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HEDGERS' DEMAND FOR FUTURES CONTRACTS: A THEORETICAL FRAMEWORK WITH APPLICATIONS TO THE UNITED STATES SOYBEAN COMPLEX†

This paper is concerned with the behavior of hedgers in commodity futures markets, particular attention being focussed on the way in which hedgers make decisions as to the size of their holdings of futures contracts. The recent proliferation of futures markets in the United States suggests that these institutions will increasingly come under public scrutiny and for this reason it is important that their peculiar resource allocative functions be well understood. Yet one can reasonably claim that this is not true even amongst professional economists; the literature is rather sparse and is often disjointed in the sense that few authors build on the work of others. With several notable exceptions no attempt has been made to test the various theories which have been proposed.

A rather heroic attempt is made to survey the literature and to develop a theory of hedging which at least partly reconciles the differing views which this survey uncovers. The theory so devised is then tested empirically using data relating to the United States soybean complex. No claim to originality is made as regards the theoretical discussion as all the concepts discussed may be found elsewhere in the literature, although not always in the context of futures markets. The empirical work, however, does break some new ground and, while not entirely successful, it yields encouraging results. All in all, the paper should be regarded as exploratory in nature, and in this spirit a number of suggestions for further work are incorporated into the final section.

TWO DEFINITIONS OF HEDGING

A survey of the literature on commodity futures markets reveals two views of hedging which appear to conflict quite sharply. On the one hand is what might be labelled the "risk reduction" view of hedging according to which the hedger uses futures markets solely as a means of reducing the price risk associated with

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† Based in large part upon the author's (unpublished) Ph.D. dissertation, "The Relationship Between Prices and Hedging Patterns in the U.S. Soybean Complex," Stanford University, December 1970.

a commitment he has made in the cash market[1] in the course of conducting his normal business operations. A cocoa grinder, for example, who has acquired stocks to meet future processing needs may sell an equivalent volume of cocoa futures in order to reduce his "price exposure." In similar fashion a grain exporter may reduce the price risk associated with a forward export sale by the purchase of futures

The risk-reduction concept of hedging has mustered considerable support amongst economists. In particular it has provided the point of departure for several empirical studies designed to test the effectiveness of futures markets for hedging (i.e., the extent of risk reduction afforded by hedging). The degree to which hedging reduces price risk depends, of course, on the notion of risk which one employs. The underlying principle, however, involves measuring the extent to which cash and futures prices move together.

The general tenor of the results of these studies, together with an indication of the concept of risk which they use, may be illustrated by B.S. Yamey's conclusions with respect to the Liverpool cotton market: "Though there were variations from sample to sample . . . the general conclusion is that . . . the hedging imperfections tended to balance out to a considerable extent . . . on balance the loss in the spot market remained a loss, but a very much smaller loss, with hedging" (*21*, p. 318). In passing we note that these studies implicitly regard risk as being equivalent to expected loss, whereas economic theory more commonly characterizes risk as variability about the expected return.

Acceptance of the risk-reduction view of hedging has by no means been unanimous. A considerably different view has been advanced by several writers, among them Holbrook Working (*18, 19*). They have noted that since cash and futures prices do not move completely in parallel, hedgers are in fact arbitraging the two markets and may be able to profit from fluctuations in the cash-futures price spread. The very source of unavoidable risk in the eyes of the risk-reduction school is transformed into a source of potential hedging profits. The empirical studies mentioned above show that *routine* arbitrage between cash and futures prices generally results in losses. Consequently this alternative view of hedging presupposes that hedgers have the ability to predict basis[2] fluctuations and that they hedge accordingly.[3] Working extended his definition to include "selective" and "anticipatory" hedging (*20*): here again the emphasis is on expected return from hedging.

The contrast between the assumptions about hedgers' attitudes towards risk involved in the above concepts of hedging may be illustrated by the following statement by T. F. Graf: "Businessmen who are primarily engaged in processing or distributing commodities are . . . anxious to avoid as fully as possible risks associated with price changes. This they do by shifting the risks to others who are willing and able to assume them. Hedging is theoretically one method of

[1] In this paper the term "cash price" will refer to prices pertaining to transactions in the physical market and will be regarded as synonymous with the term "spot."

[2] The term "basis" in the context of futures markets means the difference between the cash and futures prices.

[3] Evidence that hedgers may reasonably be expected to possess this type of forecasting ability may be found in *4*; *18*; *19*.

accomplishing this" (*3*, p. 398); and on the other hand by a statement by Working: ". . . any curtailment of risk may be only an incidental advantage gained, not a primary or even a very important incentive to hedging" (*19*, p. 561).

In each view the hedger bases his hedging decision on one variable. In the first he chooses that position which minimizes his risk, however defined; in the second, that which maximizes his expected return. These two variables have assumed increasing importance in economic theory as *joint* determinants of individual behavior under uncertainty. We shall argue in the following section that a plausible model of hedging behavior can be developed in which both risk and expected return are used as decision-making variables.

HEDGING AS BEHAVIOR UNDER UNCERTAINTY

In this section the Markowitz theory of portfolio selection (*11*) is used to develop a model of hedging behavior—one which goes at least part of the way toward resolving the conceptual differences outlined in the previous section. In particular we shall show that a hedger who is risk averse (in a sense yet to be defined) may adopt market positions consistent with those suggested by Working. Several previous studies, notably those of L. G. Telser, L. L. Johnson and J. L. Stein (*8*; *15*; *16*) have incorporated portfolio selection theory into the analysis of hedging behavior. However, none of these writers attempted to relate the portfolio model to earlier studies of hedging behavior; *a fortiori* they did not discuss the relevance of the model to the controversy alluded to in the previous section. A brief discussion of the model is therefore warranted, especially as it provides the foundation for the empirical work to be reported in later sections of this paper.

