheat Price Movements Before and After "Indication" of Episodic Cycles Since 1930*



* Data are Friday closing prices, in cents for bushel, for 30 consecutive weeks, beginning 6 weeks before the week of "indication." See text for indication criterion, and note the reasons given for considering that in 1936 the indication was reversed by occurrence of a culmination in August, and accompanying circumstances.

cycles have occurred in several earlier years, and once before (in 1890) with the second peak in August. Empirical evidence and theory both seemed to indicate that no special consideration should be given to the timing of a price culmination reached under those circumstances. We therefore expected substantial price decline from this August peak.

The next clear indication[3] of the beginning of an episodic cycle (skipping the rejected indication of 1936 already discussed) appeared in July 1937. Its occurrence tended to confirm our identification of conditions under which such price cycles were likely to occur. Nearly two months earlier, in May, we had written (*9*, p. 399):

> Serious damage to spring wheat in the United States and Canada may threaten at any time between late June and harvest, or earlier if June rainfall should be scant. . . . A rapid price advance of 20–30 cents from whatever level might have been reached at its beginning, could easily develop form threats of serious spring-wheat crop damage. The background of recent high prices and the absence of significant reserve in the world carryover offer favorable conditions for a sharp price advance in North America.

In the full price record, the episodic cycle of 1937 appears superposed on a downward trend, though I would not know just what shape to give the trend line.

The last two curves in Chart 2 present examples of the sort of abrupt price decline which previous observations had indicated was likely to occur in episodic cycles generated by developments other than crop news. In 1931 and 1933, crop news was lacking to explain the price advances, and they seemed to rest mainly on speculative buying in anticipation of the beginning of a sustained recovery of commodity prices generally. It was easy in the autumn of 1931 to think that the depression had lasted long enough, and to be encouraged by what seemed evidence of impending business recovery. In the summer of 1933, such hopes found even more encouragement. In both instances it was logical to expect business recovery to be accompanied by strong advances in commodity prices, and so buying of wheat futures found encouragement.

The record in Chart 2, it should be recalled, is not the main evidence of occurrence of episodic cycles in wheat prices, but is supplementary evidence confirming conclusions drawn from records for earlier years. In more than 50 years, ending with the 1930s, the frequency of incidence of recognized episodic cycles was rather less than once in two years, on the average. The average duration of an episodic cycle was between three and four months. Thus it appears that the identified episodic cycles occupied only one-sixth to one-eighth or less of the total number of weeks in more than 50 years ending at 1940. The intervening five-sixths to seven-eighths of the time was occupied, so far as had been determined,

[3] A price advance that fell slightly short of meeting the specification, "17 cents (of 1913 purchasing power) or more from the low price of any one day to the high price of any day less than 6 weeks later," occurred in September–October 1935, and was followed by price decline in a typical episodic cycle pattern. At about the time the price advance was starting M. K. Bennett had written: "Severe damage to Southern Hemisphere crops during the coming months might generate widespread fear of world shortage of wheat and induce such active public participation in speculative trading as to carry prices to excessive heights from which a sharp reaction would occur, . . . (*1*, p. 28).

by price fluctuations very closely resembling the random fluctuations of the curves in Chart 1.

A person who has not given much study to the behavior of wheat prices may have a first reaction that the foregoing account gives an adequate summary description of wheat price behavior, and that it poses no very pressing questions. One who studies the evidence closely, however, may wonder whether all of the significant characteristics of wheat price behavior have been revealed. An attempt to explain *why* wheat prices follow one pattern of behavior most of the time, but shift to another pattern of behavior over scattered short intervals, raises other questions. Explanation is complicated by the fact that no clear cycles have been shown to exist in prices of any commodity other than wheat. I have looked for such evidence in corn and oats prices, and have failed to find it. Perhaps the cycles exist in corn and oats prices, but are more obscure and can be brought to light only by use of more sensitive methods than proved adequate to reveal the cycles in wheat prices.

To help in answering all of these questions, we need more knowledge of statistical characteristics of the peculiar sorts of behavior found in wheat prices. This is the thought which turned me to consideration of the general question of cycle theory, and thence to new ideas arising from Yule's cycle concept.

TYPES OF CYCLES

In considering cycle theory I shall examine five different types of cyclical or cycle-like fluctuations, including the episodic cycles and the false cycles of which mention has already been made. Each type will be considered only in its simplest form as described in the diagram shown as Chart 3. The five types, with possible variants of each, and combinations of two or more types, cover an immense range of possible sorts of variation in a time series, yet the different types will be found so closely related that each appears only a special case, or class of cases, in a more general cycle theory.

Harmonic Cycles

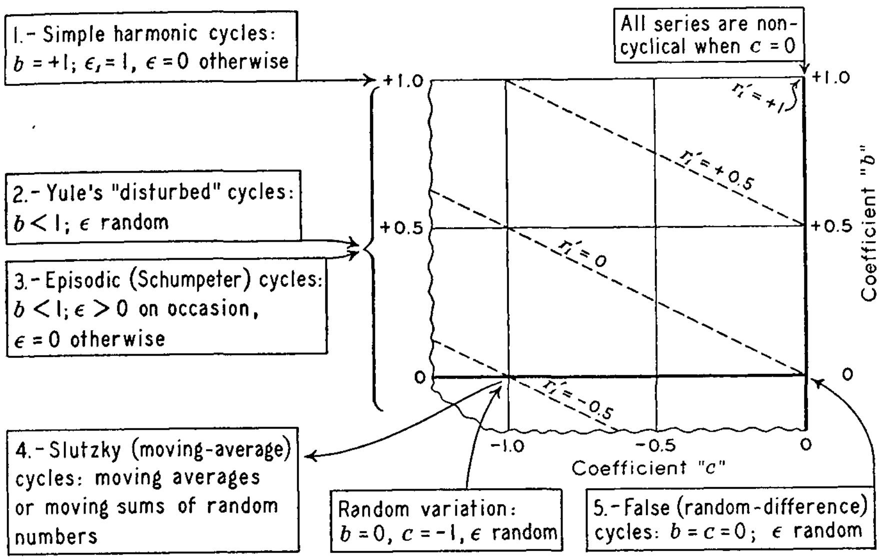
The basic concept of cyclical movement, in both mathematics and statistics, is that of simple harmonic cycles. The classic example is that of the movement of an ideal pendulum—a pendulum swinging without friction, without disturbance, and without change in the tempo of its motion. An ideal pendulum does not exist in fact, but the ideal is closely approached in any good clock.