Individual Behavior Under Uncertainty: A Single Period Analysis

Consider an individual faced with the opportunity of buying or selling short one or more of n risky assets. It is assumed that he has a subjective probability distribution defined on the vector $(\Delta p)' = (\Delta p_1, \Delta p_2, \ldots, \Delta p_n)$ of asset price changes in the period under consideration. Assuming, furthermore, that he receives no income other than that earned from asset price changes, his wealth W_1 at the end of the period is

$$W_1 = W_0 + X' (\Delta p)$$

where W_0 is his initial wealth and X is the vector of asset holdings at the beginning of the period. ($X_i > 0$ indicates that the individual holds a long position in asset i.) It is assumed that the individual's utility function u has W_1 as its argument. The expected utility theorem then states that the individual will choose that portfolio X which maximizes $E\{u(W_1)\}$. In this framework the assumption that the individual is "risk averse" means that u is a concave function of W.

Much of the early work on portfolio selection was carried out under the assumption that the individual can choose among portfolios on the basis of their means and variances. It has since been shown that this assumption is valid only under the restrictive conditions:

(i) u is quadratic; or

(ii) the subjective probability distribution is normal.[4] Despite its weaknesses, mean-variance analysis has been found useful both as a pedagogic device and as a basis for empirical analysis. It will be used in the following discussion for these reasons and also because it provides a particularly convenient framework for some of the peculiar features of the hedging problem. Using the mean-variance approach the individual's decision procedure is to maximize his expected return for any given risk (i.e., variance) or, alternatively, to minimize his risk for any given expected return. Denoting by μ and V the mean and variance-covariance matrix of the probability distribution, this decision rule may be written

$$\max X'\mu \quad \text{subject to } X'VX \leqq k_1 , \quad \text{for all values of } k_1 .$$

In order to deal with questions which arise in the following section the additional constraints are imposed:

$$0 \leqq k_2 \leqq X \leqq k_3 , \quad X'e \leqq k_4 , \quad X'(-e) \leqq k_5 ,$$

where e is the unit vector, $e' = (1, 1, \ldots, 1)$.

Redefining X and the k_1 appropriately the problem can be restated as

$$\max_{x} X'\mu$$

$$\text{subject to } X'VX \leqq k_1 , X'e \leqq k_2 , X'(-e) \leqq k_3 , X \leqq k_4 , X \leqq 0 .$$

This is a well-known problem in mathematical programming and its solution[5] is obtained by forming the Lagrangian

$$L = X'\mu - \lambda_1(X'VX - k_1) - \lambda_2(X'e - k_2) - \lambda_3 \{X'(-e) - k_3\} - \lambda_4(X - k_4)$$

where $\lambda_1 , \lambda_2 , \lambda_3$ are scalars and λ_4 is an n-dimensional row vector.

The Kuhn-Tucker conditions are

$$\frac{\partial L}{\partial X} \leqq 0 , \qquad \lambda \frac{\partial L'}{\partial X} X = 0 , \qquad X \geqq 0$$

$$\frac{\partial L}{X\lambda} \leqq 0 , \qquad \lambda \frac{\partial L}{X\lambda} = 0 , \qquad \lambda \geqq 0$$

where $\lambda = (\lambda_1 , \lambda_2 , \lambda_3 , \lambda_{41} , \lambda_{42} , \ldots , \lambda_{4n})$.

For those $X^* \neq 0$ which satisfy the above conditions,

$$\frac{\partial L}{\partial X^*} = 0$$

i.e., $$\mu - 2\lambda_1 VX^* - \lambda_2 e + \lambda_3 e - \lambda_4 = 0$$

[4] Mean-variance analysis of portfolio selection was pioneered by H. Markowitz (*11*). For an application to economic theory, see W. S. Feldstein (*2*). Discussion of the conditions under which expected utility maximization coincides with mean-variance analysis may be found in Feldstein (*1*).

[5] For details see *8*.

i.e., $$X^* = \frac{1}{2\lambda_1} V^{-1}(\mu - \lambda_2 e + \lambda_3 e - \lambda_4)$$

or $$X_i^* = \frac{1}{2\lambda_1} \sum_{j=1}^{n} v^{ij}(\mu_j - \lambda_2 + \lambda_3 - \lambda_{4j}) \; i = 1, \ldots, n . \tag{1}$$

We now provide examples of hedger decision-making and attempt to give equation (1) some intuitive meaning.

Hedging as Decision-making Under Uncertainty

The relevance of the preceding model for the analysis of hedging behavior is perhaps best illustrated by example. In general the "assets" referred to in the above discussion become cash or futures market positions and the return to asset holding is the ensuing price change in the corresponding market.

It is customary in discussing the theory of commodity price storage to attribute a convenience yield to stocks. Merchants may hold some minimum level of stocks S_1 so as to meet unforeseen demand; processors may hold some raw material inventory to facilitate the efficient management of their physical plant. The consequent yield from stock-holding arises independently of any price appreciation. It is introduced into the present analysis in the form of a simple constraint $X_1 + X_2 \geqq S_1$ where X_1 denotes unhedged stocks and X_2 denotes hedged stocks. We also introduce a capacity constraint of the form $X_1 + X_2 \leqq S_2$, where S_2 represents the upper limit on stocks.

The hedging decision can now be expressed in terms of the portfolio model:

Case A: The "Storage Problem."

A merchant who owns storage space (or who can rent storage space) may adopt one or both of the following courses of action:

(i) Purchase a quantity X_1 of the commodity in the cash market and hold it unhedged. The net return earned during the period is then $X_1(\Delta p - c)$ where Δp is the cash price change and c is the unit carrying cost.

(ii) Purchase a quantity X_2 of the commodity and short hedge it. In this case the hedger's net return is $X(\Delta b - c)$ when Δb is the basis change.

Applying equation (1), the solution to this hedging problem is seen to be

$$X_1 = \{v^{11}[(\mu_1 - c) - \lambda_2 + \lambda_3] + v^{12}[(\mu^2 - c) - \lambda_2 + \lambda_3]\}/2\lambda_1 \tag{2}$$

$$X_2 = \{v^{12}[(\mu_1 - c) - \lambda_2 + \lambda_3] + v^{22}[(\mu^2 - c) - \lambda_2 + \lambda_3]\}/2\lambda_1 . \tag{3}$$

It is instructive to consider for a moment the case in which λ_2 and λ_3 are zero (i.e., the capacity and convenience yield constraints are not binding). Noting the simple relationship between the elements of a 2×2 matrix and its inverse, equations (2) and (3) may be rewritten

$$X_1 = \{v_{22}(\mu_1 - c) - v_{12}(\mu_2 - c)\}/(v_{11}v_{22} - v_{12}^2) \tag{4}$$

$$X_2 = \{v_{12}(\mu_1 - c) + v_{11}(\mu_2 - c)\}/(v_{11}v_{22} - v_{12}^2) \tag{5}$$

where v_{11}, v_{22} are the variances of cash price changes and basis changes respectively and v_{12} is the covariance between them. Then it may be seen that if

$(\mu_1 - c)$ and $(\mu_2 - c)$ are both positive, a necessary condition for all stocks to be hedged $(X_1 = 0)$ or all stocks to remain unhedged $(X_2 = 0)$ is that cash price changes and basis changes be positively correlated $(v_{12} > 0)$. Alternatively if basis changes and cash price changes are negatively correlated it is necessary that either $\mu_1 < c$ or $\mu_2 < c$ for all stocks to be either hedged or unhedged. In particular, all stocks will be hedged (routine hedging) only if

$$\frac{v_{12}}{v_{22}} = \frac{\mu_1 - c}{\mu_2 - c} .$$

If the capacity and convenience yield constraints are binding the usual "shadow price" interpretation may be given to λ_2 and λ_3. They represent the marginal returns to the merchant of relaxing the constraints.

Case B: The "Exporter Problem."

This may be viewed as the dual of the storage problem. The exporter has the following alternatives available:

(i) He may make a forward sale of X_1 units and leave it unhedged.

(ii) He may make a forward sale of X_2 units and long hedge it.

The two "price" variables which concern the exporter are the cash price and the basis as in *Case A*, with one important difference—the relevant cash price is now the forward cash price. With this alteration, however, the solution to the "exporter problem" is identical to that of the "storage problem."

Case C: The "Processor Problem."

When futures markets exist for a commodity in both its raw and processed forms, interesting hedging problems arise for entrepreneurs who handle the commodity at both stages of production. An obvious example is that of the processor himself. Consider a processor who holds stocks of the raw material. He is assumed to have expectations about

(i) the cash price of the raw material (Δp)

(ii) the raw material basis (Δb)

(iii) the futures price spread between the raw material and the product (Δs), and

(iv) the product basis (Δb^*).

He has the following alternatives:

(i) Hold X_1 of the stocks unhedged. The net return is then $X_1(\Delta p - c)$ where c is the unit carrying cost.

(ii) Hold X_2 of the stocks hedged in raw material futures. The net return is $X_2(\Delta b - c)$.

(iii) Hold X_3 of the stocks hedged in product futures. The net return is $X_3(\Delta b + \Delta s - c)$.

(iv) Hold X_4 of the stocks hedged by forward sales of the cash product. The net return is then $X_4(\Delta b^* - \Delta s - \Delta b - c)$.

Nothing is gained in this case by explicitly stating the mathematical solution to the problem. Analogy with the "storage problem" is sufficient to allow the inference that in general the optimal portfolio will contain all four assets.

These three examples show that a risk-averse merchant may hold some stocks unhedged or that a risk-averse processor may leave some of his stocks unhedged, and hedge some part of them in product futures (if available), in addition to hedging some stocks in raw material futures. They also demonstrate that the decision to hedge is in general not made independently of that to hold stocks or make forward sales, i.e., that hedging is in fact an integral part of the merchandising operation, not simply an auxiliary device for reducing business risks.

The above analysis has several weaknesses. Certain of these arise from the use of mean-variance analysis. In particular, mean-variance analysis implies

(i) portfolio "separation"; that is, in the optimal solution, the ratio of asset holdings is independent of the initial wealth;

(ii) portfolio "myopia"; that is, the optimal solution for the multi-period problem coincides with the sequence of optimal solutions for the corresponding single-period problems.

In addition, the above analysis assumes

(iii) that delivery does not occur and that only one futures contract is traded at any point in time. This assumption is not particularly restrictive, however, and its relaxation would only complicate the exposition;

(iv) that the hedger earns all his income from basis movements and price movements, or at least that the income so earned is the only variable which determines hedging strategy. Incidental to this is the model's neglect of the possibility that hedged stocks receive preferential treatment from banks or other lending institutions;

(v) that the only admissible portfolio members are market positions (cash or futures) in the commodity. In practice other asset variables usually enter. For example, a grain merchant will normally manage a portfolio consisting of several grains; an exporter's portfolio will normally include positions in foreign exchange and ocean freight in addition to commodity positions.

These assumptions, especially the first two, reduce the power of our formal analysis somewhat. They do not, however, detract significantly from the value of the mean-variance approach as a framework in which to discuss hedging behavior. In the following section, we proceed to develop the empirical content of this approach.

HEDGERS' DEMAND FOR FUTURES CONTRACTS

The portfolio model of hedging as outlined in the previous section establishes the proposition that the volume of hedging will be responsive to changing market expectations concerning basis and price movements and their covariance structure. In the present section we attempt to measure the direction of this response—this is in effect an attempt to estimate a hedger's demand schedule for futures contracts. An estimation procedure is outlined and applied to data on soybeans, soybean oil and soybean meal.

By way of introduction to this section we present, in Tables 1 and 2, estimates of first and second order moments of cash price and basis changes in the soybean complex. The basis is calculated as the cash price minus the July futures price, whereas the "board conversion" (the difference between the futures price