The equation for simple harmonic cycles may be written in several different ways. The most common form of the equation runs in terms of trigonometric functions, but we shall find most useful a form which may be called an equation of motion:

$$\Delta_i = b\Delta_{i-1} + cu_{i-1}, \ (\Delta_i = u_i - u_{i-1}) \qquad (1)$$

Time, if we think of a pendulum, may be supposed measured in tenths of a second. Then Δ_i represents the horizontal distance which the pendulum has moved in the last tenth of a second, ending with a given time, i. The equation, then, states that the (horizontal) distance travelled by the pendulum in the last tenth of a second depends on the distance travelled in the previous tenth of a

CHART 3.—SYNTHESIS OF CYCLE CONCEPTS: TYPES OF CYCLES DESCRIBED BY THE AUTOREGRESSION EQUATION
$\Delta_t = b\Delta_{t-1} + cu_{t-1} + \varepsilon_t, \ (\Delta_t = u_t - u_{t-1})$*



* r_1' = correlation of adjacent first differences.

second (Δ_{t-1}), and on the position of the pendulum (u_{t-1}) at the beginning of the last tenth of a second.

One advantage of considering the equation in this form is that it involves coefficients, b and c, which are easily interpreted. The coefficient b expresses the combined effects of inertia (popularly, momentum) and friction. In simple harmonic motion, $b = 1$, expressing the fact that a perfect pendulum tends, as a result of inertia, to travel the same distance in one short interval of time as in the previous equal interval. In the presence of friction, b is less than unity, and the pendulum tends to come to rest, as an actual pendulum does if power is not supplied more or less regularly to offset the friction.

When $b < 1$, and the pendulum tends to come to rest, the resulting motion is *damped* harmonic motion. If the damping is slight, an initial impetus given to the pendulum results in a long series of oscillations, each passing through a range only slightly less than the previous one. When the damping is extreme, as in the case of a pendulum suspended in a heavy oil, there may be no true oscillation; the displaced pendulum may then merely settle gradually back toward the vertical. That case is represented by a value of b close to zero.

The coefficient c in the equation of motion of an ideal pendulum expresses the force with which gravity acts to affect the motion of the pendulum. With a short pendulum, swinging in a sharply curved arc, c is large. With a long pendulum, swinging in a flat arc, c is small. The value of c depends also on the length of the time-interval used. Moreover, with a pendulum of given length, the damping associated with friction modifies c as well as b.

In the foregoing equation, b and c coefficients have been used out of consideration for previous practice. That equation is a transformation of what has come to be known as the *autoregression equation.*

$$u_t + au_{t-1} + bu_{t-2} = 0 \qquad (2)$$

The coefficient b takes the same value in equation (1) as in equation (2). The other coefficients have the relation $c = -a - b - 1$.

The concept of simple harmonic motion has been applied in cyclical studies both in simple form and in terms of combination of several superposed cyclical tendencies with differing phases or differing periods, or both. Cycles in the ocean tides, for example, have been shown to consist of a considerable number of simple harmonic components.

The harmonic cycle concept provided the basis for W. Stanley Jevons' studies of a supposed relation of sunspots to commodity-price swings (*10*). The harmonic cycle concept was likewise the basis for H. L. Moore's studies of the business cycle, which led him to the odd conclusion that business cycles somehow depended on the motion of the planet Venus in relation to the Earth and the Sun (*16*). It is the ruling concept in the recent, widely-read, and highly informative book on *Cycles* by E. R. Dewey and E. F. Dakin (*7*).

The harmonic cycle concept is demonstrably invalid and misleading as applied to many cycles in economics and in biology. The principal forces involved in some economic and biological cycles are well known, and clearly do not tend to produce uniformity in period of the cycles. The cycles observed in prices of certain kinds of livestock, for example, are produced mainly by an interaction of supply on price and of price on supply, the price effect on supply being a delayed one. Certain biological cycles are well known to have a similar origin: as the populations of certain species of insects or animals increase, so does the prevalence of their parasites or diseases; and presently the parasites or diseases decimate the population of their hosts, and so destroy much of their own means of existence. Such cycles can continue indefinitely, one cycle leading into another, but they have no tendency to strict regularity in length. If high prices of hogs happen to coincide with a shortage of corn, expansion of hog production is delayed, the period of high prices tends to be prolonged, and the whole course of the hog-price cycle is somewhat altered; there is no tendency for the cycle movements to "get back into step" as they do when harmonic cycles are affected by superposed disturbances.

The serious aspect of application of the harmonic concept to economic and biological cycles, however, is not the fact that it leads to a mistaken belief that the cycles are periodic, but the fact that the concept tends toward wrong explanation of the cycles. Belief in the harmonic concept leads to search for some cosmic explanation. Thus Jevons was led to his sunspot theory (it was long after Jevons' time that Yule found that the sunspot cycles themselves are not harmonic cycles). Moore's search for a cosmic cause led to his theory of interposition by Venus. Others have sought explanations in terms of some imagined cosmic effect operating through temperature or cosmic rays.

It is probably demonstrable that only a few kinds of clear cycles in either economic or biological phenomena can possibly have a dominantly cosmic origin. Diurnal and seasonal cycles, of course, do have such an origin, but apparently most recognized cycles longer than a year cannot. In order to produce a well marked cycle in economic or biological data, cosmic influences must be strong and fairly direct. Most of the hypotheses which have been advanced on the supposition of cosmic origin involve chains of causation which are so tenuous that they could not produce the effects which they seek to explain.

One consequence of the tendency of the harmonic cycle concept toward false and irrational theories has been to discredit studies of cycles, and to encourage belief that the apparent cycles are somehow imaginary. This is unfortunate, because cyclical fluctuations which are not harmonic are none the less real and deserving of study.

Yule's "Disturbed" Cycles

In 1927, as we noted earlier, Yule undertook to work with a new concept of cyclical behavior, seeking thereby to explain peculiarities of the sunspot cycle. The observational data, expressed in terms of Wolfer's sunspot numbers, show strikingly smooth cycles; but the intervals between sunspot minima have varied from nine years to as much as thirteen years, and the amplitude of the cycles has varied much more; the widest swing is more than three times as great as the narrowest.

Yule's thought may well be stated in his own words (*26*, p. 226):

> If we observe at short intervals of time the departures of a simple harmonic pendulum from its position of rest, errors of observation will cause superposed fluctuations. . . . But by improvement of apparatus and automatic methods of recording, let us say, errors of observation are practically eliminated. The recording apparatus is left to itself, and unfortunately boys get into the room and start pelting the pendulum with peas, sometimes from one side and sometimes from the other. The motion is now affected, not by *superposed fluctuations* but by true *disturbances*, and the effect on the graph will be of an entirely different kind. The graph will remain surprisingly smooth, but amplitude and phase will vary continually.

The equation for disturbed cycles of the sort Yule considered may be written by simply adding to the equation which we have already used for harmonic cycles one new term to represent the disturbance function. The equation then becomes

$$\Delta_i = b\Delta_{i-1} + cu_{i-1} + \varepsilon_i \qquad (3)$$

In one respect, Yule's statement of the concept of disturbed cycles may be misleading; it may give the impression that in order to have disturbed cycles, the pendulum must first be set to swinging uniformly. This is not the case. The same result will be produced if the boys shoot peas at a pendulum which is initially at rest. Yule's original treatment of the concept required emendation also as regards his assumption that the coefficient b in the foregoing equation may be unity.[4] If the disturbance function, ε, is a random variable, it is necessary to have $b < 1$, else the swings of the pendulum will tend, in irregular progression, to become more and more violent; as Wold expresses the matter, if $b = 1$, the system is "evolutive" (*19*, p. 96).

[4] Because Yule wrote the equation in another form, he expressed the assumption somewhat differently.

In the statistical literature which has developed from Yule's suggestion, statistical series with the characteristics he described have been called *autoregressive*, a term due, apparently, to Wold. The adjective is a good one, expressing the fact that values of terms in such a series may be "predicted" by a regression equation involving only previous terms in the same series; but I prefer here to keep the thought implied by Yule in treating the fluctuations in such series as disturbed cycles.

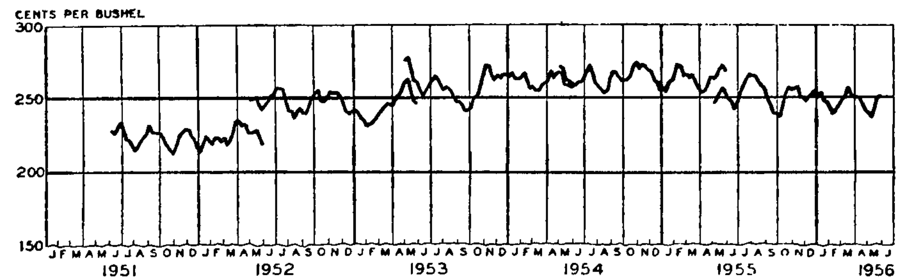
A little reflection will show that Yule's concept of disturbed cycles, with the superficial amendments noted above, may be exactly what is needed to account realistically for cyclical behavior in many sorts of economic and biological phenomena. The essentials of the concept are: (a) existence of an elastic system—that is to say, a system that will tend to react, following a disturbance, with a force which may or may not be sufficient to generate oscillation in the absence of subsequent disturbances—and (b) the presence of disturbances affecting the system.

Those are conditions known to underlie the cycles in certain livestock prices, for example, and the cycles in certain insect and animal populations.

Statistical study is required, of course, to determine whether the characteristics of a particular elastic system are adequately described by an equation of cyclical motion so simple as the one here used. Kendall applied Yule's concept in the study of fifteen British agricultural series and found no reason to think that a more complicated form of the equation was needed (*12*, *13*).

The range of different sorts of fluctuation which is comprised within the concept of disturbed cycles is immense. Yule gave one illustration in the form of an artificial series constructed with assumed values of the coefficients and values of the disturbances obtained by throwing dice. Kendall has provided some further illustrations by publishing tables showing series resulting from use of other values of the coefficients and disturbances derived from the table of random numbers which he and Babbington Smith published. One of Kendall's series, with changes of scale and of origin to give it maximum similitude to a record of wheat futures prices, is reproduced in Chart 4. No experienced student of wheat futures prices could be led to think the curves of that chart an actual record of wheat prices,

CHART 4.—EXAMPLE OF DISTURBED CYCLES*



* The equation for the curves shown is $\Delta_t = 0.4\Delta_{t-1} - 0.2u_{t-1} + \varepsilon_t$, with ε random and u representing deviations from a mean; the means change from year to year. If the coefficient of Δ_{t-1} were close to unity, the cycles would be smooth and nearly uniform in length, but their amplitude would still vary substantially. The coefficient -0.2 for u_{t-1} indicates a "natural" period of oscillation of about 14 weeks, the week being the time unit.

but we shall see later that a series could be constructed in the same plan as that one, but with different, and changing, values of the autoregression coefficients, which would reproduce very closely the actual characteristics of wheat price fluctuations.