TABLE 1.—MEANS AND VARIANCES OF CASH PRICE AND BASIS, FIRST DIFFERENCES*

	Means						Variances					
	Beans (*cents per bu.*)		Oil (*cents per lb.*)		Meal (*$ per ton*)		Beans (*cents per bu.*)		Oil (*cents per lb.*)		Meal (*$ per ton*)	
	Cash	Basis	Cash	Basis	Cash	Basis	Cash	Basis	Cash	Basis	Cash	Basis
1955–56	3.06	0.48	9.59	−3.12	0.29	−0.05	147.29	7.49	55.23	7.85	8.62	2.87
1956–57	0.74	1.44	−3.65	−1.71	−0.06	0.11	44.92	11.33	22.58	12.79	3.64	1.89
1957–58	0.38	1.43	−8.82	0.41	0.79	0.02	8.72	1.84	8.63	5.86	6.85	5.31
1958–59	1.09	1.09	−5.88	−4.00	0.50	−0.05	6.07	5.84	7.65	4.22	20.59	15.79
1959–60	0.29	1.12	0.65	0.18	−0.38	−0.34	7.97	5.40	5.56	2.40	3.49	3.19
1960–61	3.59	1.04	11.06	−3.06	0.91	0.45	269.92	18.16	38.32	6.91	18.32	3.97
1961–62	0.53	0.92	−16.88	2.00	0.65	0.09	5.40	5.82	9.24	1.35	3.42	3.27
1962–63	1.47	0.46	2.24	−1.71	0.21	−0.14	18.19	6.24	4.94	0.96	2.88	0.95
1963–64	−1.56	0.71	−6.59	5.06	−0.35	0.34	29.26	5.15	22.14	1.12	6.05	2.39
1964–65	1.53	0.46	−2.53	−2.59	0.44	0.22	66.37	5.36	30.86	3.91	4.32	2.10
1965–66	7.79	1.18	4.06	−9.71	2.12	−0.01	158.21	8.10	31.59	7.98	39.57	20.28
1966–67	−0.26	1.05	−9.41	0.88	−0.24	−0.22	13.65	9.60	4.95	1.48	6.50	4.50
1967–68	0.97	1.43	−7.88	3.06	0.68	0.19	6.72	5.33	5.62	0.60	3.38	1.83

* See Appendix Note II for sources of data. Basis is calculated as cash price minus futures price. Meal is in short tons of 2,000 pounds.

Table 2.—Simple Correlation Coefficients Between First Differences of Cash Price, Basis, and Board Conversion*

	Soybeans		Soybean oil		Soybean meal	
	Cash price and basis	Futures price and board conversion	Cash price and basis	Futures price and board conversion	Cash price and basis	Futures price and board conversion
1955–56	0.0756	−0.8086	0.2296	−0.7501	0.7196	0.0543
1956–57	0.4469	−0.7030	0.5012	−0.4261	0.7584	0.1985
1957–58	−0.1490	−0.0415	0.6491	−0.4762	0.7716	0.8798
1958–59	0.6219	0.0854	0.7986	0.0276	0.9563	0.9735
1959–60	0.6167	−0.1091	0.8280	0.1798	0.8652	0.7864
1960–61	−0.0537	−0.7925	0.1172	−0.7830	0.5312	−0.2986
1961–62	0.7827	−0.5325	0.2600	−0.3697	0.8194	0.8966
1962–63	0.2235	−0.5418	0.4577	−0.5512	0.5698	0.5141
1963–64	−0.0679	−0.5535	0.5951	−0.8154	0.6622	0.6547
1964–65	0.0934	−0.7935	0.5526	−0.7833	0.5727	0.2409
1965–66	−0.4684	−0.2251	0.6674	−0.5516	0.8690	0.6139
1966–67	0.6697	−0.1634	0.4764	−0.1263	0.8728	0.8144
1967–68	0.7208	0.0612	0.5929	−0.5263	0.8018	0.8210

* See Appendix Note II for sources of data. Basis is calculated as cash price minus futures price. "Board Conversion" (the difference between the futures price of a bushel of soybeans and the corresponding value of the oil and meal it contains) is calculated as shown in Appendix Note I.

of a bushel of soybeans and the corresponding value of the oil and meal it contains) is calculated as shown in Appendix Note I. Cash and futures prices are semimonthly quotations from October 15 to June 30, so that there are 18 observations in each season.[6] By calculating the statistics in this fashion we are implicitly assuming that the underlying distributions are stationary within a season but that they need not be so from one season to the next. In particular, we abstract from possible intraseasonal changes in price volatility.[7]

For soybeans, the means of cash price changes (P_c) and basis changes (B) are both generally positive, reflecting the fact that soybeans are carried from harvest throughout the remainder of the season. For soybean products the pattern is less clear: soybean meal and soybean oil are generally not held in large volume by merchants or processors so that period-to-period variations in cash prices or the basis will not reflect carrying costs. For all three commodities the variance of P_c is considerably larger than that of B.[8] P_c and B are generally positively correlated, although not always significantly so. Soybean price changes tend to be negatively correlated with changes in the board conversion while meal price changes are generally positively correlated with the board conversion. More interesting, perhaps, is that changes in oil prices are generally negatively correlated with changes in the board conversion, which suggests that during the period under consideration soybeans have been a "mealseed" rather than an oilseed.[9]

The General Form of the Equation to Be Estimated

Initially our discussion will relate to short hedging, but the argument applies *mutatis mutandis* to long hedging. Taking the portfolio model of the preceding section as our point of departure we write

$$X_1 = \phi_1(M, V)$$
$$X_2 = \phi_2(M, V)$$

where

X_1 is the level of unhedged stocks held by the hedger;
X_2 is the level of hedged stocks held by the hedger;
M is the mean vector of expected cash price and basis changes;
V is the variance-covariance matrix of expected cash price and basis changes.

From these microrelationships we infer an analogous macrorelationship which is to be estimated. An aggregation procedure such as this is, of course, strictly valid only under rather restrictive conditions. In the empirical analysis which follows we aggregate only over the class of "large" hedgers in order to keep our

[6] Further details on all data used in the study and their sources are provided in Appendix Note II.

[7] See P. A. Samuelson for a theoretical rationale of increasing price volatility during the life of a futures contract (*14*, pp. 44–46).

[8] The exceptions, in the case of soybeans are years of heavy loan activity. The loan program reduces cash and futures price variability leaving basis variability relatively unscathed. Consequently the loan program has a twofold impact on short hedging—it reduces the volume of free stocks and it reduces cash price variability relative to basis variability. The peculiar characteristics of the soybean meal market which lead to high basis variability are discussed in T. A. Hieronymus (*5*, pp. 20–31).

[9] This hypothesis draws additional support from the observation that the value of oil in a bushel of soybeans was 112 percent of the value of meal in a bushel of soybeans in 1955–56 but that this figure had declined to 50 percent in 1967–68.