Since autoregressive series—that is to say, disturbed cyclical series—take different forms as the coefficients of the equation are changed, it is useful to consider some of the possibilities in terms of a diagram in which combinations of values of the coefficients b and c can be plotted as ordinate and abscissa respectively. Chart 3 provides such a diagram. Sloping lines on the chart indicate how the correlation between adjacent first differences in the series is affected by change in the coefficients b and c. Comments which surround the chart describe the characteristics of fluctuations associated with different values of b and c, and with certain characteristics of the disturbance function ε. For example, the first such comment notes that if $b = 1$ and the only disturbance is a single initial impulse, the resulting fluctuations are simple harmonic cycles. This is qualified by the general observation that no series is cyclical if $c = 0$. The shocks or impulses which initiate fluctuation may have systematic characteristics, but for present purposes we shall associate the concept of disturbed cycles with random disturbances.

We pass now to brief consideration of some special types of fluctuation that have not hitherto been discussed in connection with the disturbed-cycle concept, but which deserve to be related to it.

Episodic Cycles

Episodic cycles in wheat prices, of which we saw some examples in Chart 2, represent an extreme form of a general type which may be given the same name. Those in wheat prices are extreme in the sense that individual cycles are isolated rather than connected in chains. This is a consequence of having the coefficient b close to zero.[5]

J. A. Schumpeter's theory of business cycles (*17*) may also be called a theory of episodic cycles, but it involves the supposition that an initial disturbance produces a chain of cycles, presumably with the successive swings tending to diminish in magnitude. Schumpeter assumes conditions in the economic system which are equivalent to assuming that in the equation of cyclical motion, b is fairly large—large enough to produce oscillation.

The characterization of episodic cycles in the diagram of Chart 4, specifying $\varepsilon = 0$ except at the times of episodic disturbance, is the simplest possible condition. In the case of the actual wheat price cycles, at least, the disturbance function never vanishes.

Slutzky (Moving-Average) Cycles

In the same year that Yule advanced the idea of treating statistical series by use of the concept of disturbed cycles (1927), Eugen Slutzky published his demonstration that a series with conspicuously smooth cyclical appearance may be

[5] An autoregressive series with the equation we use is non-oscillatory when $4b < (b + c + 1)^2$. The limiting condition for isolated cycles is thus $4b = (b + c + 1)^2$. This is the equation for a sheared parabola. The curve it describes may be approximated fairly well through its most interesting range by drawing a quadrant of a circle on Chart 4, with radius of unity and center at $b = +1$, $c = -1$.

produced merely by taking moving sums (or moving averages) of a random series (*18*). This possible source of cyclical fluctuations deserves to be kept in mind, even though, so far as I know, it has not been proved the true source of noteworthy cyclical fluctuations in any observed series. The concept is a realistic one which may find important use, and it is pertinent as a warning against misuse of moving averages and against misinterpretation of results obtained through their proper use.

For our present purposes we need particularly to notice *why* the process of taking moving averages or moving sums of random numbers produces a strong appearance of cyclical character in the resulting series. The taking of moving averages does only what it is generally understood to do, namely, smooth the fluctuations in the original series. The smoothing produces a recognition of cyclical character in the resulting series, not because of some peculiar property of the smoothing process, but because a random series has an essentially cyclical attribute. The smoothing modifies the form in which that attribute appears, changing it from a form which we do not associate with cycles to one which we do.

Let us seek first to define essentials of the cyclical concept. One simple course, which has some distinguished support, is to define the concept narrowly, restricting its application to harmonic series. Such a definition may be good in mathematics, but it is not acceptable for economics. The economist must follow established usage and discuss under the term "cycles," kinds of variation which are not harmonic.

What, then, are the essentials of the prevalent concept of cyclical variation? They appear to me to be two. First, the cyclical concept involves existence of possibility of prediction of future occurrences from knowledge of occurrences that have preceded them; not necessarily perfect prediction, but prediction with some degree of reliability. And second, the cyclical concept involves the idea of completion of a circuit; the movement, or chain of occurrenes, must pass through a "full cycle" so that there is in some sense a return to an initial status.[6]

There is a third element in the cycle concept as it is most commonly employed, namely, the idea of propagation of cycles; we usually think of cycles as connected in chains, one cycle following directly on another. In that case cyclical movement is oscillatory movement. The chain effect, however, seems to me not an essential feature of the concept as it is actually applied. For example, we find it seeming quite logical to speak of the *cycle* of mechanical operations set in motion by inserting a coin in a "juke-box," even though one cycle does not necessarily lead into another. (We may sometimes be grateful that the juke-box mechanism does not produce chains of cycles.) Isolated cycles represent an extreme case of episodic cycles.

If some degree of predictability of change and tendency to completion of a circuit are the only requisites for cyclicality, a random series should be recognized as cyclical. Contrary to a widespread impression, *changes* in a random series are

[6] It should be noted that in simple harmonic cycles conceived to start from zero, the first return to zero does not satisfy this specification; the position of a pendulum is the same at one passage of the vertical as at the previous passage, but the direction of its momentum is the same only at alternate passages of the vertical. On the other hand, in sufficiently damped harmonic motion, as in the case of a pendulum suspended in heavy oil, the specification for completion of a full cycle is satisfied at the first return to the zero position.

predictable to a quite substantial extent. The coefficient of correlation between a known change in a random series and the next change is $r = -0.50$; and the coefficient of correlation between a known *value* in a random series and the succeeding *change* is $r = -0.71$.

In a random series, only one change can be predicted at a time; each change is predictable only from the value with which it starts or from the change which led up to that value.[7] But when a random series is smoothed by an equally weighted moving average, the predictability is spread over a series of changes in the moving average. That is the source of the easily recognized Slutzky cycles.

With this observation we may drop the provocative question whether a random series should be called cyclical, lest it divert attention from the one fact in this connection that is important for our present purposes, namely, the fact that *changes* in a random series are *predictable.* By contrast, we turn next to what is for us the very important sort of variation in which the changes are unpredictable.

False (Random-Difference) Cycles

When the coefficients b and c in the equation for disturbed cycles become zero, the equation reduces to $\Delta_t = \varepsilon_t$. Then if the disturbance function, ε, is a random variable, the equation describes a series composed of random changes. Such a series may be called descriptively a *random-difference* series (23).