TABLE 3.—BASIC STATISTICS ON U.S. SOYBEAN INDUSTRY, SELECTED YEARS 1954–68*

	1954/55	1959/60	1967/68
Soybeans (*million bushels*)			
Production	341.1	532.9	976.1
Domestic crush	249.0	393.4	572.4
Exports	60.6	141.4	266.6
Soybean oil (*million pounds*)			
Production	2,711	4,338	6,032
Exports	50	953	992
Soybean meal (*thousand tons*)[a]			
Production	5,705	9,152	13,660
Exports	272	649	2,899
Soybean futures (*million bushels*)			
Annual volume of trading	4,952.2	5,612.5	4,805.4
Annual average month-end open contracts	88.1	134.6	174.5
Soybean oil futures (*million pounds*)			
Annual volume of trading	4,318.5	8,123.8	16,039.7
Annual average month-end open contracts	205.0	427.7	647.3
Soybean meal futures (*thousand tons*)[a]			
Annual volume of trading	5,741.3	17,499.0	33,284.6
Annual average month-end open contracts	397.1	577.9	1,139.0

* Data on production and distribution are on a crop-year basis from U.S. Dept. Agr., *Agricultural Statistics*, various issues. Data on futures markets are on a July 1–June 30 basis from U. S. Dept. Agr., Commodity Exchange Authority, *Commodity Futures Statistics*, various issues.

[a] Tons of 2,000 pounds.

sample as homogeneous as possible. When this is done the following system is obtained:

$$HS = f_1(M^*, V^*) \tag{6}$$
$$S - HS = f_2(M^*, V^*) \tag{7}$$
$$0 \leqq HS \leqq S$$

where HS is total short hedging, S is total ("free") stocks carried by hedgers, M^* and V^* are "market expectations" corresponding to M and V.

For purposes of estimation, however, there are a number of difficulties associated with equations (6) and (7) in their present form. The first of these difficulties stems from the tremendous growth of the United States soybean industry in the past twenty years. As can be seen from Table 3, production, domestic crushing capacity, and exports have all expanded steadily during this period, with several consequences for the optimal volume of hedging. In particular,

(i) an increase in storage capacity (a "relaxation" of the capacity constraint) allows more stocks, both hedged and unhedged, to be held;

(ii) increases in crushing capacity, and volume of soybean trade generally, require more stocks, hedged and unhedged to be held to meet unforeseen needs—amounting to a "tightening" of the convenience yield constraint; and

(iii) an increase in the total level of free stocks allows the restriction $0 \leq HS \leq S$ to be relaxed.[10]

The net effect of these factors is that a given expected basis change will induce more short hedging in, say, 1970 than in 1950. There are several ways in which this problem can be tackled. An "index of industry growth" such as annual production or total soybean disappearance, could be devised and introduced into the estimating equation as an independent variable. Alternatively one can notice that equations (6) and (7) can be rewritten

$$\frac{HS}{S} = g_1(M^*, V^*) \tag{8}$$

$$\frac{S - HS}{S} = g_2(M^*, V^*). \tag{9}$$

The functions g_1, g_2 are more likely to be stable through time than are f_1, f_2 and our attention will be confined to estimation of (8).[11]

$$\frac{HS}{S} + \frac{S - HS}{S} = 1 .$$

The discussion so far has been deliberately vague with respect to the question of the time horizon involved in hedgers' decision making. The formal model developed above assumes that the hedger makes all his decisions on a "single period" basis. But what is the length of this period? And, even if a single period analysis is appropriate, is it reasonable to assume that hedgers can adjust their (often very large) futures and cash positions to the desired levels in relatively short periods of time? In the present study the length of the time horizon is largely dictated by the availability of data: published data on the size of hedging commitments are available on a semimonthly basis and this is the time unit employed.

We deal with the question of rigidities which prevent immediate adjustment to the desired market position by introduction of a Koyck lag. Let $[HS/S]_t^*$ denote the desired aggregate ratio of short hedging to stocks at time t. This is the short hedging ratio which we would observe at time t if all hedgers could adjust their portfolios without cost to the optimal level. Our portfolio model would thus suggest estimation of a relationship of the form

$$[HS/S]_t^* = g(M_t^*, V_t^*) + \varepsilon_t$$

where ε_t is a stochastic disturbance term.

In fact we do not observe $[HS/S]_t^*$, but we postulate a relationship between the desired and observed variables of the form

$$[HS/S]_t - [HS/S]_{t-1} = \lambda \left[[HS/S]_t^* - [HS/S]_{t-1} \right] .$$

[10] Some elaboration is perhaps necessary on the role of stocks in the analysis. The model as developed in the previous section incorporates stocks as an endogenous variable but this refers to stocks held by hedgers (and "potential" hedgers). In practice, then, we are talking about *commercially held* stocks. Many farmers store soybeans without examining the possibility of hedging them, and the loan program is, on occasion, a great absorber of soybean stocks in the United States.

[11] By restricting our attention to equation (8) we effectively ignore the constraint

This leads to a distributed lag model,

$$[HS/S]_t = \lambda g(M_t^*, V_t^*) + (1-\lambda)[HS/S]_{t-1} + \lambda\varepsilon_t . \qquad (10)$$

No complication is introduced if $[HS/S]$ is replaced by log $[HS/S]$ in equation (10). For purposes of estimation log g will be assumed to be a linear function of its arguments. Several other functional forms were briefly examined and the results were very similar to those presented below. The estimating equation is thus obtained as

$$\log [HS/S]_t = \alpha_0 + \alpha_1 M_{1t} + \alpha_2 M_{2t} + \beta_1 V_{1t} + \beta_2 R_t + \beta_3 V_{2t} + \gamma \log [HS/S]_{t-1} + \varepsilon_t , \qquad (11)$$

or alternatively,

$$\Delta \log [HS/S]_t = \alpha_0 + \alpha_1 M_{1t} + \alpha_2 M_{2t} + \beta_1 V_{1t} + \beta_2 R_t + \beta_3 V_{2t} + \gamma \log [HS/S]_{t-1} + \varepsilon_t , \qquad (12)$$

where M_{1t}, M_{2t} denote the mean expected cash price and basis change respectively, V_{1t}, V_{2t} denote the variances of expected cash price and basis changes respectively, and R_t is the correlation coefficient between expected cash price and basis changes.