The condition thus described is that which was considered at the outset of our examination of characteristics of speculative prices. As was there observed, and illustrated in Chart 1, it is a condition which gives rise to fluctuations which appear irregularly cyclical. The apparent cycles in such a series must be regarded as false cycles because they are wholly unpredictable from any knowledge of the previous course of the series, and because the concept of cycles, however broadly interpreted, requires some degree of predictability of change. As we noted earlier, this condition of unpredictability is the theoretical ideal in a speculative price.

We can now suggest a simple logical interpretation of the occurrence in wheat prices of what seemed to be two distinctly different types of behavior: random, or nearly random, change during most of the time, and a sort of true cyclical movement in isolated occasions. Both sorts of behavior represent special cases under the general condition for disturbed cycles, the only necessary difference between the two special cases is a difference in value of the coefficient c in the equation of cyclical motion. The corresponding economic interpretation is that wheat prices usually behave in a manner approximating the theoretical ideal, corresponding to $c = 0$; but that occasionally something happens to throw prices out of proper adjustment, whereupon readjustment occurs gradually. During the readjustment, $c < 0$.

MEASURES OF WHEAT PRICE CYCLES

The foregoing cyclical theory does not directly solve any of our problems of measuring cycles in speculative prices, but it puts us on a promising track toward solutions. For example, it has been the source of a great practical improvement

[7] That is, Δ_t $(=x_t - x_{t-1})$ is correlated with the antecedent x_{t-1} and Δ_{t-1}; and of course it is equally correlated with the succeeding x_1 and Δ_{t+1}.

in the primary criterion for indication of occurrence of an episodic cycle in wheat prices.[8]

Because I worked initially with weekly averages, the criterion was originally found in those terms. The quantitative specification, on the basis of prices deflated to the 1913 price level, was a 14-cent price advance between weekly averages. Imagine the dissatisfaction of a trader with that specification. On a Monday, perhaps, a price advance has gone to a point which he thinks likely to mean that the weekly average price will show a 14-cent advance; but he has to wait until Saturday to be sure. As a mere academic student of wheat prices, I also found that specification unsatisfactory.

I would have liked, in 1931, to have modified the criterion of price advance to read in terms of an extreme price range; but it seemed to me quite clear, on the basis of conventional interpretation of the character of price changes, that a criterion based on extreme price range would be affected too much by accidental, erratic, fluctuations. Then as this paper was being written, it occurred to me that the theory here developed led to an entirely different conclusion: if price changes closely approximated random walk, it followed logically that extreme price range within a specified period ought to provide just as reliable a criterion as did amount of change between weekly average prices.

I must admit that, despite the force of logic, I tried the experiment with misgiving. The outcome was a vindication of the theory. It turned out that using weekly averages gave no advantage; a criterion of 17-cent price advance between extreme quotations produced exactly the same indications of episodic cycles, in the 60-year price record, as did the criterion of 14-cent price advance between weekly averages. The advantages, therefore, are all on the side of using the criterion based on extreme price range. A false concept of the nature of price fluctuations had long prevented discovery of that fact.

A further implication of the cyclical theory here advanced is that measurement of cycles in wheat prices calls for deriving estimates of the coefficients b and c in the equation of cyclical motion, and requires means for measuring the characteristics of the disturbance function in that equation. Another contribution of the theory is that it helps toward understanding how the behavior pattern of wheat prices can shift from a pattern of purely false cycles to one of true episodic cycles, and then back again. From a statistical standpoint, as we have just noticed, such shifts may involve only rather moderate changes in value of the coefficient c in the equation of cyclical motion.

Two general methods have been used in study of statistical series believed to fit the concept of disturbed cycles. One is to make direct calculation of the autoregression coefficients by the principles of the method of least squares. The other is to calculate a great number of serial correlation coefficients (called also "autocorrelation coefficients") and to form from them either a "correlogram" or a "lambdagram." Both of the latter are graphic analytical devices suggested by Yule. The laborious calculations which they require are resorted to partly for the sake of evidence they give as to whether the series is really autoregressive (i.e., cyclical, as we used the term here), and partly because they may lead to a more reliable estimate than the autoregression coefficients of the "natural period" of

[8] The improvement was mentioned earlier, with explanation of its source deferred to this point.

any inherent cyclical tendency in the series; the least squares estimates of the autoregression coefficients tend to be strongly biased in the presence of superposed random variations in the series.[9]

Neither of these general methods, as hitherto applied, appears usable with speculative prices. Both require preliminary fitting of trends, unless the series under study may be considered trend-free, and it seems quite impossible to fit trends to the wheat-price data which would merit any confidence. I would not know how to choose among several possible trend curves whose use would lead to widely differing outcomes from analysis of the deviations-from-trend. Prices for any speculative commodity would present similarly insuperable difficulties of trend-fitting. I imagine that security prices would also, unless attention were confined to the long swings associated with business cycles, in which case simple trends might be usable with some confidence.

Faced with this problem, I saw no good solution unless it should be possible to make the analysis wholly in terms of changes in the series—that is to say, without any use of deviations from a supposed norm. In other classes of problems involving correlation of time series, the use of first differences has been found an effective device for circumventing the difficulties of trend-fitting, and the present problem appeared capable of solution by that route *if* a way could be found to make the analysis in terms of first differences alone.[10]

CONCLUSIONS

This new approach to the study of speculative price behavior has not as yet been carried much beyond the stage of developing the theory and overcoming some problems of practical application. I cannot say yet that the new approach will yield much advantage beyond greatly improved understanding of the nature of the statistical problem. Two fairly important conclusions have emerged, however, and I shall merely state them briefly.