Equation (12) very closely resembles the function form estimated by F. de Leeuw for aggregate financial behavior in the Brookings econometric model of the American economy (*10*). The level of reported short hedging replaces holdings of financial assets and the stocks variable is the hedging analog of wealth. Equation (12) includes second-order moments which do not appear in the Brookings equation, while the latter includes short-run financial constraints which have no obvious counterpart in the hedging model. This overall resemblance reflects the fact that both equations are derived from portfolio theory; it also serves as a reminder of the essential similarity between commodity futures markets and other financial markets.

There remains some doubt as to whether or not equation (12) is identified. Although we have failed to confront the fact squarely, it has been implicit in the preceding discussion that the hedging demand equations which we have derived are embedded in a system of equations, the other equations being demand (or supply) equations of speculators and spreaders. Since we have not been able to disaggregate traders' positions according to contract maturity, we are obliged to abstract from spreading operations, but the possibility remains that the hedging demand schedules are less stable than the speculative supply schedules, in which case the parameters of the hedging schedule cannot be identified. There is considerable evidence, however, that the speculative schedule is perfectly elastic with respect to price expectations.[12] In this case the system may be regarded as recursive so that the parameters of the hedging demand equation are identified and may be estimated without introducing simultaneous equation bias.

The final question to be treated here concerns the specification of the variables

[12] In particular, see Telser (*17*).

M_1, M_2, V_1, V_2, and R. Since these variables denote expectations they cannot be directly observed and consequently a new set of issues is raised. The most commonly employed solution to this problem is to incorporate into the model an assumption as to how price expectations are formed, usually that expected price is some function of past prices. This procedure has *not* been used in the present paper. There is convincing evidence that apart from a significant seasonal component both cash prices and the basis behave like random walks.[13] Models which incorporate distributed lags on prices will consequently misspecify the mechanism which generates prices. Furthermore, there is now evidence to the effect that "the actual realization can be fruitfully employed as an implicit proxy for *ex ante* variables in econometric applications whenever anticipations are not directly observable."[14] This procedure is used here. Actual price and basis changes are regarded as observations on the variables M_1 and M_2 respectively and the variances and correlation coefficients reported in Tables 1 and 2 are regarded as observations on V_1, V_2, and R.[15]

The Estimation Procedure

Since the hedging series are themselves highly autocorrelated there is a strong presumption that the error term $\{\varepsilon_t\}$ in equation (12) exhibits significant autocorrelation. Consequently parameter estimates obtained by applying ordinary least squares to equation (12) will in general be inconsistent. Furthermore, it is not possible to gauge the full extent of this autocorrelation from the Durbin-Watson statistic calculated from ordinary least squares residuals since it too is biased.

If, however, we assume that the error term in equation (8) is generated by a first order autoregressive process, we can obtain maximum likelihood estimates of the parameters by the following procedure. First define the quasi-difference operator

$$D_\rho(Z_t) = Z_t - \rho Z_{t-1}\,.$$

Then for any given value of ρ, ordinary least squares regression of $D_\rho(\Delta \log [HS/S]_t)$ on $D_\rho(M_{1t})$, $D_\rho(M_{2t})$, $D_\rho(V_{1t})$, $D_\rho(V_{2t})$, $D_\rho(R_t)$, and $D_\rho(\log[HS/S]_{t-1})$ will yield conditional maximum likelihood estimates of α_0, α_1, α_2, β_1, β_2, β_3, and γ. By iterating over ρ in the range $-1 < \rho < 1$ in search of the minimum sum of squared residuals, global maximum likelihood estimates may be obtained.

Using this procedure, hedging demand schedules are estimated for soybeans, soybean oil, and soybean meal for the period 1955–56 to 1967–68. There are several variations from one estimating equation to the next as regards specification of the independent variables, however. For example, while S (stocks) is a

[13] This proposition more commonly takes the form of asserting that futures prices behave like a random walk whereas cash prices have a significant seasonal component (see *13*, *17*). W. C. Labys and C. W. J. Granger (9) have recently produced evidence that both cash and futures behave like random walks. This only serves to strengthen the present argument.

[14] The quotation is from A. A. Hirsch and M. C. Lovell (*6*, p. 170) although much of the original work in this field is due to E. S. Mills (*12*).

[15] Price and basis changes are in fact measured as $\log\left(\frac{P_{t+1}}{P_t}\right)$ and $\log\left(\frac{B_{t+1}}{B_t}\right)$ so as to yield constant elasticity demand schedules.

satisfactory proxy variable for long cash commitments in the short hedging equations, its use in the long hedging equations would be inappropriate. What is required in this case is a proxy for short (forward) cash commitments. Unfortunately, as no obvious counterpart to the stocks variable is available, resort has to be made to an artificially constructed variable. For soybeans this variable is formed by lagging by two months the sum of domestic crushings and soybean exports. For each of the soybean products it is formed by lagging by two months the sum of domestic disappearance and exports of that product. The other variation from the basic form of equation (12) is the inclusion of the board conversion and its second moments to take account of cross-hedging by soybean processors. In several equations (short soybean hedging and both equations for soybean oil) no board conversion variables appeared as significant explanatory variables. In such instances they are omitted from the reported results. The results are summarized in Table 4.

Discussion of the Results

Generally speaking, the results accord quite well with prior expectations. While neither the values of $\bar{R}^2$ nor those of the estimated standard errors should be taken too seriously, since their significance under this estimation procedure is dubious, they are certainly no cause for concern. More important than goodness of fit is the plausibility of the pattern of estimated coefficients and, with two exceptions to be discussed below, the results are satisfactory in this regard. Expected price or basis changes appear as explanatory variables in all equations except the short hedging equations for the products and in all but two cases have the sign which is suggested by intuition.[16] Furthermore, the coefficient of expected basis change is invariably greater than one, whereas that of expected flat price change is always less than one. Since the variables M_1 and M_2 are defined in terms of logarithms the demand equation is effectively of the double-logarithmic form as far as these variables are concerned and consequently the estimated coefficients can be interpreted as elasticities. This is to say that hedgers' demand for futures is elastic with respect to basis expectations but inelastic with respect to flat price expectations. This result is in accordance with the emphasis that Working's early writings on hedging placed on "arbitrage" hedging as opposed to "anticipatory" or "selective" hedging: even though hedgers do take flat price expectations into account when formulating their plans they tend to be more responsive to basis expectations (*18*, *19*).