One of these concerns the nature of the price advances which initiate the identified episodic cycles in wheat prices. My initial interpretation was that these arose from emergence of what might be described as a predominance of bullish *sentiment* in the market. That explanation would imply that the upward movements have the character of short-time trends. It appears that that is not the case. The evidence is that the upward movements in such cycles arise only from a *chance* predominance of bullish *influences*, in circumstances which make the price unduly sensitive. The temporary undue sensitiveness causes a proper price advance to go too far. This conclusion seems to accord with a hypothesis advanced elsewhere to account for the peculiar circumstance that episodic cycles have been found in wheat prices, but not in the similarly speculative prices of corn and oats. That hypothesis was that the excesses in wheat price movements may have arisen from a tendency of traders in the United States to attach undue

[9] The position of random variation in the diagram of Chart 3 helps to explain this.

[10] There followed in the original manuscript three paragraphs and an appendix concerned with estimation of the parameters of an autoregression equation from the serial correlations of first differences of a series. Those paragraphs and the appendix are omitted here inasmuch as further study of the autocorrelations in speculative prices quickly produced evidence that the statistical analysis had to proceed otherwise than through that approach.

importance to developments in North America, and particularly to developments in their own country, losing sight of the extent to which the wheat market was international (*22*, pp. 150–66).

The other conclusion relates to a special sort of behavior which is found in "microscopic" study of speculative prices, looking at the full detail of the fluctuations. In the Chicago wheat market there may be a dozen or more quotations recorded within the space of one minute, and so refined is the price-recording technique that these quotations are usually recorded properly in the right order. These tiny fluctuations merit study from the standpoint of the question whether the professional pit traders known as "scalpers" operate in such a way as to restrict price fluctuations, as they are supposed to do in theory; or whether, as several students have claimed, the scalpers actually tend to "ride the waves" and to magnify price fluctuations within the day. The answer from statistical study is quite clear: the predominant effect of the trading of scalpers is to restrain price fluctuations.

EPILOGUE

1. Pursuit of the ideas embodied in the foregoing paper quickly produced evidence that required a drastic change in the research plan. In the years that have followed several avenues of statistical research have been pursued, but with attention to autocorrelations or autoregressions, variously measured, retaining a central place in the statistical analyses. Also retained has been the concept that the autocorrelations arise from operating characteristics of a mechanism subject to impulses that may be regarded, for most purposes, as occurring randomly. The following summary of leading conclusions rests in part on results of recent research not yet published.

2. A speculative market is an intricate mechanism, with complex operating characteristics incapable of being usefully represented by any such simple model as a pendulum. The impulses acting on it may be treated as random, with zero mean, but not as having a constant variance. It is necessary to recognize the impulses as falling into two general classes. The impulses of the first class generate, through the market mechanism, price changes among which all non-zero autocorrelations, under most circumstances, are positive. The impulses of the second class generate price changes among which all non-zero autocorrelations tend to be negative.

Impulses of the first class vary greatly through time in force and frequency, whereas impulses of the second class vary much less in those respects. There exists, consequently, a *tendency* for the generated price changes to show fairly strong positive autocorrelations during some intervals of time, and fairly strong negative autocorrelations during other intervals. Observed variation in that respect, however, is much restricted by existence in the market mechanism of an analog to the governor of an engine, which, in the case of the market mechanism, operates to hold the autocorrelations among price changes within a narrow range, not far from zero.

3. Each of the foregoing conclusions rests in large part on statistical evidence, most of it accumulated in the course of testing a series of tentative speculative-market models. The first of these was that of the "ideal futures market," con-

sidered early in the preceding paper. The operation of that *IM* (ideal market) model is described by the equation

$$\Delta_{j \cdot t} = x_t - x_{t-j} = \tau_{j \cdot t},$$

where x_t is the price at time t, $t = 1, 2, 3 \ldots$, with t measured in units chosen in accordance with the minimum value to be assigned to j; j may be assigned any value within (or perhaps beyond) the range 10 min. $\leqq j \leqq$ 6 months; and $\tau_{j \cdot t}$ is a random variable, with zero mean, and with variance that changes, more or less systematically, through time.

In the foregoing equation, $\tau_{j \cdot t}$ is an impulse acting on the market mechanism and $\Delta_{j \cdot t}$ is the price change induced thereby. The equation specifies that the market mechanism transmits the impulses instantly, and without modification, to the price.

4. The *IM* model serves, first, as definition of the impulses $\tau_{j \cdot t}$, which are impulses of the "first class" referred to above. They are changes in observable conditions, of any kind relevant to price prospects, measured in terms of the amount of price change warranted by them. In order to have price effect, any $\tau_{j \cdot t}$ must be known to traders, hence the mechanism by which information on $\tau_{j \cdot t}$ is gathered and disseminated must be treated as a part of the market mechanism.[11]

The *IM* model has served, secondly, to pose sharply the major problem to be solved. In the *IM* model, $\Delta_{j \cdot t}$ is a random variable *because* of the perfection with which the assumed market functions. In actual speculative markets, which are surely imperfect, the imperfections of their functioning should be reflected in nonrandomness of $\Delta_{j \cdot t}$. Yet, oddly, attempts to detect nonrandomness in the changes of speculative prices had produced little evidence that $\Delta_{j \cdot t}$, in actual speculative markets, was otherwise than purely random.[12]

5. C. S. Brinegar (*2*, *3*), using new statistical tests that I had devised for the purpose, found in futures prices of three commodities some evidence of negative first-order autocorrelations in $\Delta_{j \cdot t}$, for $j \leqq 1$ week; and clear evidence of weak positive first-order autocorrelations for j in the range $2 \leqq j \leqq 8$ weeks.

6. Brinegar's evidence suggested trial of a dispersed effect (*DE*) model, defined by the equation

$$\Delta_{1 \cdot t} = b(\tau_{1 \cdot t} + \tau_{1 \cdot t-1} + \ldots + \tau_{1 \cdot t-s+1}), \ b = 1/s.$$

A. B. Larson (*14*, *15*), tested this model and found, among other things, that it provided a substantial approach toward reality; and evidence that the major requirement for closer approach was a model that included a source of some propensity toward negative first-order autocorrelations in $\Delta_{j \cdot t}$, as well as of the positive autocorrelations generated by the *DE* model.

7. Larson's results led to design of a model that may reasonably be considered a first approximation (*FA*) to reality. Its equation may be written, for present purposes, in the general form

$$\Delta_{j \cdot t} = \Phi(\tau_{1 \cdot t}, \tau_{1 \cdot t-1}, \ldots) + \Psi(\varepsilon_{1 \cdot t}, \varepsilon_{1 \cdot t-1}, \ldots),$$

[11] This is a feature of the *IM* model that I failed to recognize as necessary when writing earlier descriptions of the model.

[12] I refer here to a large body of evidence, mainly unpublished, that is not included in Cootner's valuable collection of material on the subject (*4*). Until fairly recent years, evidence of randomness in price changes was rarely recognized as deserving attention. Investigators sought evidence of nonrandomness and, failing to find it, considered their efforts to have produced nothing worth publication.

where $\Phi(\tau)$ is a function that, operating alone, would generate only positive first-order autocorrelations in $\Delta_{j \cdot t}$, and comprises, as a special case, the equation of the *DE* model; and where $\Psi(\varepsilon)$ is a function of a second class of random impulses that, operating alone, would generate chiefly or solely negative first-order autocorrelations in $\Delta_{j \cdot t}$.

This *FA* model carries implications regarding the activities of traders on speculative markets that are especially clear in the case of floor traders. An application of the model to the interpretation of data on floor trading (*24*) threw much light on their activities, and confirmed to that extent the usefulness of the model.

8. Though evidence leading to design of the *FA* model has come wholly from studies of commodity futures prices, there is reason to believe it to be equally applicable to stock prices. Long before Brinegar produced his evidence of positive first-order autocorrelation in $\Delta_{j \cdot t}$, $2 \leqq j \leqq 8$ weeks for futures prices, Cowles and Jones (*6*) had reported evidence that, except in one respect, indicated the presence in $\Delta_{j \cdot t}$ for stock-price index numbers of similarly weak positive autocorrelations. A subsequent revision by Cowles (*5*) of the earlier work brought the stock-price evidence, within the range $2 \leqq j \leqq 8$ weeks, into full accord with Brinegar's evidence. There remained an apparent discrepancy between Cowles' evidence and that of Brinegar and of Larson concerning autocorrelations of $\Delta_{j \cdot t}$, $j \leqq 1$ week; but that discrepancy can be interpreted as tending to support the validity of the *FA* model, inasmuch as the use of index numbers, based on prices of a large number of stocks, would tend to obliterate evidence of the presence of the ε impulses of the *FA* model.

9. A noteworthy feature of the *FA* model is that, in the presence of sufficiently wide variation in var $\Delta_{j \cdot t}$, it leads to sinusoidal regressions of $\Delta_{j \cdot t}$ on $\Delta_{j \cdot t-j}$. Evidence that those regressions took forms such as are illustrated in Chart 5 provided part of the basis for adoption of the model.[13]

The joint frequency distributions are also strongly heteroscedastic, as a necessary consequence of the changes through time in var $\tau_{j \cdot t}$.

10. None of the models considered above can generate price movements resembling the episodic cycles in wheat prices described and illustrated in the foregoing paper. To include the generation of such price cycles in a speculative-market model it is necessary to adopt a model with at least two different operating modes. The *FA* model, with constant parameters, appears to represent reasonably well the operation of the market mechanism in its normal (*N*) mode. Its operation in its episodic cycles (*EC*) mode may be represented fairly realistically by a two-phase model, taking $j = 1$ day, and

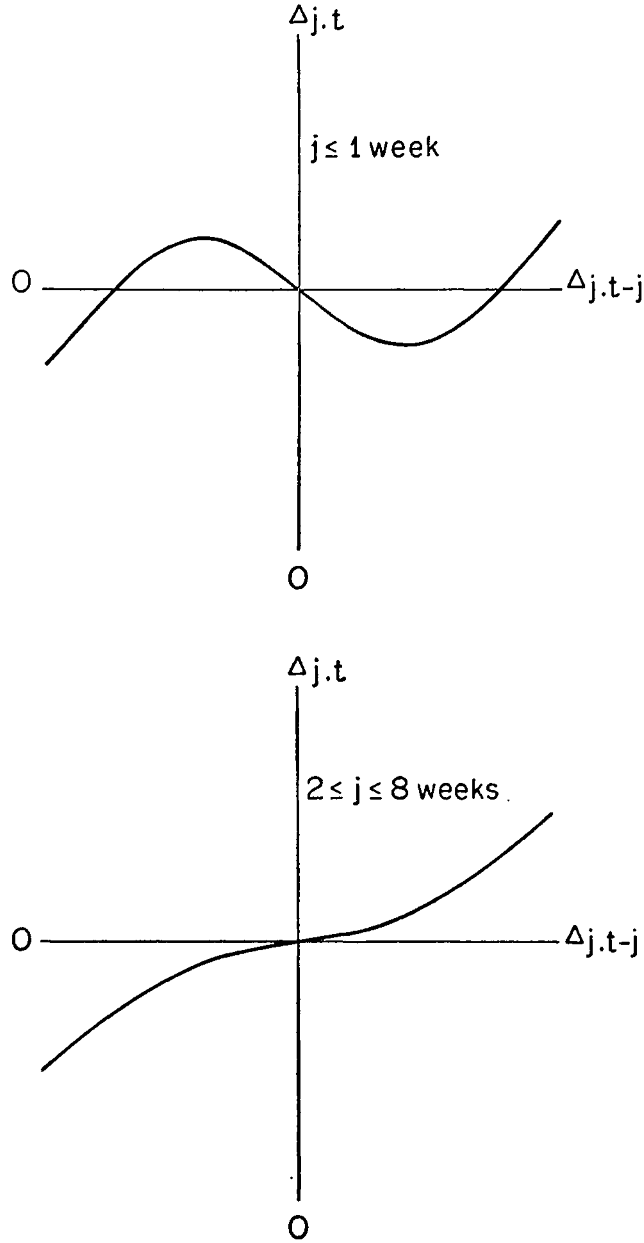
$$\Delta_{1 \cdot t} = b^* \tau_{1 \cdot t},$$

where $b^* > 1$ during the primary phase of the cycle. That condition persists until the occurrence of a series of mainly negative values of $\tau_{1 \cdot t}$. When such a series occurs, b^* takes different values according to whether $\tau_{1 \cdot t}$ is positive or negative, not far from unity in either case, but larger for $\tau_{1 \cdot t} < 0$ than for $\tau_{1 \cdot t} > 0$.