The two estimated equations which were mentioned above as yielding unsatisfactory results are the short hedging equations for soybean oil and soybean meal. In both these equations the estimate of γ is less than minus one, which implies that the value of the coefficient of adjustment λ is greater than one, a result which is completely unacceptable. While the problem could be circumvented by iterating also with respect to γ (in addition to ρ) to find a constrained maximum likelihood estimate of the parameters, this has not been done as the coefficients of the other variables in these equations do not conform to the pat-

[16] Nor are the two exceptions cause for dismay. Feldstein has shown that intuition may be misleading with respect to the sign of these coefficients (2, pp. 186–87).

TABLE 4.—ESTIMATED HEDGERS' DEMAND EQUATION

Equation description / Independent variables	M_1	M_2	M_3	V_1	V_2	V_3	R_1	R_{13}	R_{23}	log [HS/S]	log [HL/F]	$\bar{R}^2$
Short hedging—Soybeans	−1.762	−0.773	—	−0.046	0.005	—	0.308	—	—	−0.973	—	0.990
Estimated S.E.	0.918	0.277		0.014	0.001		0.140			0.012		
Long hedging—Soybeans	5.137	—	—	—	—	−78.640	—	—	−0.437	—	−0.336	0.198
Estimated S.E.	3.696					58.686			0.230		0.070	
Short hedging—Soybean oil	—	—	—	5.118	—	—	—	—	—	−1.069	—	0.990
Estimated S.E.				1.646						0.011		
Long hedging—Soybean oil	1.266	0.976	—	−5.412	0.973	—	—	—	—	—	−0.992	0.968
Estimated S.E.	1.439	0.555		2.019	0.470						0.020	
Short hedging—Soybean meal	—	—	—	—	—	−99.140	1.883	—	—	−1.111	—	0.952
Estimated S.E.						46.986	0.981			0.027		
Long hedging—Soybean meal	−1.627	−0.926	0.045	—	—	—	—	—	—	—	−0.987	0.918
Estimated S.E.	0.598	0.380	0.027								0.034	

M_1, M_2, M_3 denote the expected basis, cash price, and board conversion changes respectively.
V_1, V_2, V_3 denote the variances of the basis, cash price, and board conversion respectively.
R_1 denotes the correlation between the expected basis change and the expected cash price change.
R_{13} denotes the correlation between the expected basis change and the expected board conversion change.
R_{23} denotes the correlation between the expected cash price change and the expected board conversion change.
Blank (—) indicates that the variable is omitted from the equation.

tern of those in the remaining equations. For short hedging in soybean products our model is apparently not well specified.

Returning now to the other equations, the coefficients of V_1 and V_2 (basis and cash price "variability") do form a consistent pattern. An increase in anticipated basis variability or a decrease in anticipated cash price variability will lead to a decrease in the size of hedging commitments, short or long, relative to the size of cash commitments. There is also some evidence that basis and price variability play more of a role in determining the behavior of hedging in soybeans than in soybean products.

Finally, no board conversion variables enter significantly into either of the soybean oil equations. This is surprising in view of the fact that certain board conversion variables do enter the soybean and soybean meal equations. A possible interpretation is that the oil "leg" of the beans-oil-meal straddle is placed more often in the (forward) cash market than in the futures market. Another seemingly incongruous result is that long hedging in soybeans should be responsive to variability in the board conversion but not to monthly changes in the board conversion. There are, however, two plausible explanations for this phenomenon. Firstly, soybean processors may be more concerned with capturing a satisfactory margin by putting on crush[17] than with profiting from month-to-month changes in the board conversion. Alternatively, soybean processors may attempt to forecast fluctuations in the board conversion, but as a group are neither consistently successful nor unsuccessful in this attempt.

CONCLUSIONS

This paper has attempted to illustrate the power of the portfolio management concept as a framework within which different views of hedging behavior can be welded together. In addition to demonstrating the main theoretical contribution of portfolio analysis, namely that expected return and risk may be viewed as joint determinants of hedging behavior, we have developed numerical estimates of hedgers' responsiveness to basis and price expectations. The empirical work, while yielding encouraging results, did not manage to overcome all the obstacles which were encountered. The first of these difficulties relates to the appropriateness of the available data for measuring the variables which theory specifies. Published data on hedgers' holdings of futures contracts are highly aggregative and fail to distinguish between different classes of hedgers such as merchants, processors, or exporters and between hedgers' holdings of futures contracts with different delivery dates. This means in the first instance that we are forced to aggregate over individuals who are managing essentially different portfolios, and in the second that we can only approximately determine the prices at which hedgers' purchases and sales of futures contracts take place. Data on cash commitments are also difficult to obtain, especially in the case of forward short positions. On this point we may note that aggregate data on stocks and domestic disappearance are much more readily available for the soybean complex than for other major commodities traded on futures exchanges.

[17] The term "putting on crush" refers to the purchase of soybean futures and simultaneous sale of product futures by soybean processors in an attempt to ensure a processing margin.

This suggests that any major breakthrough in the construction of aggregate models of hedging behavior is most likely to be made in the soybean complex. But at the same time the data difficulties referred to here suggest that it is towards disaggregated models that further research should be directed. Other benefits would arise from models of individual firms or relatively homogeneous groups of firms. In particular, the supply of speculative services to an individual hedger is bound to be at least as elastic as it is to hedgers as a group so that the question of identification would be less troublesome.

These technical difficulties aside, what are the broad implications of portfolio theory for our understanding of the functions of commodity futures markets? In the first place it emphasizes that risk is inherent in all marketing and processing strategies, not only those in which hedging does not take place, i.e., that futures markets facilitate "risk management" rather than "risk transferral." Furthermore, since stock levels, processing rates, and similar physical variables are determined endogenously, the portfolio model implies that futures markets affect the allocation of resources over time and are not simply ancillary marketing institutions. Insofar as portfolio concepts have relevance for commodity futures markets, the welfare effects of these markets go far beyond those of simple risk reduction.

CITATIONS

1 M. S. Feldstein, "Mean-Variance Analysis in the Theory of Liquidity Preference and Portfolio Selection," *Rev. Econ. Studies*, Jan. 1968.

2 ———, "Uncertainty and Forward Exchange Speculation," *Rev. Econ. Stat.*, May 1965.