11. The condition $b^* > 1$, required during the primary phase of an episodic

[13] Such evidence is present, mostly in obscure form, in the results obtained by both Brinegar and Larson.

CHART 5.—REPRESENTATIVE PATTERNS OF REGRESSION OF $\Delta_{j\cdot t}$ ON $\Delta_{j\cdot t-j}$*



* For any large sample as a whole, var $\Delta_{j\cdot t} =$ var $\Delta_{j\cdot t-j} \simeq Cj$, with C a constant for that particular sample.

cycle (according to the foregoing model), is one that is met only when the $\tau_{1 \cdot t}$ impulses are changes in prospect of a sort widely and immediately recognized as deserving to affect the price, and that are widely publicized among interested persons. That at least is my reading of the wheat-price evidence. What other circumstances may be necessary for emergence of $b^* > 1$ is not clear. The special characteristics of the pre-1940 wheat market that are noted earlier as perhaps explaining the occurrence of episodic price cycles in that market, despite their apparent absence in certain other speculative markets, may deserve a different interpretation than is there suggested; they may not have been requisites for the occurrence of such cycles, but conditions permitting such cycles to attain an amplitude that made them clearly distinguishable from such patterns of price movement as are generated when the market is operating in its N mode.

CITATIONS

1 M. K. Bennett, "World Wheat Survey and Outlook, September 1935," *Wheat Studies of the Food Research Institute*, XII, 1 (Stanford, Calif., 1935).

2 C. S. Brinegar, "A Statistical Analysis of Speculative Price Behavior" (unpub. Ph.D. diss., Stanford Univ., 1954).

3 ———, "A Statistical Analysis of Speculative Price Behavior," revised, *Food Res. Inst. Studies*, IX, Supp., 1970.

4 P. H. Cootner, ed., *The Random Character of Stock Market Prices* (Cambridge, Mass., 1964).

5 Alfred Cowles, "A Revision of Previous Conclusions Regarding Stock Price Behavior," *Econometrica*, Oct. 1960: 909–15; reprinted in *4*.

6 Alfred Cowles and H. E. Jones, "Some A Posteriori Probabilities in Stock Market Action," *Econometrica*, July 1937: 280–94.

7 E. R. Dewey and E. F. Dakin, *Cycles, The Science of Prediction* (New York, 1947).

8 Helen C. Farnsworth and Holbrook Working, "World Wheat Survey and Outlook, September 1936," *Wheat Studies of the Food Research Institute*, XIII, 1 (Stanford, Calif., 1936).

9 Helen C. Farnsworth, Holbrook Working, and M. K. Bennett, "World Wheat Survey and Outlook, May 1937," *Wheat Studies*, XIII, 8 (Stanford, Calif., 1937).

10 W. S. Jevons, *Investigations in Currency and Finance* (London, 1884).

11 M. G. Kendall, "The Analysis of Economic Time Series—Part I: Prices," *J. Royal Stat. Soc.* (London), CXVI, 1953: 11–25; reprinted in *4*.

12 ———, "On the Analysis of Oscillatory Time Series," *ibid.*, CVIII, 1945: 93–129.

13 ———, "Oscillatory Movements in English Agriculture," *ibid.*, CVI, 1943: 93–129.

14 A. B. Larson, "Evidence on the Temporal Dispersion of Price Effects of New Market Information" (unpub. Ph.D. diss., Stanford Univ., 1960).

15 ———, "Measurement of a Random Process in Futures Prices," *Food Res. Inst. Studies*, I, 3, 1960; reprinted in *4*.

16 H. L. Moore, *Generating Economic Cycles* (New York, 1923).

17 J. A. Schumpeter, *Business Cycles* (New York, 1939).

18 Eugen Slutzky, "The Summation of Random Causes as the Source of Cyclic Processes," *Econometrica*, V, 2, 1937. Revision and translation by Slutzky of his article in Russian originally published in *Problems of Economic Conditions*, III, 1, ed. by The Conjuncture Institute (Moscow, 1927).

19 Herman Wold, *A Study in the Analysis of Stationary Time Series* (Uppsala, 1938).

20 Holbrook Working, "Cycles in Wheat Prices," *Wheat Studies of the Food Research Institute*, VIII, 1 (Stanford, Calif., Nov. 1931).

21 ———, "How Far Will History Repeat Itself in Wheat"? *Barron's Financial Weekly*, Nov. 2, 1936.

22 ———, "The Investigation of Economic Expectations," *Amer. Econ. Rev.*, May 1949.

23 ———, "A Random-Difference Series for Use in the Analysis of Time Series," *J. Amer. Stat. Assn.*, Mar. 1934.

24 ———, "Tests of a Theory Concerning Floor Trading on Commodity Exchanges," *Food Res. Inst. Studies*, VII, Supp., 1967.

25 ———, "A Theory of Anticipatory Prices," *Amer. Econ. Rev.*, May 1958.

26 G. Udny Yule, "On a Method of Investigating Periodicities in Disturbed Series, with Special Reference to Wolfer's Sunspot Numbers," in *Philosophical Transactions of the Royal Society of London* (London, July 1927).

27 ———, "On a Method of Studying Time Series Based on Their Internal Correlations" (with a note by M. G. Kendall), *J. Royal Stat. Soc.* (London), CVIII, 1945: 208–30.