3 T. F. Graf, "Hedging—How Effective Is It?" *J. Farm Econ.*, Aug. 1953.

4 R. G. Heifner, "The Gains From Basing Grain Storage Decisions on Cash-Future Spreads," *J. Farm Econ.*, Dec. 1966.

5 T. A. Hieronymus, "Forecasting Soybean Meal Prices," *Commodity Year Book 1961* (New York Commodity Res. Bur., 1961).

6 A. A. Hirsch and M. C. Lovell, *Sales Anticipations and Inventory Behavior* (New York, 1969).

7 L. L. Johnson, "The Theory of Hedging and Speculation in Commodity Futures," *Rev. Econ. Studies*, June 1960.

8 H. W. Kuhn and A. W. Tucker, "Nonlinear Programming," in *Proceedings of the Second Berkeley Symposium on Mathematical Statistics and Probability*, ed. by J. Neyman (Berkeley, Calif., 1950).

9 W. C. Labys and C. W. J. Granger, *Speculation, Hedging and Commodity Price Forecasts* (Lexington, Mass., 1970).

10 F. de Leeuw, "A Model of Financial Behavior," in *The Brookings Quarterly Economic Model of the United States*, ed. by J. S. Duesenberry, G. Fromm, L. R. Klein, and E. Kuh (Chicago, 1965).

11 H. Markowitz, *Portfolio Selection—Efficient Diversification of Investments* (New York, 1959).

12 E. S. Mills, *Price Output and Inventory Policy* (New York, 1962).

13 L. H. Rocca, "Time Series Analysis of Commodity Futures Prices" (unpub. Ph.D. diss., Univ. Calif., Berkeley, 1969).

14 P. A. Samuelson, "Proof that Properly Anticipated Prices Fluctuate Randomly," *Indus. Mangmt. Rev.*, Spring 1965.

15 J. L. Stein, "The Simultaneous Determination of Spot and Futures Prices," *Amer. Econ. Rev.*, Dec. 1961.

16 L. G. Telser, "Safety First and Hedging," *Rev. Econ. Studies*, XXIII, 1, 1955–56.

17 ———, "The Supply of Speculative Services in Wheat, Corn and Soybeans," Supplement to *Food Res. Inst. Studies*, VII, 1967.

18 Holbrook Working, "Futures Trading and Hedging," *Amer. Econ. Rev.*, June 1953.

19 ———, "Hedging Reconsidered," *J. Farm Econ.*, Nov. 1953.

20 ———, "New Concepts Concerning Futures Markets and Prices," *Am. Econ. Rev.*, June 1962.

21 B. S. Yamey, "An Investigation of Hedging on an Organized Produce Exchange," *The Manchester School of Econ. and Soc. Studies*, Sept. 1951.

APPENDIX NOTE I

CALCULATION OF THE BOARD CONVERSION

The board conversion C is calculated as follows:

Let P_B = Price of beans (in cents per bushel).
P_M = Price of meal (in dollars per short ton).
P_O = Price of oil (in cents per pound).
Then: $C = 2.4\,P_M + 11\,P_O - P_B$ (in cents per bushel).

This formula reflects crushing yields of 48 lbs./bu. for meal and 11 lbs./bu. for oil. Both these yields overstate actual yields but are used by the trade in calculating the board conversion. The formula consequently ignores any seasonality in crushing yields.

APPENDIX NOTE II

DATA SOURCES

This appendix provides details on sources of data used in this paper together with some discussion of the difficulties associated with their use.

1. *Reported hedging commitments* in soybeans, soybean oil, and soybean meal were obtained from United States Department of Agriculture, Commodity Exchange Authority, *Commodity Futures Statistics*, annual. The minimum size of position in soybean meal to be reported was increased from 1,500 tons to 2,500 tons on February 3, 1959. No obvious discontinuity was introduced into the series by this change and it was ignored in the analysis. Also ignored was the fact that reported hedging commitments cover *all* contract markets. During the period under consideration, soybean oil and meal were traded to some extent on the Memphis Board of Trade and soybeans on the Chicago Open Board of Trade, in addition to the major part of the trade which was carried out on the Chicago Board of Trade.

2. *Futures prices* were also obtained from United States Department of Agriculture, Commodity Exchange Authority, *Commodity Futures Statistics*, annual. The only discontinuity in these series which arises during the period under consideration is associated with the change in the billing of deliverable soybean meal from unrestricted to Eastern Trunk Line, which took effect with the May 1961 contract. This presents no problem for the empirical work in this study, however, since we are concerned with futures prices within each season or with basis *changes*.

3. *Cash prices* used are as follows:

(a) *Soybeans*: Track Country Station (Illinois points), No. 1 Yellow. These data were provided by Professor T. A. Hieronymus of the University of Illinois, Urbana.

(b) *Soybean Oil*: Crude, tank cars, f.o.b. Decatur, Illinois.

(c) *Soybean Meal*: 44 percent protein, unrestricted billing, bulk; Decatur, Illinois.

The oil and meal cash prices for the period January 1960 to June 1968 (inclusive) were obtained from Chicago Board of Trade, *Annual Reports*, in the following fashion:

If a single price is quoted, that price is used. If a range (bid-asked) is quoted, the mid-range is used.

For the period October 1955 to December 1959 (inclusive) the oil and meal cash prices were obtained in the same fashion from Chicago Board of Trade unpublished records.

4. *Stocks* data were obtained as follows:

Stocks of soybeans at mills, stocks of crude soybean oil at mills and stocks of soybean cake and meal at mills were obtained for the period 1955–58 (inclusive) from United States Department of Commerce, Bureau of the Census, *Facts for Industry: Animal and Vegetable Fats and Oils*, annual; and for the period 1959–68 (inclusive) from United States Department of Commerce, Bureau of the Census, *Current Industrial Reports, Series M 20 J, Fats and Oils, Vegetable Oil Crushers*, annual.

5. *Exports*

(a) Soybean exports were obtained from Chicago Board of Trade, *Annual Reports*.

(b) Soybean oil and soybean meal exports were obtained from United States Department of Agriculture, Economic Research Service, *Fats and Oils Situation*.

6. *Soybean crushings* were obtained from the sources listed in 4. above.